Probability Method in Civil Engineering

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Lecture No.# 37

Hypothesis Testing

Helloandwelcometothislecture,thisisbasicallythird lecture of this module.Andin this lecture, we will be discussing about hypothesis testing.You know that in the last lecture, we actually continued thathypothesistesting, we started that one.So, in this lecture, we will continue the samethe hypothesis regardingone mean orconcerning one mean, so one mean means here, we are having one sample.So, we will do sometesting, basically, this hypothesis testing are drawing some inferenceprobabilistically from thesample mean with respect to thepopulation mean,those we discussin this last lecture.

So, our outline for today'slecture is hypothesisconcerning one mean, hypothesis concerning two means, hypothesis concerning one variance and hypothesis concerning two variance.And if time permits,we will also seethe probability paper that we aretalking aboutthatonce the data is available, generally so far what we have done?We have assumed thatit is following some particular distribution.So,this probability paper is basicallythatfirst step of for a graphical inspectionto test thatwhat the data possibly couldfollow.

So, this is basically the test of this sample data that what is thedistribution that population may follow, so that if the time permits after this hypothesis testing,we will do that one also.

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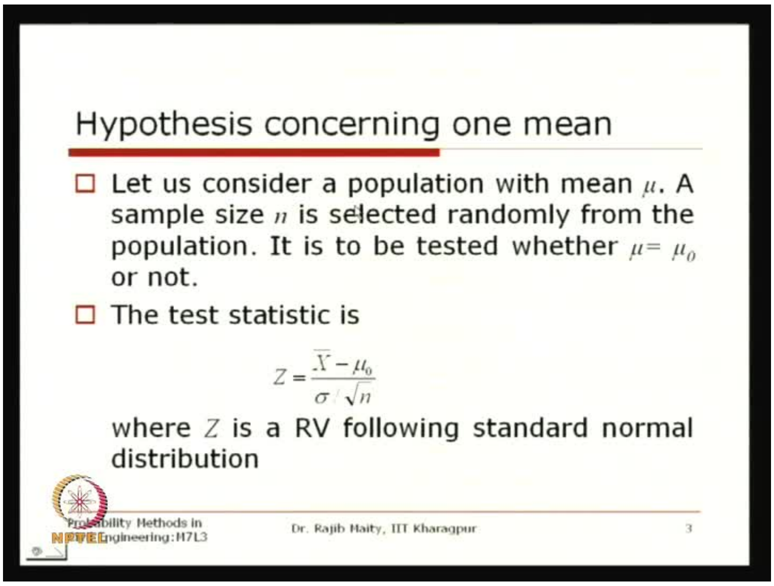

Well, so, hypothesis concerning one mean,let us consider a population with mean mu a sample size n is selected randomlyfrom the population.It is to be tested whether this mu is equals to mu naught or not, so this mu naught is basically one threshold value that weneed to test whether from the sample whatever.The mean that we are estimating from thisestimation technique that we discussearlier, whether that is having some relationshipso that whether probabilistically, we can infersomething about it some specific value.

So, that specific value is your mu naught,what is ourtest,what is our goal to testthis population mean.So, here the test, sothat from our last lecture, we have to define some test statistics, so here the statistics is that Z equals to X bar minus mu naught divided by sigma by squareroot n.This sigma by squareroot n thatis thestandard deviation for the sampling distributionof this mean and this X bar is your mean and thismu naught is the valuewhich we are expecting that populationmean might be this mu naught.

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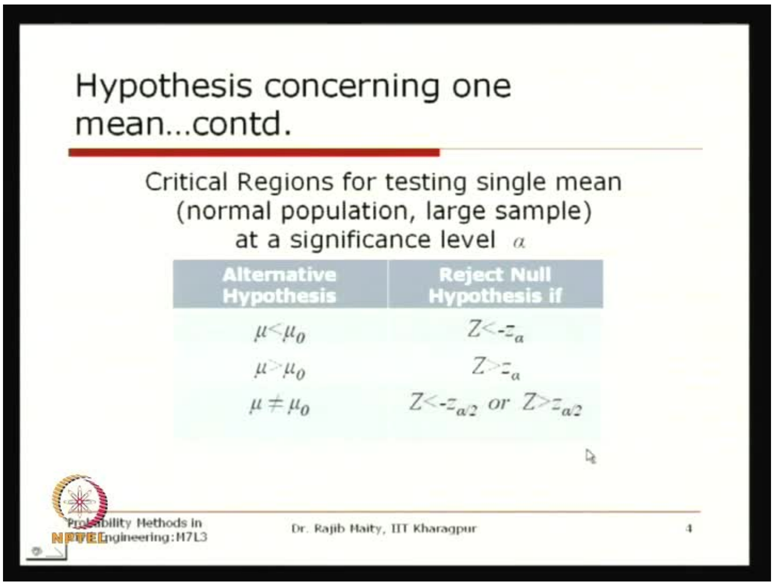

So, hereyou know that this is a reduce standard variate now, so this z isa random variable which is following the standard normal distribution.And the critical regionsfor testing this single mean, and again, here theassumption is that the population is normal and it is a large sample.This large means, here you can say that approximately if thesample size is more than 30, we can consider that this is a large sample.So, this critical region at the significance level alpha, and the discussion on the significance level also we have covered in this last lecture.

So,if it there are two possible case it can be one sided or it can be two sided; for this one sided, it can be the left hand side or the right hand side, and reject the null hypothesis if this Z is less than minus Z alpha.In this case, when the alternative hypothesisis mu is greater than mu naught, then Z is greater thanZ alpha.And if it isa two sidedtesting,mu is not equal to mu naught then, whether Z isless thanminus Z alpha by 2 or Z is greater thanZ alpha by 2 andthat this Z alpha by 2 is the quintile for which thatremaining probability is youralpha by t 2 or the right hand side of this one that we have discussed while discussing that critical zone.

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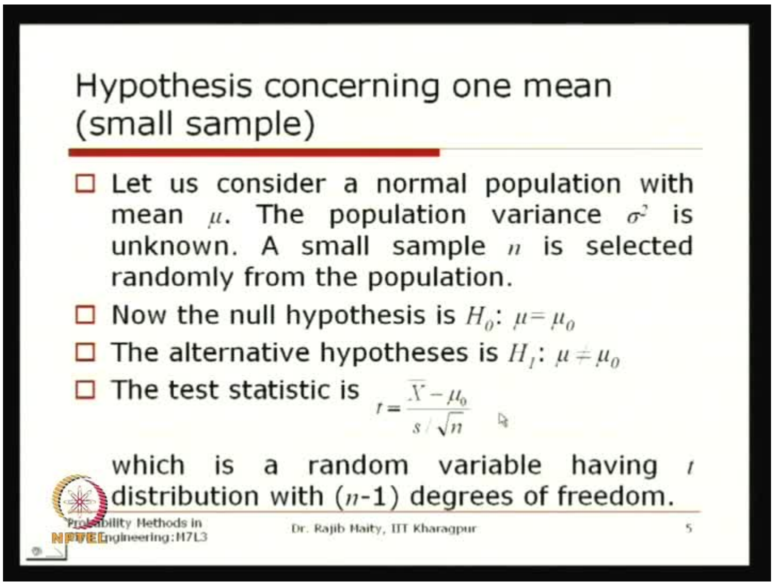

So, if that test statistics falls in thiscritical zone then, we have to reject the null hypothesis.So, let us consider a normal population with the mean mu,the population variance sigma square is unknown a small sample n is selected randomly from thispopulation.

Now, in this case, what we are doing so far,what we are telling that, if thissample is large, nowwe are discussing if the sample is small.And as I was telling that, if the sample size is less than 30 then, we canwe have to consider that this is a small sample.And here,if thatthe test statistics for this null hypothesis and the alternative hypothesis, if we consider is to be this central one or in fact, it is not only for the central, it can be one sided also.

So, this quantity now,which we were just estimating as a Zvalue that we have seen, now this s is replacedhas replaced that sigma.So, sigma by squareroot n that sigma was a standard deviation for the population.Here, it is a small sample, so we do not, and thevariance is unknown - population variance is unknown - so we have to estimate again, this one from the sample.

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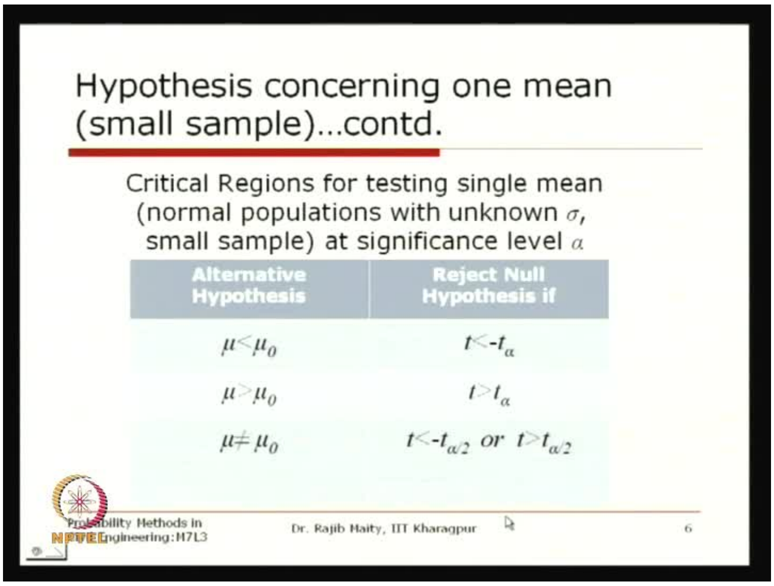

So, this s is basically the standard deviation estimated for this sample and so this quintile now is a is arandom variable having t distribution with n minus one degrees of freedom.And for this case, the critical regions will be for this one sidedtest, it will bet less than equal to minus t alpha, and if it is right hand side then, it is t greater than t alpha and for the central test, it is less than minus t alpha by 2 or greater than t alpha by 2.

So, now, you can see that for the large sample, we are approximatingit with the standard normal distribution and if it is ansmall sample, we have the approximated to thet distribution with the degrees of freedom n minus 1, where n is thesample size.

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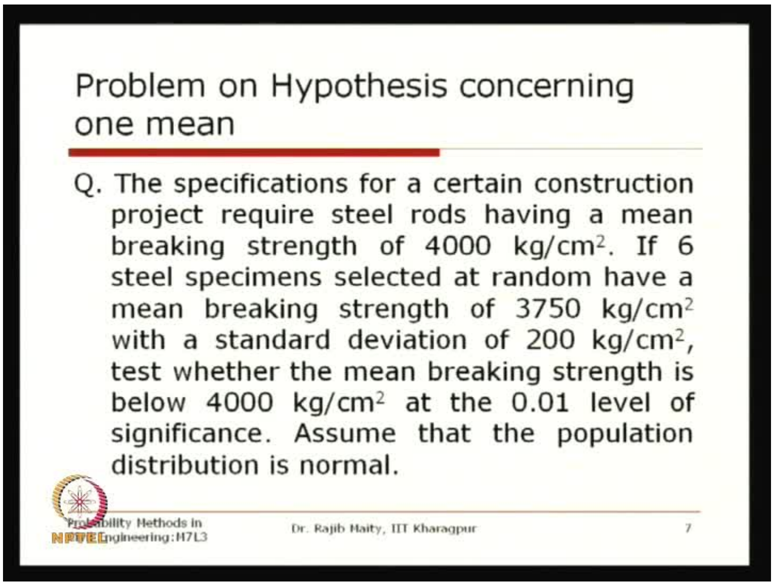

Well, take up one problem here,the problem states the specificationsfor a certain construction project require the steel rods having a mean breaking strength of 4000kg per centimeter square.If6 steel specimen selected at random have a mean breaking strength of3750 kg per centimeter square with a standard deviation of 200 kg per centimeter square,test whether the mean breaking strength is below 4000 kgper centimeter square at the 0.01 level of significance.Assume that the population is distribution normal.

So, you see that one small sample, which issample size is 6 and it is showing that mean strength is3750, which numerically if we test that it is below this4000 kg per centimeter square, but this differencewhether should we consider that whatever thespecimens that is being supplied for that project is really less than thespecification, so that statistically we have toinfer about that.

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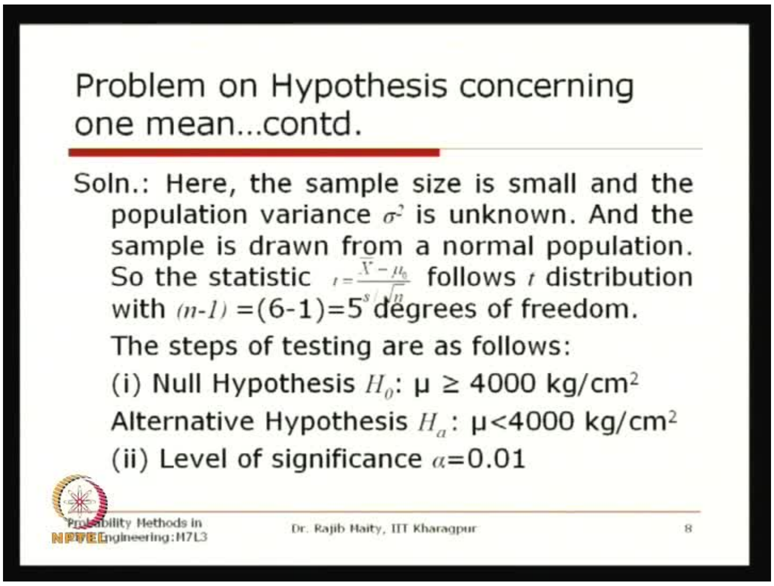

So, here thesample size is small and this population variance is unknown and the sampleis drawn from a normal population.So, the test statistics as we have discuss so far is this, t equals to X bar minus mu naught divided by sby squareroot of n, which is following a t distribution with n minus 1,n is equals to 6 years, so 5 degrees of freedom.So, our null hypothesis is, whether this is greater than or equal to this4000 kg per centimeter square which is ourspecification.So, we have to test that whether, it is below that our specification, so that is why in the alternative hypothesis, we put that mu is less than 4000 kg per centimeter square and this alpha is equals to 1.01.

