Probability Methods in Civil Engineering
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Lecture No. # 32

Probability Models using Log Normal and Exponential Distribution

Hello and welcome to this second lecture of this module. This module is on common probability models; and you know in the last class, we have covered the normal distributions, some models on this normal distribution. We have we have already discussed this common probability distributions in earlier module, in module three. And from where you have to, actually this module, this particular, this current module and that module three has to be a referred together. Because in module 3, we have discussed about the basic properties of all this distributions, and here we are utilizing those distribution, properties of those distribution to address some of the real problem in civil engineering.

Now, depending on the nature of the problem, depending on the nature of the random variable, we are selecting which distribution it could follow. Depending, once we have decided that, this will follow that particular distribution and depending on that, we are answer, we are trying to answer several questions that is associated with that problem. But still, I should mention here that all this, throughout this this this module, you will see that in the question itself, we are trying to give that assumption that if this random variable follow that type of distribution.

Now, these are all means, whatever the example is used and whatever the correspondent distribution is used, is from the experience and that will follow that particular distribution. But there are methodologies how from the data we we can determine that yes, this data set will follow that particular distribution, so that so that goodness of fit that is called the statistical test from the data. How we can determine that which distribution it is following, that will be taken in the next module, that is module seven.

In this module, what we will do, whatever the standard distribution, whatever the standard probability distribution we have discussed earlier. We will be using those distributions to address some real problem of this of related to the civil engineering problems. Now in the last lecture, we have covered that normal distribution and we have also taken some of this, some of the problems, which can be solved through the normal distribution. In this lecture also, we will take two more continuous distributions and will continue to the next lecture as as well for another a continuous distribution. After that, we will also take a that discrete distribution, and that will be, we will trying to cover that whatever the standard distribution that we have that we have studied and their related problems in this module.

So, in this lecture, we will be covering that probability models using log normal and exponential distribution, you know these are the two distribution, which are also important and we have seen the many applications are there, related to these two distribution. So, these two distribution will be mainly, is in our focus for today's lecture.

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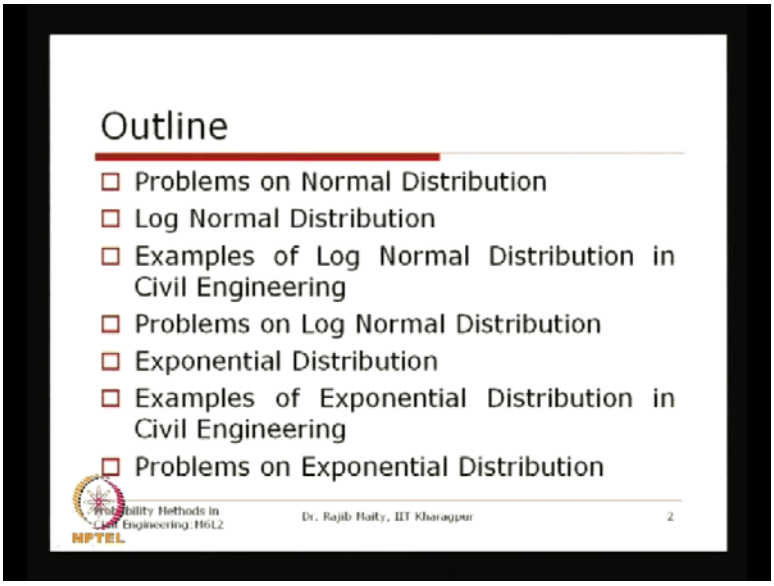

To start with, as you know that we in the last class, we have discussed about the several properties and we have also discussed some problems on this normal distribution. We will also take some more problems of this normal distribution. As you know that, we are giving the that impression that this normal distribution is one of the most common and very widely used distribution. So, whatever the problem we have discussed in the last lecture, we will try to take some more problems here, because the application of normal distribution for the several problems in the civil engineering is many folds. This in this particular lecture, we will be taking some problem from the Geo technical engineering problems. After that, after discussing that problem, even though this lecture is mainly on this log normal and exponential distribution, but to complete that normal distribution, we will take up two more problems on the normal distribution.

After that, we will go to the log normal distribution and we will see that, what are the examples of the log normal distribution in civil engineering. Also, some specific problem that we can solve from this log normal distribution, and then we will take the exponential distribution. Similarly, what are the different examples that can be modeled through this exponential distribution, we will discuss in civil engineering and we will also take up some numerical problems also, which we will solve in through this exponential distribution.

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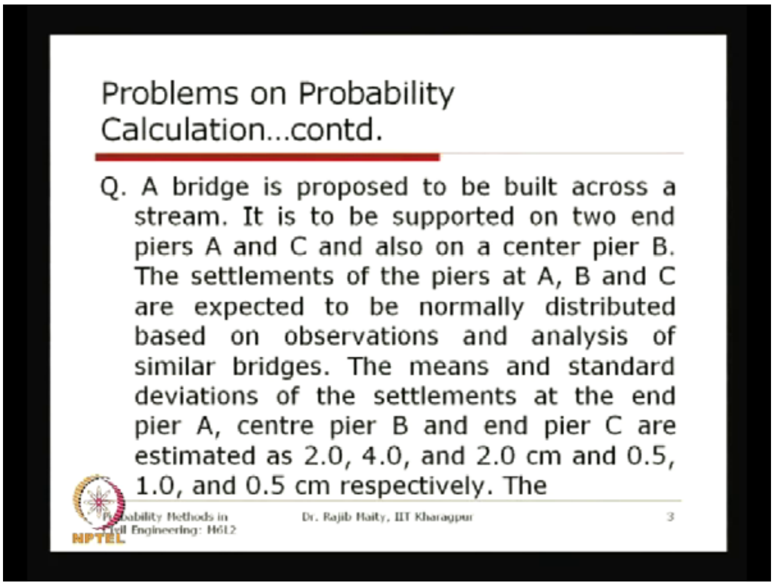

So, we will start with the some two more geo technical problem in a normal distribution. Other problems we discuss in this last class and today's class, this geo technical problem, this problem is mainly related to the settlement. First one it is, a settlement of a bridge piers. So, a bridge is proposed to build across a stream. It is to be supported on two end piers A and C and also on a center pier B. The settlement of the piers at A, B and C are expected to be normally distributed based on the observation and analysis of the similar bridges.

So, this is what we are just mentioning that in the problem here, we are we are declaring that this kind of data set is following that normal distribution or whatever the distribution that will be taking up. So, as it is mention here, based on the observation and analysis of the similar bridges, so some observation was recorded, data was collected; and from the data, we have satisfied that this data set is following normal distribution; how to satisfy that particular need, that we will take up in this next module through this goodness of fit and different probability papers and all, we will we will discuss that one later. But for the time being for this, for the discussion for this module, we will be assuming or we will be declaring that this data set is following this particular distribution. Based on that, we will solve those problems.

So here also, the settlement of the piers A, B and C are expected to be normally distributed based on the observations and analysis of the similar bridges. The mean and standard deviation of the settlement at the end pier A and center pier B and end pier C are estimated as 0.2 0.4 and sorry 2, 4 and 2 centimeter and 0.5, 1 and 0.5 centimeter respectively. So, once we are declaring that this is following certain distribution, so so that means, we are we also know what are the parameters for those distribution. For the normal distribution, we know it should have two parameters; one is the mean and other one is the standard deviation. So, these mean and standard deviation for all these three, support all these three piers are shown here. For the end pier A and the end pier C, if this are same, the mean is 2 and and their standard deviation is 0.5, and for the center one the mean is 4 centimeter and the standard deviation is 1 centimeter.

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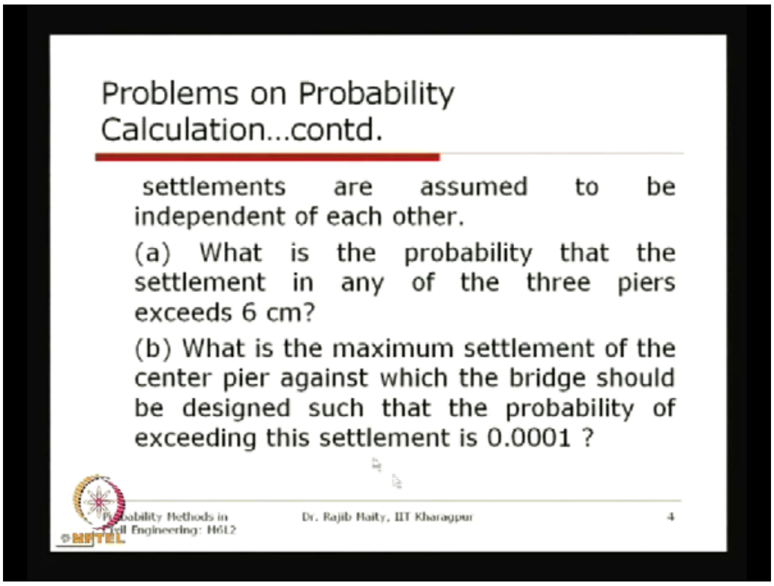

So, the question is, the settlement are assumed to be independent of each other means, one th the settlement of one th support is independent of the what is happening for the other support. So, we have to determine, what is the probability that the settlement in any of these three piers exceed 6 centimeter? So, what is the probability that any of these thing will exceed will exceed 6 centimeters.

So, it could be it is a, so this question is having several events; say that pier A exceeds 6, other two does not or pier B exceeds 6, other two does not; similarly, pier C exceeds 6, centimeters other two does not or any of these two, say A B exceeds 6, but not C B exceeds 6, not not the other one. Like that 2 out of 3 we can select and we can say that those are exceeding or all three can also exceed. So, what is so, this is basically the collection of so many events. So, what is the probability that the settlement in any of these piers will exceed 6 centimeter?

And the second question is, what is the maximum settlement of the center pier against which the bridge bridge should be designed such that the probability of exceeding this settlement is 0.0001. So, very less probability is assigned and what is that settlement that we should consider to design that structure and what is that particular settlement. So, that the exceedence probability of that particular settlement will be 0.0001.

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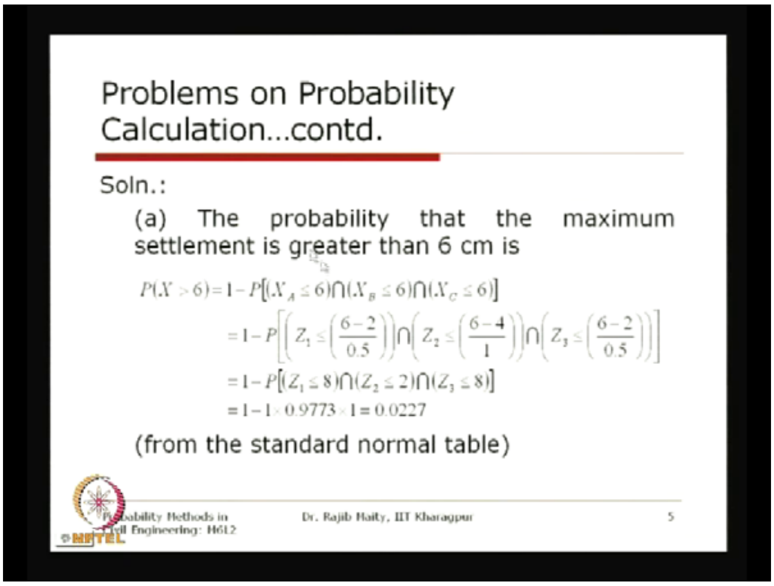

So, we will take this first one; first problem to answer this one is the probability that the maximum settlement is greater than the maximum settlement any of these three, one will be greater than 6 centimeter is that probability of x is greater than 6, this x is now a vector. So, any of this things can occur or the combination of their their is combination of their. So this, what we can say is that we will just reverse this problem that 1 minus this one is the total probability, 1 minus this, none of this piers is exceeding 6 centimeter. So, that is the option that to exclude or to group all the events that will basically taking care of this whatever, this, any of this or either of this or a combination of these three will exceed 6 centimeter.

