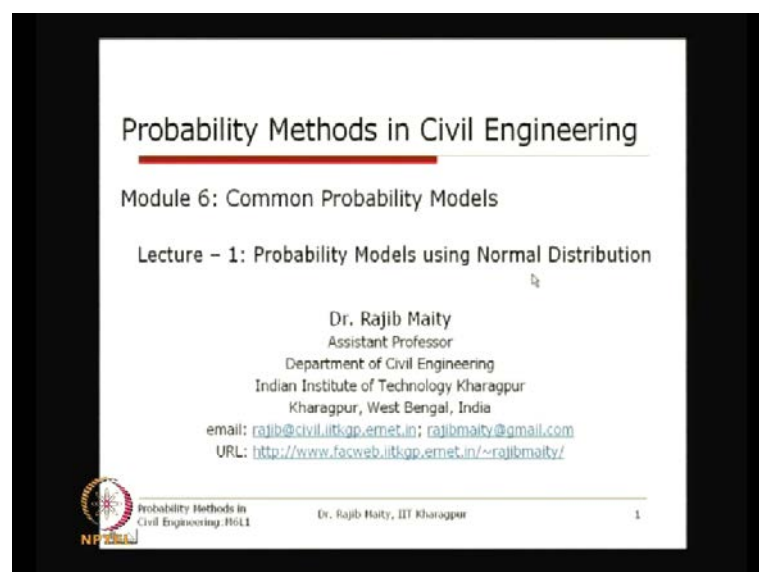


Probability Methods in Civil Engineering
Prof. Dr. Rajib Maity
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 31
Probability Models using Normal Distribution

Welcome to this course on probability methods in civil engineering. Today, we are starting a new module and this module is on common probability models and in this module, we will take whatever the standard distribution that we discuss in module 4. We will take those models, those distributions and we will show some practical application in the different fields of civil engineering. So, today that this lecture mainly we will be talking about the probability models using normal distribution and this distribution when we are taking the normal distribution, you know that its different, it is different properties and so far as this is p d f and c d f is concerned, we have discussed in earlier module and in this lecture also, we will just briefly see those things and mainly our focus for today's lecture will be show some practical civil engineering related problem, in which this properties of this normal distribution has been used. That we will see.

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Probability Methods in Civil Engineering

Module 6: Common Probability Models

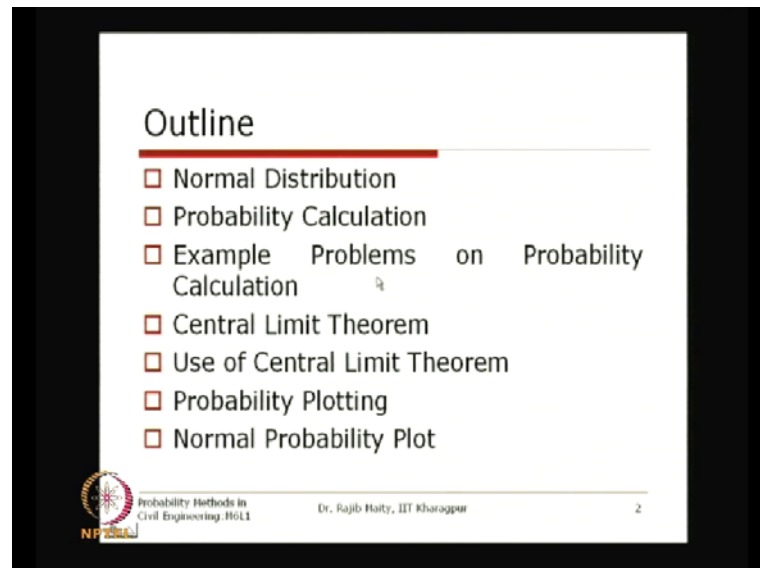
Lecture – 1: Probability Models using Normal Distribution

Dr. Rajib Maity
Assistant Professor
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Kharagpur, West Bengal, India
email: rajib@civil.iitkgp.emet.in; rajibmaity@gmail.com
URL: <http://www.facweb.iitkgp.emet.in/~rajibmaity/>

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Basically in this module, we will take up both the discrete random variable as well as continuous random variable, both and their standard distribution, that we discussed earlier. So, we are starting with the normal distribution, because this is so far as the concept of that continuous random variable, will be easier for to start with this normal distribution.

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So, our the outline for today's lecture will be, we will first briefly discuss about this normal distribution and how we calculate the probability because you can recall from our earlier lecture, that for this normal distribution, the calculation of probability or the integration, the integration form is not available in the closed forms. So, we have to go for some numerical integration and there are some standard tables that we introduced. So, those tables have to be used. So, basically after that, we will discuss about some of the problems related to the civil engineering, gradually we will also see the other, which are some of the very important theorem in the context of this normal distribution for example, the central limit theorem. That theorem we will discuss, may be not in this lecture, but the subsequent lectures of this module.

