**Probability Methods in Civil Engineering**

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**Lecture No. # 33**

**Probability Models using Gamma and Extreme Value Distribution**

Hello and welcome to this lecture.We are in now module six and where we are discussing about some of the probability models.Using those standard probability distributions, we are trying to solve some of the civil engineering real life problems.This is our third lecture of this module and in the earlier two lectures, we have seen some of the models for example, that normal distribution, then log normal distribution. So, this lecture will be taking some more continuous random variables; for example, we will take the gamma distribution, then extreme value distribution.All this distribution is similar to the other engineering or other field of application.In civil engineering area also, there are tremendous application of this type of distribution. So, we will take up this in this lecture also, we will take you through some of the applications of the civil engineering problems through these probability models.

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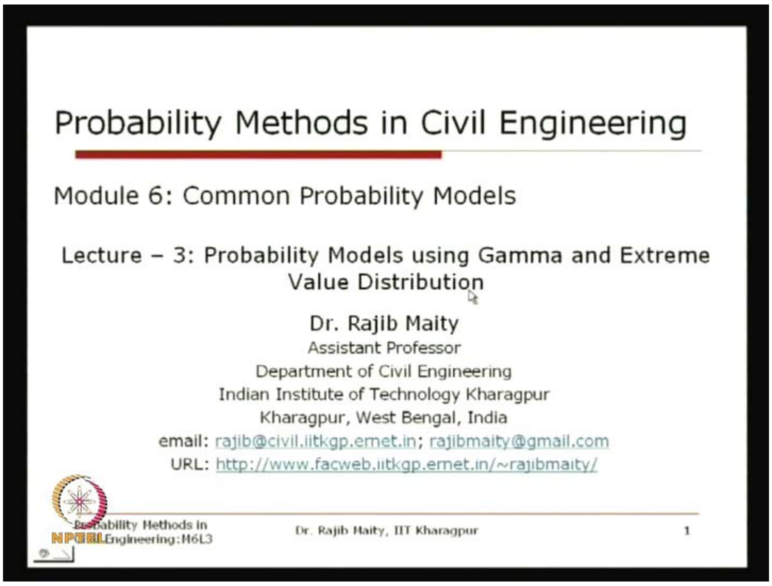

So, our today's lecture is on this probability models using gamma and extreme value distributions.

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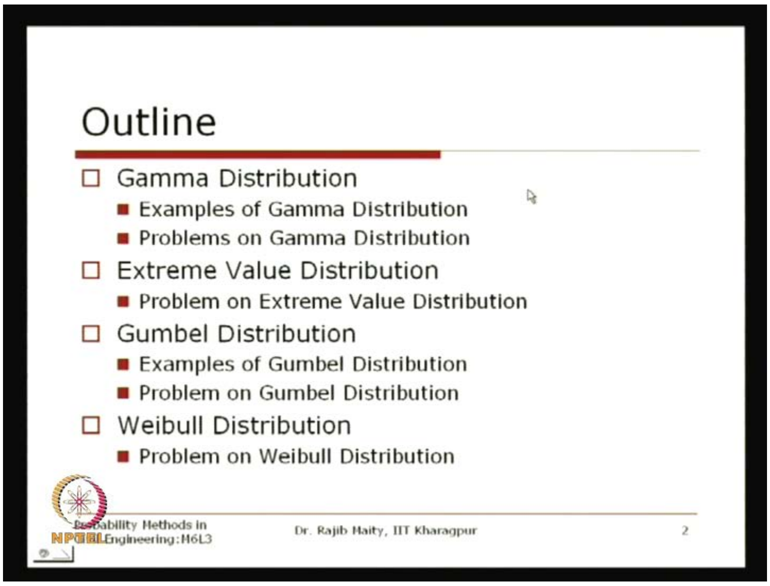


We will start with the gamma distribution, where we will first discuss about what are the basics of this distribution function and all.You know that these distributions are discuss earlier in details.Now we will not go into that detail, but, we will just briefly, overall we will mention those distributions and their few properties.We will, basically, this module is focused to its application through this type of probability models, this type of models.

So, we will we will go to some of the examples of this gamma distribution.There are some problems that we can address through this gamma distribution; obviously, this problems are related to the civil engineering problem, that we will we will discuss.After that, we will take another distribution which is extreme value distribution and we will discuss some of this basic problems first on this extreme value distribution.We will discuss, what are the different types of extreme value distribution?There are basically three types are there. So, we will discuss which type is generally is used. We will also see what are the generalized extreme value distribution form and how we can take those distribution of this different types of information. And how those different types of extreme value distribution can be applied to different types of problems.

Then, we will take that Gumbel distribution, which is also that extreme value type one distribution you also say. So, this Gumbel distribution having a very wide application,particularly, when we are talking about some extreme events, in particularly in the area of the hydrology and water resource, which is one of this discipline of the civil engineering.That we will we will discuss through what are the different application problems that we can address with this distribution,Also, we will see some of the examples using this distribution.

After that, another important distribution is a Weibull distribution.This Weibull distribution generally is used to address some of the structural engineering problem for its failure and all.We will see, we will discuss that distribution along with some of this problems also. One thing, that this there is one distribution called the reverse Weibull distribution, which is also that one of this type of this extreme value distribution.This is known as the type three distribution.We will see different types, as I told that while, when we are discussing, when we will be discussing this extreme value distribution.

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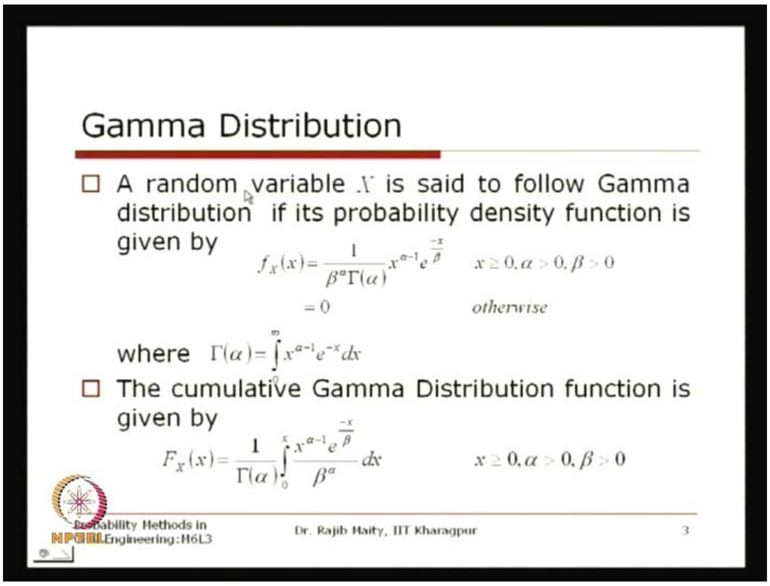


So, to start with, we will start with this gamma distribution and we know from the earlier module and our earlier discussion, that a random variable x is said to follow the gamma distribution.If its probability density function is given by this, in this form, which is that f x x equals to 1 by beta power alpha gamma alpha x power alpha minus 1 e power minus x by beta.You know that this support of this distribution is non negative, that is greater than equal to 0 that support of this random variable x.This alpha and beta are non negative are greater than 0. So far, as their this these are the two parameters of this distribution.This this gamma function,this is this is a gamma function which is its notation is gamma alpha and which can be expressed through this one, there is from 0to infinity x power alpha minus 1 e power minus x d x.

Now, this gamma distribution, this is a pdf, that is probability density function and the cumulative gamma distribution function is given by, you know that we have to integrate it from this left extreme, that is 0to that particular value x. It will come like this one, by gamma alpha which is; obviously, a constant and we can integrate this one to get its cumulative distribution.

Now, this integration, that is this gamma alpha,is generally available from this standard table.Now, we also discuss earlier, that if if alpha is an integer, then that gamma alpha can be expressed at this alpha minus 1 multiplied by gamma alpha minus 1. So, in that way, if we know that this values for the 0to 1 only for this alpha, then we can get any any value of this gamma function for for any number so that, we can get from that table.This integration we can get from this, either from this gamma distribution table from this standard text or from this numerically integration.Now, there are different soft wares are available from which we can use this, use, we can use the numerical integration to obtain thesethis values.

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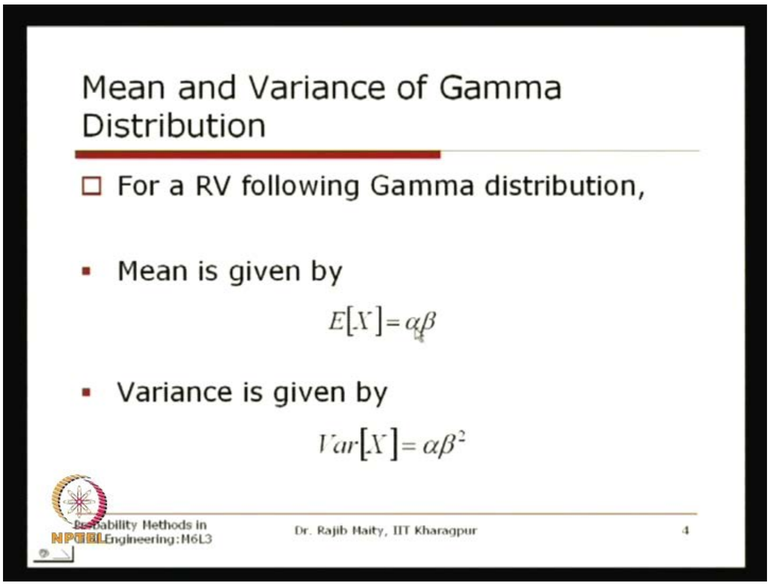


Now, from, we have also seen that this parameters, that is the mean of this random variable of which follows this gamma distribution can be expressed through its parameter.That is, mean is that the product of two parameters alpha and beta and the variance is alpha beta square.