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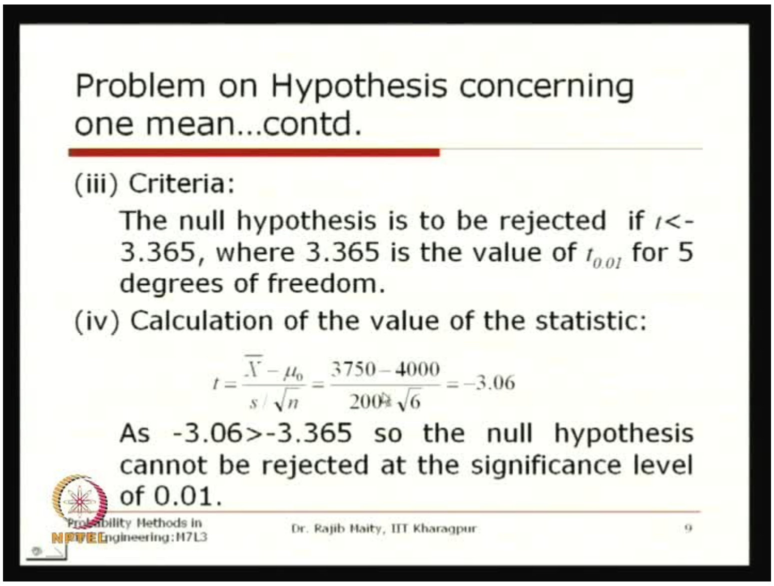

So, the criteria here is that asthis depends on this, what is the level of significance, so this t 0 alpha and this is an one sided test.So, this t 0 alpha for 5 degrees of freedom from thestandardtable from the standard text book if you see, it is 3.365.So,that test statistics if it is greater than this then, we have to reject the null hypothesis.If we calculate this one then, we see that,so this is t is less than minus 3.365.So, this test statistics now from the data that is availableX bar is 3750 and s equals to 200, we are getting that it is minus 3.06.

So, as this minus 3.06 is greater than this minus3.365, so the null hypothesis cannot berejected at the significance level of 0.01.So, we cannot claim, we fail toprove that this mean isreally less than the specification which is 4000 kg per centimeter square even though the mean that we have seen for this 6 specimen is 3750.

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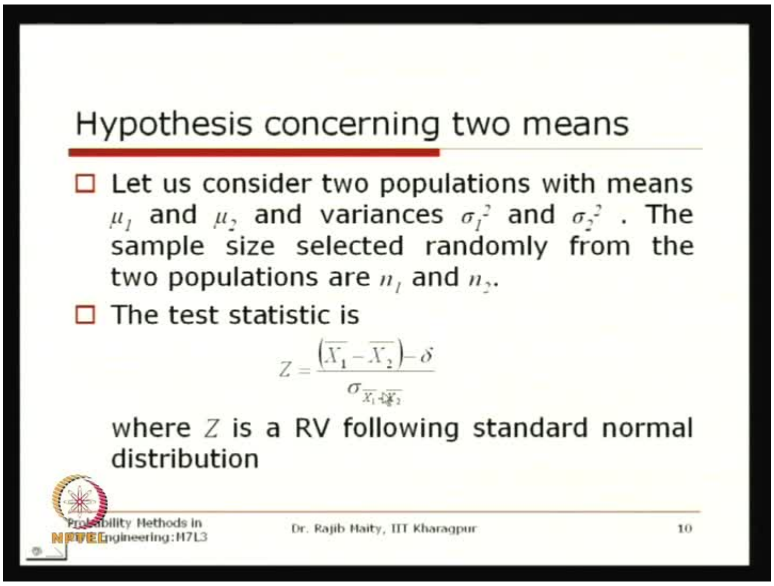

Well, now,we will take that hypothesis concerning two means, now this two means there are two samples are there and two samplemeans are there.So, we can test whethermany questions you can ask that whether both thesample is having the same mean.Even though that so far as the samplemeans should not be exactly same, that we can test and then, we can test that whether the one is really greater than the other one, that also we can test.

So, here what we aretaking is that basically the difference between this two means, that is,X1 is the mean for the first population and the X2 is the mean for the second population.So, if we take this difference then the mean, basically, now it is becoming a singlerandom variable with that delta value.Now, if I put that delta equals to 0; that means, both the means are affectively what we are testing whether the means are same or not ordepending on what value of this delta that we are selecting,depending on the whether we are testing that whether X1,the mean for the first population is greater than the second population and so on.

And this sigmathisdenominator sigma X bar minusX2 baris the standard deviation for thisdifference andthat this Z is now the random variable which is following again the standard normaldistribution, in case that this both this n 1 and n 2 which is the sample size are large enough. You can say thatthe if it is greater than 30 then, we can say that it is a standard normal distribution.

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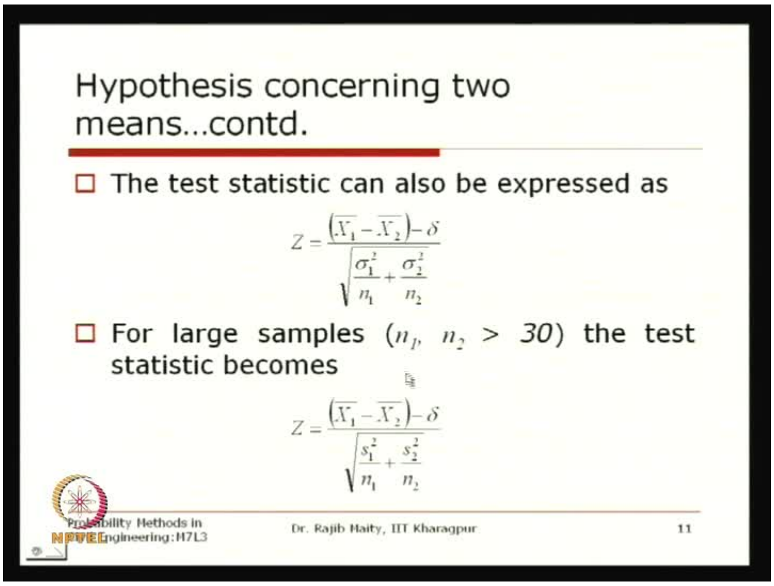

Once we get this one we can inferabout this,about their sample means, so thatteststatistics here can be the this one, this X1 bar minus X2 bar minus delta and that standard deviation that we aregetting for this difference is basically a sigma 1square by n 1 plus sigma 2square by n 2 and their whole squarerootwhich is giving you this estimate for thisstandard deviation for this difference.

And for the large sample that I was telling that if they are greater thanthis30, then also we can say that this can be easily replace by their samplestandard deviation, which is again that standard normal distributionwe can say.

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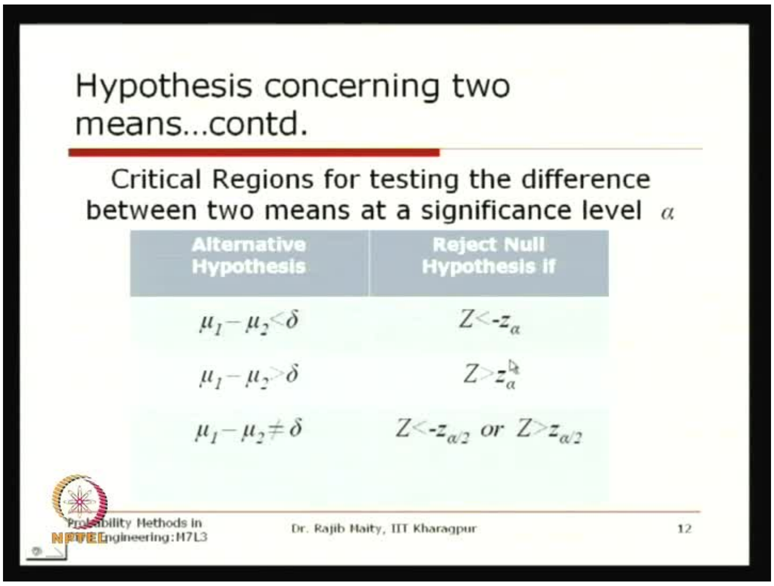

This is for this large sampleas we haveshown and then after this one we will also see that if it is the small sample. Now, so far as this is large sample, this will follow a standard normal distribution and the critical regions are now same for this,this are the different alternative hypothesis whether it is…whether.Now, this case is basically showingthat whether this mu 1 isless than mu 2 and this is showing that whether mu 1 is greater thanmu 2 and this is a two sided test whether mu 1 isnot equal to mu 2.

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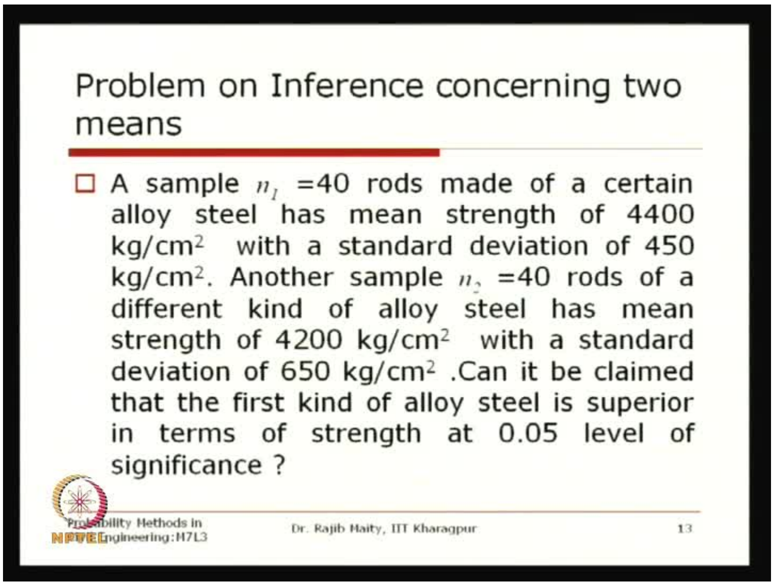

So, these are the critical regions as samethat we discuss earlier for this single samplemeanfewslides before.And the second thing is thefirst,we will take one problem on this if the sample size is large.So, asample n 1 is equals to 40 rods made of the certain alloy steel has mean strength of 4400 kg per centimeter square with a standard deviation of450 kg per centimeter square.The another sample n 2 equals to 40 row rods havingthis different kind of alloy steel which is having a mean strength of 4200 kg per centimeter square with a standard deviation of 650 kg percentimeter square.

Can it be claimed that the first kind of alloy is superior in terms of this strength at 0.05 level of significance. So, numerically if I just take this mean then, I can say that this 4400 is greater than 4200,but if we see their spread their standard deviation,so this is 450 and here it is 650.So, the statistically we have to test whether really that firstalloy steel is superior than the second one.

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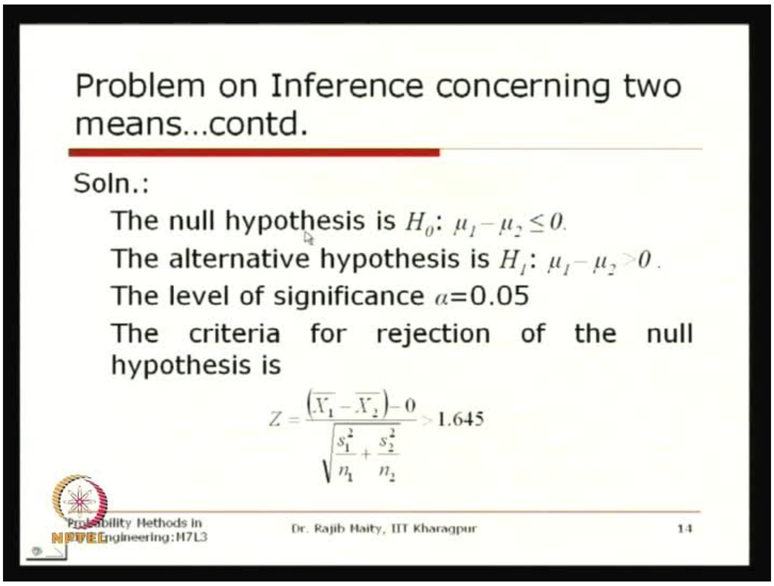

So, at the null hypothesis is that whether mu 1 minus mu 2 is less than0, here we place is delta equals to 0.And this alternative hypothesis basically what we are testing is this, mu 1 minus mu 2 greater than 0;that means, we are testing whether mu 1 is greater than mu 2 or not and the significance level is 0.05.The test statistics the criteria here it should be at 0.05, you knowthat this Z quintile is 1.645 at 95 percentprobability level, for this standard normal distribution the value is 1.645.

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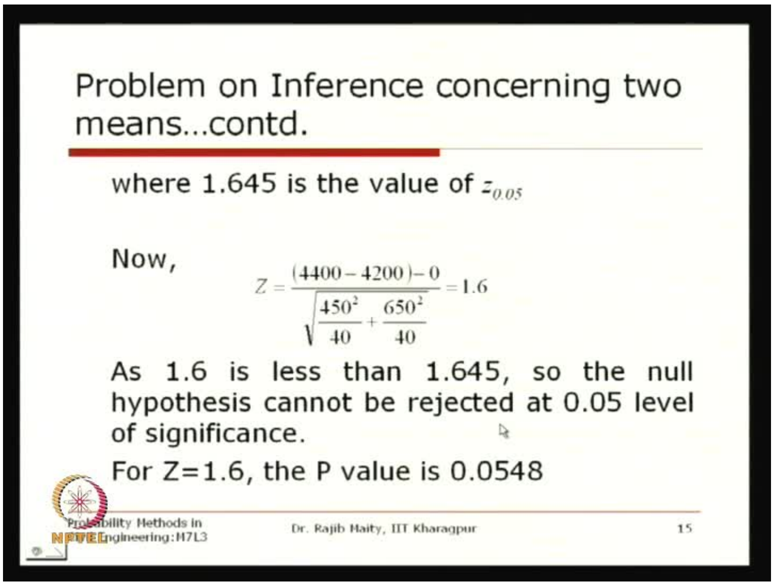

So, this is ourcritical value, and greater than this value if we get that z then, we should reject thenullhypothesis,so this is the value of this Z at 0.05.So, this Z now, if we just use thosethe sample statistics and we will get that is 1.6, so which is 1.6 is less than 1.645, so the null hypothesis cannot be rejected at 0.05 level of significance.

So, we cannot say that whether the sample one is really superior than the sample two, because we have seen that we failed to reject thenull hypothesis.As I mention earlier also we should not say that null hypothesis is accepted,we generally we should say that either the null hypothesis isrejected or it cannot be rejected with the sample data, because we have to take then larger sample and forfurther test.