And after that, we will take up this probability plotting, basically when we see that probability plotting, that in that probability plotting, what we do? We generally test. This is not specific to the normal distribution, this can be for any distribution and probability plotting means, we generally plot the probability on a special paper just to investigate, whether the data that we are handling with, whether that is following a particular distribution or not. Basically, in this in this lecture, while discussing those problems, we

generally assume that that data set is following a normal distribution. So, that means, the problems are started with the assumption of this normal distribution.

So, the question that remain unanswered is that how do you test that whether really the data that we are talking about is following a particular distribution, in this case, the normal distributions. So, that is generally tested through different statistical test and those test, we will take up in this next module and in this one, what we will do, we will show some the concept of this probability plotting and so, through which, we can test that from which distribution the data set is drawn. So, here in the context of this normal distribution, we will we will focus on this normal probability plot, but this will be taken in the subsequent lectures. Today, we will specifically concern about that normal distribution at their some of the specific application in the different field of civil engineering.

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Normal Distribution


□ Normal Distribution is a continuous probability distribution function with two parameters – mean (generally denoted by μ) and variance (generally denoted by σ^2).

The probability density function is given by

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty$$

The cumulative distribution function is given by

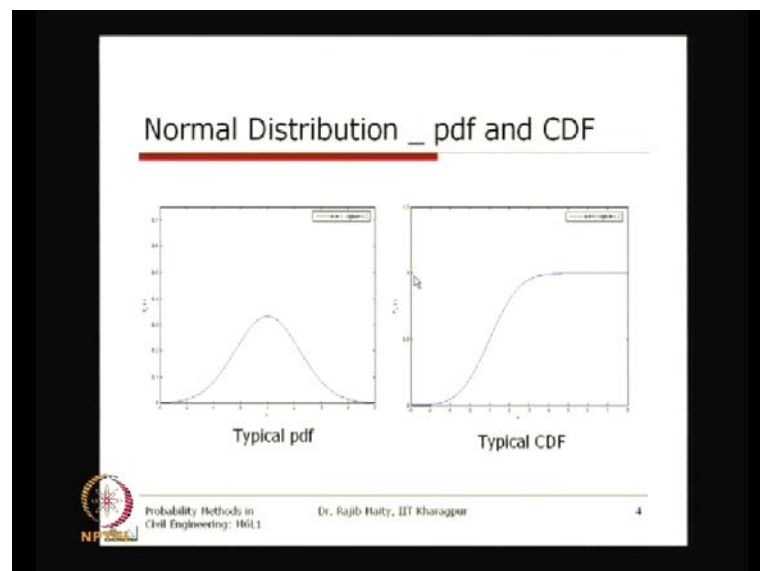
$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$$

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So, as you know from the earlier lecture, that this normal distribution is having two parameters, one you know, that is the mu and another parameter is called the variances. So, using those two parameters, this is a bell set curve and the p d f of that normal distribution is from this one, where these two parameters are there mu and sigma square. So, and this is a continuous probability distribution you know that and this continuous probability, density function with two parameters, one is that mean, which is denoted by mu and other one is variance which is denoted by this sigma square.

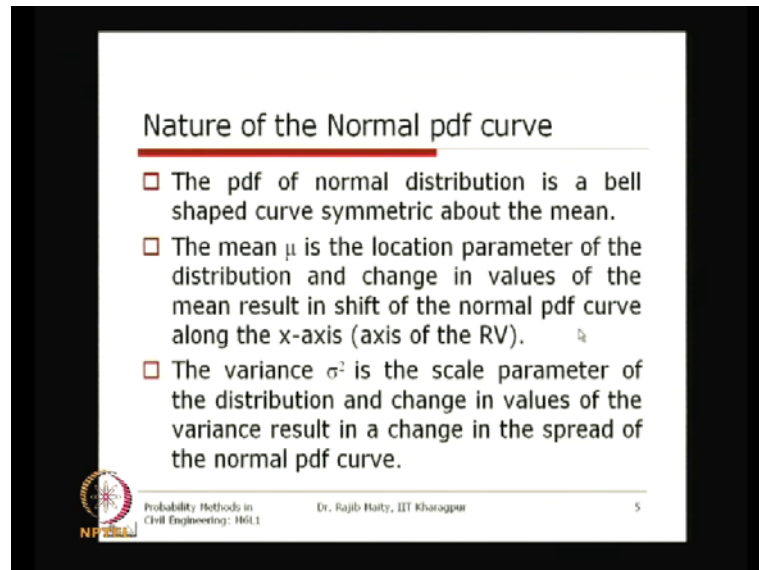
And now, this probability density function is given by in this form, that is $f(x)$ equals to $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. And this is a support of this distribution which is from minus infinity to plus infinity. And, if we want to know that that cumulative distribution function, which is $F(x)$ at any specific for a specific value, which is small x , you know that this is a integration from this minus infinity to that specific point and this integral from. And this is the integration which, for which that any close form solution or the analytical solution is not available and we have also discussed that, there are some different, that the numerical integration is done and that is presented in terms of some tables, that table also we will see in a minute.