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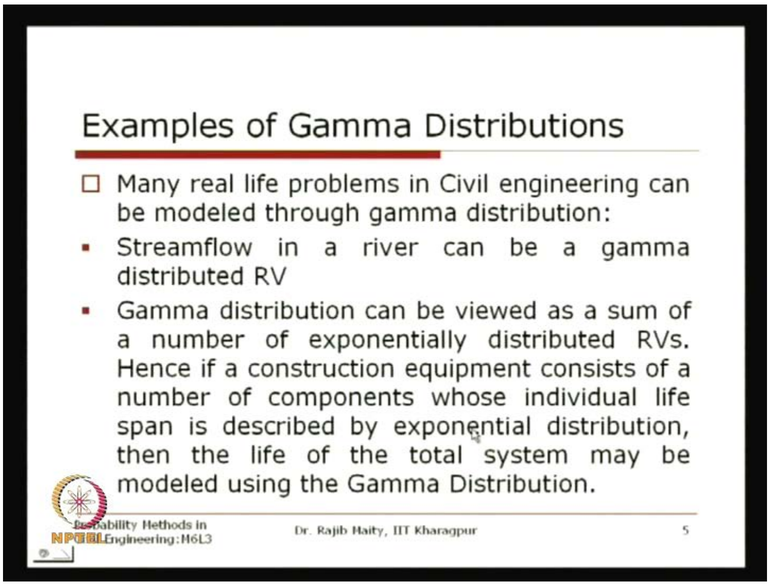


The other parameters are also discussed earlier.Now, we will see some of this.As our main goal of this lecture is to just see, how we can use this probability model for different problems of this civil engineering.So first, we will first, we will discuss some of the real life problem which can be modeled through this gamma distribution. So, there are, as I told, that this this is one of the very important potential distribution because of these two parameters. Most of the cases, when we can say that this random variable having a range from 0to the infinity. So, the the negative side is excluded. Then, the two parameters, there is alpha and beta are there, by changing their values, it can take a wide range of safe of the distribution.

So…So, many random variable related to the civil engineering can be modeled through this this this distribution just by adjusting their their parameters.This we have discussed earlier also.We have we have shown through the graphical presentation that how this, the change of this parameters can change the shape of this, shape of the distribution function, sothat a wide range of random variable can be modeled through this distribution.For details, you can refer to the module three, when we discuss this distribution.So, many real life problems in civil engineering can be modeled through this gamma distribution.For example, that stream flow in a river can be a gamma distributed random variable. So, most of this large river basins, where this you know, this this stream flow can is always a positive number 0to infinity. So, this this stream flow can be modeled through this gamma distribution.Again,one anothermost important feature of this gamma distribution, that we have also shown in the earlier, that if you just adjust the parameter, this is that exponential distribution is a special case of this gamma distribution.

Basically, if we just take few of their, few that exponential distribution and we can define another random variable which is a summation of those exponential distribution.Then that summation is basically a gamma distribution. So, this is what, using this properties this properties, there are many problems can be modeled. So, the gamma distribution can be viewed as a sum of a number of exponentially distributed random variables.

Hence, if a construction equipment consists of a number of components whose individual life span is described by the exponential distribution, then the life of the total system may be modeled using the gamma distribution. So, there are different components and each component is having a life span of some x 1 x 2 x 3.All these, if all these variables generally can be modeled, you know that this time to the first occurrence of one event can be modeled through this exponential distribution. So…So, now, the full system can consist of this summation of this this distribution. So, the total system the life of the total system can be modeled using this gamma distribution.

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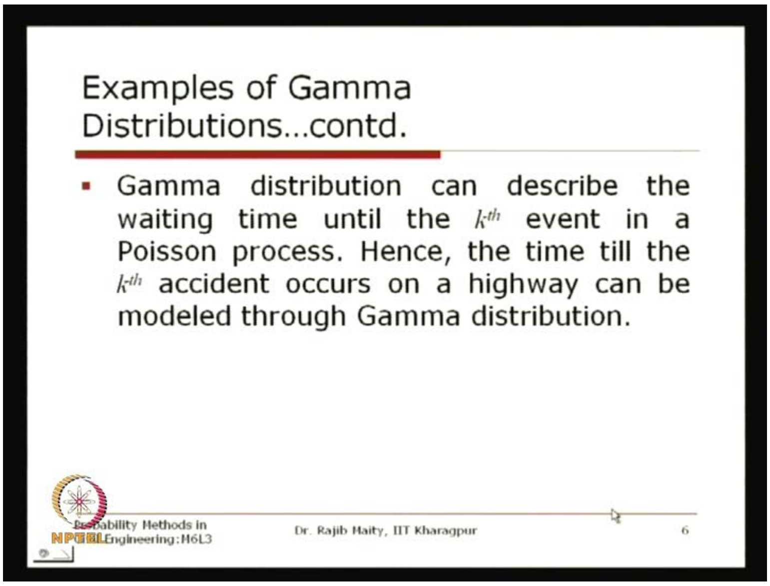

Another thing is that, this is this is mostly used to address different problems of this transportation engineering.That this gamma distribution can describe the waiting time until the k-eth event in a poisson process.Hence, the time till the k-eth accident for example, one example is taken from this transportation engineering, that there are the accidents take place either on the railway or on the highway.

So that, you,we can we can take that what is that what is the waiting time till that k-eth accident can take place. So…So, that k-eth accident take place means, there are basically the summation of this k interarrival times of the of the accidents. So, that it is a summation of those that exponential distribution, which is the interarrival time is modeled through.

So, this one, so, this time till the k-eth accident, which is summation of the kindividual exponentially distributed random variables. So, this can be modeled through the gamma distribution as it is explained in the last point.

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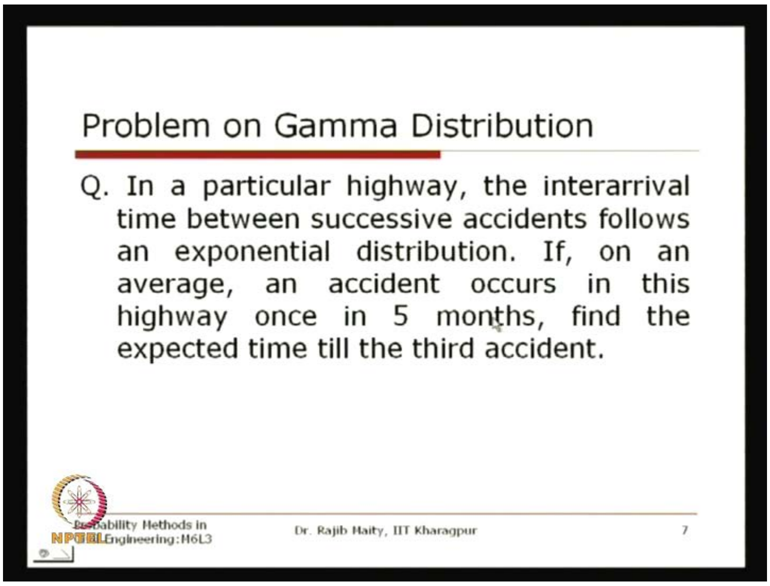


So, we will take a similar problem here.The problem is easy, but, it is very interesting to understand that how it is linked to this gamma distribution? So, in a particular highway the interarrival time between the successive accidents follows an exponential distribution.If, on an average, an accident occurs in this highway once in five months, find the expected time till the third accident.

Now, thing is that, so, this is known, that we know that the exponential distribution that we have describe earlier that and it is given that the average time between the accident is five months. So, that the parameter of this exponential distribution which is lambda is equals to 1 by x bar. So, this one by five months.

So, this is the parameter for this exponential distribution.Now we have to find out what is the, what are the parameter associated parameter for the gamma distribution.We know that the expected time is you looked for. So, just the multiplication of that two parameters that is alpha and beta. So, that will give you the expected time for till the third accident.

Now, the straight forward thing here is that,one accident in the five months and we are assuming that these are the dependent events.That is, the interarrival time between the first and between the, between now and the first accident and then, from the first accident to the second and then, from the second to the third. So, all these three interarrival times are independent to each other. So that, if,for the one inter arrival time, the expected time is five months, then the expected time for the three such events can be easily multiplied by this 3 and by this five months, we can get the answer, that is thetime is the fifteen months.

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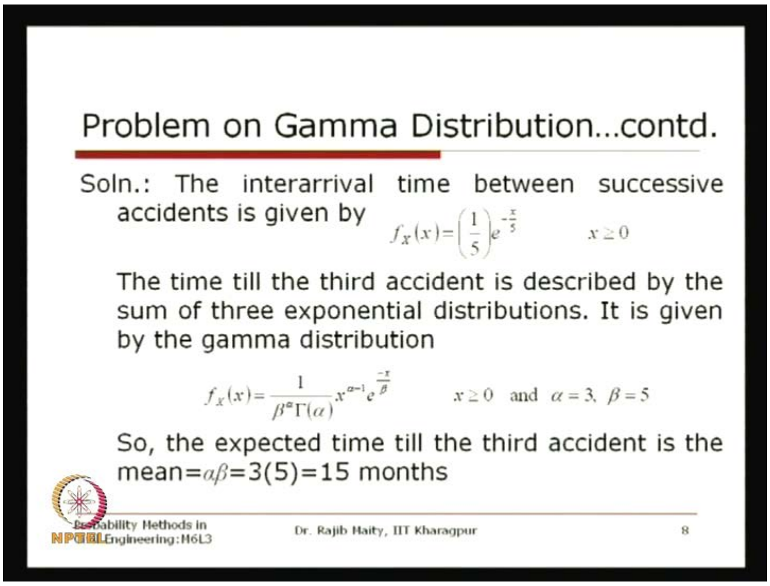


Now, if we want to convert it from the exponential distribution to the gamma distribution, then first of all, the interarrival time between the successive accidents is given by that distribution of this exponential distribution, as it is mentioned in the problem.That is f x x equals to 1 by, that is lambda e power lambda x and this lambda equals to 1 by 5. So, 1 by 5 e power minus x by lambda; obviously, x is greater than equal to 0.

So, if you get this one, then the time till the third accident is described by the sum of the three exponential distribution. So, thus 3 means from today. So, from today, I am starting what is the time till the first accident, this is a one inter arrival. So one, the first incident occurs, then from the first incident to the second incident then second to the third. So, the three exponential distributions are there.

So, the, so the gamma distribution will have the parameters, x is greater than equal to 0and the alpha should be equals to 3 and beta is equals to is 5. So, beta is nothing but, you know that beta is the your that 1 by beta is here that that lambda. So, this beta is 5 and there are three interarrival times. So this one,so the, and so, we have seen that parameters are alpha equals to 3 and beta equals to 5.

So, the expected time till the third accident is the mean that is alpha into beta. So, 3 into 5. So, fifteen months as we have just expecting just by looking the problem itself. So, this is how as how many exponential distribution we are adding and what is the parameter for the exponential distribution. So that, which come from this lambda and that number of those distributions are being added, from which we are getting this gamma distribution.