So, this event, the probability of this event that is x A less than 6 means, settlement at the pier A is less than 6 centimeter, intersection for the B less than 6 intersection with this C less than 6. You know that we have discussed earlier, this means that so, all these things is occurring simultaneously. The x that settlement at A does not exceed 6 and settlement B does not exceed 6 and settlement C does not exceed exceeds 6 centimeter of the settlement. If we deduct it from this one, which is the total probability, then what we get any of this thing of this support or the their combination there of whether they are exceeding 6 or not.

Now, once we have decided this this line, that this is the way, we will calculate the probability. Rest of the things are you know that from the normal distribution is simple, for this x A. First of all, we have to get that reduced variate. Basically, we are transforming from the any normal distribution to the standard normal distribution, because we know those values from the probability table and normal standard normal probability table is available. So, this is that so, 6 is that value minus mean for that support A divided by the standard deviation. So, 6 minus 2 by 0.5 is the reduced variate. Similarly, for this support B 6 minus 4 by 1 and support C 6 minus 2 by 0.5.

So, if we do all this and so, this Z 1 less than 8 Z 2 less than 2 and Z 3 less than 8. You know that for this Z 1, this 8 is for the standard normal distribution, this 8 is very high value to easily declare that this probability is equals to 1. For this 2, we can get it from this table that this probability will be 0.9773 and this one this probability Z is equals to minus 8, Z 3 minus 8 is again 1. Now, as the assumption was that these settlements are independent to each other. So, we can get that joint probability by their a by their multiplication. So, that is why we are using 1 multiplied by 0.9773 multiplied by 1. So, the probability of any of this, or any combination of this three support will exceed 6 centimeter is 0.0227.

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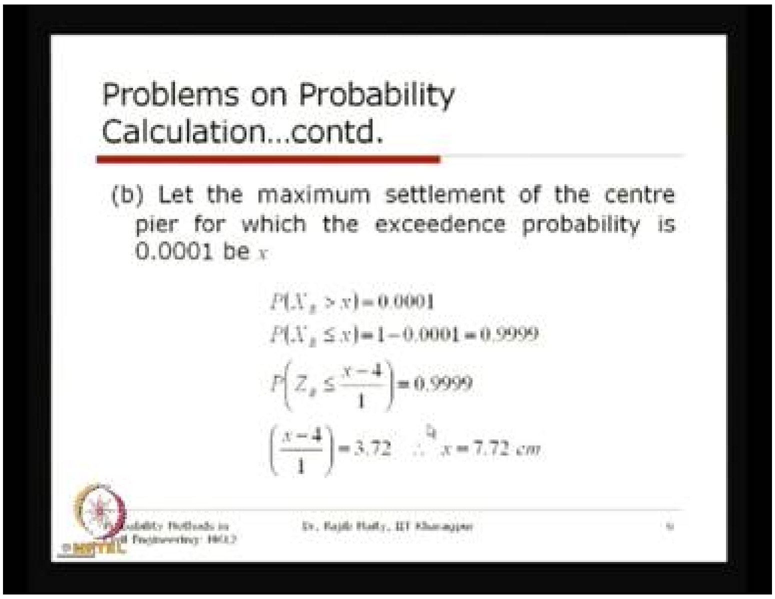

The second question was on that, on that the maximum settlement of the centre pier for which the exceedence probability is 0.0001. Let this is x. So, we have denoting this x. So, that the exceedence probability; that means, what we look for is the probability of this x B, that is the center one is greater than 6 is equals to 0.000 0001. Now, this is the settlement. So, what we can write is that we can just convert it to this less than equals to, so the, total probability 1 minus this probability. So, that will be 0.9999. So, now we can reduce, obtain its reduced variate, x minus 4 is the mean divided by 1 for the centre support, and this is equals to this value. We know that for this value, from the table, we can get that the value of the z is equals to 3.72. So, thus the x minus 4 by 1 is equals to 3.72. So, the x is equals to 7.72 centimeter.

So, the the settlement for which the exceedence that non exceedence probability is 0.00 sorry, the non exceedence probability is 0.9999 and exceedence probability is this one, 0.0001 is equals to 7.72 centimeter. So, if we take this one this settlement into that design, then we can assure that this is, with some reasonable probability, reasonably low probability we can design that center support. Similarly, we can answer, similar answer for the other other piers as well for this one.

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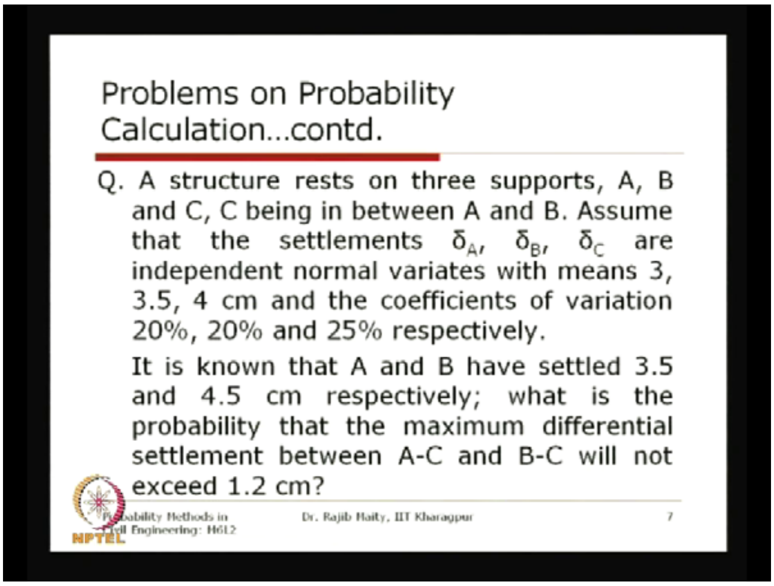

We will take another problem, which is also the similar, that is a this is a structural problem here. Second, the settlement here, what is happening is that there is one thing called the differential settlement. You know this differential settlement is very important for the structural design, which will basically induced some moments to the structural joints. So, here we will see that how to determine, how to see, how to control or how to get the probability of the events of the differential settlement. So, their structure rest on three support, A B and C; and C being the, C being in between A and B.

So, the sequence of the support is the A, then C then B. Assume that the settlement delta A, delta B and delta C are independent normal variates with mean 3, 3.5 and 4 centimeter and their coefficient of variation is 20 percent, 20 percent and 25 percent. Now, it is known that A and B have settled 3.5 and 4.5 centimeter respectively. What is the probability that maximum differential settlement between A C and B C will not exceed 1.2 centimeter?

So, we know that, so several things has been declared, one is that the settlements are normally distributed as was also for the last problem. As we have given this, it is the normally distributed then their then their parameters are also declared. There is a mean, is 3, 3.5 and 4 and their coefficient of variation is also given; that means, you know that this standard deviation divided by mu is the coefficient of variation. So, from this information, we can get their standard deviation as well.

Now, the settlement of two support, basically two end support A and B are given here. So, we have to find out the differential settlement, what should be the maximum probability, that the maximum differential settlement will not exceed 1.2 centimeter. So, to seek this one, first of all we have to find out what is the settlement range that we are talking about for the support C. That we have to find out; once we find out that, what is the range of the settlement at C, then from the information supplied, we can get that what is the probability of that particular event.

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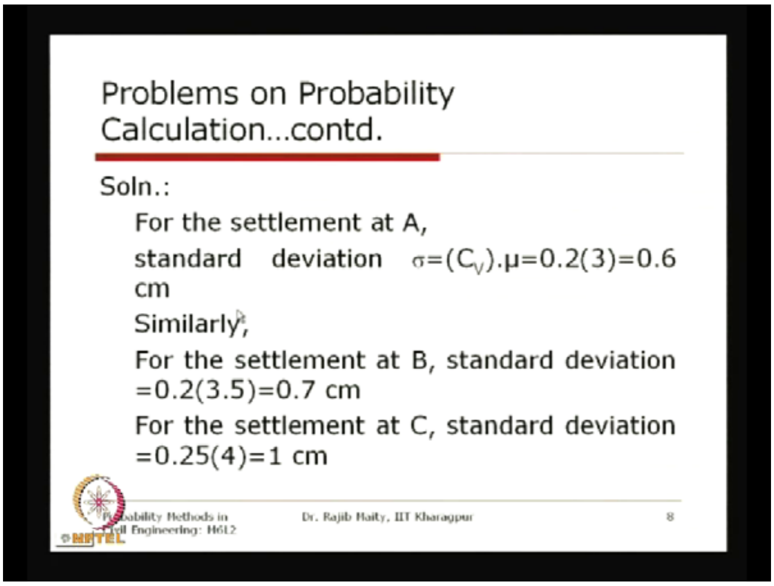

So, for the settlement at A, the standard deviation should be, you know that C v multiplied by mu. As we as we have told that this is the standard deviation divided by mu is the co efficient of variation. So, this C v multiplied by mu, so 20 percent multiplied by its mean is 3 centimeters, so 0.6 centimeter is the standard deviation for the support A. Similarly, for B it is that 0.7 centimeter, and for the C it is 1 centimeter standard deviation; just we are getting this information from their co efficient of variation.

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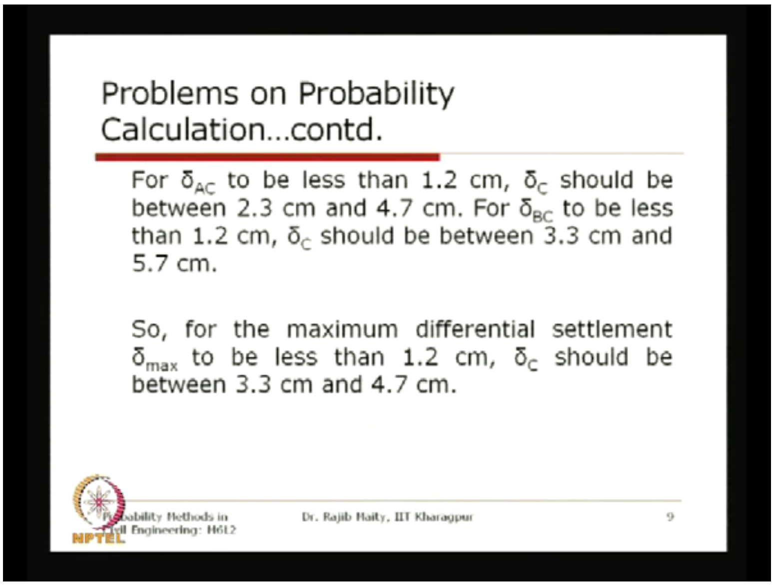

So, now, for that delta A C to be that less than 1.2 centimeters. So, what that delta C should be what? So, we know that know that what is the settlement of this A. Now the settlement of the A is, is as explained, the settlement of the A is 3.5 centimeter.