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Before that, these are you know, that this is a typical p d f of a normal distribution, how the p d f looks like, this is a this is bell shape curve and it is symmetrical with respect to its mean and its typical c d f that is distribution, that cumulative distribution function, which is starts from 0 at minus infinity and go gradually increases and it is asymptotic to 1 at plus infinity.

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Nature of the Normal pdf curve

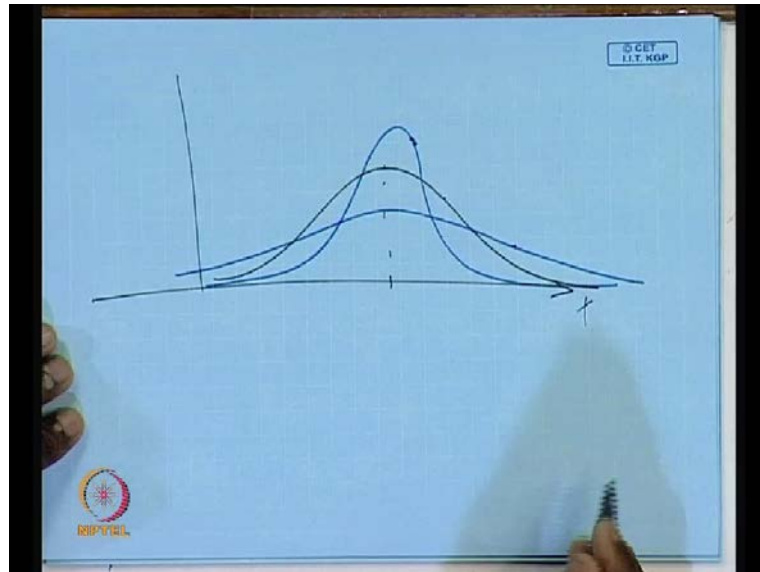
- The pdf of normal distribution is a bell shaped curve symmetric about the mean.
- The mean μ is the location parameter of the distribution and change in values of the mean result in shift of the normal pdf curve along the x-axis (axis of the RV).
- The variance σ^2 is the scale parameter of the distribution and change in values of the variance result in a change in the spread of the normal pdf curve.

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So, now the p d f of the normal distribution is a bell shaped curve, that you have seen, that it is symmetric about its mean and the mean μ is the location parameter of the distribution and the change in the values of the mean results in the shift of this normal p d f curve along the x axis. So, that what happens is that? So, here you can see that may be the mean is around. So, mean is at 1. So, if we just change it. So, at this mean, this is the maximum, you can say that the value of this probability density function is maximum at this means, so if we do not change its sigma. So, this over all shape of this curve will not change, only that that depending on what the mean is this curve will be, will be relocated or this will be shifted.

So, that is why, this μ is known as your, that location parameter and whereas, that this variance, that is sigma square, this sigma square is the scale parameter, why this scale parameter, because it is generally controls that how what is the shape of that distribution and if we change this parameter, that is if the variance is changed, then the that it will result in the change in the spread of the normal p d f curve. So, the change in the spread of this normal p d f curve means. It will be, so, if the mean is same. So, its maximum value of this p d f will be same at this location only, but its spread will be changed. So, you have seen it earlier. So, the curve will look like this.

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So, if this is your x axis, then if the 1 normal distribution looks like this and if this its mean and if the scale parameter changes, then the then depending on whether it is increasing or decreasing, this will be either, it will be flatter or it may be even the even be more, the spread will be much lesser. So, this is for the minimum, among these 3 curves, this is for the minimum variance and this is for the medium and this is the maximum variance is here, but all, for all these the mean is same, the location parameter is same. So, this is what is explained here, that variance sigma square is the scale parameter of this distribution and change in values of the variance, will result in a change in the spread of the normal p d f curve.


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Probability Calculation for a RV with Normal Distribution

- If a RV X follows normal distribution with parameters μ and σ^2 , then the probability of X being less than or equal to a particular value x is given by

$$P(X \leq x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

- The equation for normal probability density cannot be integrated analytically in closed form, thus the values of probability is to be computed by numerical integration.

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Now, as we have shown earlier also, that for this random variable x , which follows a normal distribution with parameter mean is μ and the other one is σ^2 , then the probability of x being less than or equal to a particular value x is given by the probability x less than equals to that particular value, which is the integration from this left extreme, that is minus infinity to x and from 1 by this form, that $p d f$ form, $\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$. So, this integration, if we do it for any specific value, putting any specific value x , then we will get its $c d f$, that is cumulative distribution function.