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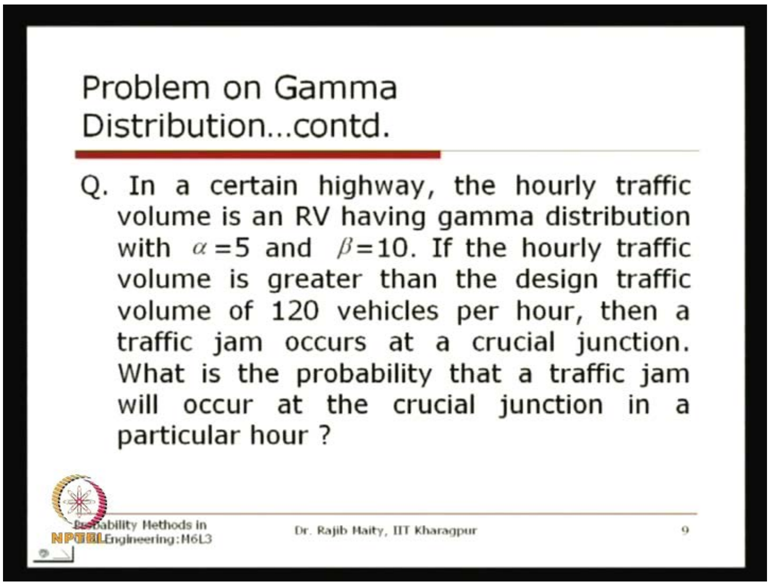


So, we will take another one another problem.This is also related to the transportation engineering.Sometimes, we design the road network for certain traffic volume and then we can see, we can estimate that how this traffic volume can be modeled through.This traffic volume also can be modeled through this gamma distribution.From there, we can answer some of the real life problem.That, what is the probability of the traffic jam and all. So, this type of problem is taken here once. So, in a certain highway, the hourly traffic volume is an random variable, is a random variable having gamma distribution with alpha equals to 5 and beta equals to 10.If the hourly traffic volume is greater than the design traffic volume of this 120 vehicles per hour, then a traffic jam is possibly can occur at a at a crucial junction of the road network.What is the probability that a traffic jam will occur at the crucial junction in a particular hour.

So, first of all, so it means traffic jam will occur.That is, the random variable that I have to define, that is what is the hourly traffic volume, that is the random variable here which is a gamma distribution parameters are given. So, now, we have to find out what is the probability that particular random variable will be greater than equal to 120.

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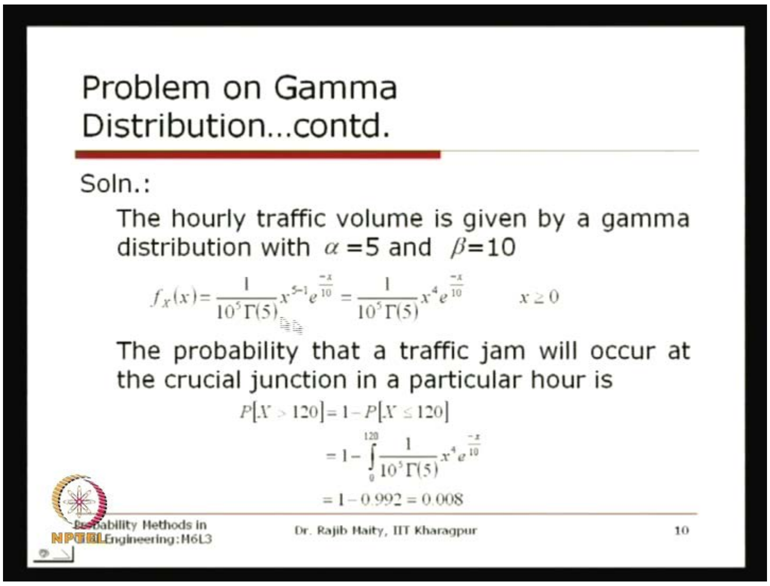


So, that is the overall problem is given here. So, the hourly traffic volume denoted by this random variable x is given by the gamma distribution with alpha equals to 5 and beta equals to 10. So,the distribution that we can get from this gamma distribution, which is that your f x of x equals to 1 by 10 power 5, then gamma 5 and then x power 5 minus 1. So, these are just we are using this, those parameters alpha minus 1 and e power minus x by beta.

So, we will just put this value here to get that complete form of this gamma distribution.Now the probability that a traffic jam will occur at the crucial junction is a is a in in a particular hour is given by this one, that is probability of x greater than 120.Now x greater than 120; obviously you know that, by this time that it can be the total probability minus x less than 120 which is 1 minus 0to 120 of these integration of this full this distribution and; obviously, d x.

Now, if you do this integration, you can use some of the software to do this one, this numerical integration.Or, you can use some table also to get the value, this value will come as 0.992. So, 1 minus 0.992 gives you the 0.008. So, the probability of this traffic jam is a very less here, you can see.This also can be inferred from the data that is that is given, that is alpha equals to 5 and beta equals to 10; that means, the expected time expected traffic volume is that 10 multiplied by 5. So, which is the 50 vehicles per hour and we are looking for the probability which is exceeding 120 vehicles per hour.So, the probability that is there, so is, obviously, should be very less which is 0.008.

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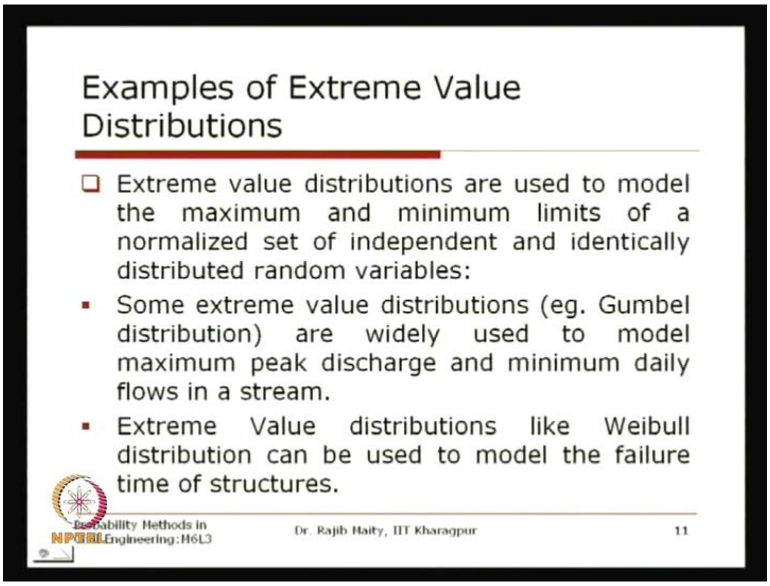


Now, we will go through some of this extreme value distributions.First of all, we will just see what is this extreme value distribution and what are this its possible application.This extreme value distributions are used to model the maximum or minimum limits of a normalized set of independent and identically distributed random variables.

So, there are some random variables is available to us and we are interested only for their extremes.Extremes means, here either we are interested to know the nature of its maximum side or the nature of it is the minimum side. So this,when we are when we are interested for this type of variables, that is suppose that one data set is available to me and I am just interested to know its maximum one, the maximum value out of this data set and what is the distribution of the maximum. So, that is the extreme value that we are referring to and its distribution, its associated distribution is refer to as this extreme value distribution.

Similarly, for the minimum also we will see and there are different types, basically is available that we will discuss through. So, the extreme value distribution when we are when we are talking about, we are basically referring to the event where out of the set of from this data or out of the set of this random variable, we just want to pick up its one of the extreme, either maximum or the minimum and want to know its probabilistic distribution.

So, if you see from the example, from the civil engineering, one is that, for anything suppose that we are talking about the stream flow then whether the low flow is the one extreme and the the maximum flow is the another extreme.Similarly, if we see that the structural components so its failure due to the several reasons are there.One of the thing is that fatigue, the failure due to fatigue. So, how many times it can, what is the total time it can go through and what is the maximum time that it can pass through.That can be also be modeled through this extreme value distribution.

Similarly, in the other application also from this environmental engineering or the transportation engineering, whatever the data that we are having, if we are interested to model only its extreme side either maximum or the minimum one, then we will refer to this type of distribution.

So, some of the extreme value distribution that is the Gumbel distribution is one that I was also mentioning at the beginning of this lecture.This this extreme type distributions are widely used to model the pick discharge and minimum daily flow in a in a stream.There are other extreme value distribution, like the Weibull distribution which can be used.Basically, this is the reverse Weibull type distribution, which is the type three distribution, which can be modeled for this failure of this structure also.

So here we will we will take that first.We will take that generalized extreme value distribution, then we will discuss about its different type.Then we will go through this.Will take into details as to what is this Gumbel distribution and how to model this peak discharge and this minimum flow in a stream.

So, this is important because this extreme value distribution means, so far, as this water resource or hydrology is concerned, we are always interested to know that either the very low flow or the or the very maximum flow, because both are is needed to be explode for its for is the societal impact. So, this we will see.

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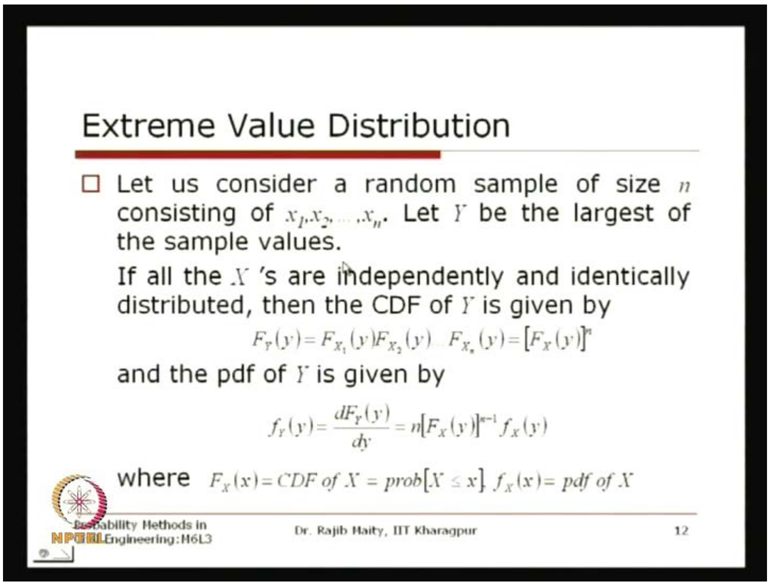


So that, when we are talking about that, there is a, maximum or minimum limits of a normalized set of this independent and identically distributed random variables. So, we will just see the, first of all we will see, what is the specific distribution that we can that we can take you through through.Whatever the knowledge that we have seen from this CDF and the p and the pdf,of the if if the assumption on the assumption that it is independently and identically distributed.

Let us consider a random sample of size n consisting of this x 1 x 2 and up to x n. So, this is basically the data that is available to us.Let y be the largest of this sample value. So, we are interested to know out of this n, which one is the maximum. So that, we are denoting as this y.Now if all the x's are independently and identically distributed, then the CDF of y is given by, that we know from our earlier discussion is that the C D F, that is, the f y of y should be the product of their individual individual cumulative distribution.