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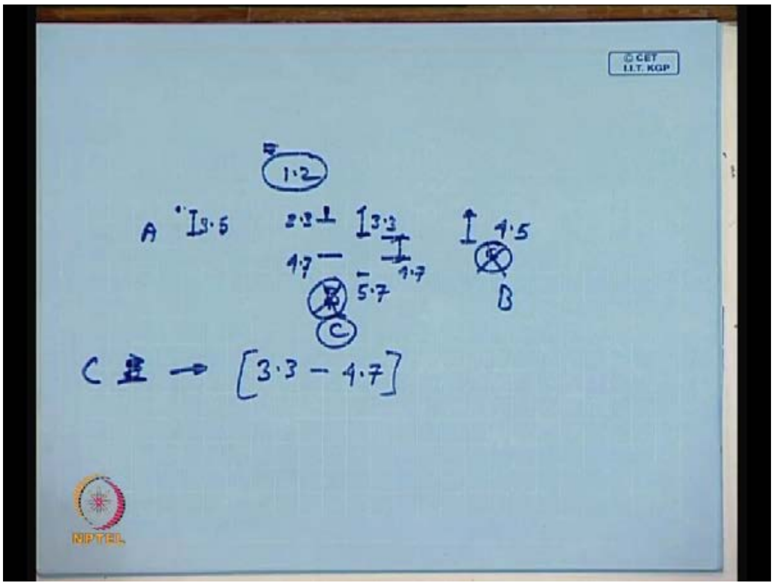

Now, if you just see it here, so suppose that this is your support A, then this has already has come down to this 3.5 centimeter. Now we are, what is that that 1.2 centimeter is the differential settlement. So, it should be either so, from this range to this range, this support B can settle for which the differential settlement will be 1.2; that means, it can minimum settlement for this one is that what we are talking about, that this minus this 1.2, that is 2.3. If support B settled 2.3 that means, the differential settlement will be 1.2 or else if it is settle more and if it goes up to say 4.7, and then also we can say that the differential settlement between this A and B will be 1.2. So, this is a minimum settlement possible to to maintain that differential settlement 1.2 and this is the maximum possible settlement at B.

Similarly, if we if we see from this support C and this support C has already settled, that settlement for the support C is is 4.5 centimeter; so it has settled 4.5 centimeter. Now, if I want to see that to maintain this 1.2 differential settlement between B and C also, then then this will be 3.3. So, this can minimum settlement here should be 3.3, and the maximum settlement that it can come is that 5.7. Now, you see now what we have to find out is that the maximum differential settlement should be limited to this 1.2.

So, what should be the settlement of this? Considering both the end support, what should be the range of the possible settlement of B? So, that the maximum differential settlement should be within this 1.2 centimeter. So now, to answer, to to see this one, we see that what is the overlap zone for this two possibilities, and the overlap zone is 3.3 to 4.7, so this 3.3 to 4.7.

So, if the support B settle 3.3 centimeter minimum and maximum is 4.7 centimeter, then we can say that differential settlement should be within 1.2 either between A B or that B C. So, we know this range now, and we know the property of this; what is the property of the settlement probabilistic, property of the settlement at B? We have to find out what is the probability of this support B, settlement at support B would be within 3.3 centimeter to 4.4 sorry 4.7 centimeter.

So, this is how the problem reduces to; so which is explained here. For delta A C to be less than 1.2 centimeter, delta C should be should be between 2.3 centimeter and 4.7 centimeter and for delta B C to be less than 1.2, delta C delta C should be C should be between 0.3 to 5.7. I did what I have shown here in this diagram is that I just shown that this is that support B and this is support C. But anyway you have seen that this is not the notation that we have used. This is the support B and this is the support C. So, C is between in between A and A and B.

So, the settlement for the C, settlement for the C is between this 3.3 to 4. 7. So, so delta C, for the delta B C to be less than 1.2, delta C should be between 3.3 and 5.7. So, for the maximum differential settlement delta max to be less than 1.2 centimeter, delta C should be between 3.3 centimeter and 4.7 centimeter.

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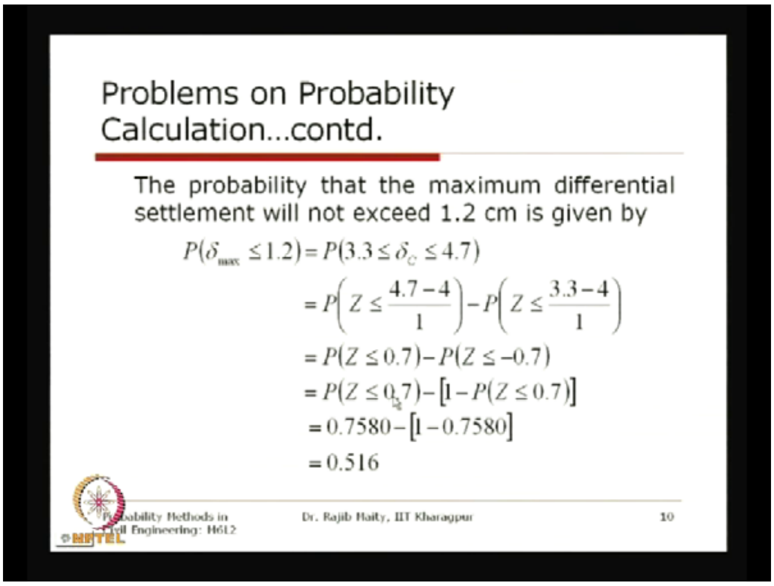

So, by this one, so, the problem now is reduces to the thing that the delta max is the differential settlement to be less than 1.2 centimeter. The probability that delta C, the settlement at C should be between 3.3 and 4.7. Once we have decided this one, the rest of the problem becomes straight forward. So, so this one we can just write their probability of Z or delta C less than equals to 4.7 minus probability of delta C less than equals to 3.3. So, after that we cannot make their reduced variate 4.7 minus 4 is the mean divided by standard deviation 1 then P Z less than equals to 3.3 minus 4. Let me just check whether this two values are mistakenly given.

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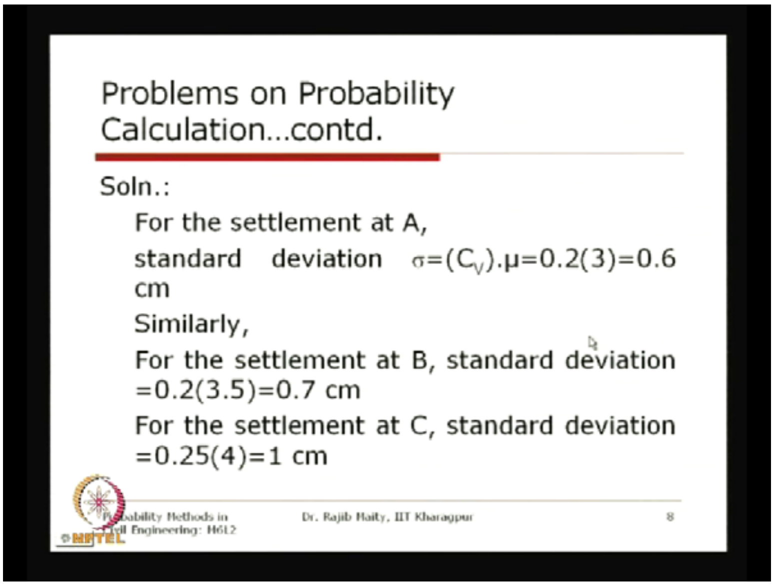

So C, for the C, the standard deviation is 1 and the mean of this C delta C is 4 centimeters. So, this one, the minus mean divided by standard deviation. So, which is Z is less than equals to 0.7 minus this Z equals to minus 0.7. Now, just to refer that one value, we know that from the symmetry, we can we can change this one to 1 minus probability of Z less than 0.7, which is the same value. So, 7.7580 minus 1 minus 0.7580 which is equals to 0.516.

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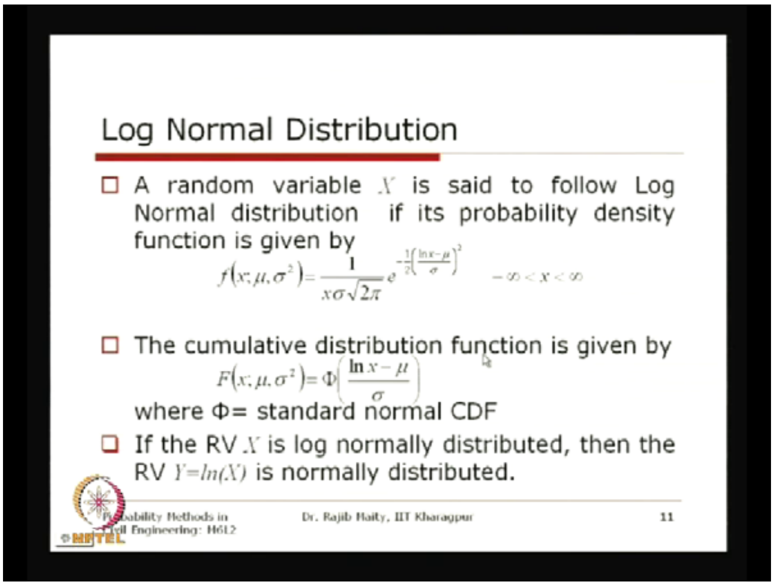

So now, what we can take is that log normal distribution, as we are discussing that we will be taking this log normal distribution here. You know that we have discussed this, their basic properties in the earlier module as well, is that a normal variable x is said to follow a log normal distribution, if its probability density function is given by f x with the parameter mu and sigma square 1 by x sigma square root 2 pi exponential minus half log natural of x minus mu divided by sigma whole square. And this, and the range of this x is from 0 to infinity sorry this is a mistake. This should be 0 to infinity minus infinity to plus infinity is the property for this normal distribution you know, so this one is the is the limit from this 0 to infinity.

The cumulative distribution function is that this phi log x minus mu by sigma, where this phi is the standard normal CDF. You know that if the random variable x is log normally distributed, then if I take their log natural of that data, then that log of that x is normally distributed. So, basically when we are addressing some problem through this log normal distribution log normal distribution, then if we can if we can if we can take, if you convert that variable through this transformation, then it essentially reduces to a normal distribution. Once it reduces to the normal distribution, then the everything remains same. And we can refer to the again that normal distribution table to find out what are the different answers that we are looking for for the different problems.