Now, this equation, for this normal probability density, cannot be integrated analytically as I told in the in the close form. Thus the value of the probability is to be computed by numerical integration. So, once this μ , these parameters are known, that is the μ and this σ is known, then using this one for a specific value of this x , if we do this integration numerically, then we will get what is the probability from minus infinity to that particular, to that specific value.

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Probability Calculation for a RV with Normal Distribution...contd.

- The probability of a RV X being less than or equal to a particular value x is equal to the area under the normal pdf curve bounded by $X=x$.
- The probability is also equal to the ordinate corresponding to $X=x$ in the normal CDF curve.

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So, every time the numerical integration instead of doing that one, there are some standard tables are also available, that we will discuss in a minute. So, we can make use of that table, but that table is generally for the standard normal distribution and we can use that standard normal distribution for the calculation of this probability and that we will discuss, while we are applying to a different problems.

Problems on Probability Calculation...contd.


From the standard normal table, the value of Z corresponding to 0.95 is 1.645. So, the value of Z corresponding to 0.05 is -1.645.

So,

$$\left(\frac{x_{95} - 24.9676}{4.1594} \right) = -1.645$$

or, $x_{95} = 18.125 \text{ N/mm}^2$

As x_{95} is less than 20 N/mm^2 , it is not acceptable as M20 concrete.

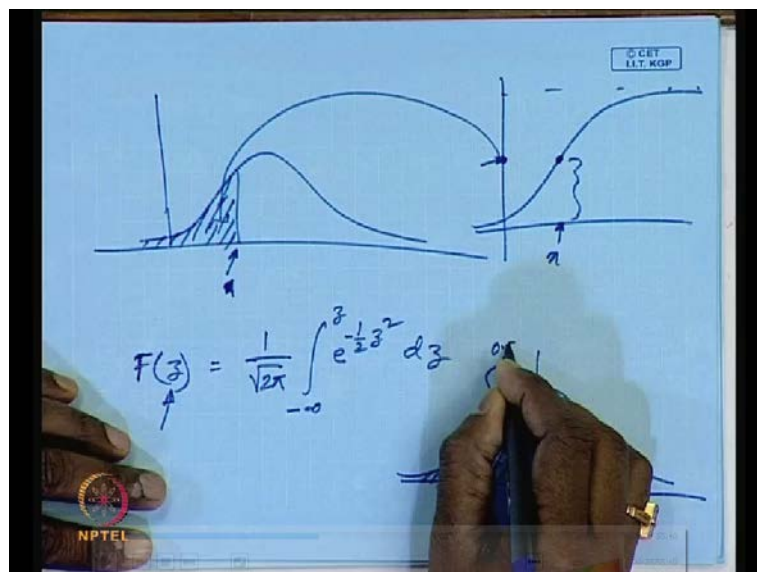
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So, the probability of a random variable, that is x being less than or equal to a specific value x is equal to the area under the normal p d f curve bounded by x equals to that specific value x and the probabilities also equal to the ordinate of the corresponding to the x equals to this x in the normal c d f curve. So, what is mean is that. So, this is the

total area, that we are talking about from this integration, that what we are getting, if we just refer to the, those curves that is so, this is.

So, up to any specific point, whether it is from any specific value if we come, so, it is the total area up to that point below this curve and so, and that for that specific point, whatever the area is obtained, that is the probability, which is indicated here by its ordinate value. So, what we are referring here is that is that this area that is for say this is your p d f. So, any specific value that we are talking about that x, so, up to this, whatever the total area is covered here, so, this area for this value x, will be reflected in the c d f. So, if this x corresponds to here, this x, now the c d f curve, that we are, that we have obtained is so, this ordinate, so, this value, this value here, this height or this value is nothing, but equal to this area.

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So, this is what how you know that this p d f and c d f is related to, so, this integration when we are talking about, this integration when are talking about, this integration is basically referred to the area and that the curves. So, far as p d f is concerned and the ordinate value in case of the c d f. So, now we will see that that a standard normal distribution. So, you can you can now understand, that even though this integration, it can be done numerically. Now, depending on the choice of this mu and sigma so, that curve will change.

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Probability Calculation using Standard Normal table

- The standard normal distribution has mean $\mu=0$ and variance $\sigma^2=1$.
- The Standard Normal table lists the cumulative probability values of the reduced variate $Z = \frac{X - \mu}{\sigma}$
- The table may be used conveniently for calculating the probabilities associated with any normally distributed RV after normalizing it with appropriate parameters.