Now, when we are taking that taking that data data dataset, so, each and every observation can be can be treated as one random variable. So, like that this is the this is the distribution of this the first one, that is x 1 x 2 and this is for the x n.Now, as they are they are as they are independent, that is why we have we got their product as to get their joint distribution. As they are identical, that is, if we just say that all these are identical and equal to that f x, then we can say that f x of now it is expressed through the y, is power n. So that, there are n different random variables are there. So, power n will give you that CDF of that the largest value of the sample.

And now; obviously, once we get that CDF, we can get its pdf also, probability density function that by differentiating that. So, this F y of y is equals to that differentiation with respect to y which is nothing, but n, the cumulative density power n minus 1 multiplied by its pdf of the individual random variables. So, where this f x x is a CDF of x, that is probability of x less than a specific value of x and this f small f x x is the pdf of that x.

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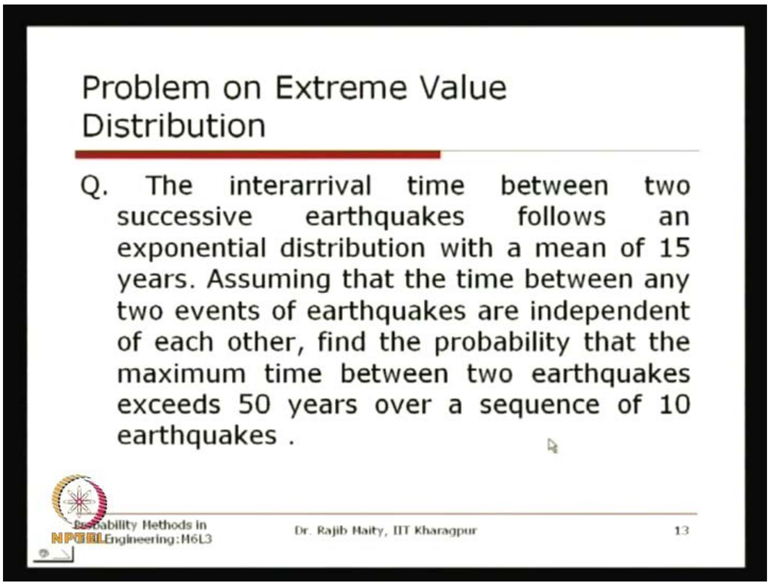


Now, these are all means obviously, express through that one variable which is y. Now,suppose, we will just see the same thing through 1 example, that is a interarrival time between two successive earthquakes. So, again we know that these are this can be easily treated as this independent and this can also be treated as this identical. So, both aresame distribution, if we assume, whether we can answer this one, there is a maximum inter arrival time. So, the problem related to that one.

So, the inter arrival time between two successive earthquakes follows and exponential distribution with a mean of 15 years. So, we know that mean between two successive earthquake is 15 years, assuming that the time between any two events of the earthquakes are independent of each other. So, this is that that first assumption. So, find the probability that the maximum time between two earthquakes exceeds 50 years over a sequence of 10 earthquakes.

So, the interarrival time having a mean of 15 years, we can model it through this exponential distribution.Their independence, we get their joint distribution by their product and then what we are interested to know is that what is the maximum inter arrival time. Then what is the probability that the maximum time will exceed 50 years over a sequence of 10 successive earthquakes.

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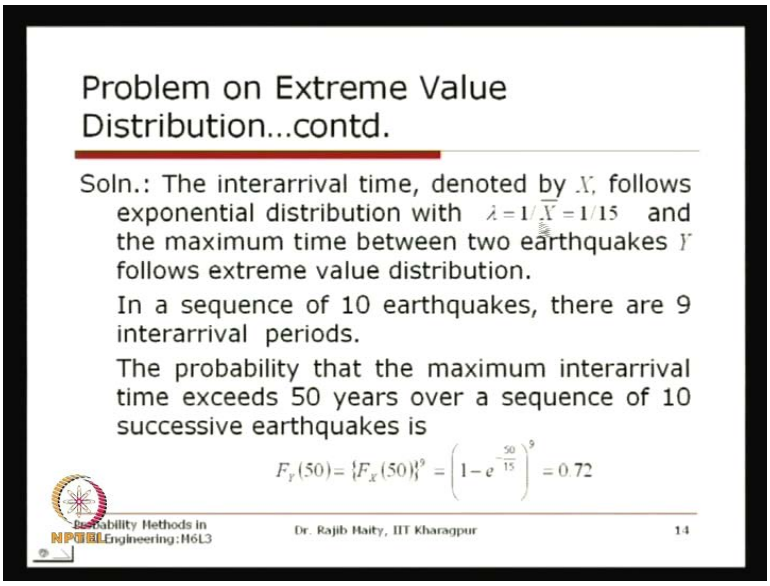


So, the inter arrival time which we can denote by x follows an exponential distribution with lambda equals to 1 by x bar that is 1 by 15.The maximum time between two earthquakes that is denoted by y, follows an extreme value distribution.

Now, in a sequence of 10 earthquakes, there are 9inter arrival inter arrival periods.Now, the probability that the maximum interarrival time exceeds 50 years over a sequence of 10 successive earthquakes is that, is this, that is f y of 50 is equals to f x 50 power 9. So, this is basically, we are taking it from that this expression.

So, this n is now here is total number of interarrival times which is 10 minus 1; obviously, that is 9. So, this 9 if we take and this is, that this is their 1 c power minus 50 by 0.15 which is equals to 0.72. One correction, this is not the maximum interarrival time exceeds 50 years.Rather, this 0.72 is the probability of maximum interarrival time be less than 50 years.Now, if you want to know the maximum interarrival time exceeds 50 years, then it should be subtracted from total probability 1. So, the maximum inter arrival time exceeds 50 years should be 1 minus 0.72 that is 0.28.

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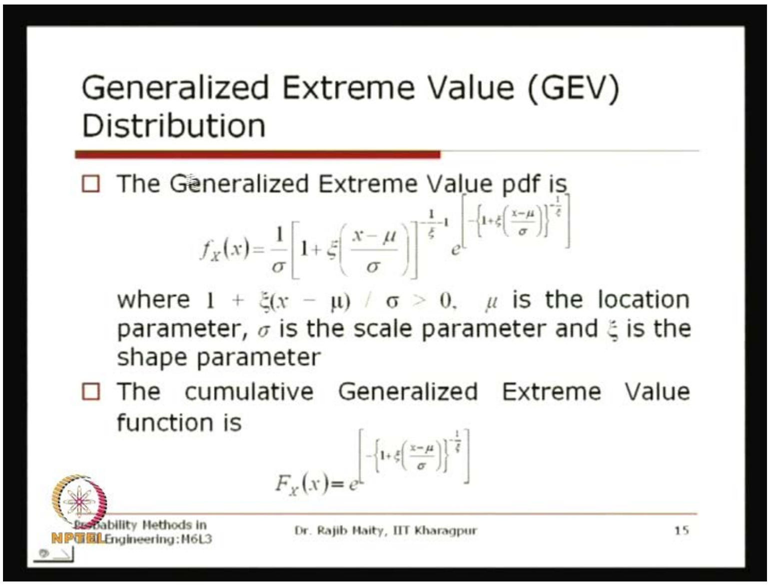


Now, we will see the generalized extreme value distribution.First of all, this is a, this this distribution that we are going to discuss it is, it it it is a generalized frame work.Now, this generalized extreme value pdf is expressed through a little cumbersome expression, where this f x is equals to 1 by sigma into 1 plus xi x minus mu by sigma power 1 by xi minus 1.That exponential power again, the same expression that is minus 1 plus xi x minus mu by sigma whole power minus 1 by xi.

Now, if we just see that their individual element then, it will be more more meaningful.That the first thing is that, this this distribution is is the, its range, its its support should be such that that 1 plus xi x minus mu by sigma should be greater than greater than 0. Where this mu and sigma that we know know already, which is similar to our that other distribution, that is mu is your the location parameter and sigma is your scale parameter.Basically, both are location parameters, means where it is, means it is it is giving the information about its mean and this is giving its information about its variants.Now this xi is another parameter which is the shape parameter.

Now, this xi is one of this crucial parameter in this distribution.In the sense, that it generally controls the tail behavior.Now depending on this, whether this tail behavior, how it will behave that we can classify into three different types.

The first thing is that if this xi is tending to 0, or whether this is negative or this is positive. So, this three cases can lead to three different types of the of the distribution that we will discuss in a minute.For this one, if we just follow the same principal of getting its the cumulative distribution function, then it can be shown that this f x x is equals to exponential of minus 1 plus xi x minus mu by sigma power 1 by xi.

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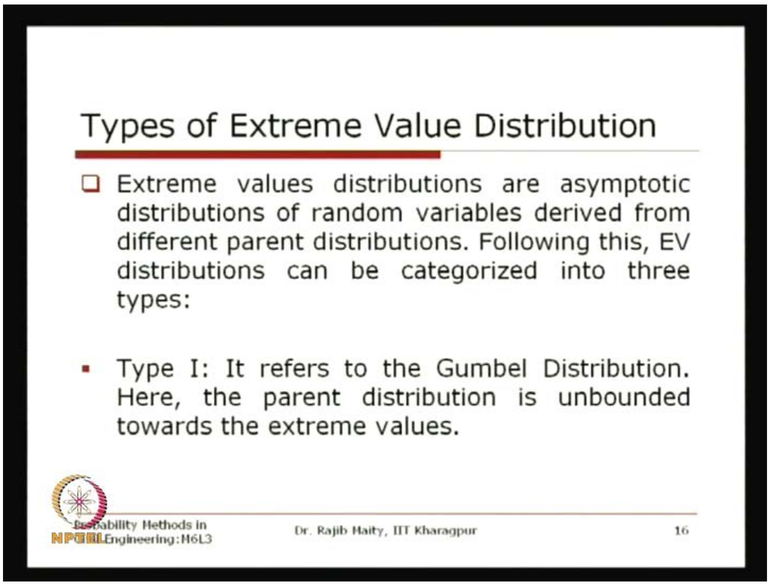


Now, as I was telling that this xi is a shape parameter, depending on which it is either tending to 0 or it is positive or it is negative. So, in these three cases, there are three different types are there.So, this extreme value distributions are asymptotic distribution of the random variable derived from the different parent distribution.Following this extreme value distributions, can be categorized into three types.The the first type, that is type one, it refers to the Gumbel distribution.As I was also mentioning earlier, this type, this Gumbel distribution is also known as these type one distribution.Here, the parent distribution is unbounded towards the extreme value.Now, unbounded towards the extreme values means, if you are interested, suppose that the maximum one or the largest one, if we tell, then we can see that.This for example, that this normal distribution towards the positive extreme is unbounded and even the gamma distribution towards the positive boundaries unbounded, like that.