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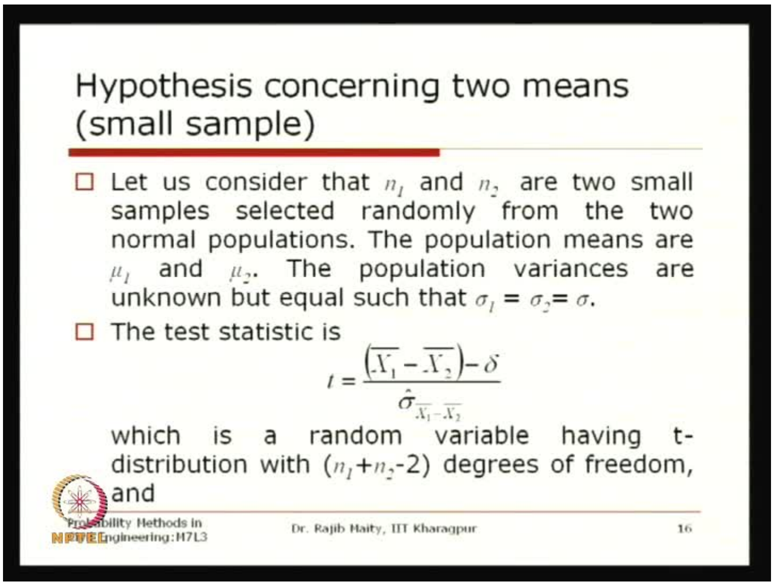

Well, so, regarding this two sample means, again,ifeither of these twosample the sample size is less than 30 then, this should be consider as asmall sample and this small sample as we have seen in thissinglemean also it follows a t distribution.

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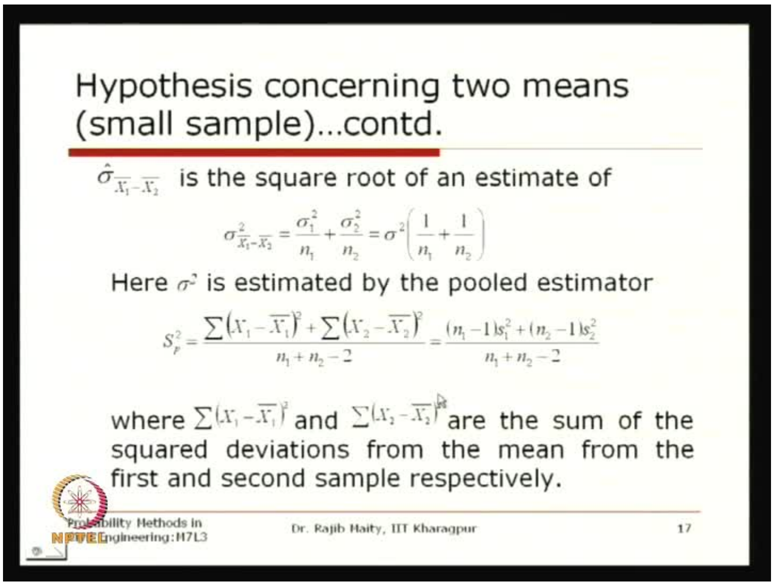

So, here, the statistics is your X1 bar minus X2 bar minus delta and this sigma hat X1 bar minus X2 hat, this is a estimate of thisdifferencebetween two mean standard deviation.So, which is athis This test statistics is a random variable having a t distribution with n 1 plus n 2 minus 2 degrees offreedom.Andthis estimate for this standard deviationis a squareroot of an estimate of this one, so this variance is sigma 1square by n 1 plus sigma 2square n 2 like this.

So, if this population variance is known and if it is not known then, it should be estimated by the pooled estimator, it is called thepooled standard deviationorpooled variance Spsquare,which is,you will get from this one.So,X1 minus X1 bar square plus the summation of X2 minus X2 bar square divided by n 1 plus n 2 by 2, which is basically that n 1 minus 1 times of this standard variance of this first sample and n 2 minus 1 times the variance of second sample divided by n 1 plus n 2 by 2, where these two quantity as the sum of the square deviation from the mean of the respective samples.

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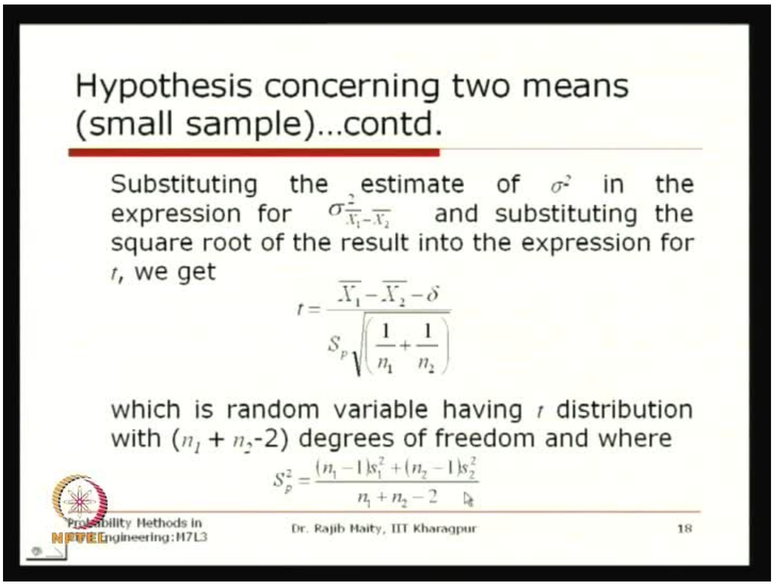

So, if we substitute the estimate of this sigma square in the expression of thisstandard this variance then, we get that this t is equals to X1 bar minus X2 bar minus delta divided by S t multiplied by squareroot1 by n 1 plus 1 by n 2, which is a random variable having t distribution with n 1 plus n 2 minus 2 degrees of freedom.And this S pcan be obtained as this S p square equals to n 1 minus1 into S1square plus n 2 minus 1 into S2square divided by n 1 plus n 2 minus 2.

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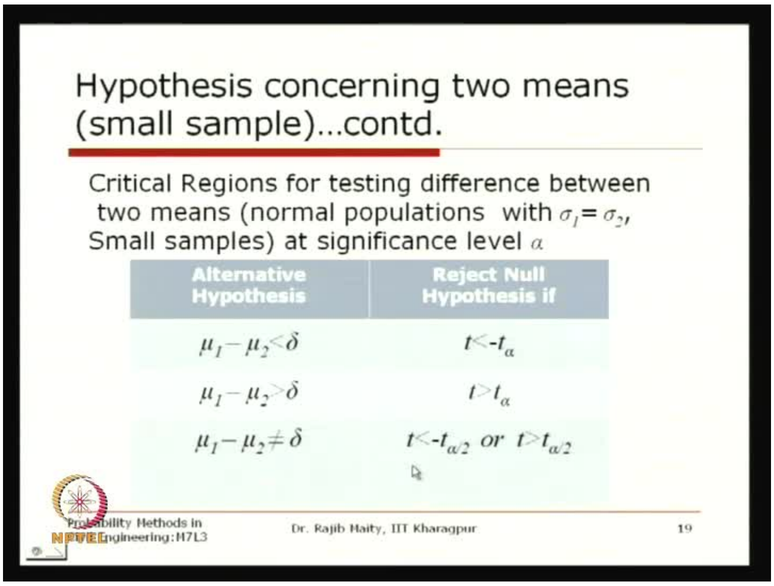

And the criticalregions are these for that teststatistics, for thisleft sided test, that is, whether the mu 1 isless than mu 2, then t whether the t is less than minus t alpha then this case t is greater than t alpha and two sided test whether t is less than minus t alpha by 2 or t is greater than plust alpha by 2.

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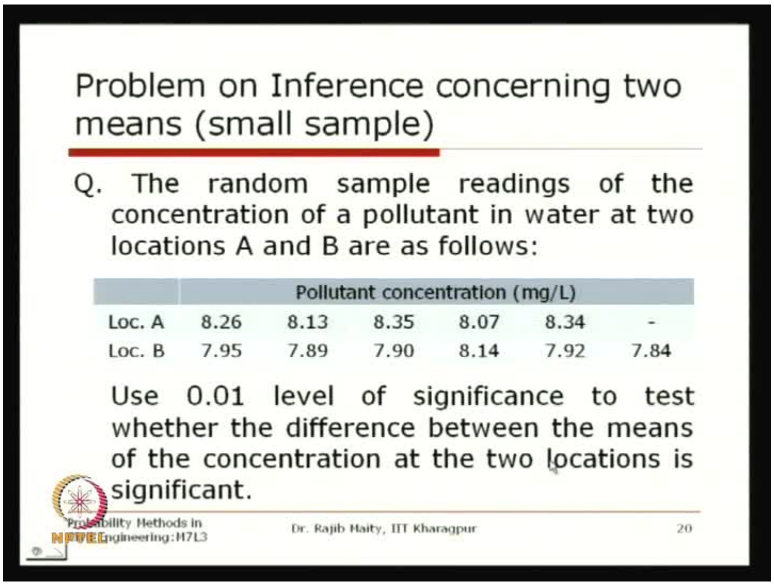

So, will take one problem on thissmall sample test islike this,that therandom sample readings of the concentration of the pollutant in the water at two locations A and B are as follows.So, there are two locations one is that location A and other one is this locationB;

and for this location A we are having 5 samples, and for this location B we are having 6samples.So, this problem is taken because, you see that it is not required that we should have this same length of this data, so that n 1 and n 2 need not be same.

So, for the sample one we are having5 samples and for thesample two we are having 6 samples.So, usethe 0.01 level of significanceto test whether the difference between the means of the concentration at two locations issignificant or not.

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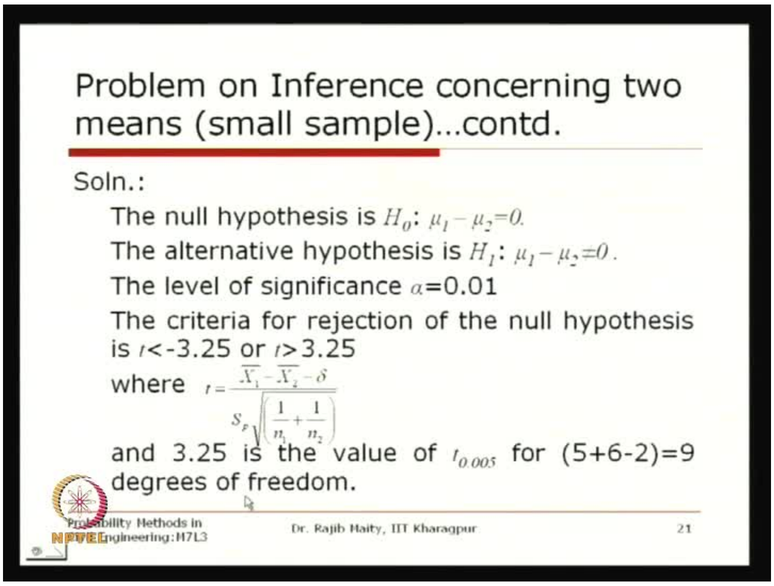

So, then what we have to do?We have to first,weformulate that hypothesis then, the null hypothesis is that, whether the difference is significant or not.So, this is a two sided test and the null hypothesis, we put that mu 1 minus mu 2 is equals to0, because we are trying to test whether these are different or not.So, mu 1 minus mu 2 is not equal to 0 that is what ourtest goal, so this we have put in this alternativehypothesis and this mu 1 minus mu 2 equals to 0 is your null hypothesis and here the level of significance alpha is your0.1.

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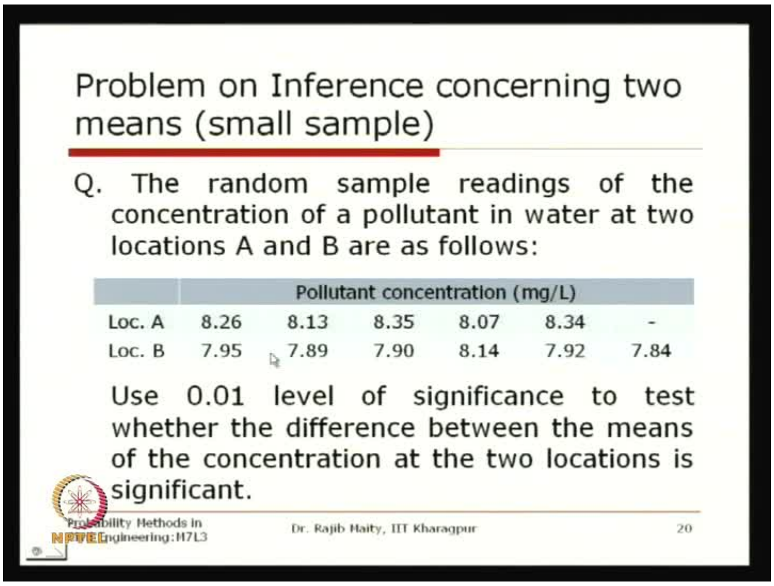

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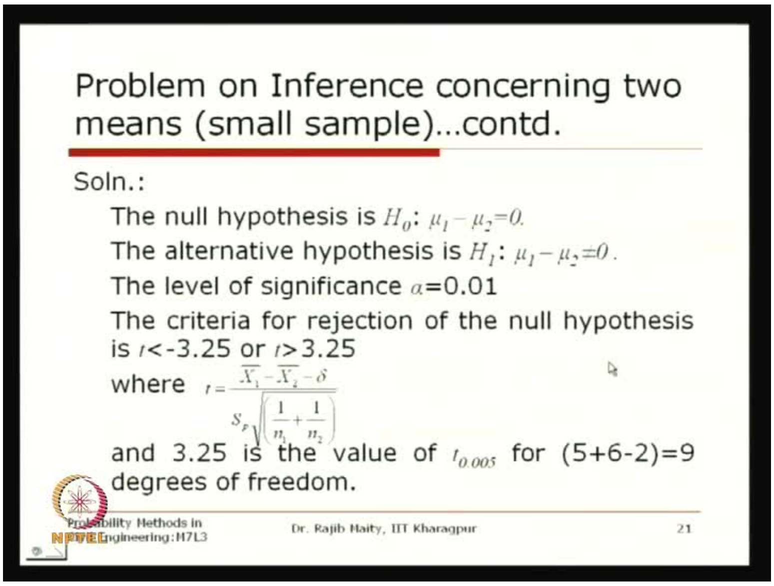

The criteria forrejection of the null hypothesis is that if this t is less than minus 3.25 or t is greater than3.25.So, these two values are basically the statistics value at the significance level 0.01 and for thet distribution having the degrees of freedom equals tothissample lengths,say,this is your 5, this is your 6, so 5 plus 6 equals to 11 minus 2 so 9.So, with the degrees offreedom 9at significance level 0.01, we get these two values that 3.25 and 3.minus3.25 and 3.25.