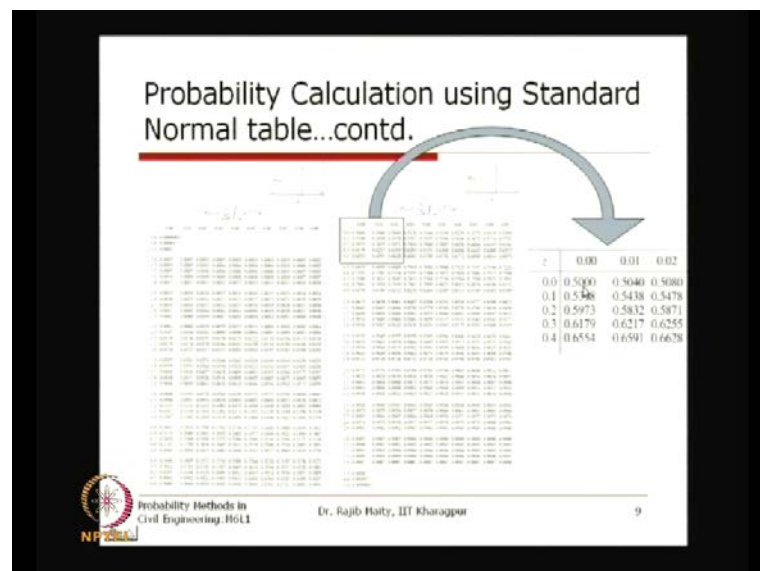
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So, once. So, there can be some many means infinite numbers of the combinations so, where for a specific value of mu and the specific value of sigma square, so depending on this mean and sigma square, the curve will change. So, every time you are supposed to do that numerical integration, if you want to do that integration for the specific normal distribution. So, instead of that, if we use the concept of this standard normal distribution, then for the standard normal distribution, thus mean is 0 and the variance is 1, So, for that specific curve, if we can do this integration and if the value is available, then for any such distribution, we can use the same table to know what is that integral value or what is the probability, corresponding probability. So, this is what is explained here, this standard normal distribution as you know, has the mean, mu equals to 0 and variance sigma square is equals to 1. So, the standard normal table, lists the cumulative probability values of the reduced variate z equals to x minus mu by sigma.

So, this is important that when, so, if this x is following a normal distribution, which is having the parameter mean mu and variance sigma square, now you we have also seen in this last module also in the function of random variable, you can refer to that. This is a 1 to 1 transformation and this is a linear transformation as well. So, if we do this type of transformation, then this is also another random variable, which is denoted here z and is generally known as their reduced variate, now this z will have, will also follow a normal distribution, which will have your that mean is equals to 0 and the variance is equals to 1.

So, once we from any random variable, if it follows the normal distribution, now if we do this type of transformation, that is the transformation is that x minus μ by σ , then the resulting random variable will have a, will also have the normal distribution, but its parameter will be mean 0 and variance 1. Now, the table may be used conveniently for calculating the probabilities associated with any normally distributed random variable after normalizing it with the appropriate parameters. Now, with the appropriate parameters, what is mean is that. So, with the mean of the original random variable, that is x and divided it by the standard deviation of the original random variable x . So, before you discuss further on this we will just see for the standard normal distribution how the integral values or the standard statistical table is available to us.

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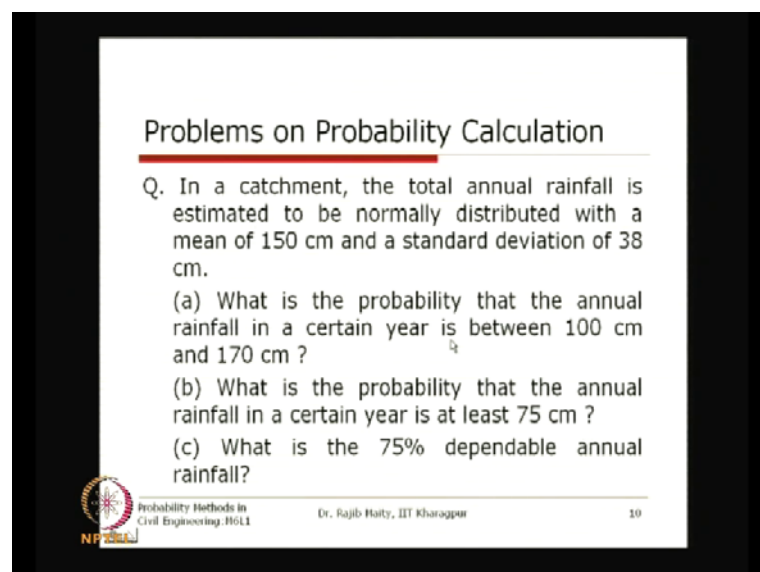
This, we have we have discussed that if so, this is the bell shaped curve, this is the p d f of this normal distribution, basically this is for the standard normal distribution and this point is indicating that this is 0. So, which is the mean of this of this distribution and this, for this standard normal distribution you know, how this forms look like. So, this is the cumulative form that is shown here, which you can see which you know that that is your $F(z)$, that is a reduced variate z and this is capital F ; that means, we are referring to that the cumulative distribution function for this standard normal distribution, which is equals to $1/\sqrt{2\pi}$, we are supposed to multiply it by σ , but σ you know that this is 1. So, now, this is integration, integration from the left extreme, which is minus infinity up to the specific value of that z and the integral form will be that e power half, I

can write half, then that this is that z minus 0 divided by sigma whole square. So, sigma is 1 mu equals to 0. So, it will be only z square d z .