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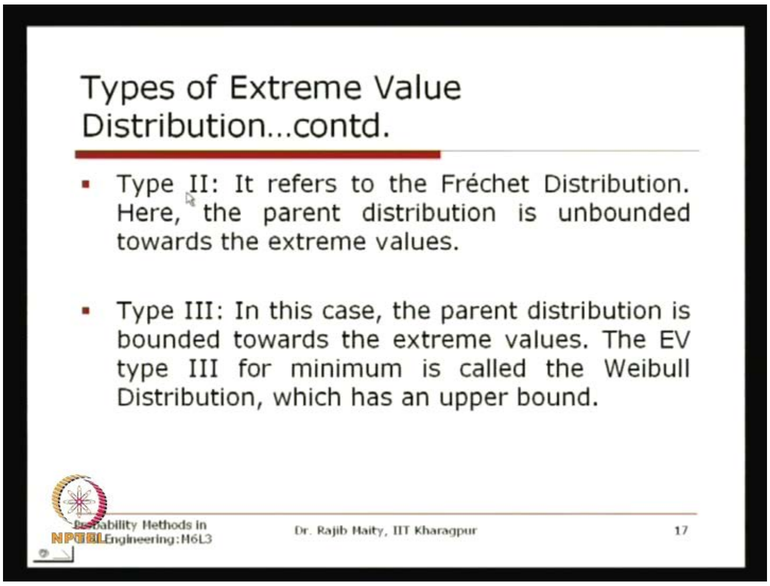


Similarly type two, it refers to the Frechet distribution.Here, the parent distribution is unbounded towards the towards the extreme values.The type three in this case, the parent distribution is bounded towards the extreme values.The extreme value type three of the minimum is called the Weibull distribution, which has the which has an upper bound.

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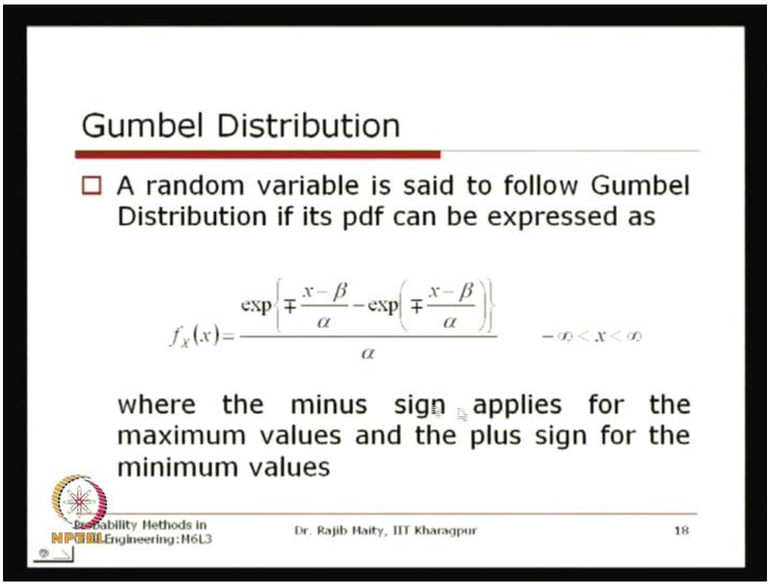


So, with these three types of this distribution, now we will see, one of this the type one distribution which is the Gumbel distribution.As I told, that this is having a very wide application. So far as the hydrology and water resource is concerned, to analyze the the extreme values of the stream flow either it is low flow or it is the high flow.

So, a random variable is said to follow the Gumbel distribution if its pdf can be expressed as like this; that is, f x x is equals to exponential of minus plus x minus beta by alpha minus exponential of minus x minus beta by alpha divided by alpha.This x has a limit from this minus infinity to plus infinity.Now this both, this signs have some means, we have to use one of these signs as a means of the minus and plus,one we have to use for the maximum and other one we have to use for the minimum one.

So, here it is mentioned that where the minus sign applies for the maximum values and the plus sign is applicable for the minimum values. So, if you are interested to model thethe higher side of the extreme, that is the maximum values, then we have use this minus sign.This is, then the, and if it is for the, if it is for the minimum one, then you have to use the plus sign.

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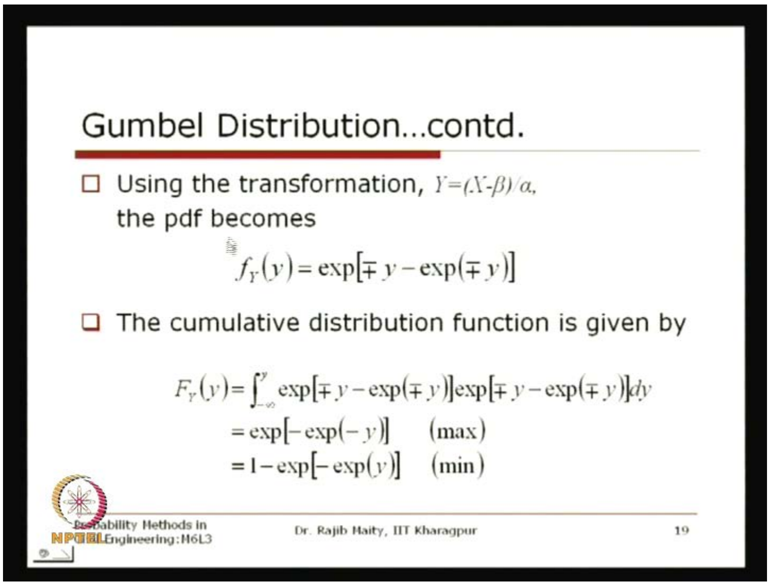


Now, if we use this transformation that is y equals to x minus beta by alpha then, the pdf can be again expressed, this one just we are transforming that x minus beta by alpha equals to that y. So, this pdf can be expressed by exponential of this minus plus y minus exponential of minus plus y.Again that same, that sign convention, that minus is for the as I told, the minus is for the maximum and the plus is for the minimum.

The cumulative distribution function is given by; we can do this integration from this minus infinity to this specific value of this y.We will get that for the maximum one, it comes at exponential of minus exponential of minus y and for the minimum one it is 1 minus exponential of minus exponential of y. So, even though so,with the complicate, we started with the complicated or little bit looking cumbersome distribution. But, whenever we come to this cumulative distribution, it generally becomes simpler and very easy to remember also.

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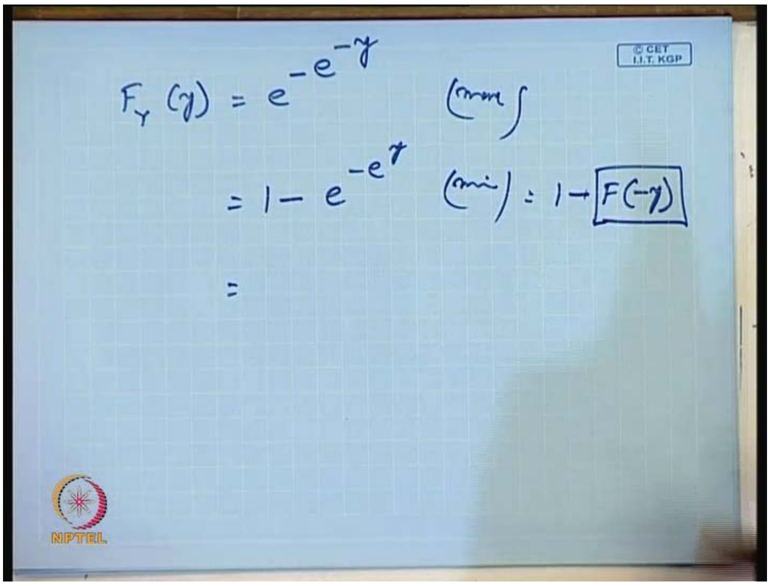

This means, this is thatF y of this y is equals to, generally we remember it like this, e power e power minus minus y and this is for the maximum. Similarly, 1 minus e power minus e power y this is for the minimum. So, you can see that that one, that is, if we just write that this can be for this minimum one, the minimum one can be written as 1 minus f of minus y. So…So, this one is basically for the for the maximum one. So, generally this tables are available for the, for one of this thismaximum. If it is available only for the maximum also, then also you can use the same table for the minimum through this this one.Because this, if I change this attribute to this minus y, then that one is subtracted from 1 will give you the required value for this minimum one.

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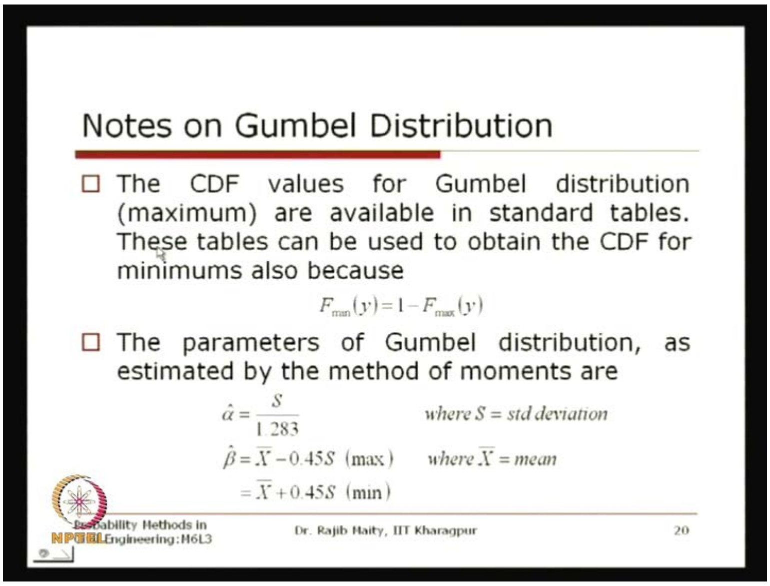


So, we will see one thing that is some notes first. That is, if the the CDF values for the Gumbel distribution for the maximum are available in the standard tables, this tables can be used to obtain the CDF for the minimum also.Because, as I was telling that F minimum is equals to f maximum, this will be minus y for this maximum one.This will be minus. So, the parameters of the Gumbel distribution as estimated by the method of moments are this alpha beta. So, from this method of moments how it is; so, we have not yet so far in this course, we have not discussed about this parameter estimation is in different methods.As I mentioned, that this parameter estimation will be covered in this module 7. So, till that you can just remember that these are one of the methods of this parameter estimation which is known as method of moments. So, we will be discussed these things, different methods of this parameter estimation in the next module.