So, if ourtest statistics is fallingin this region, then we shouldreject the null hypothesis.And the teststatistics thatis, that X1 bar minus X2 barminus delta divided by S p square root of 1 by n 1 plus1 by n 2.So, this X1 bar is the sample mean for the first sample,X2 bar is the sample mean for this second sample that we can get whatever the data that we are having.And this S p, that is, the pooled variance,sorry, pooledstandard deviation we should also calculate whatever we have discussed so far.

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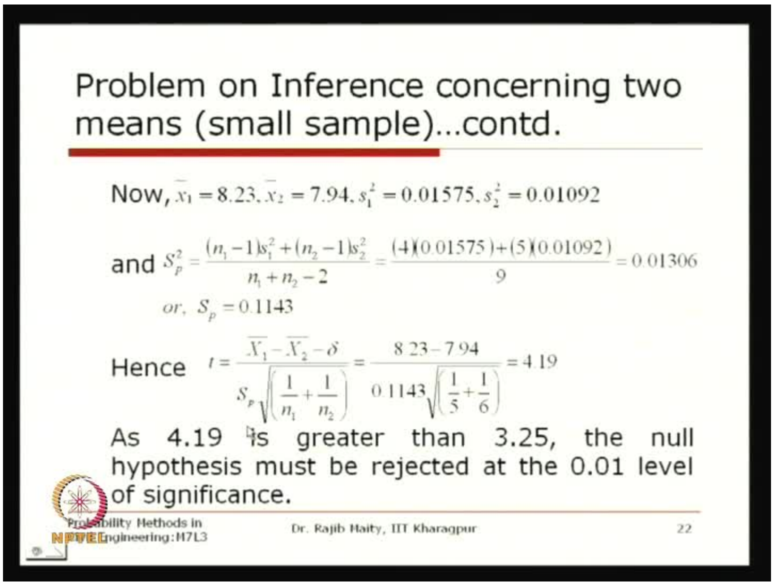

So, here, are the calculations, this X1 bar which is your mean for the first sample which is 8.23 and mean for the second sample is your 7.94,variance for this first sample is 0.01575 and variance for this second sample is 0.01092. So, thissample estimates we have already learned, so we can apply that one for whatever the data is given and we will get thissample estimates.

Now, this pooled variance is that n 1 minus 1 times thisstandard deviation of this first sample, sorry,variance of this first sample multiplied byplus that n 2 minus 1 times of this variance of second sample divided by n 1 plus n 2 minus 2.So,we will just use this one, so this first sample asthat there are 5 samples that is 4 multiplied by it is variance.And then second sample size is 6, so 6 minus 1is5 times, it is variance divided by 5 plus 6minus 2 which is 9, so we will get that 0.01306, so the S p here is equals to your 0.1143.

Hence, the t isequal toX1 bar minus X2 bar minus delta divided by S t square root of 1 by n 1 plus 1 byn2, so if we put this one,we will get the statistics as 4.19.

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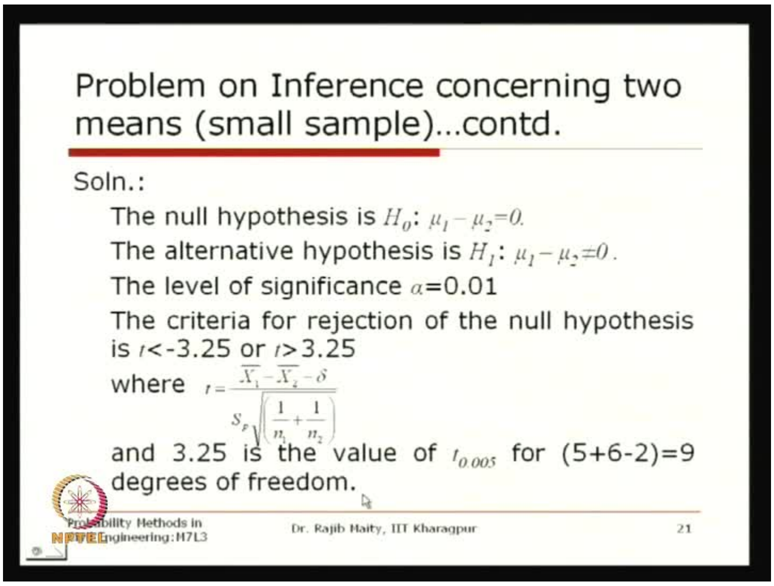

Now, this 4.19 which is greater than your 3.25, it is falling in thisright side of their distribution and which is obviously greater than the criticalvalue.So, the null hypothesismust be rejected at this0.01 level of significance, we can say so, in this null you can say that it wasthere that this both this sample means aresame.So, we shouldmust reject this null hypothesis; that means, we can infer that themean from both the locations are not same.

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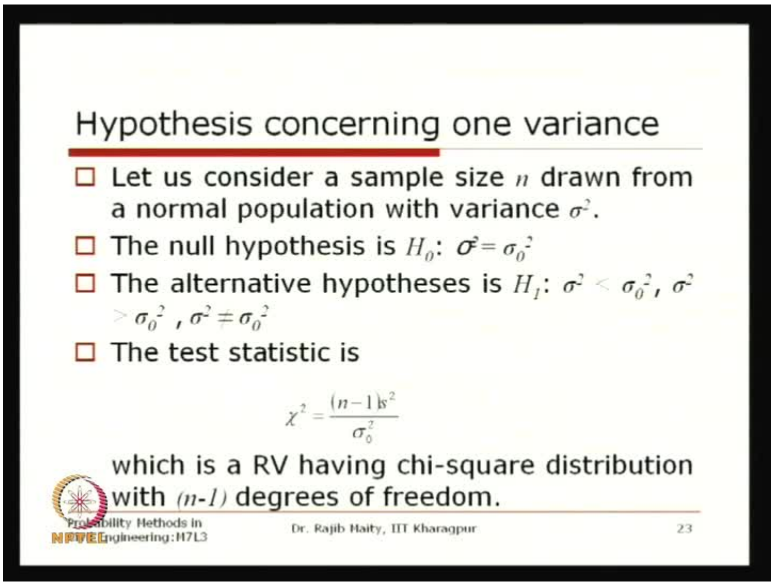

Well, so far we have seen that what is thetest for theone mean and two means and now we will see thatthevariance.First,we will start with this one variance and thus then we will go for the second variance.And let usconsider a sample size ndrawn from a normal population withvariancesigma square and this null hypothesis is h naught equals to sigma square is equal tosigma naught square.

And the alternative hypothesis is H1can be, say that less thansigma 2square, this is greater than sigma naught square and this is naught equal to sigma 2square, so anything this can happen so.And depending on this null hypothesis also will change, so this is basically for this alternative hypothesis when this is naught equal to this.

And in ourlast lecture, when we aredoing thissampling distribution also, we have seen that this variance,that is, n minus 1 where n is the sample size that times they are variance divided by sigma naught square this ratio is basically following a chi square distribution. So, this chi square, so this statistics we can use to testfor the single samplevariance, so this is whether one variance and this sigma naught is your somethreshold value for this which we are inferring from this population.And this quantity, this statistics is followingwhich is having a chi square distribution with n minus 1 degrees of freedom.

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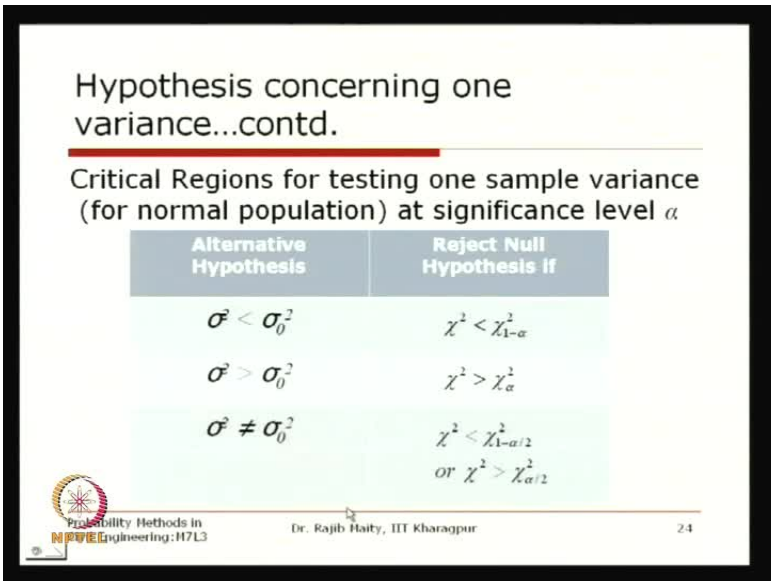

Now, the critical region for testing this one sample variance andhere theassumption is that, this population from where the sample is drawn is having a normal distribution.At this significance level alpha, if it is one sided test and if it is left sided test then,whether the chi square the statistics that we have discuss so far, that is, n minus 1 time variance divided by sample variance divided by this population variance, that statistics if it is less than thischi square valueat1 minus alphacumulative distribution,if it is less than that then, we should reject the null hypothesis.Andif it is for this right side test, if it is greater than thatchi square alpha then,we will reject this null hypothesis.And for the two sided test, if it is less than 1 minus chi square 1 minus alpha by 2 or greater than chi squarealpha by 2, then we should reject this null hypothesis.

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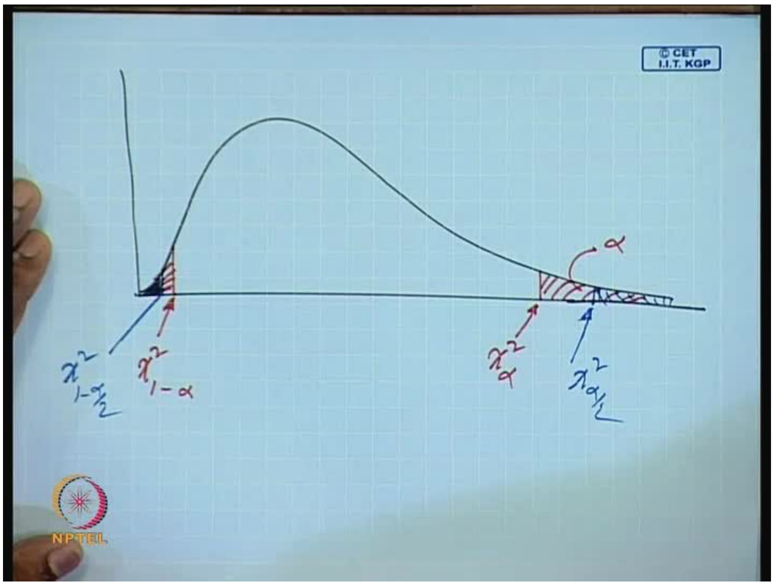

If we just see quickly thatthis typical shape of 1 chi square distribution it looks like this, so this is not symmetrical.So, here what we are referring to that this left side, so this area is basically what we are telling, so this value what we are referring to as this chi square 1 minus alpha and the same area for this thatwhen we are doing for this right handside test, suppose, that this is yourthis area if is your what is that?Alpha, then this value what we are saying that is the chi square alpha.So, here the chi square 1 minus alpha means, the total area below this curve is one, so this full area apart from this red one, this starting from the white area as well as this shaded area is your 1 minus alpha.

And these are for thisone sided test, now at this significance level of this alpha if we go for this two sided test then, obviously we are testing that 1 minus alpha by 2 which is obviously will belittle smaller thanthis 1.So, this 1 and this will be somewhat greater thanhere, so these two values which is your chi square 1 minus alpha by 2 and this is your chi square alpha by 2.

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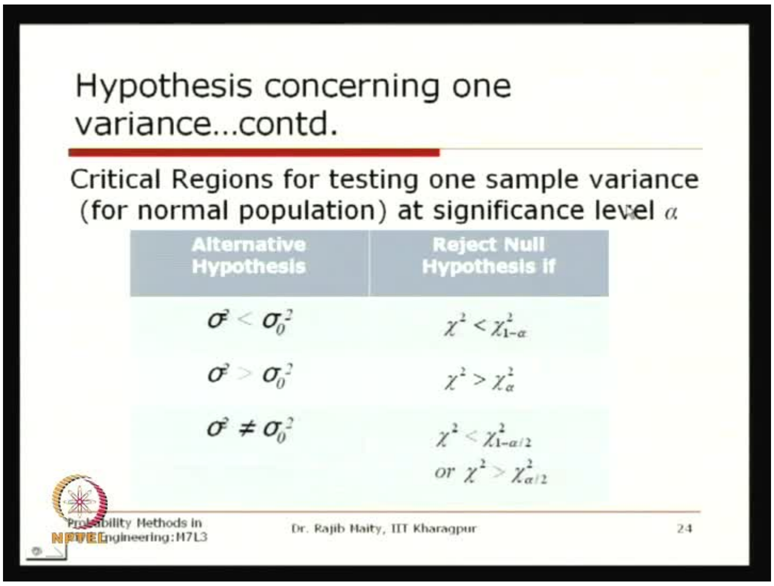

And thisarea, this now,what I am now, I am shedding the blue area here plus the blue area here should be equal to alpha.And as this chi square distribution is not symmetrical,so here these twovalues that we are looking for this critical value will not besymmetrical, as we have seen so far in case of thatstandard normal distribution and as well asin the t distribution.So, these are the critical regions for those for the one sample for the chi square distribution forone sample variance.