So, this is the form, what is the value is shown in this table. Now, for a specific value of z this value refers to this area, if this parameter is mu equals to 0 and standard deviation equals to 1. So, here that probabilities have shown here, you can see that this z values are starting from this minus 1. So, this is zoomed here for. So, this is actually the entire table, which is actually starting from minus 5 to plus 5. So, here some portion is zoomed for the easy visibility. So, here you can see that, when the z is equals to 0. So, this column will just read that 1, that is z equals to 0 and this row is for the second decimal. So, this point corresponding to, so 0.5832 that you can see here, this is basically corresponds to z equals to 0.2, 1. So, this is the second decimal point. So, at z equals to 0.2, 1 the value is 0.5832.

So, this is the value is given. Now, you know from the symmetry, if this is the standard normal distribution. So, this is your 0 and as this is symmetric, you know that due to the symmetry, So, what should be this area up to this 0 means, the total area under this curve is 1. So, up to this 0, it would be 0.5, this is what is also shown in this curve, that is when z equals to 0, the integral value is 0.5.


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Problems on Probability Calculation

Q. In a catchment, the total annual rainfall is estimated to be normally distributed with a mean of 150 cm and a standard deviation of 38 cm.

- (a) What is the probability that the annual rainfall in a certain year is between 100 cm and 170 cm ?
- (b) What is the probability that the annual rainfall in a certain year is at least 75 cm ?
- (c) What is the 75% dependable annual rainfall?

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So, we will now see a specific problem on this, how we can utilize this property, is of this normal distribution for some specific problem in civil engineering. So, the first

problem, in a catchment, the total annual rainfall is estimated to be normally distributed with a mean 150 centimeter and a standard deviation of 38 centimeter now. Here lies the question, that we just started with this class, that whenever we are starting this in this lecture, whenever the problem we are starting with, we are stating that a particular random variable is following a normal distribution. So, once we say the normal distribution, that means, that parameters are also we have to know.

So, here the mean and standard deviation we know, that for the normal distribution there are two parameters. So, those two parameters are also known, but the, what is unanswered is that how to test that, that here it is referred to that annual rainfall, how to know that weather that is really a normal distribution or not. So, that question will be taken in the subsequent lecture, how to test that its normality that we will see. But, here we are starting the problem with the assumption that this follows a normal distribution with some parameter value.

So, in a catchment the total annual rainfall is estimated to be normally distributed with a mean of 150 centimeter and a standard deviation of 38 centimeter. So, what is the probability that the annual rainfall in a certain year is between 100 centimeter and 170 centimeter. Second is what is the probability that the annual rainfall in a certain year is at least 75 centimeter and the third is, what is the 75 percent dependable annual rainfall? So, there are 3 questions is put and the 3 specific goal is kept here. The first question reads that, what is the probability that, it will be between 100 centimeter to 170 centimeter. So, may be for the policy making in some cases, I need to know, that what is the probability that it will be a normal year, normal year means, that the rainfall is near normal and what is that that probability. So, if I say that, that probability is very high, then we will be more confident to state. So, that particular year, the probability of this much rainfall, that we get is having high probability and the second type of question, what is thrown is that what is the probability that the annual rainfall in a certain year be at least 75 centimeter.

This kind of issues comes, when we are interested about that minimum requirement of the catchment. So, if this is. So, some stake holders for the water resource, there are a different stake holders for example, the for the reservoir operation, we are supposed to give the water for that multipurpose, one may be the power generation or for the irrigation or for the industrial use. So, there are different users are there, whatever the

water is available, now the question is that to sustain the whatever the need to the community, sometimes we need to ask the question, what is the probability that at least this much rainfall or at least this much water is available to me. So, this is of that type of question that at least 75 centimeters we can get from the thing.

And the last one is the what is the 75 percent dependable annual rainfall? That means, that I want to be confident, I want to be confident that the with the 75 percent confidence level, that I need to know that, yes, this much rainfall is certain at some probability level. So, at 0.75 probability level, what is the value of this rainfall that I can expect. So, all these answers will or based on the assumption that, at that point the rainfall is following certain probability distributions. So, we are first developing, that probability e thing, that probability model and from that whatever the parameter that we are estimated, from that we have to answer these two question in terms of a probabilistic manner.

So, we will take these problems now one after another. The first one is that 100 centimeter to 17 centimeter, now again if you go back, what is the information that is supplied to us, the first is that it is, mean is that 150 centimeter and this variance, the standard deviation is given 38 centimeter. So, you know that for this one, we do not have any statistical table if of this of any normal distribution which is having mean 150 and this standard deviation is 38. What we are having is that? We are having the table for this 0 mean and standard deviation equals to 1, means variance equals to 1 means, that standard deviation is also 1.