So, using that method of moment method, this alpha can be that estimated value of these alpha.This is the parameter of this Gumbel distribution is equal to s by 1.283 where s is the standard deviation.

This beta cap is the x bar minus 0.45 s for this maximum and x bar plus 0.45 s for the minimum, where this x bar is the mean. So, by knowing the, if the data is available, we can calculate what is its sample estimate of this standard deviation, sample estimate of the mean and we can use that some of those s and x bar information to get that parameter alpha and beta.

So, once we know this parameter alpha and beta, we know what is y, and why because is that x minus alpha by beta. So, from there we we know what are the probability or what are the particular, the question that is asked for.

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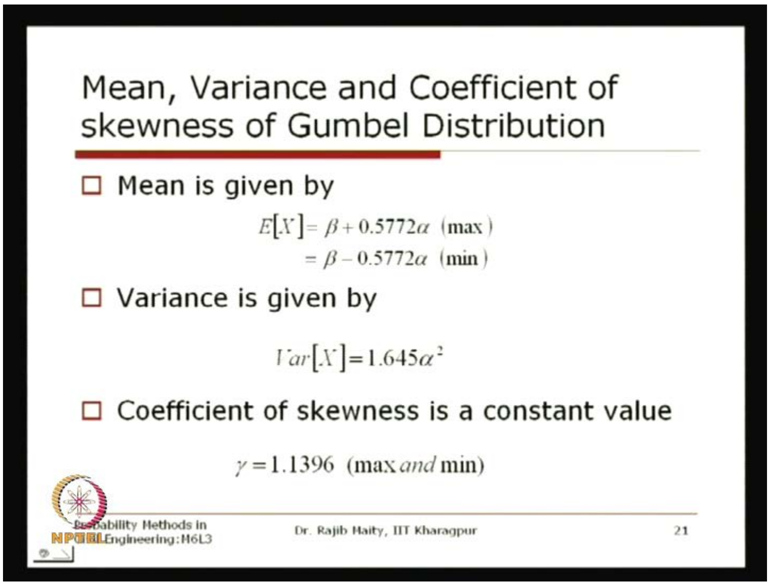


The mean of this extreme value distribution is given by this beta plus 0.5772 alpha for the maximum one and beta minus 0.5772 alpha for the minimum one. Variance is given by 1.645 alpha square. The Coefficient of skewness is a constant value for both this maximum and minimum it is 1.1396.

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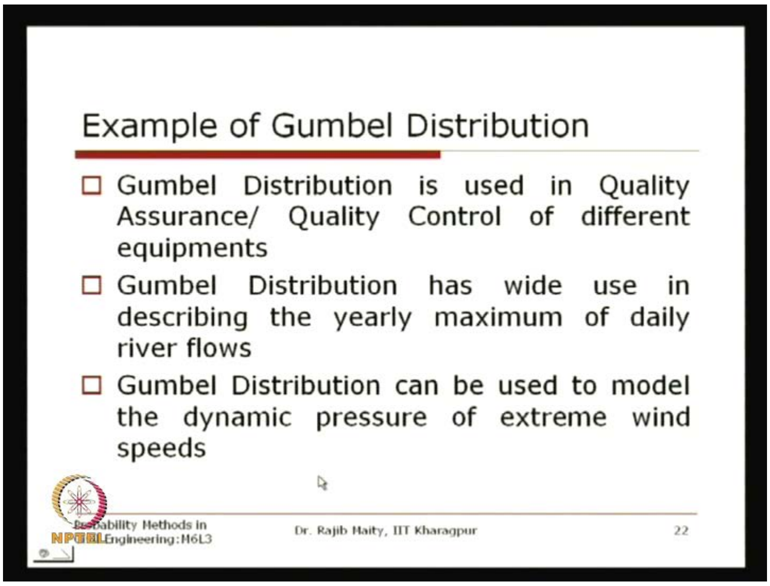

The example of Gumbel distribution, this Gumbel distribution is used in the quality assurance or quality control of different equipments.This Gumbel distribution has a wide use in describing the yearly maximum daily river flows.Gumbel distribution can be used to model the dynamic pressure of the extreme wind speed. So, these things can be always means, whenever as I was mentioning, always whatever the random variable that we are talking about, if we are looking for one of its extreme, then this distribution can be used.These are very, these these are the cases where we have seen in this in the civil engineering, where the wide application of these distribution is there.

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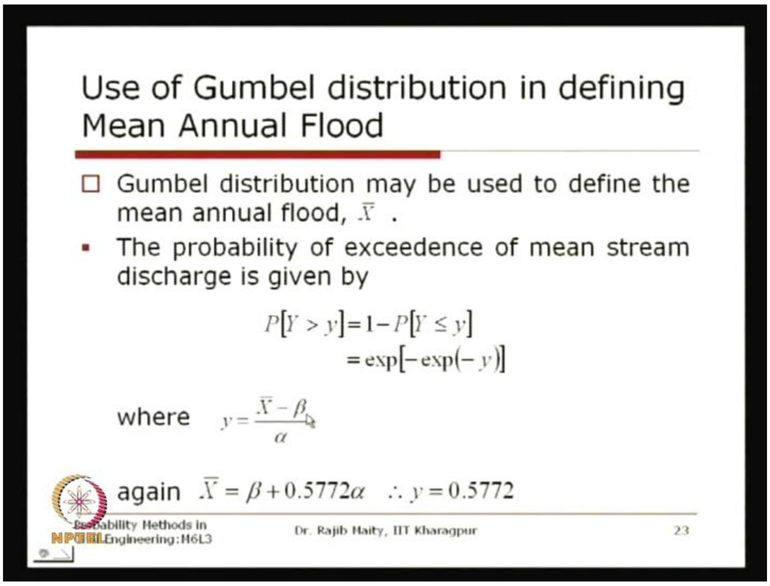


Now, this one example we will see here, that is use of this Gumbel distribution in defining this mean annual flood. So, mean annual flood, the Gumbel distribution we used to define its mean annual flood. So, what is, that is the probability of exceedence of this mean stream discharge is given by this probability of y greater than y which is basically 1 minus probability of y less than y.This one we know that it is one exponential of minus exponential of minus y.

So, this one again we know that this y is equals to x bar minus beta by alpha. So, basically what we are looking for is that, the mean annual flood.What is its, how can we define that? So, from the from the extreme value distribution. So, with respect to its return period and all that we will see.

So, this y is equals to that x bar minus beta by alpha again. So, this we we know that x bar equals to that beta plus 0.5772 alpha, which we have seen from this earlier from from the estimate. So, that y is equals to your 0.5772. So, whatever the this this probability, will get the probability of exceedence, that will if you put this value of 0.5772, then we will get what is this that probability.

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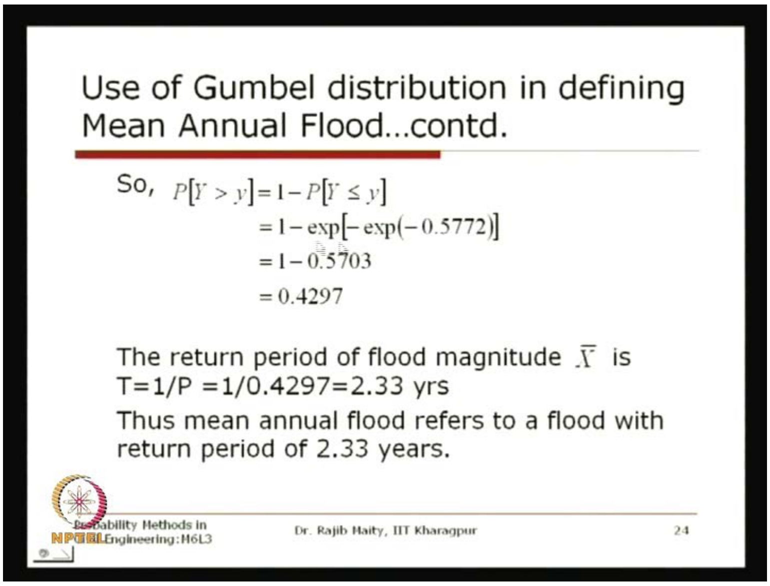


If you put in in that expression then, it will come that that probability of y greater than this distribution is equals to 1 minus probability of y less than equals to this this y.That is the cumulative distribution here. So, that is that exponential of minus exponential of minus y and y in that, as we have seen here is the distribution for that 0.5772. If you solve this one, it will see that it is 1 minus 0.5703 which is the 0.4297.

Now, once you get this probability, what we can define is that,is the is what is its return period. So, the return period we have seen in this, in one of this lecture, previous or previous to previous lecture that this is the 1 by p. So, 1 by the probability of that particular event. So, if we get this one, we will get the return period of that particular event.That is, T equals to 1 by p. So, 1 by 0. 4297 is 2.33 years.

Thus the mean annual flood refers to a flood with the return period of 2.33 years. So, that mean annual flood is having a return return period of 2.33 years. So, this information is is being, is can be used to find out what is the mean annual flood.

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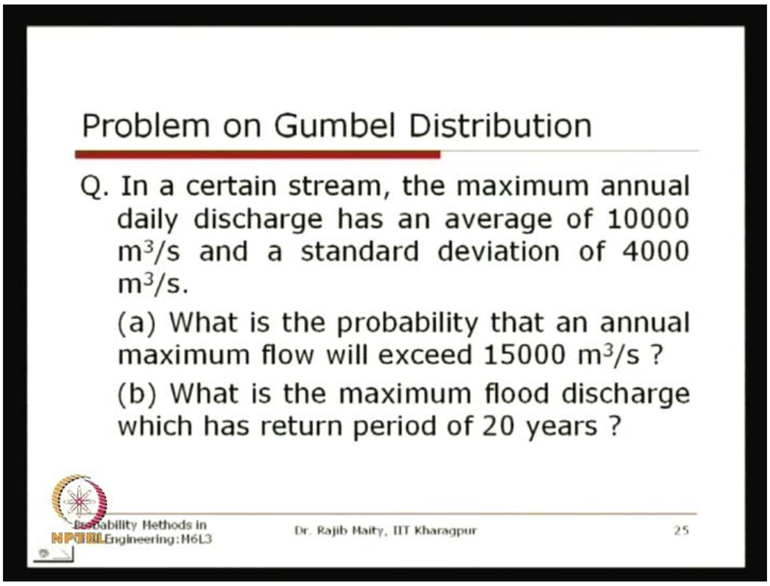


We will take one more problem on this, because that is just one of this application that we have shown.Now, we will take one example on this Gumbel distribution, where we will see some of these answer for the maximum annual daily discharge. So, in a certain stream, the maximum annual daily discharge has an average of this 10000 meter cube per second and a standard deviation of 4000 meter cube per second.