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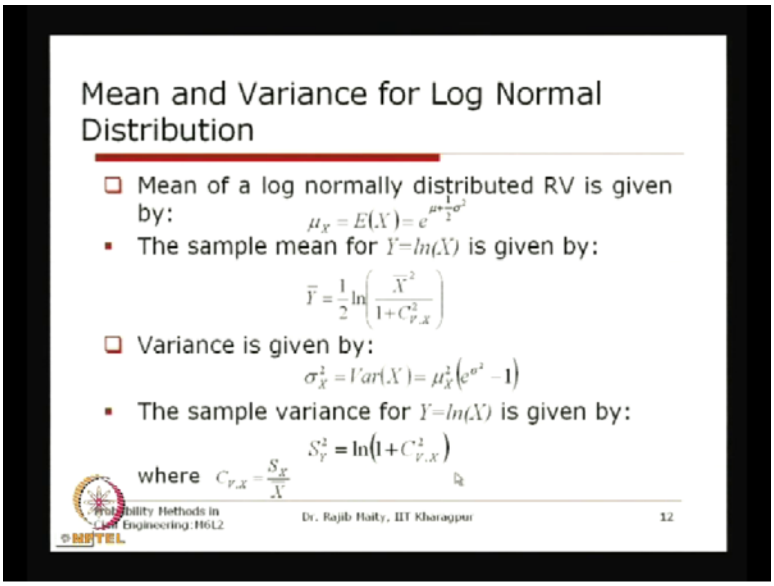

So now, basically, to this transfer, when we are taking that if we if we transfer that whatever the data that we have observed and for that data we can calculate, what is its mean and what is its standard deviation. So, from there, there are two possibilities now; one is that one data is available to us and that we have that we have somehow satisfy that this follows a log normal distribution. So, what we can do from the original data, we can calculate its mean and standard deviation and through some transformation equation, we can calculate what should be the mean for the for the transfer transferred random variable; transform means that through that log natural of that of that original data set.

We can calculate that one or what we can do is the original data set itself, for each and every data point; we can convert through this log transformation. We will get a new series and for the new series we can calculate its mean and standard deviation. Solve the problem and get the answer and that final answer, we should again transform back to the original scale that is, by taking the exponential of that particular value to get the answer in the original scale.

So, those are the properties; these are also discussed in the earlier module as well. That is mean of a log normally distributed random variable is given by the mu x equals to exponential of x equals to e power mu plus half sigma square. The sample mean for the y equals to log x means, after we transform it is given by y bar equals to half of log natural x bar square; this x bar is the basically is the original scale, what is the original data set. That mean, that square mean of that original data set that square divided by C v x. C v is nothing but the coefficient of variation for that data set x, that is the original, which is log normally distributed this x. If we follow this transformation, then the the value that we get, that will be the mean for this transformed variable.

Similarly, the variance is that sigma x square square is equals to the variance of x, which is the mu x square e power sigma square minus 1. The sample variance for this this transformation y equals to log natural of x is given by s x square equals to log natural of 1 plus C v x square. So, C v x is you know that this is the co efficient of variation for the data x, where this C c v x is the standard deviation of the original data, that is the x divided by its mean.

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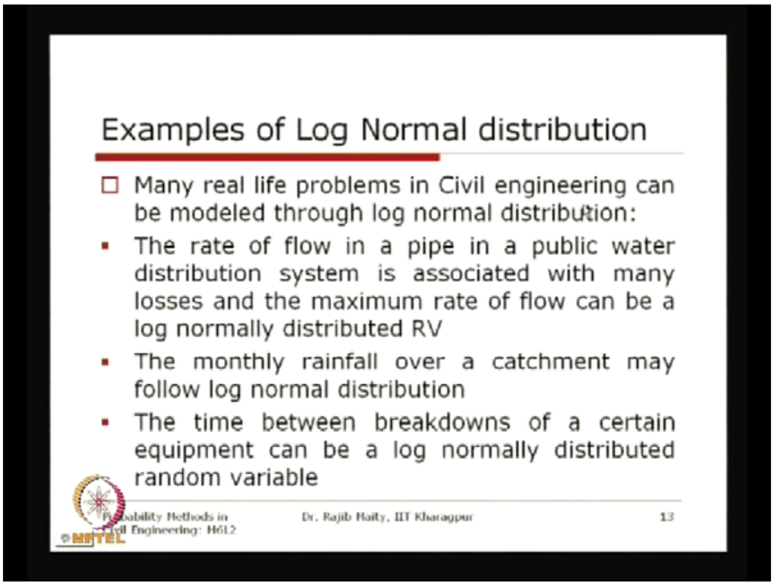

Now, the many real life problems in civil engineering can be modeled through this log normal distribution. The rate of flow in a pipe in a public water distribution system is associated with a many losses and the maximum rate of flow that can be the log normally distributed random variable. The monthly rainfall over a catchment may follow a log normal distribution. The time between the breakdowns of a certain equipment can be a log normally distributed random variable.

Basically so, these as I told that once we have the data set, we can test it whether it is following this that particular distribution or not. Here what you can say is that you know that support for this log normal distribution is 0 to infinity. So, first of all, the example that we are taking care, whether we have to see that whether the support is, can go or that value of that random variable we are talking can go to the negative side or not.

If it can go to the negative side obviously, that we will never follow the log normal distribution. Now, if it is within this only, within this positive side that is greater than 0, then there is a possibility that it can follow a log normal distribution, because there are other distributions as well with this same support. So, we have to taste first of all, but whatever is listed here is from the experience that these are the dataset, which may follow the log normal distribution.

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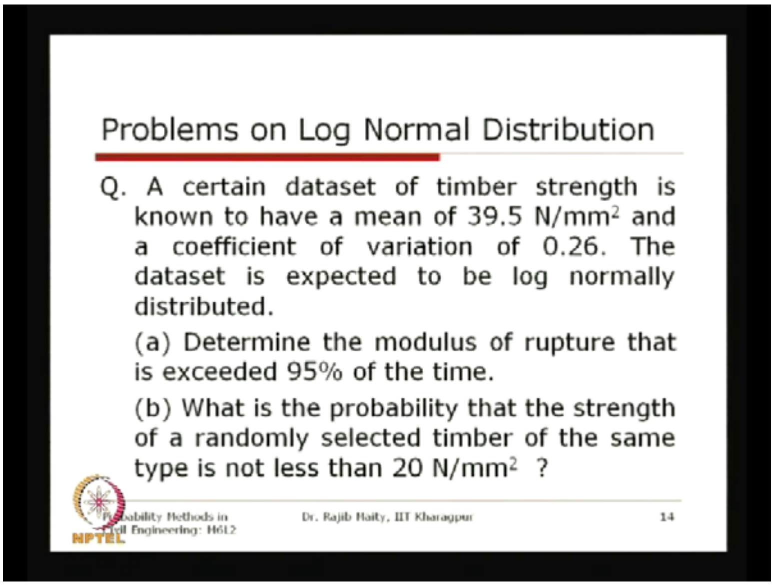

We will take one example on this log normal distribution. A certain dataset of timber strength. Here the strength of a timber, there are many structure which are a constructed with the with timber, and we need to know the properties of this strength, and that also calculated through the probability. So, one such problem is taken here on this timber strength. A certain dataset of the timber strength is known to have a mean of 39.5 newton per millimeter square and a coefficient of variation of 0.26. The dataset is to be log normally distributed.

Now, determine the modulus of rupture that is exceeded 95 percent of the time. So, this modulus of rupture, the way we define is that, that in a, in that, what is the strength that it should be exceeded. It should be exceeded 95 percent of the time; that means, the probability of one particular event for which the exceedence probability should be 0.95. So, that strength should be the modulus of rapture for that particular type of that timber. This is the first question, we have to find out from this data is what is the modulus of rupture.

Second one is the what is the probability that the strength of randomly selected timber of the same type is not less than 20 newton per millimeter square. So, this probability we have to calculate. Now as it is already given that this follow a log normally distribution. So, we can use the normal distribution table, after we can transform the data through the transformation that y equals to log natural of x. We have seen in the earlier slide that how to transform these data, because these data what is the 39.5 newton per millimeter square and 0.226; that means, that it is the information on the standard deviation. So, these two things has to be has to be first convert to that transform scale, and then we can follow and we can solve this problem. After that we will transfer back, the back transformation of that answer to this original scale to know the answer.

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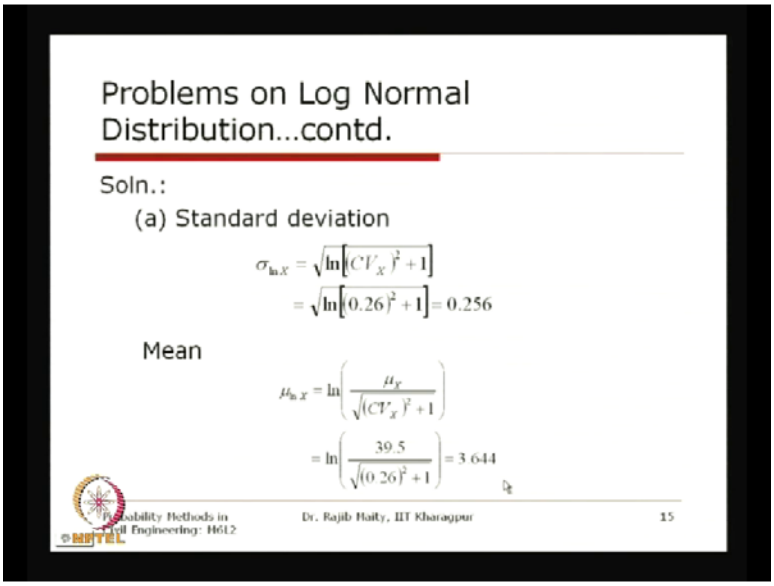

So, to this one, first of all we have to find out after the transformation, that is the log natural of this x. What should be its sigma? So, this we know that this in that coefficient of variation of the original dataset square plus 1 log natural and square root. So, it gives you the answer that 0.256 for the sigma, and the mean of that value, is that mean is that again through this equation, that is mean divided by square root of CV square plus 1 for this full quantity its log natural, which is equals to 3.644. So, so this is now the mean, and this is now the standard deviation, for which we have to, we can use that that normal distribution table to solve this problem first.

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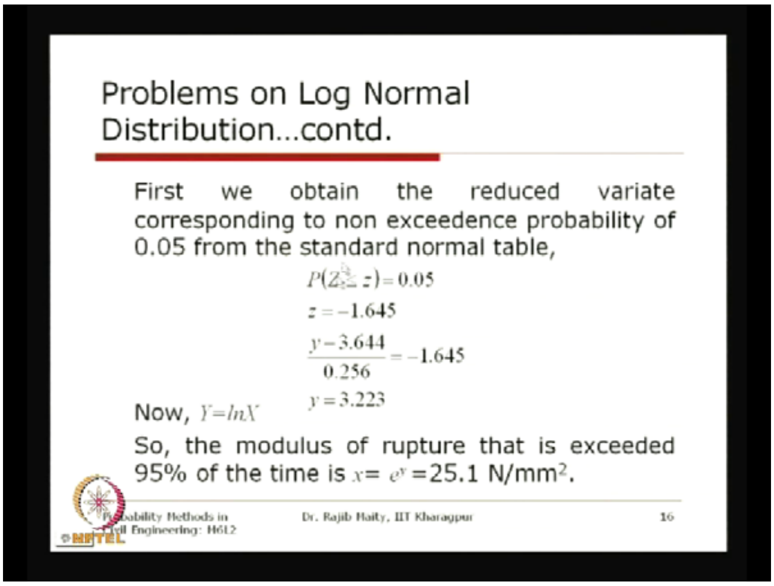

So, now you have seen that the first to obtain the reduced variate corresponding to the non exceedence probability of 0.05, so that modulus of rupture the exceedence probability is 0.95 that means, the non exceedence probability is 0.05. So, from the standard normal table; so this should be that Z less than equals to that specific value should be equals to 0.05. So, here we know that from the table, from the standard normal distribution table that Z should be equals to minus 1.645.