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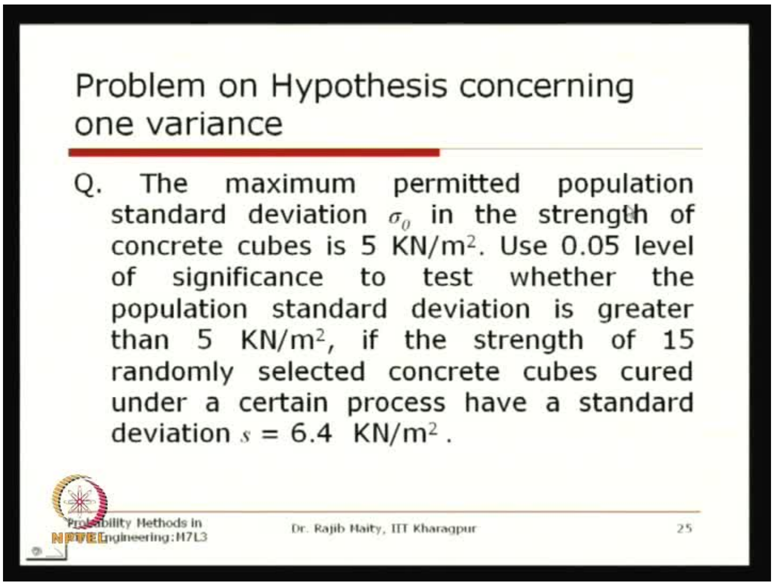

Well, take one example here,the maximum permitted population standard deviation sigma naught in the strength of a concrete cube is 5 kilo newton per meter square,so this is our requirement.Use that 0.05 level of significance to test whether the population standard deviation is greater than thisthreshold value, that is,5 kilo newton per meter square, if the strength of 15 randomly selected concrete cube cured under a certain process have a standard deviation equals to 6.4 kilo newton per meter square.So, we have taken randomly 15 samples and that sample standard deviation is 6.4, if we just compare this6.4 and 5kilo newton per meter square, this is obviously greater than that.

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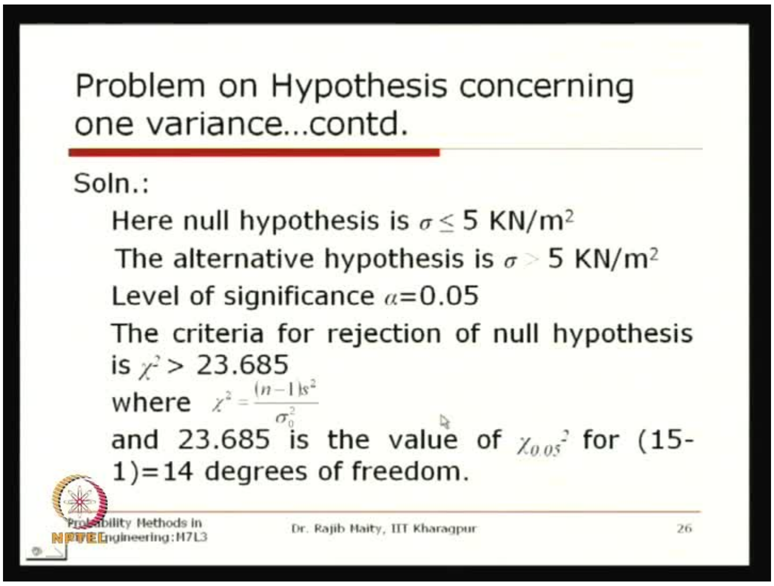

Now, through this hypothesis testing for thisvariance, one variancewe have to say that whetherprobabilistically we have to infer, whether this value is really greater than what is ourthatrequirement.So, our null hypothesis is that whether this sigma is less than equal to this 5 kilo Newton per meter square, because we are targeting to test whether thusthat populationstandard deviation from which the sample is taken is really greater than 5 kilo Newton per meter square or not.

So, the level of significance here is that 0.05, the criteria perrejection of this null hypothesis isthis chi square is greater than 23.685.Now, where from we got this23.685 is the value for the alpha 0.05, where this chi square distribution having a degrees of freedom equals to 15 minus 1 equals to 14.

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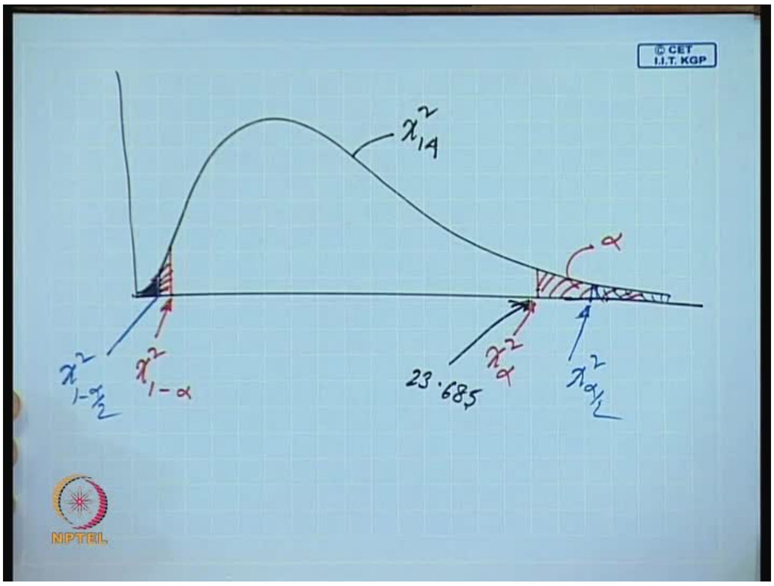

So, now, if we see that, if I say here that, this is your the chi square distribution with 14 degrees of freedom then, that this chi square alpha that we are saying that, if we just say that this red area is your 0.05 then, this value is your 23.685, which you can get from anystandard text book, these tables are this chi square tables aregiven here. So, if my test statistics falls in this one,that this is your critical zone, so if the test statistics fall in this zone, then we have to reject thenull hypothesis, and if it is belowthis one, we cannot reject the null hypothesis with the sample data that is availablewith us.

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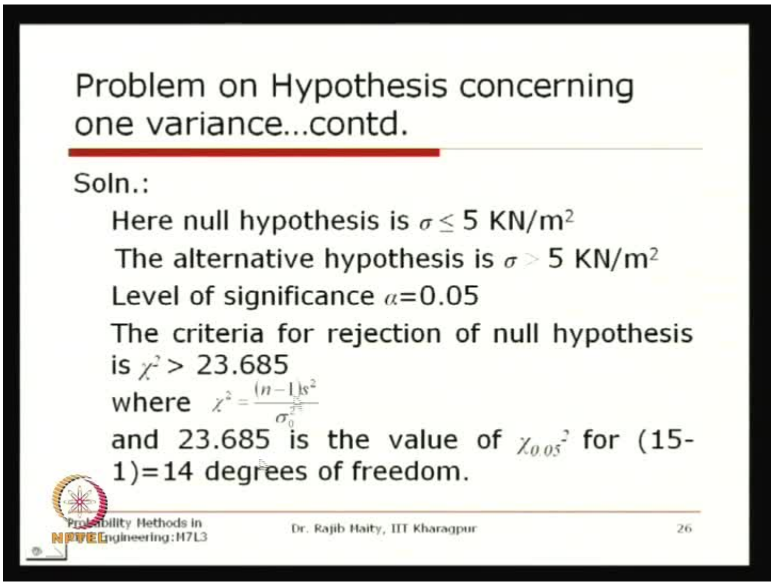

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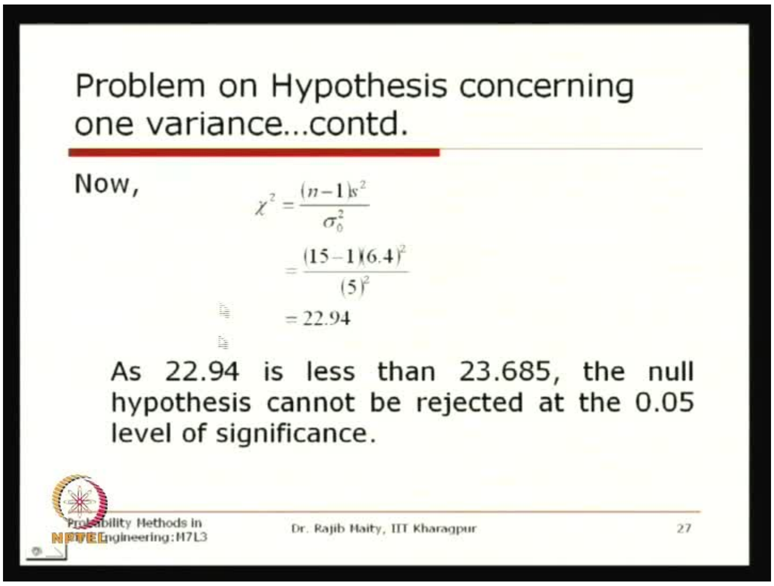

So, this is why this 23.685 is ourcritical value, and if wecalculate this teststatistics thenwith this sample data, it is now n equals to 15 minus 1 and 6.4and divided by this 5 square, then we get that22.94 which is definitely less than that our critical value.So, null hypothesis cannot be rejected at 0.05 level of significance; that means, we cannot say that whatever the sample data that we are having from that we cannot infer thatthepopulationstandard deviation is greater than ours,what is our target; that is,what is ourrequirement 5 kilo newton per meter square it is not greater than that.

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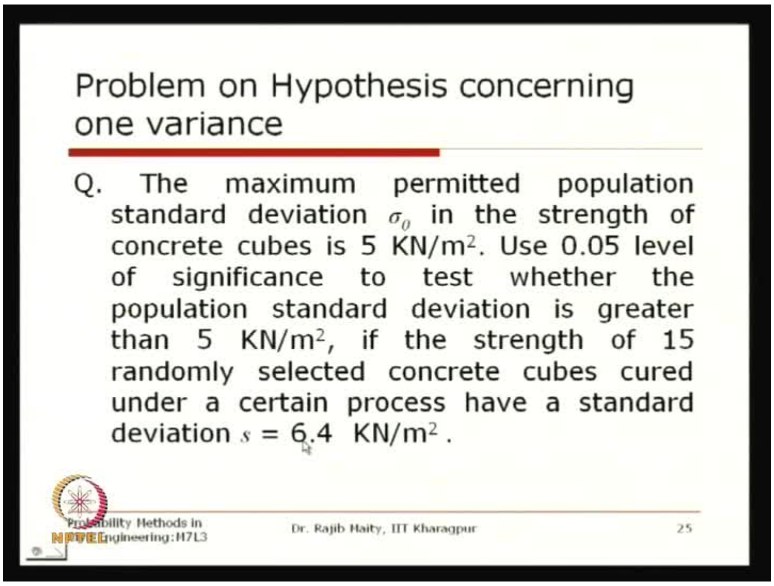

Even though we have seen that numerically, the samplestandard deviation issomething 6.4which is aclearly greater than this 5, but probabilistically we cannot infer that.

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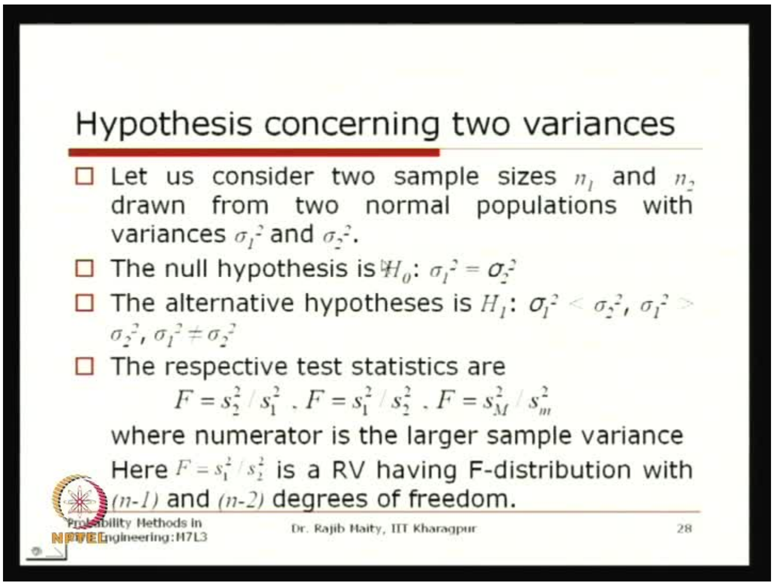

Well, now,we will take that hypothesis concerning two variances;let usconsidertwo sample size of n 1 and n 2 drawn from two normal populations, so this is thatrequirement that whatever sample we aredrawing it is from thenormal population with their variancessigma 1 square and sigma 2square.

So, here, the null hypothesis iscan be like this, so sigma 1 square is equals whether they are equal to or1 is less than or1 is less than other like that.So,here the test statistics are depending on what is your alternative hypothesis, whether the sigma 1 square is less than sigma 2 square or sigma 1 square is greater than sigma 2 square or they are not equal.Depending on that the test statistics is S2 square by S1 square or f S1 square by s 2 square or f is equal to S m square by s small m square, where the numerator is the larger sample variance.

So, you see, for the first test, when we are saying that this sigma 1 testing, we are trying to test that whether sigma 1 square is less than sigma 2 square then, we have to test that the sample variance for the second sample divided by sample variance of the first sample, this statistics we should use.

On the other hand, we should use that sigmathe variance for the first sample divided by variance for the second sample, if the first sample is greater than thesecondone.And if we are testing the two sided one then, we have to find out which one is thegreater.So, that greater value should be in the numerator and this f is having aF distribution withn minus 1 and n minus 2, so this is basically, thatthis nwhen we arewriting here,this isthere is something typing mistake here so this the first one is for the numerator.

So,whatever in this statistics whatever is there in the numerator that should be your first degrees of freedom and whatever is there in the denominator, that should be your second degrees of freedom.Now, if I just want to take all these thing in a singlestatement, then whatever we are testing always the F statistics that we are calculating, it should be always we have to keep in mind that that should be always greater than 1; that means, in the numerator always we are putting the greater value and this numeratorand thedenominator we are putting the smaller value.

So, if we can ensure thatthen we can say that f statistics is having a F distribution which is following which is following the F distribution with the first degrees of freedom is thelarger sample size minus 1 and second degrees of freedom is the smaller sample size minus 1, this is not 2, this is also 1.