Now, these two information has given. So, this can be also, can be estimated from these data also.If the data is available, we know this one.Now once we know these two information, then, basically what we can get is that the parameters of these Gumbel distribution.Just now, we have discussed that. So, those parameters can be estimated.Now, we are supposed to answer that what is the probability that an annual maximum flow will exceed that 15000 meter cube per second.

Second is the, what is the maximum flood discharge which has a return period of 20 years. Again, that we know that if the return period is given, we can calculate what is this, what is this that non exceedence and what is its exceedence probability just by getting is 1 by 20. So, basically there are structures of these related to the water resource have some specific return period we have to consider.Based on that, we have to find out what is the magnitude of the flood is coming, and with that value, we have to use that one.Because, whatever the historical data that are that is available to us, may not reflect that that complete nature.For that reason only, we are, we have to we have to do this exercise to find out what is the maximum possible event or maximum possible magnitude that can occur with that return period.With that value, we have to with this is tremendous useful for this reason purpose and all.

So, one such example is shown here. So, that two parameter that we are talking about is is obtained from the available data that is recorded for that particular sight.From there, with through the extreme value distribution and then I will try to find out these answers.

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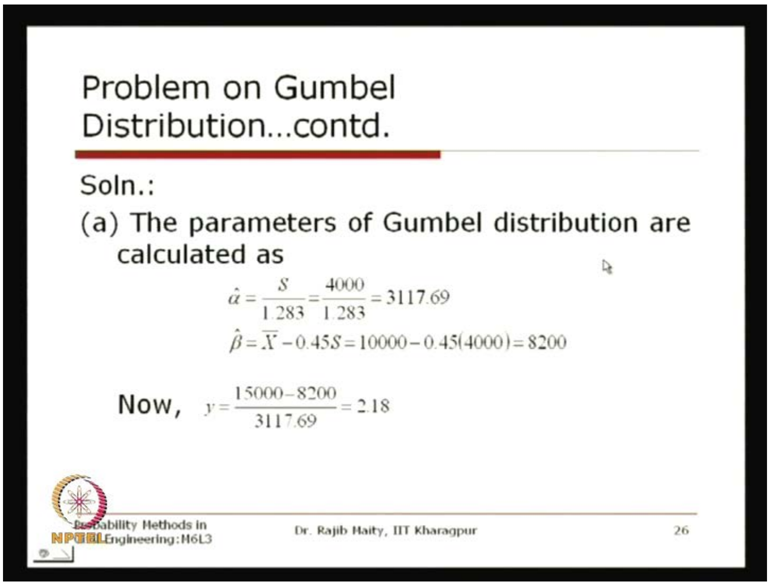


So, the parameters of these Gumbel distributions can be calculated from this, their expressions that is alpha cap is equals to s by 1.283, which we have shown just now that is it that method of moments estimated through the method of moments. So, this s is now here shown that 4000 divided by 1.283 which is 3117.69. The beta cap is also that a mean minus 0.45 times of this standard deviation s and which is a value of8200.

Now, so this, now what we will get, we will get this information on the reduced variable.We are supposed to know the question that, what is the probability that it will exceed that 15000 meter cube per second? So, this 15000 meter cube per second minus alpha divided by beta. So, that should be the reduced variate. That should be the transformation we have to do, before we can get that answers.

So, from this Gumbel distribution, basically to know, what is that what is the probability of x greater than 15000 is equal to what is the probability of y greater than 2.18. So, this is why we are just transforming this, what is the original data through this x minus alpha by beta.

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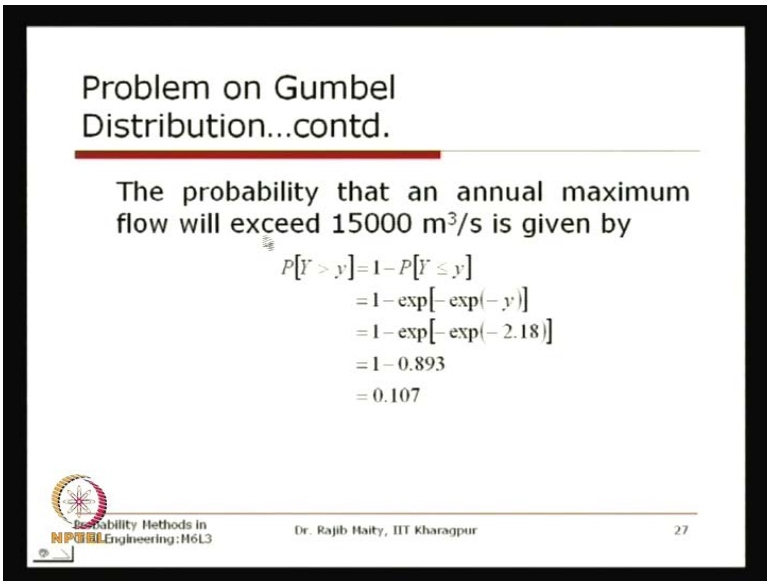


So, the probability that that annual maximum flood will be, will exceed this 15000 meter cube per second is given by the probability that y greater than this value is equal to 1 minus this cumulative probability. So, this 1 minus exponential of minus exponential y and just now we have calculated the y is equals to 2.18. So, if we put this one in this expression, then we will get this 1 is 0.893. So, the probability that we will get finally, is 0.107. So, the probability that an an annual maximum flow will exceed 15000 meter cube is is 10.7 percent you can say.

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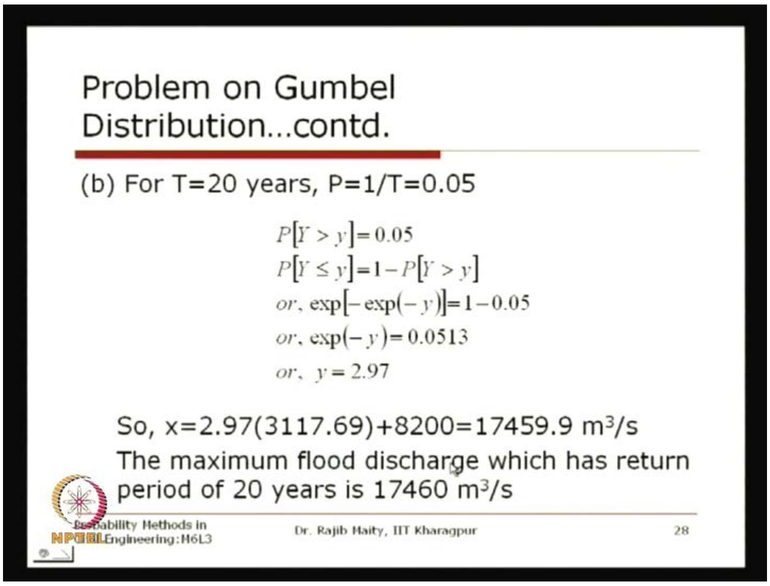


The second thing, second question that was looked for is, what is the, that what is the magnitude here. So, earlier we have calculated the probability here, the return period is is given. So, we are, So, we are supposed to find out what should be the corresponding magnitude. So, here the return period 20 years is given. So, first of all we have to find out what is this probability.That is, so, as as we have seen in this last problem also, that probability is equals to 1 by t that is 1 by period t.

So, this 1 by t if we put, it is 0.05. So, this is basically that on the on the right extreme, that is if the distribution looks like this. So, basically we are looking for this particular value where this 1 is your0.05.

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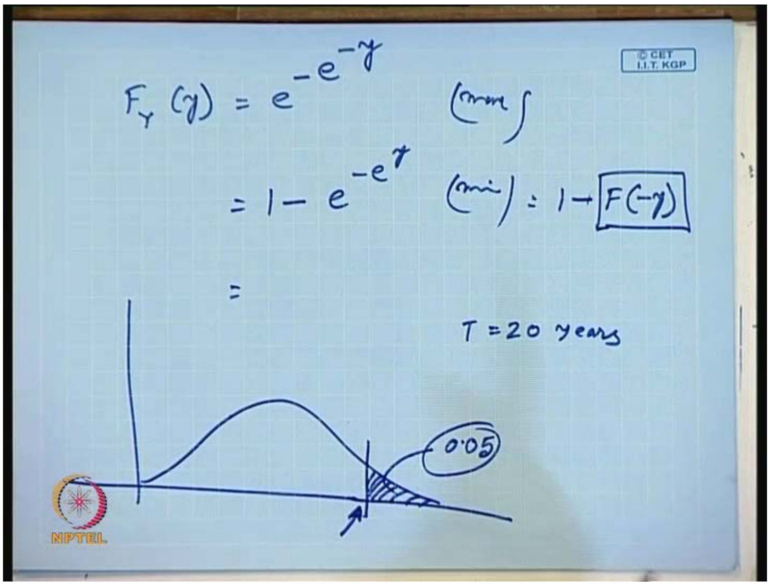


So, this particular magnitude we want to know from this one. Now, it is it is for the return period t equals to for the 20 years.Now depending on this, what is the project that is under consideration, this can change. So, this can change to even 50 years or 100 years or so.

So similarly, based on the, what is the return period we are considering, based on that this will this will change. So, once this probability, we get. But, whatever may be this return period, once we get the corresponding probability, then we will be using it from this cumulative distribution and we will get the answer.For example, here the probability is your 0.05. So, this probability of y greater than this magnitude that we looking for; obviously, this magnitude first we are looking for in the scale of y. So, that is equals to 0.05. So, this probability of y less than equals to y is equals to 1 minus probability of y greater than this magnitude. So, or what we can write that this exponential of this exponential of exponential of minus y which is equals to 1 minus this 0.05.

So, once we do this one and after this, after we solved it for the y finally, we will get the value of this y is equals to 2.97. So…So, this probability of y greater than 2.97 is basically is this probability 0.5.Now, we have to get it back to the original scale. That is, what is the value of this magnitude of this stream flow, which is now is equals to 2.97 multiplied by your that 2.97 multiplied by that that value isalpha or this this alpha plus this beta.Basically, this is just inversion of this this expression.