So, this Z is now is the y minus that mean divided by this standard deviation, which we just now we got. These values at the mean of this reduced variate of course, that is y through this transformation y equal to l n x. So, through this one what we will get that y is equals to 3.223. So, the set so, the probability of this y greater than 3.223 is equals to 0.95, it will come. But you know that what we are looking for is the strength which is in the original scale. So, so through this transformation, we have to back transform this one. So, we have to find out the value of the x for which the y is equals to 3.223.

So, the modulus of rapture that is exceeded 95 percent of the time is the x equals to e power y. So, we will put this value here and we will get that 25.1 newton per millimeter square. So, the modulus of rupture for this particular timber is your that 25.1 newton per millimeter square.

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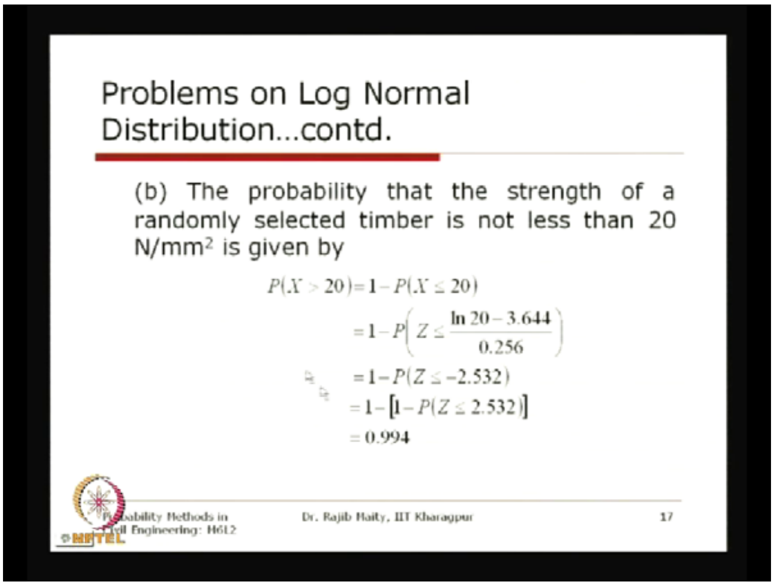

The second one, the probability that the strength of randomly selected timber is not less than 20 newton per millimeter square is given by that probability that x greater than 20. So, that we convert that 1 minus probability of x less than 20 here. What we have to do here first, in this case, is the first we have to find out the transformed variables. So, to get this reduced variate in this normal distribution is to use normal distribution table. So, this one, this 20 we are transforming to this log of that the log natural of that 20 minus its mean and standard deviation. We will get that Z less than equals to minus 2.532. After this, after doing this equation, we will get that the probability that is not less than 20 newton per millimeter square is equals to 0.994.

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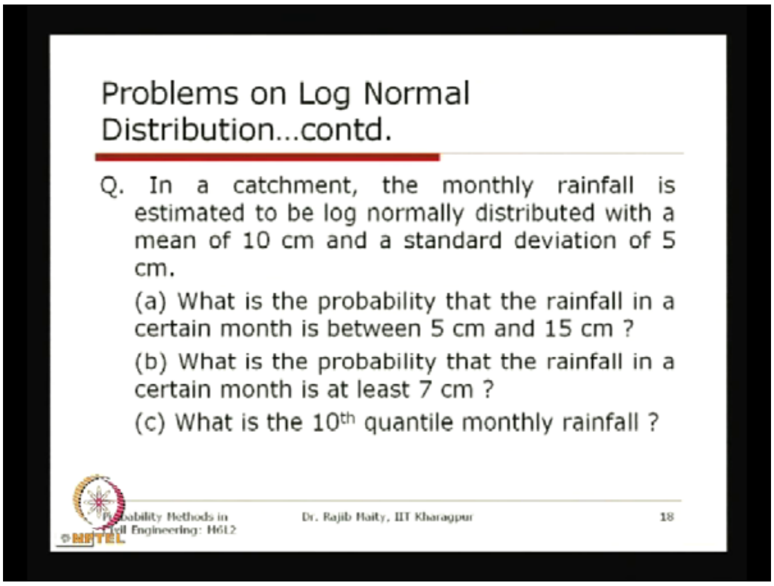

We will take another problem on this log normal distribution. This is on this monthly rainfall over a catchment. So, problem states in a catchment the monthly rainfall is estimated to be log normally distributed with a mean of 10 centimeter and a standard deviation of 5 centimeter. So again, that instead of providing the full dataset, that parameter of the dataset is supplied that is 10 centimeter is the mean and standard deviation is 5 centimeter and it is declared it is that log normally distributed. The question is, what is the probability that the rainfall in a certain month is between 5 centimeter and 15 centimeter? What is the probability that rainfall in a certain month is at least 7 centimeter and what is the 10^{th} quantile of the monthly rainfall?

So first, so, similar to the previous problem again, first of all, we have to, you know we have to transform it from this log normal to the normal distribution case. For that case, we have to transform the mean and transform this standard deviation. After we transform this one, we have to, we also have to find out the corresponding values at this normal distribution scale. What is the value for this 5 centimeter and 15 centimeter and in between that that probability has to be calculated. Similarly, for the other two problems also, this is for the 7 centimeter. We have to see that, what is exactly, look for this here, we need to know that at least 7 centimeter. So, the greater than 7 centimeter probability, we have to deduct from the total probability.

And the last one is the 10^{th} quantile. This 10^{th} quantile means that it is, that the exceedence probability should be 1 minus 0.1, that means 0.9. 10^{th} quantile means that it is the it it is that non exceedence probability is 0.1. So, that particular value we have to find out.

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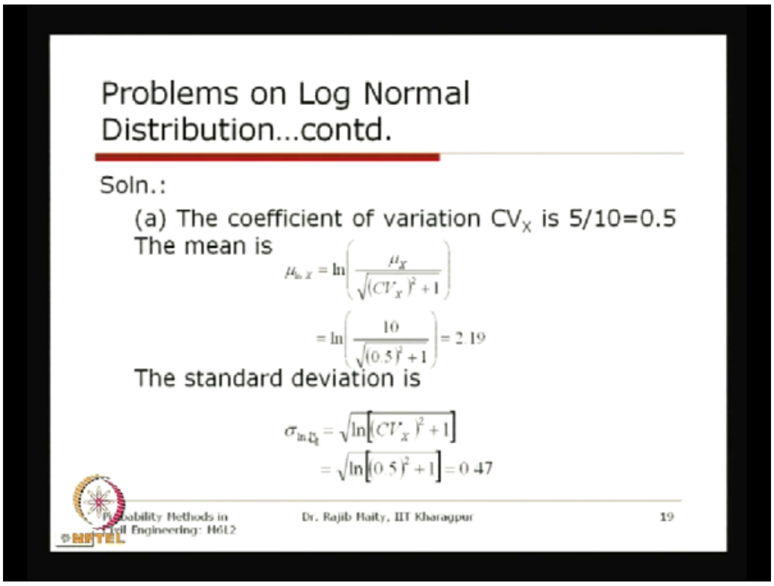

So, to answer this first one, as I have told that we have to find out that what is the mean at this transform scale, and for which we have to find out what is that C V. That is coefficient of variation, which is standard deviation divided by mean which is 0.5. Through this transformation equation, this mu of the transform variable is 2.19 and standard deviation of the transform variable is equals to 0.47.

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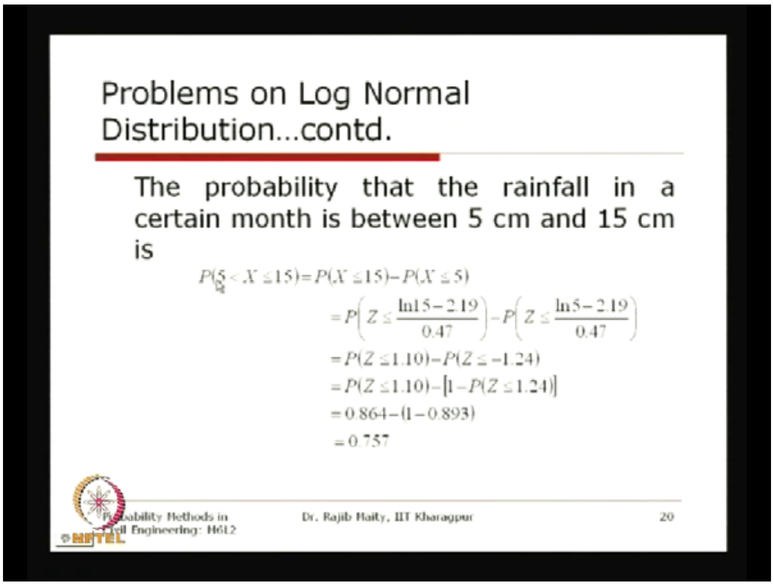

Now, using this 2.19 and that and that 0.47, we have to calculate what is the probability that that x, that rainfall value is between 15 to 5, which we can also write that probability that x less than 15 minus probability of x less than 5. So, this one I hope, that these things as I have also mentioned in this normal distribution problem as well as here also. We are using hope that you can see it here that for the normal distribution what we are…

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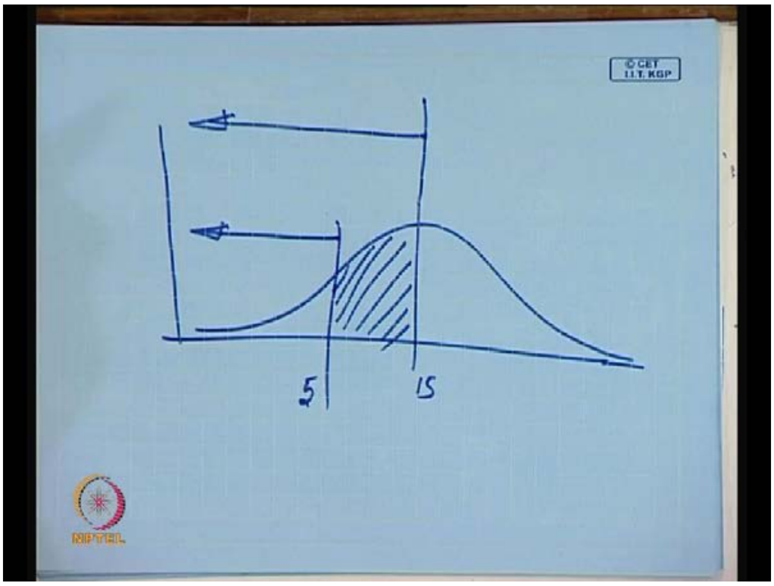

So, if this is the distribution 1. So now, if this is you say 15, and if this is you say 5, then basically we are interested to this area. Now, how we are calculating this one is that probability of this this x less than up to 15. So, this from 15 to the minus infinity minus what is the from this 5 to this minus infinity to get this area. So, this full area minus this area will give you this area. This is what this we have discussed earlier also from this area concept of this single random variable.