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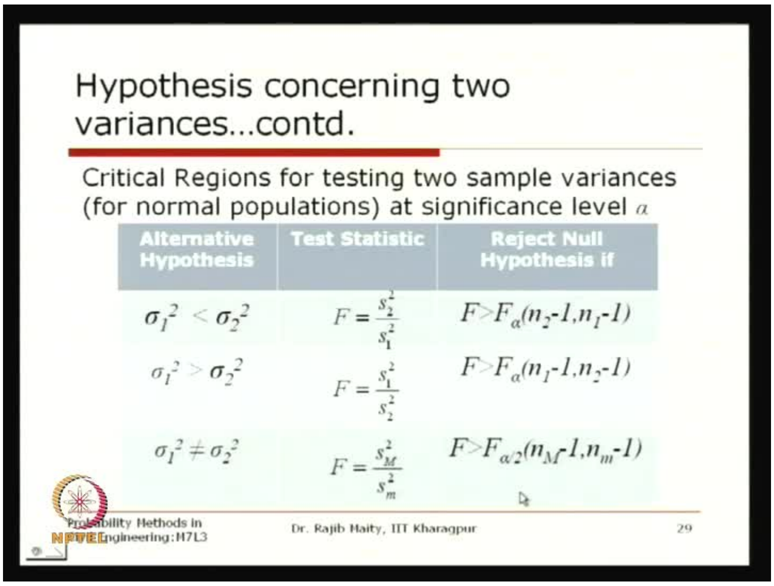

So, these two degrees of freedom with that is aF distribution,so there are again the 3 differentcases and there are 3 different teststatistics.And if we are justtaking in this way whetherwith the first one is less than the second one or first one is greater than the second one or they are not equal then,here you candepending on that you have to find out what is your critical region here.

And on the other hand, if you just want toadjust what I just now mention that, if you just simply want to use that which one is yourlarger variance, which one is your smaller variance, then always for all thesecases you can calculate these Fstatistics, which is following thatF distribution having this n m minus 1 degrees of first degrees of freedom and second degrees of freedom is n small m minus 1, where n capital subscript capital m is the larger sample size and n subscript small m is a smaller sample size.

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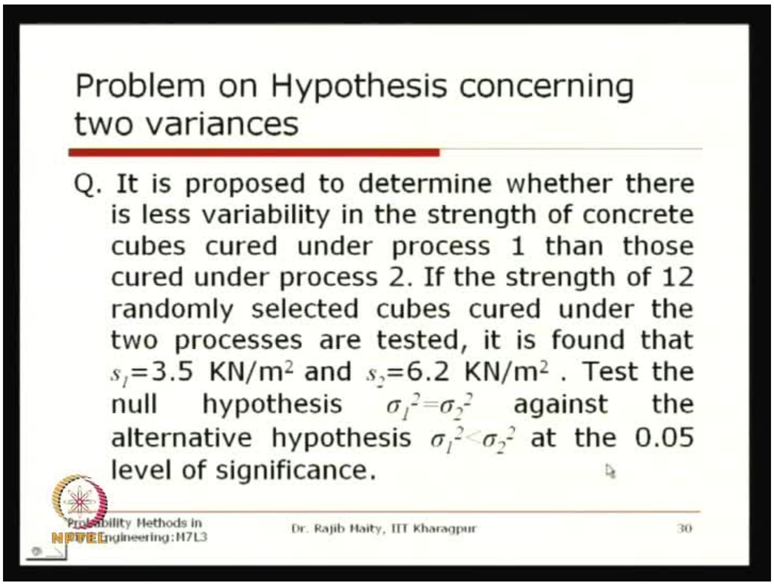

Well, take up one example here,it is proposed to determine whether there is less variability in the strength of concrete cubes cured under process one, than those cured under process two.So, there are two different process is identified and we have to test whether the strength of concrete in one process is different from theother process orhere we are testing that whether in this process one, what we are getting is the strength is less than this process two, before I can declare thatwe should follow the process two for curing the concrete,so that we can achieve that greater strength.

So, sample data istaken, so like that there are 12 randomly selected cubes are there, under two processes are tested, it is found that this s 1 is 3.5 kilo newton per meter square and s 2 is 6.2 kilo newton per meter square.Test the null hypothesiswhether thereare seen against that alternative hypothesis,whether the first one is lesser than the second one at the level of significance 0.105.Here, the null hypothesis is this and alternative hypothesis whether the sigma 1 is less than sigma 2 square level of significance is 0.05.

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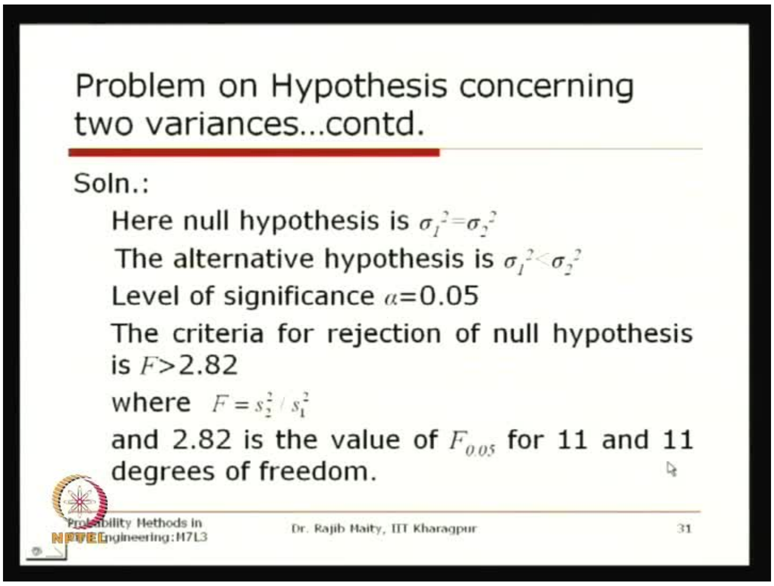

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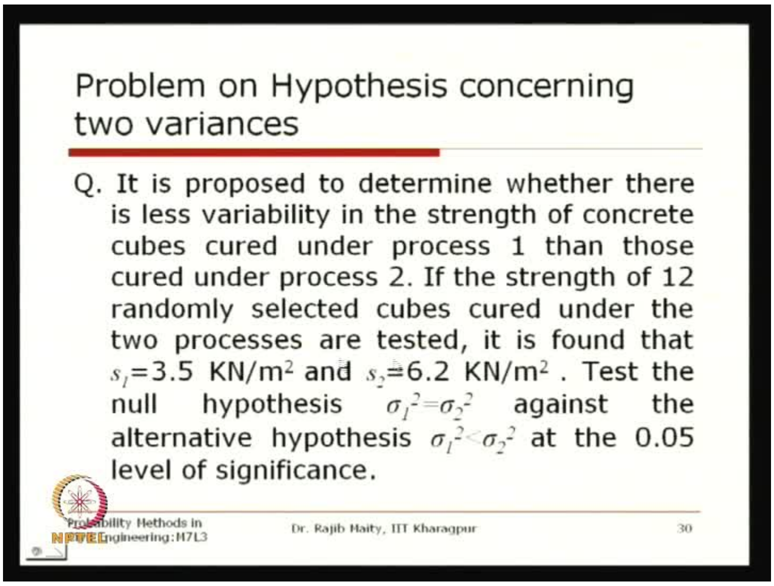

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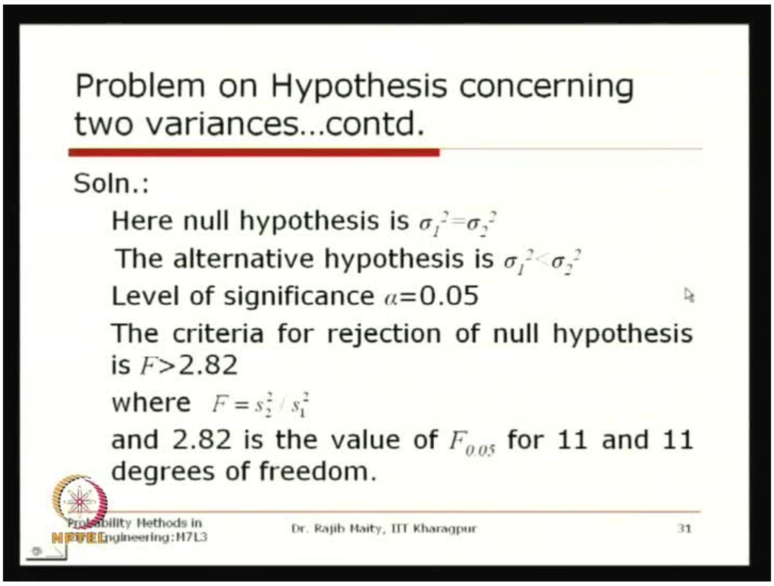

And this criteria for rejection of this null hypothesis whether if thestatistics is greater than 2.42,so how we are getting this one.So,this is the f distribution having the degrees of freedom of the sample size, that is, in both the cases the sample size is 12 here, so 12 minus 1, so 11; first degrees of freedom is 11, second degrees of freedom is also 11.

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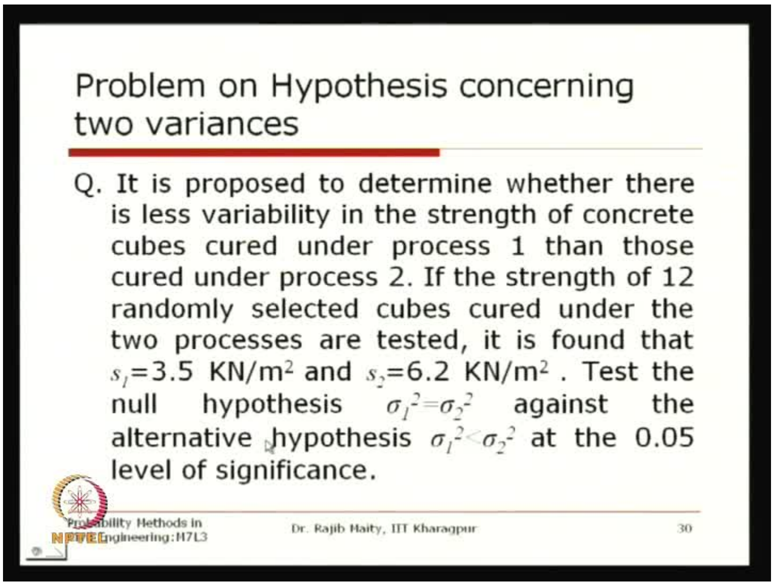

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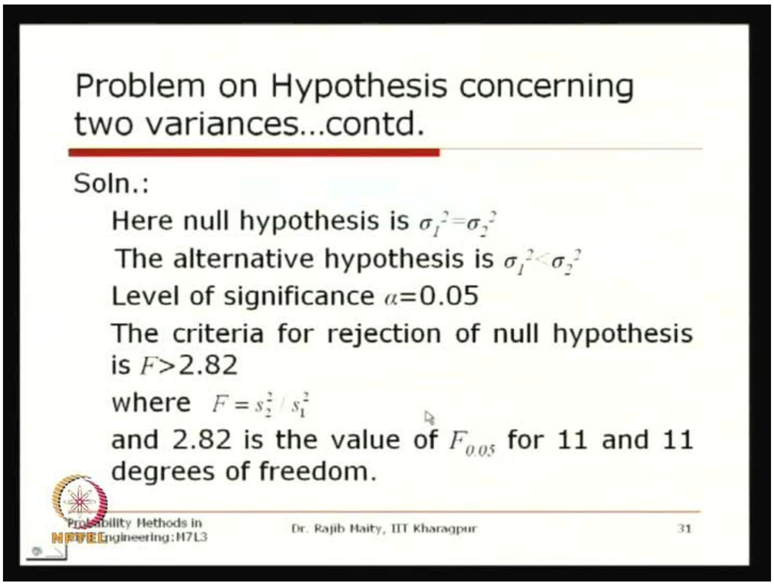

So, where this F is equals to s 2 squareby s 1; s 2 square bys 1 square and you have seen that this s 2 is your 6.2 and s 1 is 3.5, so s 2 is greater than that, so if we use this teststatistics s 2 square by s 1 square, so it isthat anyway here in this case both the sample sizes are same.

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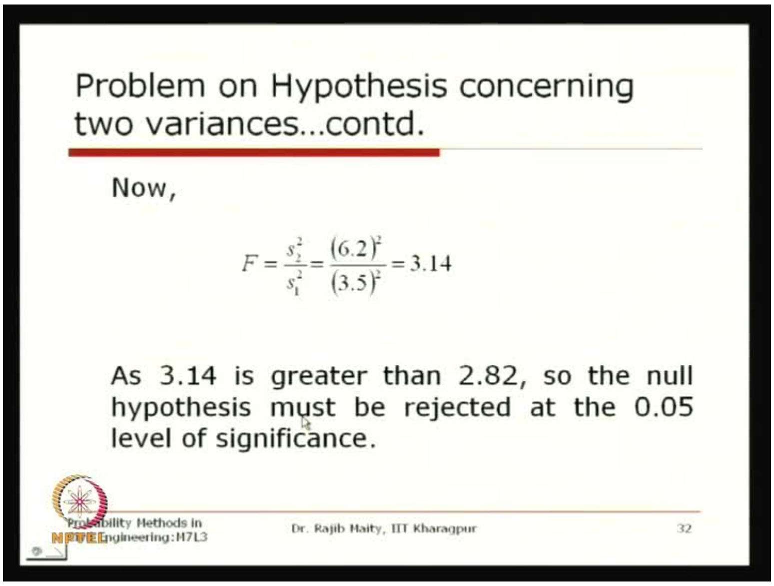

Here, the test statistics counts to be 3.14 as this 3.14 isgreater than2.82, so the null hypothesis must be rejected at 0.05 level ofsignificance.

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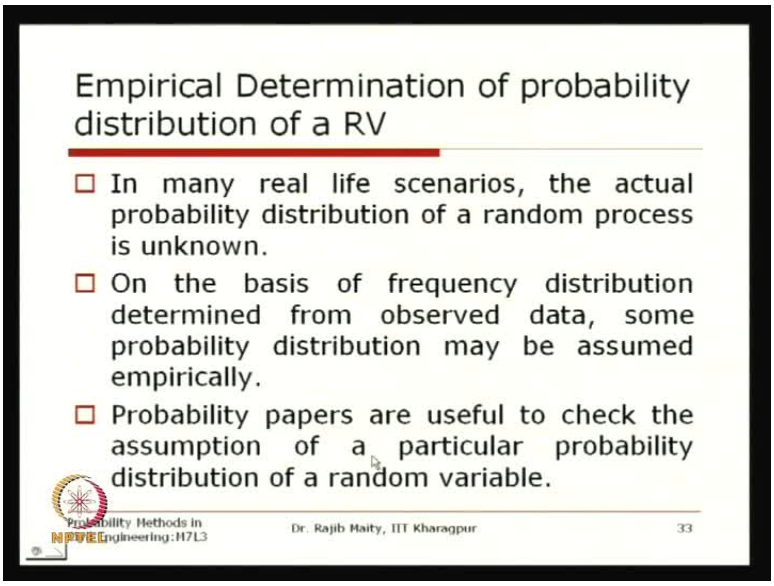

So far in thishypothesis testing what we have seen that, we have firstfound out in the earlier lecture, what are the sampling distribution of thosetestof thosethe that estimated value the sample estimation.Andin this hypothesis testing, we are basically trying to infer something aboutthe population from which the sample is wrong.Suppose, that we are having thissomesample of size n and we estimate what is it is mean and if we want to infer something from that sample estimateand we want to infer something regarding the populationthen, we have seen that, we have tested for thissingle mean,we have tested for the two means and again, we have taken the single variance, we have taken the two variances.