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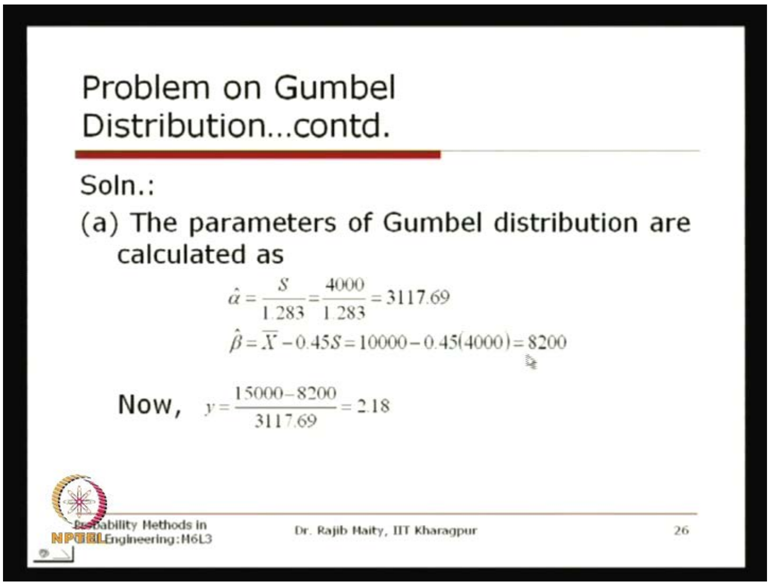


So this, if we do we will get that 17459.9 meter cube per second. So, we can say that the maximum flood discharge which has a return period of 20 years is equals to 17460meter cube per second. So, similarly, in the previous problem what we have seen is that, that mean mean flood, that mean mean annual flood that is having the returned period of 2.33 years.

So, if we calculate from this data, if we want to know, if we calculate it from here that, what is the magnitude of the return period? So, magnitude of the return period means you have to follow the first one.Magnitude of the return period for the for the mean value, then will get that and then will first get the probability, and then we make it inverse and will get that that is 2.33 years.

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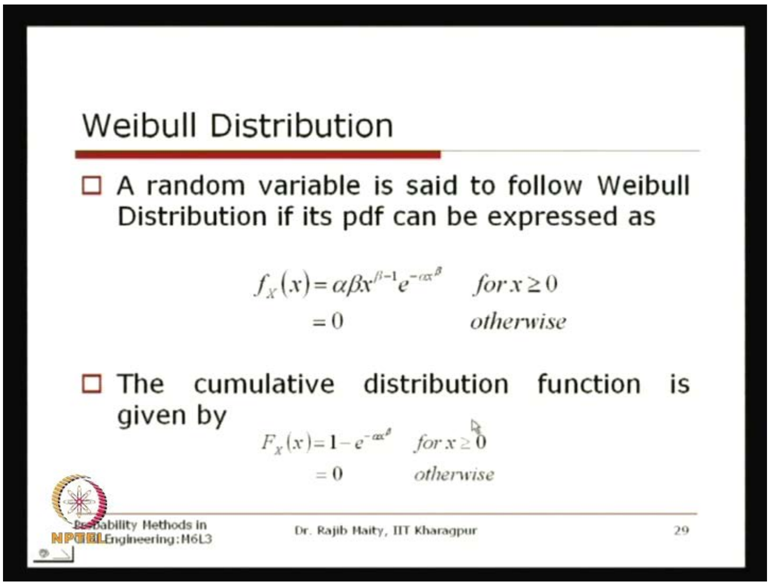


Then, we will take that Weibull distribution.A random variable is said to follow the Weibull distribution, if its pdf can be expressed as this one; that f x x equals to alpha beta x power beta minus 1 and e power minus alpha x power beta for the x is greater than equals to 0. The cumulative distribution function of this Weibull distribution is equals distribution to 1 minus e power minus alpha x power beta. So, we will use this one to get this Weibull distribution and will see some application in this civil engineering.

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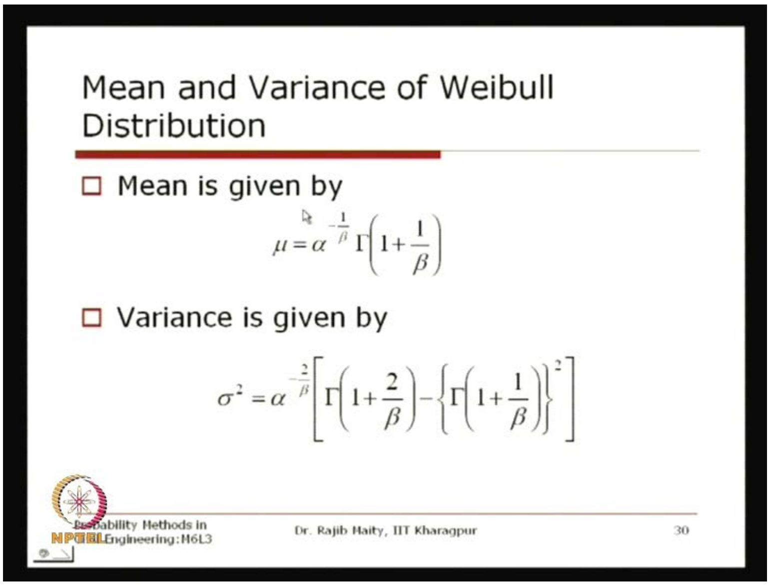

So, this mean of this distribution is mean equals to, this can be express through its parameter.That is, alpha power minus 1 by beta and gamma of this 1 plus 1 by beta. Variance is also given by this expression sigma square is equals to alpha power minus 2 by beta gamma of 1 plus 2 by beta minus gamma 1 plus by beta whole square.

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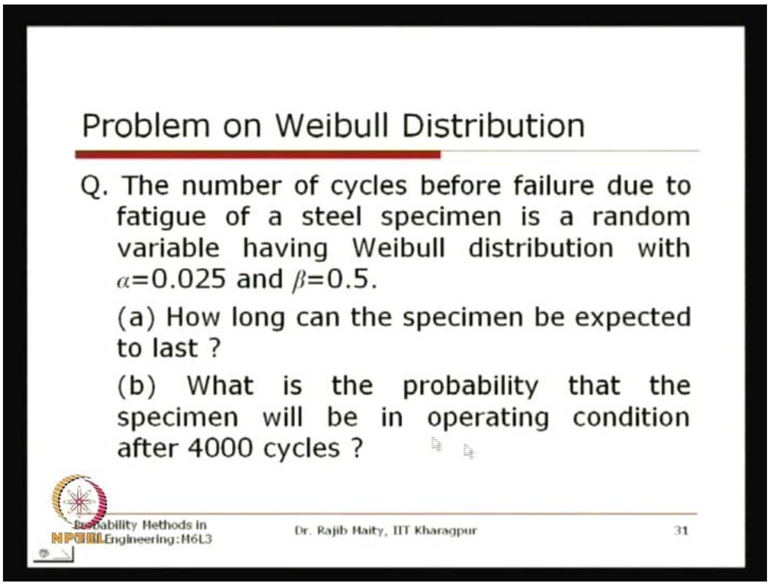


So, with this one if we just take one example of this of this Weibull distribution, as we was mentioning this is generally used for the failure of this structure. So, one such example is taken here through this Weibull distribution.

So, this number of cycles before the failure due to fatigue of a still specimen is random variables having an Weibull distribution with alpha equals to 0.025 and beta equals to 0.05.Now, I hope that you know this this failure due to fatigue means this is there are there are several experiments also can be referred to.Basically, what does it mean is that it is subject to a reversal of the force.Either it is in the compression, or it is in the it is in the tension with this force pattern is getting reversed and the load is applied which by the by the static analysis, it can be shown that this structure is safe under that load.

But, once it is go on for this reversal of force and there are certain cycle.After certain cycle, the structure may fail even though the applied load is within the within the this specified limit. So, like that one. So, how many cycles it can go through that can be modeled through this through this Weibull distribution.One such example is shown here, for which the the parameter alpha equals to 0.25 and beta equals to 0.05.The question is,how long can the specimen be expected to last and so, the expected value, we have to get and what is the probability that the specimen will be in operating condition after 4000 cycles.

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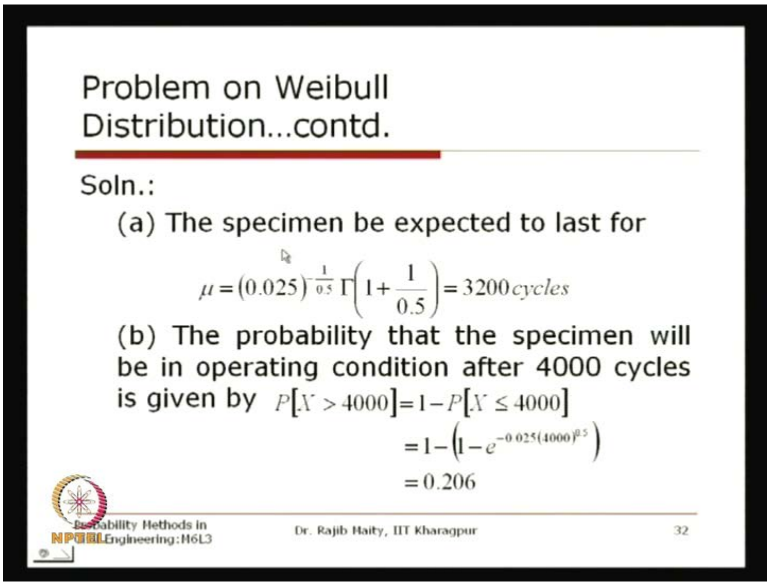


So, the first question is straight forward which is the mean that we are looking for, and this mean we can expresses through its parameters. So, this mean 0.025 power 1 by 0.05 gamma of 1 plus 1 by 0.05.This gamma value we can get from the table and we can calculate this mean, which can which is 300 sorry 3200 cycle. So, any specimen on an average that is expected value of the specimen that can last before failure is that 3200 cycles.

The second question is that probability that the specimen will be in the operating condition after 4000 cycle. Obviously, this probability will be less. So, that can be given by this probability that x greater than 4000, which is the 1 minus the probability of x less then equals to 4000, which we can get from its cumulative distribution function from this from this 1 minus c power minus alpha x power beta.

So, if u put that expression here. So, we get that probability is equals to 0.0sorry 0.206. So, this is a probability that the specimen will be in operating condition after 4000 cycles

So, in today's lecture also, we have taken some more probability models gamma distribution, extreme value distribution,Weibull distribution.In that extreme value, we have discussed in detail about this Gumbel distribution and its application to analyze the extreme value, extreme stream for values, the annual maximum and all.

So, we will be taking some more examples because these are all the continuous distribution that we have discussed so far in this module.We will also discussed that discrete random variable, which are also, there are some wide application in this civil engineering difference civil engineering problem and those distribution we will take in the next lectures.Thank you.