So, that is why from this one we are we are writing the text less than equals to 15 minus x less than equals to 5. Now, again from as we have done for this last problem, we have to first of all find out the reduced variate in this normal distribution scale. That is log natural of 15 minus its mean divided by 0.47 and similarly, log natural of 5 minus mean by standard deviation. So, these values are giving you that 1.1 and this is minus 1.24 and this 2, if we just get the values from this table, standard normal distribution table. We will get these values and the final the probability is equals to 0.757.

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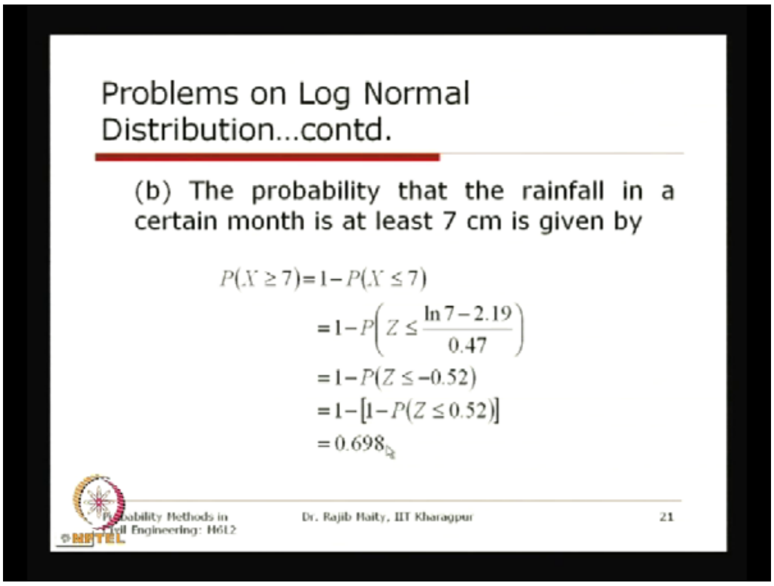

If we take the second one, the probability that the rainfall in a certain month is at least 7 centimeter that means, we are looking for the answer the x is greater than equal to 7. It can be again written that 1, that is the total probability minus x less than equals to 7. We also have discussed that this equality sign, so far as the random variable is continuous. This equality sign whether included here, as well as here does not mean anything, because that probability for the continuous random variable probability of x exactly equal to some specific value is always equals to 0 is equals to 0.

So that means, whether it is included here, the greater than equal to and here also it is less than equals to does not mean anything, but because that probability x exactly equals to 7 equals to 0. So this one, we can again, we are just obtain their reduced variate minus 0.52. From the table, we will get the answer that this probability that the rainfall in the certain month is at least 7 centimeter is 0.698.

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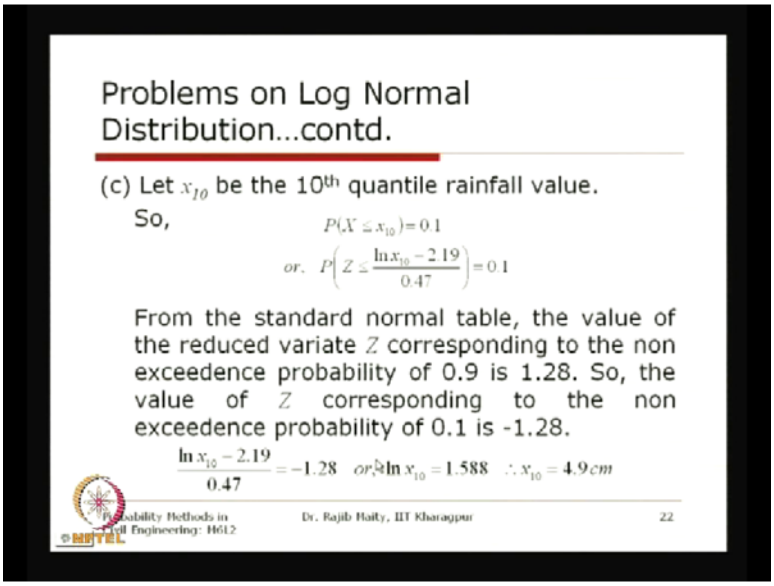

And the 10th quantile of this rainfall value. So, let us say that this value is x 10. So, as I told that this non exceedence probability should be 0.1. So, probability of x less than equals to x 10 equals to 0.1. Now, we will find out is again the reduced variate, a log natural of that value minus mean by standard deviation equals to this one.

Now, from the table, from the standard normal table, the value of the reduced variate Z corresponding to the non exceedence probability 0.9 is 1.28. So, the value of the Z corresponding to the non exceedence probability of 0.1 equals to minus 1.28 from the symmetry. Some text book will give you the standard normal distribution for both, from the negative side to the positive side. That is from the almost from the minus 3 to plus 3 it will be given. If only the positive side is given, then also using the property of the symmetry we can get these these values; that is if it is 0.9 it is 1.28, then for 0.1 it will be minus 1.28. That is what it is explained here. So, this value should be equals to minus 1.28, where we can say that log natural of x 10 is 1.58. So, x 10 is 4.9 centimeter. So, the 10th quantile of the rainfall value is 4.9 centimeter.

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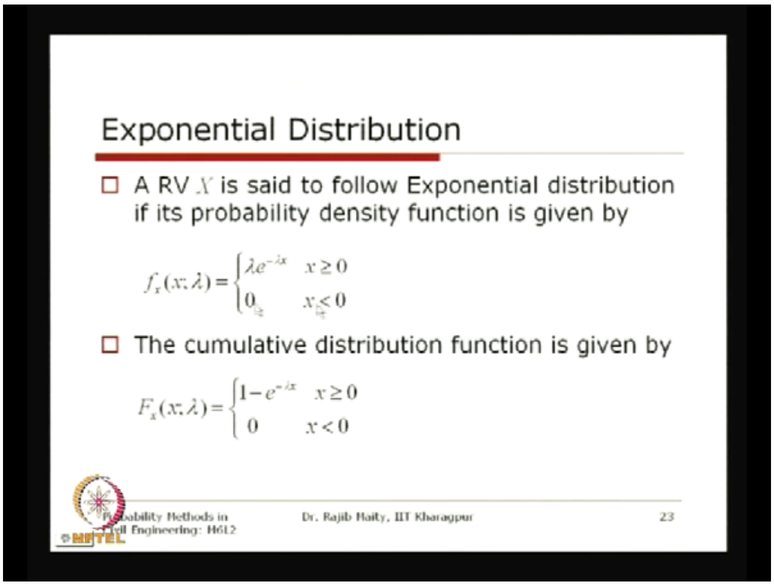

Now, we will take that exponential distribution. A random variable x is said to follow exponential distribution if its probability density function is given by this one, f x of x. This will be capital X as you know that notation that we are using the random variables is shown here. So, f x of this x with their parameter lambda is equals to lambda e power minus lambda x for the area, for the support is greater than equal to 0. For the negative side, it is 0.

So, this distribution also we have discussed, we have we have explained its properties and other things in the earlier module. We will be using these properties now to solve some of these problems in the civil engineering. We have also seen earlier that the cumulative distribution function of this exponential distribution is, again this will be capital X, and this one is 1 minus e power minus lambda x for x greater than equal to 0 and for else where it is 0.

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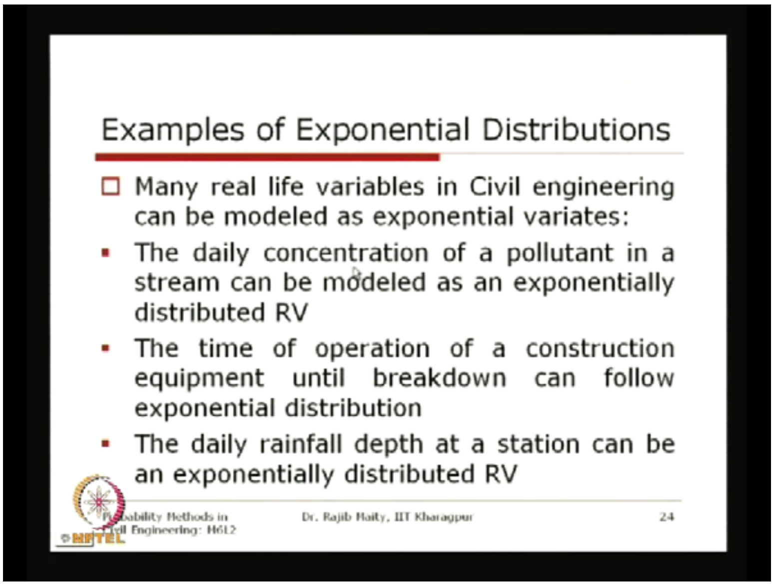

Now in the civil engineering, the different example that ca[n]-, that may follow, the exponential distribution that may follow, I can use because obviously, any dataset that you get, you have to check it first whether it is really following that distribution or not. So here, some of the possible examples are the daily concentration of a pollutant in a stream can be modeled as an exponentially distributed random variable.

This is basically the environmental engineering discipline of the civil engineering. It is the daily concentration of the pollutant, which is exponentially the concentration of the pollutant due to its natural decay. In generally, we assume it to be decay through an exponential distribution. So, the daily concentration that can be modeled through this exponential distribution.

Secondly, the time of operation of construction equipment until breakdown can follow one exponential distribution. This is that how long that one construction equipment can be utilized before it before it breakdown. That time of operation, that is that can follow the exponential distribution; and daily rainfall depth at a station can be the, can also be an exponentially distributed. But in particularly, this daily rainfall depth when we consider, it may have some significant number of 0 values also. So, even though it is exponentially distributed, but there may be a concentrated probability at 0. So, this type of distribution is known as the mixed distribution. The mix distribution is that it is a mix between this discrete distribution and the continuous distribution. We will take up a specific problem on this one and discuss this issue.

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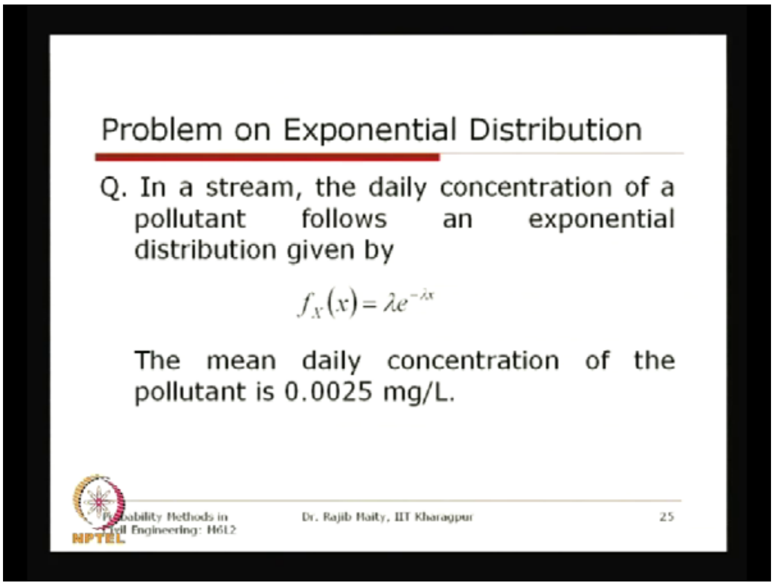

So so, first we will take one problem on this environmental engineering component, where the daily concentration of the pollutant is our concern. In a stream, the daily concentration of a pollutant follows an exponential distribution, which is given by this f x x equals to lambda e power minus lambda x, and obviously, x is greater than equal to 0 which is the support for this distribution. The mean daily concentration of the pollutant it 0.0025 milligram per liter.