So, first of all, as we are talking that, we have to do the transformation, we have to generate, we have to obtain this reduce variate first, for which we are interested to know the probability and after that we can make use of that table to give this type of answer. So, first question we are taking, what is the probability that the annual rainfall in a certain year is between 100 centimeter and 170 centimeter.


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Problems on Probability Calculation...contd.

Soln.:

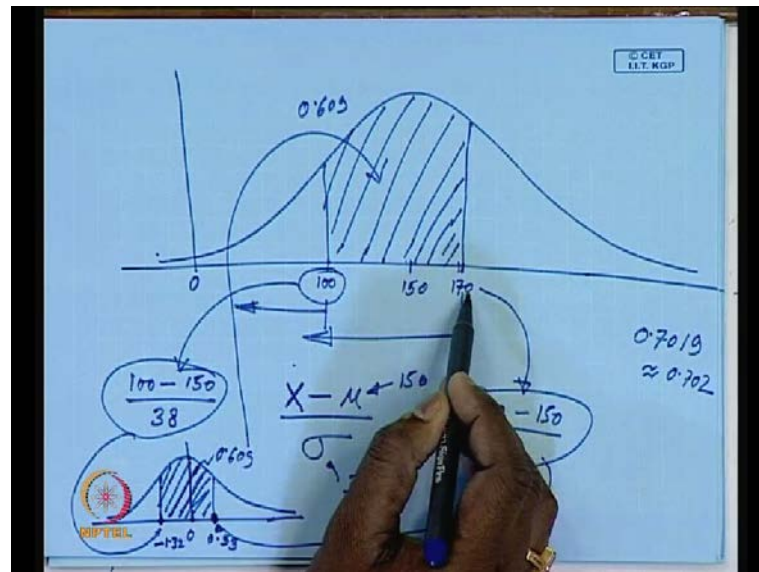
(a) The probability that the annual rainfall in a certain year is between 100 cm and 170 cm is given by

$$\begin{aligned} P(100 < X \leq 170) &= P(X \leq 170) - P(X \leq 100) \\ &= P\left(Z \leq \frac{170-150}{38}\right) - P\left(Z \leq \frac{100-150}{38}\right) \\ &= P(Z \leq 0.53) - P(Z \leq -1.32) \\ &= P(Z \leq 0.53) - [1 - P(Z \leq 1.32)] \\ &= 0.702 - (1 - 0.907) \\ &= 0.609 \end{aligned}$$

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So, to find out this probability, what basically we are looking for, is the probability of this random variable x . So, that annual rainfall, that we are getting in that catchment is, we are the random variable is denoted by this x . So, this x will be between 100 to 170. So, if we want to know this one. Then what we can say is that, so, if I just want to know that this is that distribution form, so, obviously, if this is the origin I can say that, this is 0. So, the mean of this normal distribution, which is 0, which is away from this origin, away from this 0. So, mean is shown to be that 150. So, this is your 150. So, what we are interested to know that this random variable between 100 and 170. So, that this is your 170 and this is your 100.

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So, what we are interested to know is this area. So, what we can, how to get this area, what we cannot from the c d f point of view, if you want to know, that what is this area, that is what we can get first, we can get the total area up to this 170, which will be include this area as well minus this area, this up to 100. So, that difference will give you this area. This is exactly what is written here also, that is probability x less than 1 less than equals to 170 minus probability x less than 100.

Now, from here, so, for this type of normal distribution, that numerical integrate, numerically integrated values are not available to us, we are having the standard normal distribution only. So, for that what we have to do, we have to first convert it to this standard normal distribution and we have, we know that what should be the conversion to declare that the new random variable will be having that 0 mean and standard deviation 1. So, that conversion is nothing, but the x that specific value, that is 170 minus mean and mean is 150 here, divided by the standard deviation.

So, that is what, we are just mentioning that x minus μ divided by σ . Now, x can be of anything whatever is our interest and here μ , we have defined to be 150 and σ is your 38. Now, for this 170, the reduced variate will be, 170 minus 150 divided by 38. Similarly, for the 100, this one the reduce variate will be 100 minus 150 divided by 38. Basically, when we are getting these values, so, this values what basically what we are doing it, we taking this full graph, we are basically converting it to another normal distribution, for which the mean is at 0, standard deviation is 1 and we are here

instead of referring to 170, this 170 has converted to this value and this value is somewhere here, which is showing to be this point.

And similarly, the 100 is converted to this one and this value is somewhere here, which is a here. So, now, we are interested to know that this area, so, this area is equals to this area to that original random variable scale. So, first of all, we have converted to like this and because these values are available to us, then we will be referring to this, what the standard distribution is available to us, then to get this answer that what is this area. So, here is done this one, that is that we are reducing it from 175 minus 150 divided by 38 minus probability of z 100 minus 150 divided by 38. So, this corresponds to 0.53 and this corresponds to minus 1.32.