Like this, we can test and there are several, we have even usedifferent examples from this civil engineering.There should be somemany exampleslike this, where you can use this theory to infer something about their population, because always the sample that we get is limited.So, before we caninfer something about their population, we have to use this testproperly, so that we canjudge something.

Even though we get some numericalvalues from the sample, just by this comparison of this numerical value can show you something else, but whether that is really significant, because always you take one sample and take the another sample from the same population, the statistics may not be same then, mean for both the samples may not be exactly same.

So, those differences whether they are really significant from the statistical point of view or not to test that one whatever the discussion we have done so far, with respect to the single mean,two means, single variance, two variances that we have to follow.

Now,we will take some time to spend on that,because we have seen thatin many times I mention thatlook at this data isfollowing that distribution.Now, we are having thatdata,now with the data how can we say that this data is coming from a from a population which is following this distribution.So, there are basically two things, one is that first we will do some graphical approach and there are some statistical test also for these twoinfers that whether, this is really following thatpopulation from which the sample is drawn is really following a particular distribution or not.

So,we will start that one and the first thing is thegraphicalrepresentation which may bewhich we generally test through thisprobabilitypaper.So, the construction of this probability paper and the testing that how, whether it is really following or not graphically that will see now.

So, the empirical determination of the probability distribution of a random variable, so in many real life scenario the actual probability distribution of a random process is unknown.So, on the basis of the frequency distribution of the sample data that we are having, so determine from this observed data which is the sampleavailable to ussome probability distribution may be assumed empirically.

Probability papers are useful to check the assumption of a particular probability distribution of a random variable.Say that, I have a sample I am saying thatthis sample is taken from a population which is distributed normally.Now, we have to use a normal probability paper and plot that data and then,we will just discuss how we can infer thator the visually how we can inspect that whether it is really following the normal distribution or not.And again,we will take somestatistical test to probabilistically infer whether really that is following the distribution or not.

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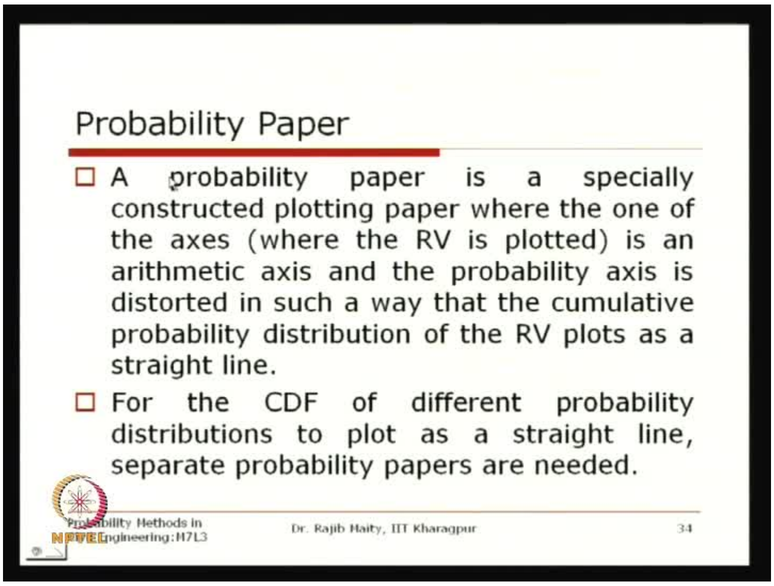

So, a probability paper is aspecially constructed plotting paper, where the one of the axis, where the random variable is plotted is an arithmetic axis and the probability axis is distorted in such a way that the cumulative probability distribution of the random variable plots appears as a straight line.So,there will be one appears as a straight line, so for the CDF- that cumulative distribution function - of different probability distribution to plot as a straight line separate probability papers are needed.

So, if I want to test that whether it is following a normal distribution then, I have to use a normal probability paper, and if I want to test whether it is following a exponential distribution, so we have to use the differentpaper.Now, how these papers areconstructedand how it is tested we will see now.

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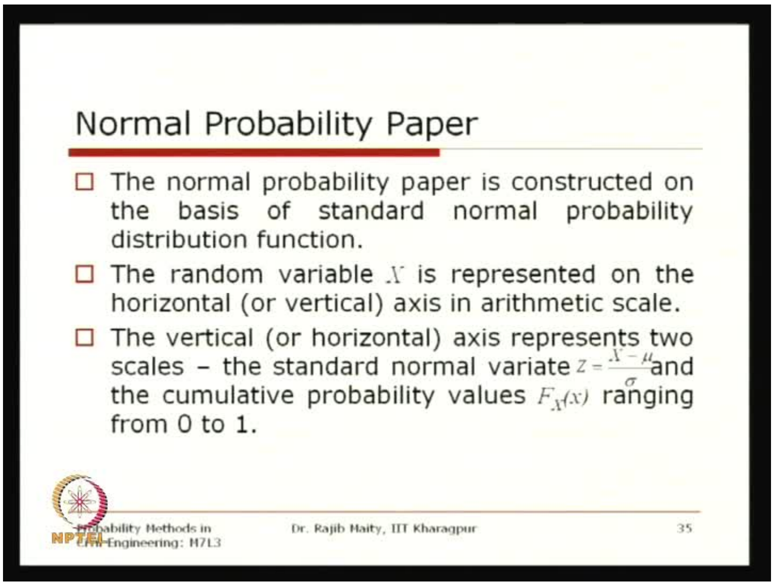

So, first,we are taking this normal probabilitypaper which is most widely used so to test that whether thesample data belongs to a population which isnormally distributed.So, thisnormal probability paper is constructed on the basis of standard normal probability distribution function. The random variable X is represented on the horizontal or sometime in some casesvertical axis also, but mostlythis random variable is generally represented on thein the horizontal axis and that axis is a arithmetic scale.The vertical axis or horizontal, if I justreverse this one as I was telling that if this X is on the vertical end, this one will be the horizontal otherwise in most of the cases it isvertical axis.So, the vertical axis represents the two scales, the standard normal variateZ equals to X minus mu by sigma and the cumulative probability values Fx ranging from 0 to 1.

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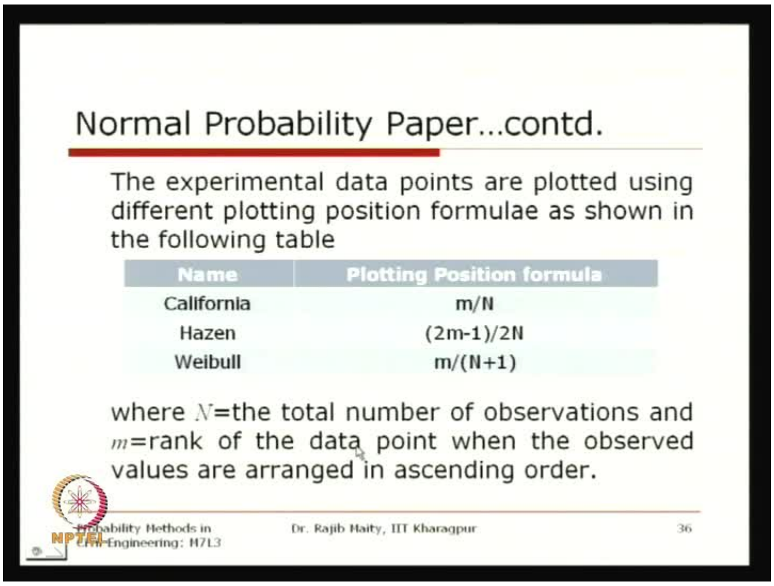

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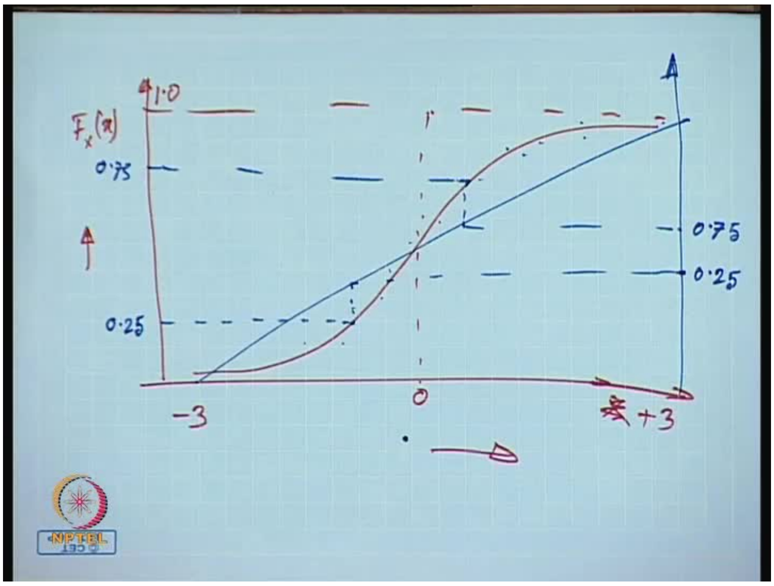

Now,before I go further, if I just seeit graphically how these things, how the concept is taken, you know thatif this is your thatXthis is the axis for your the random variable and if this is your that cumulative probability axis which is that FX for this specific value x,which is a cumulative distribution, you have seen earliertowards the beginning of this course.This distribution is generally looks like this, so which is asymptotic to 0 at this minus infinity and which is asymptotic to one atplus infinity.And we have also seen that it is from this, if it is a standardnormal distribution then, it is from this minus 3 to plus 3 almost, thatmost of this probability isexhausted here.

Now, if I justtake that this is your, say, that for this standard normaldistribution, if we just see,so here the mean is coming approximately say 0, and this is a minus 3and this is a something plus 3 or in somecomputer application, sometimes even we can go that minus 5 to plus 5 almost very closed one probability is exhausted there.

So, what we generally want to do is that,basically, whatever the sample data that we are havingnow if I just watchnow,that this is also that arithmetic scale and this is also the arithmetic scale.Now, if you plot that data, whatever the data that we are havingwith respect to their cumulative probability, how to get the cumulative probability,we will discuss.

So, if we can plot that one and if it follow approximately thisline then, we can infer that yet it is following this particular distribution, but by that I inspection, it is very difficult to say whether this particular shape is following or not.Rather, we can say, if we can distortthis axis in such a way that this will appear as a straight line then inspecting by I that whether, this is following a straight line or not that is easier than comparing that whether it is following this particularcurve linear path or not.

So, to make this axis distorted what we generally do is that, we generally take it straight line between this.And now, let us create one new axis here such that, suppose, that this I am starting from 0.25, so I am starting from thisline going up to this point and then, I am going to this straight line, and from this one I am just giving the name, giving the number as 0.25.

Similarly, suppose, that I am starting from 0.75 here,I will go to this one, first come to this straight line and go here, so that I write that 0.75.Similarly,from 0.05,0.1,0.15 like this all thesepoints whatever is there, if I just go there and distort this axis, basically, this axis I am just squeezing some part expanding some part, in such a way that this red curve linier path is becoming stretching to a straight line.

And in that axis - in that distorted axis - if I plot whatever thedata that we are having that if it appears that straight line, then we can conclude that it is following thenormal distribution.Now, with this x axis keeping in the sameaxis and this distorted probability axis, whatever the paper that we get that is known as this probability paper.

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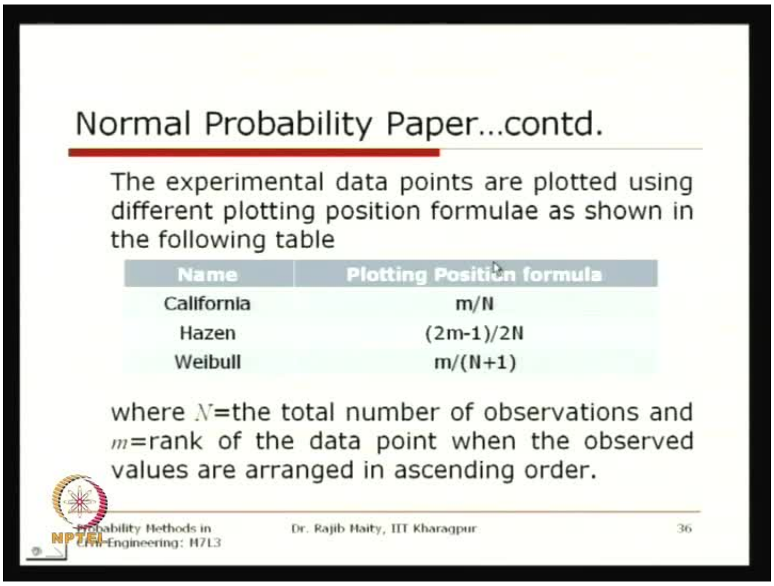

Now, this example, that I have shown this is for the normal distribution and this similarly,this can be done for this other distributions as well.Now, second question is,how we will get what is their cumulative probability?For that there are different plotting position formula is available;for example,the California gave that m by N,Hazen gave the 2 m minus 1 by 2 N,Weibull gave theformula m by N plus 1.

Where thisN is the total number of observation that is the sample size and m is the rank of the data point when the observed values are arranged in ascending order.So,thisplotting from this plotting position, generally, this Weibull plotting position is mostly used,using this one we get this cumulative probability distribution, andwe get that probabilityvalue and then, we plot it on this different probability paper.So, we can first plot it on this normal probability paper and check whether it is coming to be the straight line or not and other probability papers also from what it is following this straight line.We should conclude that this the population of this sample isfollowing that particular distribution.