So, why this information is supplied? You know that this mean and this parameter lambda is equals to 1 by x bar; that is the mean of that value of this random variable. So, this also, this we have discussed earlier how to get this parameter from this data this lambda is equals to 1 by x bar. So, that is why this mean concentration is given to get this parameter lambda.

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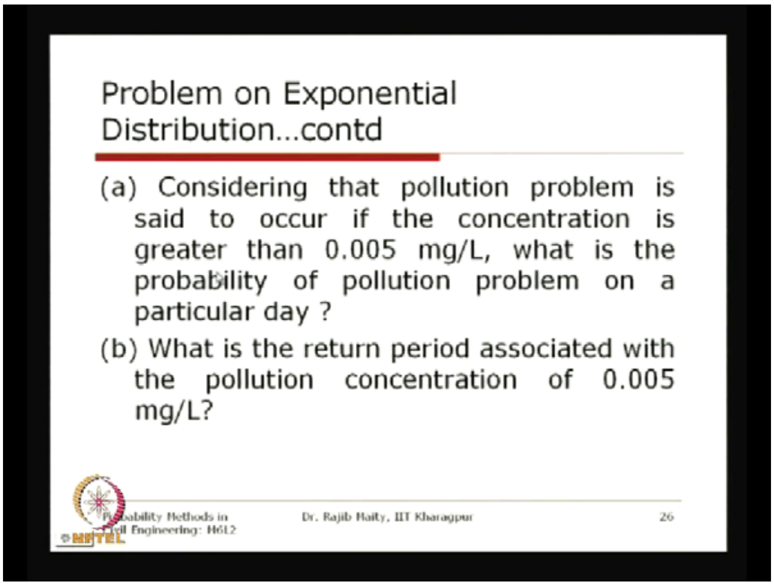

The question is consider that the pollution problem is said to occur, if the concentration is greater than 0.005 milligram per liter. So, if this is the level, if this the stressful level to declare that declare that this has followed this has the pollution problem has occurred, then you have to find out what is the probability of the pollution problem on a particular day. So, basically what we have to look is that what is the probability that the concentration will exceed this magnitude.

Second question is what is the return period of the associated, association period associated with the pollution concentration of this 0.005 milligram per liters? So, this is the return period means how frequent this problem can can occur. So, how frequent this probability can be, so this magnitude can be exceeded. That is known as this return period. So, we will just see how we can calculate the return period from the probability.

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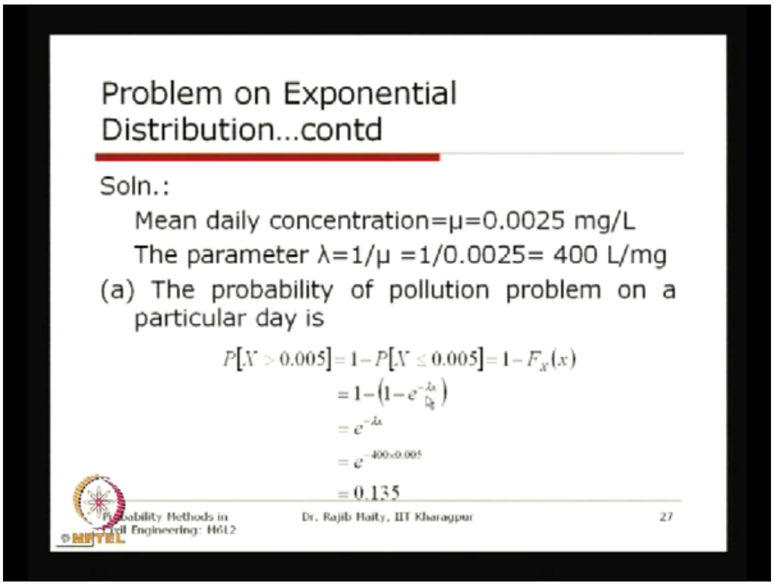

So so, this answer, this first one as I told, that the mean daily concentration is given as 0.0025 milligram per liter. So, this parameter of this lambda is 1 by x bar, that is the mu 1 by divide this one by 0.0025, so 400 is the parameter lambda. And the probability of the pollution problem on a particular day is probability that x greater than 0.005 is equals to 1 minus probability x is less than equals to this one. We know that from this, from the total probability, we can we can calculate and this pollution problem occurs, when this x, this random variable is exceeding this particular threshold value as declared.

So, this we can write that 1 minus this is that cumulative probability F x (x) and the cumulative probability is expressed that 1 minus c power minus lambda x, which is essentially coming down to this number that e power minus lambda x, which we can write that e power minus 400 is the lambda parameter multiplied by this x is 0.005. So, 0.135 is the probability that on a particular day that pollution problem can occur.

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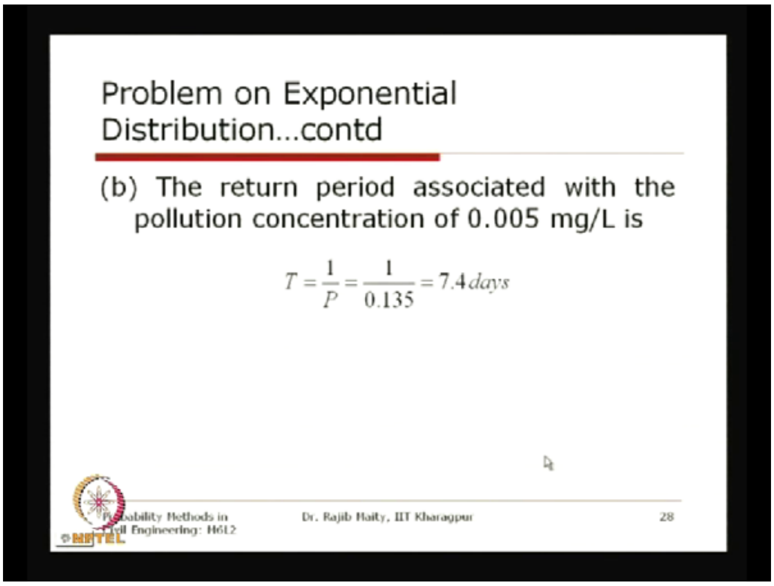

The second one, the second question is that what is the return period associated with the pollution concentration of 0.005 milligram per liters? So, this return period can be calculated through the that this is the inverse of the probability. So, what is the probability that is associated with that particular event? If we just take that inverse of that one, that will give you that what is the return return period of that particular event. So, as you have got that this probability is 0.135 here. So, if we take that inverse, then we will get that 7.4 days. So, on an average, we can approximated that on an average once in a week that pollution problem can occur depending on the data or the information that is supplied for that particular station. So, on an average, once in a week that pollution problem can occur at that place.

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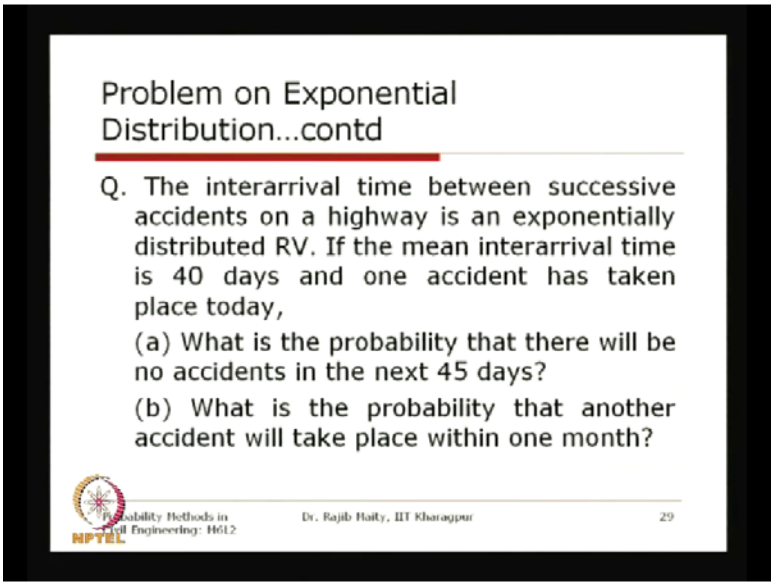

Now, we will take that another one that interarrival time between the successive accidents on a highway is an exponentially distributed random variable. So, again this is this, so far, as the accident is concerned there are two distributions. Generally, we take one is that, if we just count the number of accident over a given time, then that follows one distribution which is a discrete distribution, which is poisson distribution. We will take up that one later, when we are discussing the discrete distribution. But here, we are, what we are referring to is that interarrival time between two successive events. So, that is, that that can be modeled through this exponentially distributed exponential distribution.

The mean interarrival time is 40 days and and one accident has taken place today. If this is the case, then what is the probability that there will be no accident no accident in next 45 days. And what is the probability that another accident will take place within one month?

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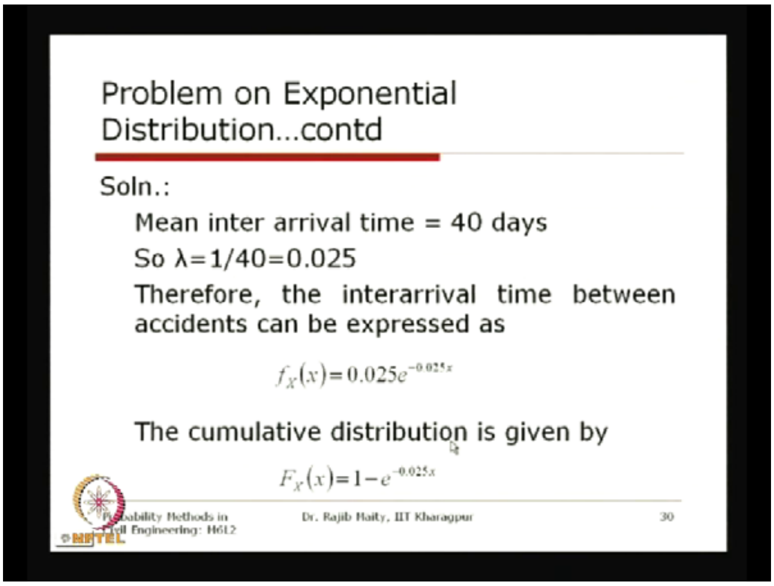

Two questions are there. And also first of all, we have to calculate, what is its parameter lambda equals to 1 by 40, which is 0.025. Therefore, the interarrival time between the accident can be expressed as f x (x) equals to 0.025 e power minus 0.25 x. Obviously, x is greater than equal to 0. Its cumulative distribution is also 1 minus e power minus lambda x, where the lambda is 0.025 x x is greater than equal to 0.