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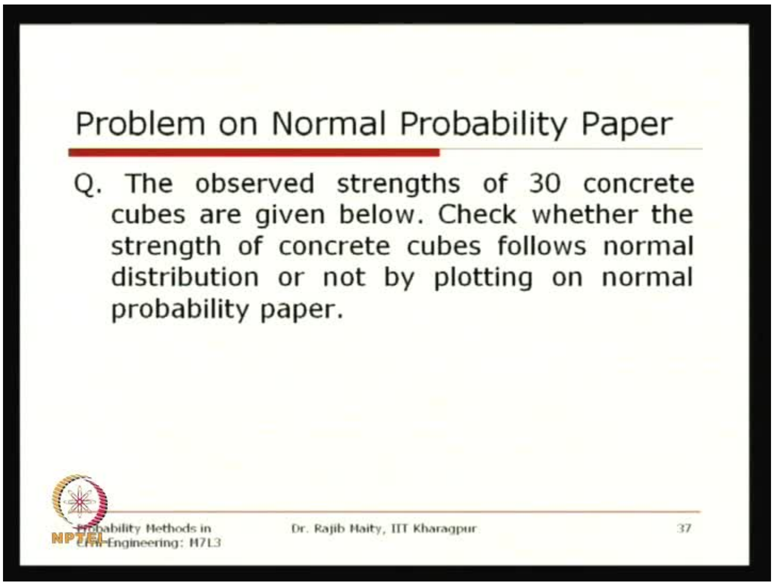

Well, take one example here, the observe strength of 30 concrete cubes is given below means, in this table next slide, checkwhether the strength of the concrete cube follows the normal distribution or not, by plotting on this normal probability paper.

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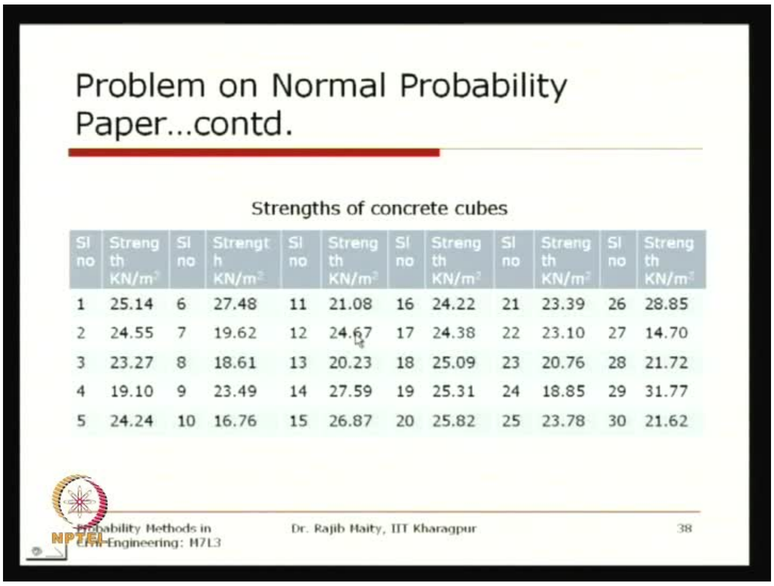

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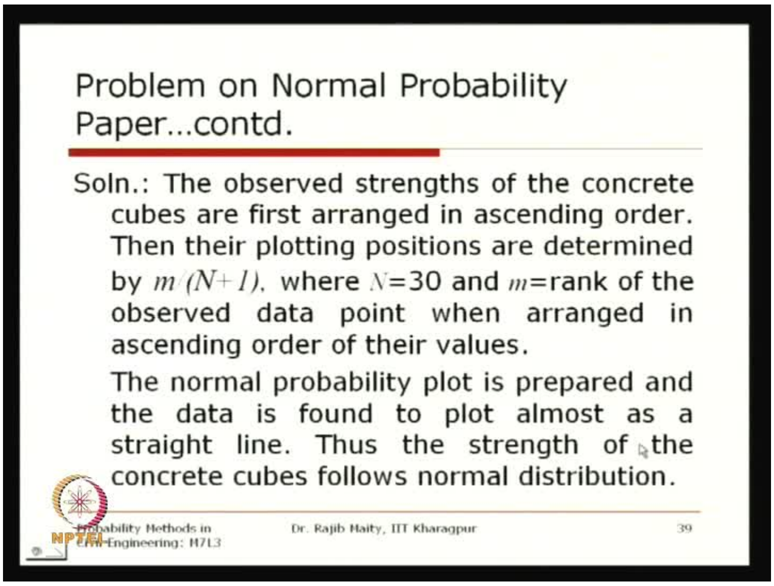

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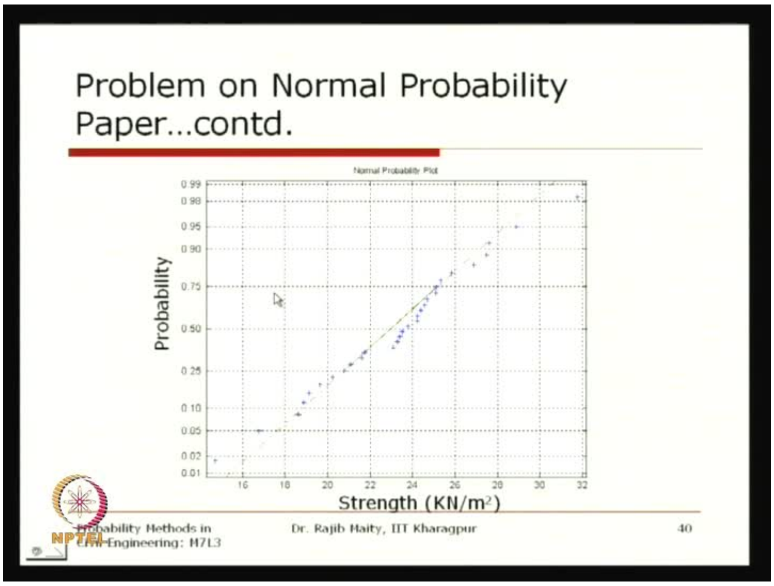

So, these are the strength of this 30 samples are given here, and after doing this using that Weibull formula for their plotting position we get all those we just arrange it in this ascending order and get their respective values and then, we plot it here.Now, you see here, this x axis is yourthat actual strength, thepictorial representation that I have given it is for thisstandard normal distribution.

Now, this axis can easily be, for this any axis that and so here that this actual axis is shown, so what it is the strength range.And this you can see that this axis is now distorted to get that what is theirprobability values.Now, this one is coming, this blue plus sign are the data and now you have to buy your judgment you have to test whether this bluelines are your that this blueplus dots are really following a straight line or not.

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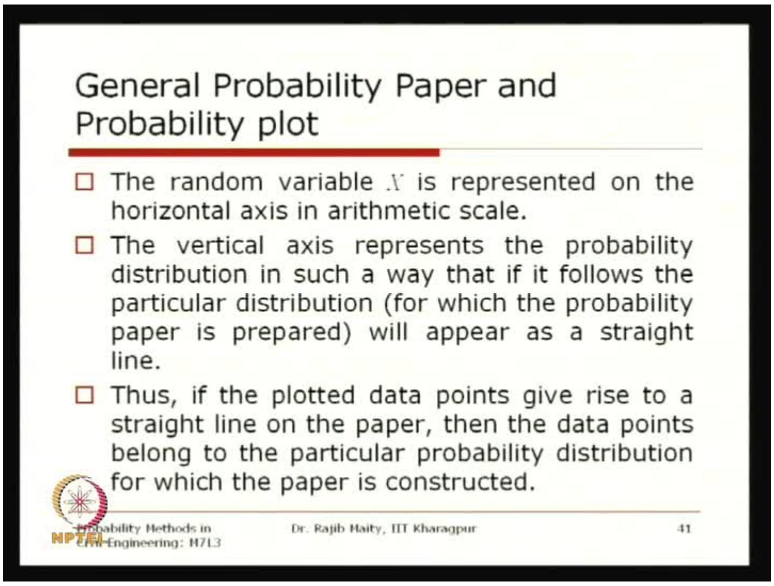

So, this is just by your Iinspection and there are some test also,statistical test that also we will see next.Thisis for this normal probability paper and that the general probability paper and the probability plot the random variable X is represented on the horizontal axis in the arithmetic scale.The vertical axis represents the probability distribution in such a way,that if it follows a particular distribution for which the probability paper is prepared, whether the probability paper is prepared for the normal distribution or gamma distribution or exponential distribution depending on that.If it appears on that paper as a straight line then, we can say that this is following that particular distribution.

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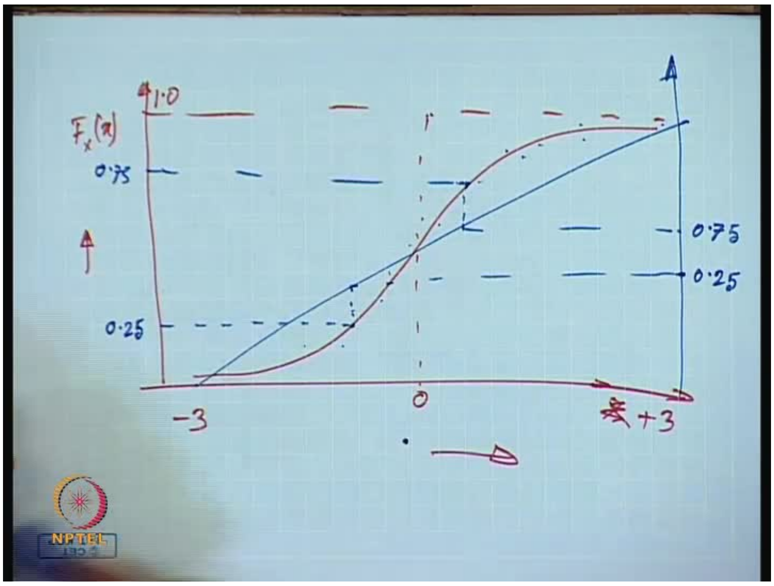

Thus, if the plotteddata pointsgive rise to the straight line on the paper then, the data points belongs to the particular probability distribution for which the paperisconstructed.Now,so this is for this general case, now say that,so this cumulative distribution that I have shown ithere,this is that, this is for this normaldistribution.

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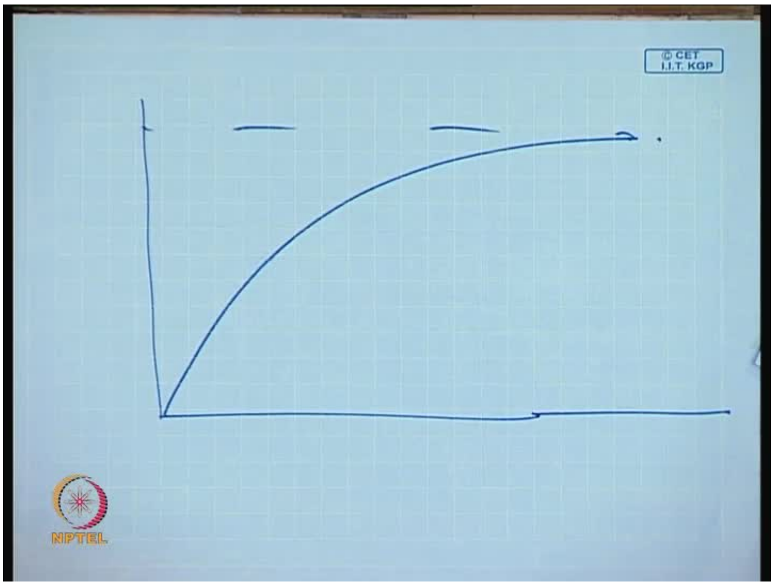

Now, I can take for this exponential distribution and this exponential distribution,alsothat will be if I just take, so this exponential distribution, the cumulative distribution looks like this, thatnow, this one also where this almost is approximately almost equals to one.So, from there if you have to draw a straight line and then, you have this probability axis, you have to distort thus within the same method, as I havediscussed now.

So, whatever the new axis you will get with respect to this actual axis thishorizontal axis of this random variable you willget, the paper which is for this exponential distribution.And on that paper if it appears to be the straight line then, we can say that this is following theexponential distribution.

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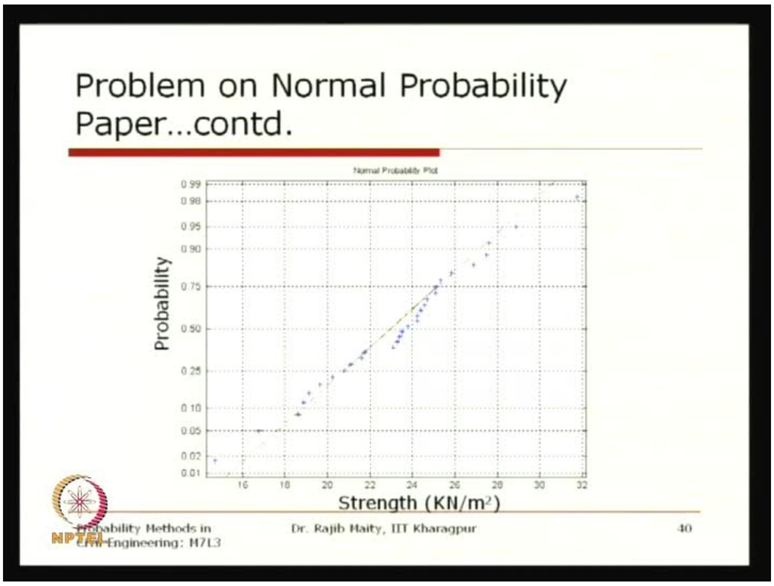

Now,one thing may ask thatNow again, I am nowgoing back to the problem we havediscussedthat.Now, this blue stars whether this is really a straight line or not.So, this is now, whether you can say just by your Iinspection you say whether it is really following the straight line or not.So,that is your, basically, a personal judgment that whether I can say that it is following the straight line or sometime we can say that no it is not following the straight line.

So, what we need actually?We need one probabilistic test forwhich, basically, we cansay, we can infer probabilistically at this significance level, this particulardata sample ordata sample fromthe population of thatdata sample is following that distribution.So, there also we need the hypothesis testing, thereare different teststatistics are there,different tests are there, the chi square test,(( ))of test, through that test we caninfer that whichdistributionthat the population is following, and that we will take up in our next lecture; thank you.