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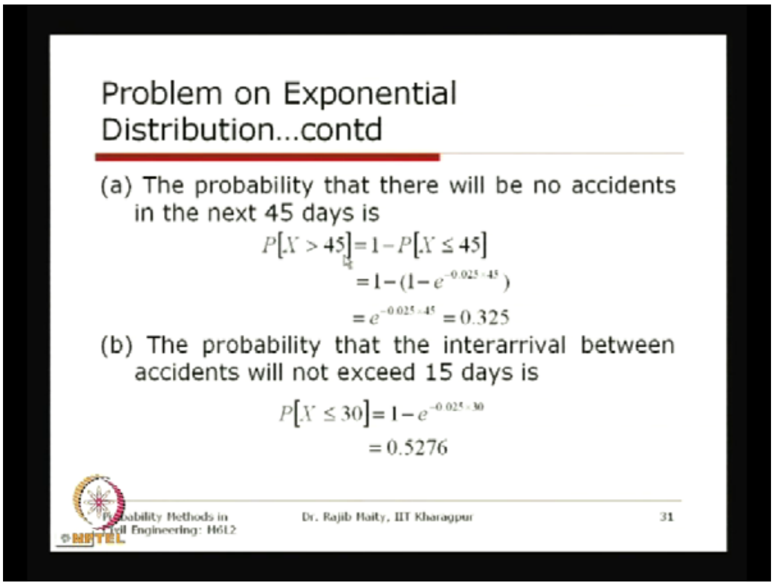

Now, once we get two information, any answer what we are looking for is that, first one is that there is no accident within next 45 days; so that means, x is greater than 45. So, 1 minus x less than 45, we can say this is a cumulative distribution, 1 minus e 1 minus lambda x, x is 45 here, which we can get that probability that there will be no accident in next 45 days is 0.325. The probability that the interarrival time, that interarrival between the accidents will not exceed 30 days. No I think this is written wrong, so the question that we are looking for is that, there will be another accident within one month. That was the second question here.

What is the probability that another accident will take place within one month; that means, 30 days. Obviously, we are considering here 30 days, one month. So, if if that is a, please ignore this whatever is written here. So, we are looking for the answer, what is the probability that one accident, another accident will occur within one month? So that means, what we are looking for is that x less than equals to 30. So, x less than equals to 30, directly we will get from the cumulative distribution, which is 1 minus e power minus 0.025 multiplied by 30, the probability is 0.5276.

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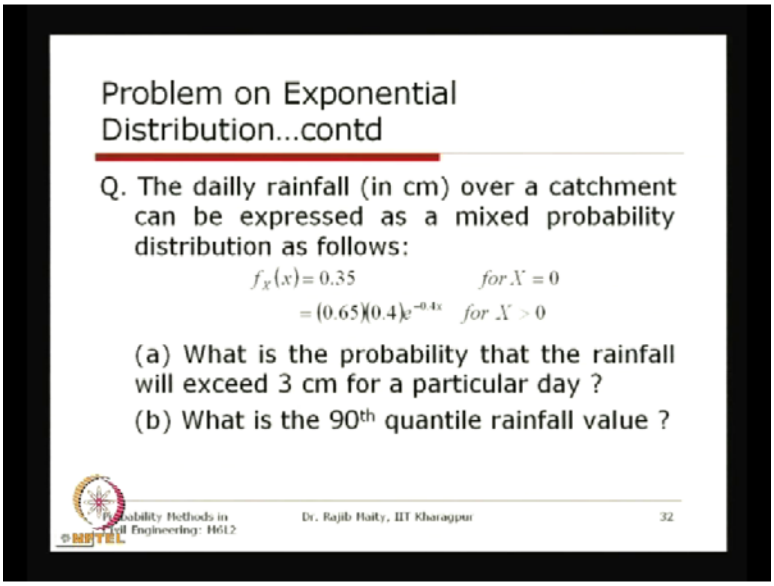

Well, so in this exponential distribution, sometimes as we are as we are mentioning that it could be a mixed type distribution. So, mixed type distribution means that at certain point for certain value, there could be some probabilities concentrated; one example is taken on this daily rainfall in centimeter over a catchment can be expressed as a mixed probability distribution as as follows.

So, this f x (x) equals to 0.35 for x equals to 0 and 0.65 multiplied by 0.4 e power minus 0.4 x for x greater than 0. Let me take some time to express express this one. How this distribution comes here? For this x equals to 0; that means, there is no rainfall on a day. There are 35. The the probability is 0.35 that whether there will be rain or not. If there is raining, then that rain is following following an exponential distribution for which the parameter lambda is 0.4. Now, we know that this lambda e power minus lambda x, the integration from 0 to infinity, which is a support of the exponential distribution will be equal to 1 will be equal to 1.

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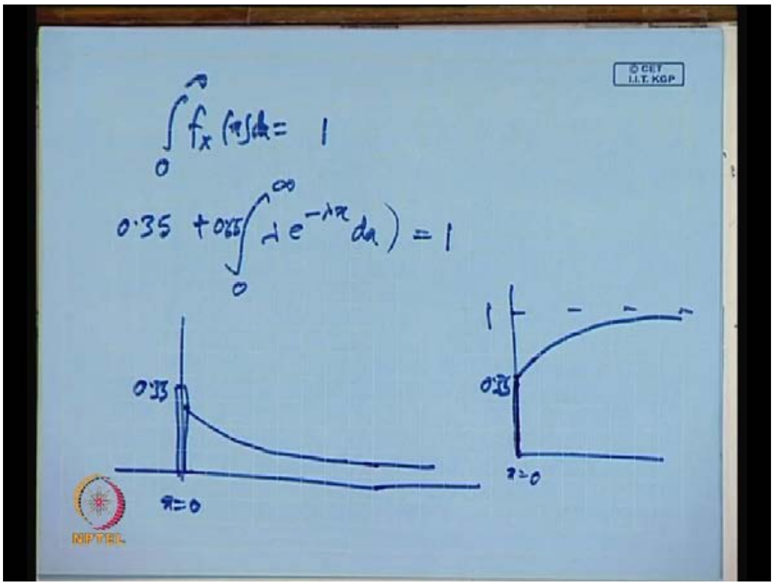

Now, as this is one mixed distributions. So, the total probability of this x starting from 0 to 1 is equals to 1.So that means, what we are trying to say is that 0 to infinity f x of x is equals to your sorry d x is equals to your 1. Now at x equals to 0, we know that already that 0.35 probability is concentrated plus in integration of that exponential distribution, whatever may be the value of this lambda, e for minus lambda x d x should be equals to 1. Now we know that, this one itself is equals to 1. So, if we multiply this one with 0.655; obviously, this total amount will become 0.65. 0.65 plus this one will be equals to 1.

So, if I just draw, the distribution that is at x equals to 0, there is a probability concentrated here which is equals to 0.35. Then that for this x greater than equal to 0, this is a, this is following one exponential distribution. The cumulative probability also will look like that it will first, it will rise to this 0.35 and then it will gradually accentuate to the value 1; it will start from here itself.

So, this is how this mixed distribution is referred here that is 0.65 multiplied by this lambda e for lambda x, the lambda is 0.4. What is the probability that the rainfall will exceed 3 centimeter for a particular day and what is the 19th quantile rainfall value?

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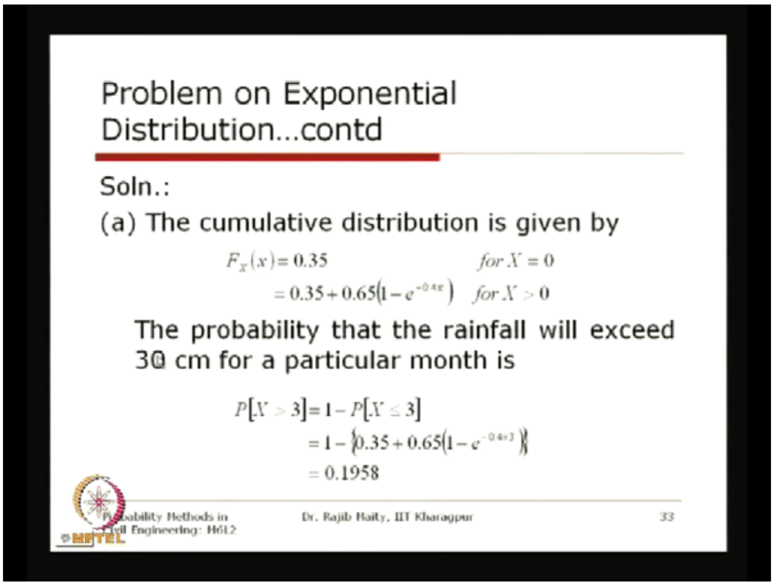

So, the cumulative distribution is f x equals to 0.35 for x equals to 0. You can refer to this diagram that at x equals to 0, that f x equals to 0.35 and it is going to the 1. So, this is the cumulative distribution and for the will exceed 3 centimeter, not 30. It is, yes it is 3 centimeter that is x greater than 3 is equals to 1 minus probability x less than 3. Just we put in this f x and we will get the value 0.1958.

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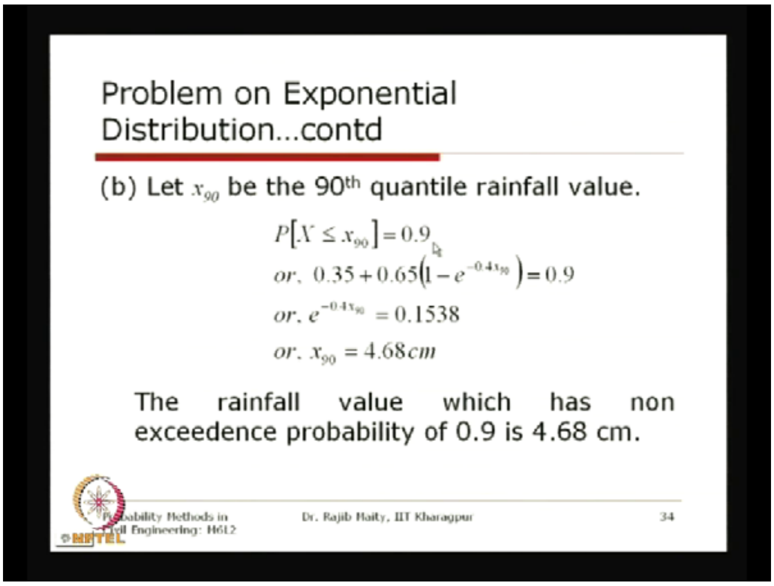

The 90th quantile value is this x 90 equals to 0.9, which is your 0.35 plus 0.65 multiplied by this one; and if we solve this one, we will get that x 90 is equals to 4.68 centimeter. So, the rainfall value for which the non exceedence probability is 0.9, that is the 90th quantile is 4.68 centimeter in a in a day.

So, today's lecture we have seen that some of these examples of the log normal distribution and the exponential distribution; we have discussed one problem on this mixed distribution as as well. And we have also taken some some more problem on the normal distribution what was discussed in the last lecture of this module also. In the next lecture, we will be taking some more continuous distribution and their… That application in different problems in civil engineering. Thank you.