Now, these values can also be directly obtained from the table, that we have shown just now. So, first of all we will take this one, we will just discuss more on this minus 1.32. Basically, we are using the symmetrical property. So, if the value is 0.53, then we have to we have to find out what is this value is corresponding to.

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Problems on Probability Calculation...contd.

z	0.00	0.01	0.02	0.03
0.0	0.5000	0.5040	0.5080	0.5120
0.1	0.5198	0.5248	0.5288	0.5328
0.2	0.5398	0.5448	0.5488	0.5528
0.3	0.5598	0.5648	0.5688	0.5728
0.4	0.5798	0.5848	0.5888	0.5928
0.5	0.5998	0.6048	0.6088	0.6128
0.6	0.6198	0.6248	0.6288	0.6328
0.7	0.6398	0.6448	0.6488	0.6528
0.8	0.6598	0.6648	0.6688	0.6728
0.9	0.6798	0.6848	0.6888	0.6928
1.0	0.6998	0.7048	0.7088	0.7128
1.1	0.7198	0.7248	0.7288	0.7328
1.2	0.7398	0.7448	0.7488	0.7528
1.3	0.7598	0.7648	0.7688	0.7728
1.4	0.7798	0.7848	0.7888	0.7928
1.5	0.7998	0.8048	0.8088	0.8128
1.6	0.8198	0.8248	0.8288	0.8328
1.7	0.8398	0.8448	0.8488	0.8528
1.8	0.8598	0.8648	0.8688	0.8728
1.9	0.8798	0.8848	0.8888	0.8928
2.0	0.8998	0.9048	0.9088	0.9128
2.1	0.9198	0.9248	0.9288	0.9328
2.2	0.9398	0.9448	0.9488	0.9528
2.3	0.9598	0.9648	0.9688	0.9728
2.4	0.9798	0.9848	0.9888	0.9928
2.5	0.9998	1.0000	1.0000	1.0000

$P(Z \leq 0.53) = 0.7019$

$P(Z \leq 1.32) = 0.9066$

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Now, you see here. So, this is, so, 0.53, as we have telling that this column is the first decimal of the reduce variate z and this row is the second one. So, we have to go to this 0.5 and here we have to go to that 003. So, this is your 0.53, the value for this 0.53, which is 0.7019. So, this is what we have written that probability z less than equals to 0.53 is equals to 0.702.

Now, this 1 that z less than equals to minus 1.32 can also be obtained from the table and can be directly used for, what is this value. Sometimes, what happens, in some text book you will find that, using the symmetrical nature of this normal distribution table only one side and preferably the positive side values are shown. So, what happens? In those cases is done, that from this 0 it starts and go up to infinity means, here only 4 or 5 and those values are given there.

So, once we know this value, that area then using this symmetrical property, it will be easier to know that what is the area that we are looking for. For example, this point, for this problem what we have seen is referring to minus 1.32. Now, we are interested to know this area, now what is so, we are interested to this area. So, what we are doing is that. So, up to this point, that is 0.53, we have, we came to know this from minus infinity to this 0.53 that total area that we have seen is 0.7019 which is approximated to 0.702.

Now, what which area we have to, we have to deduct is this white area, that is from minus infinity to point to 1.32. So, this much area we have to deduct. So, if this is the negative side is not available to you, then using simply that positive side also, we can do that analysis. So, how to do that one, so, we have to just replace this one. That is 1.32 to this positive side, this area we have to know and this remaining this, whatever the white area is there, which is equals to this side area. So, this is what is shown here that is probability of z less than equals to minus 1.32 is equals to 1 minus probability of z less than equals to 1.32. That is now we are using only the positive side of that value.

So, now that 1.32 value is also known from this table, that is this is z is 1.3 and second decimal is 2, which is 0.9066. So, that 0.9066 is a 0.907. So, this we calculate. So, we are getting the total area is 0.609. So, what is this total area, that we are getting here from this reduced variate is 0.609, which is equal to this area as well. So, this is 0.609. So, we can say that probability of the random variable x being between 100 to 170 is equals to 0.609. This is what is what is obtained here and we have explained that how we got this corresponding values.

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Problems on Probability Calculation...contd.


(b) The probability that the annual rainfall in a certain year is at least 75 cm is given by

$$\begin{aligned}P(X \geq 75) &= 1 - P(X \leq 75) \\&= 1 - P\left(Z \leq \frac{75 - 150}{38}\right) \\&= 1 - P(Z \leq -1.97) \\&= 1 - [1 - P(Z \leq 1.97)] \\&= 0.976\end{aligned}$$

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Probability Calculation using Standard Normal table...contd.



z	0.00	0.01	0.02
0.0	0.5000	0.5040	0.5080
0.1	0.5398	0.5438	0.5479
0.2	0.5973	0.5832	0.5873
0.3	0.6179	0.6217	0.6255
0.4	0.6554	0.6591	0.6628

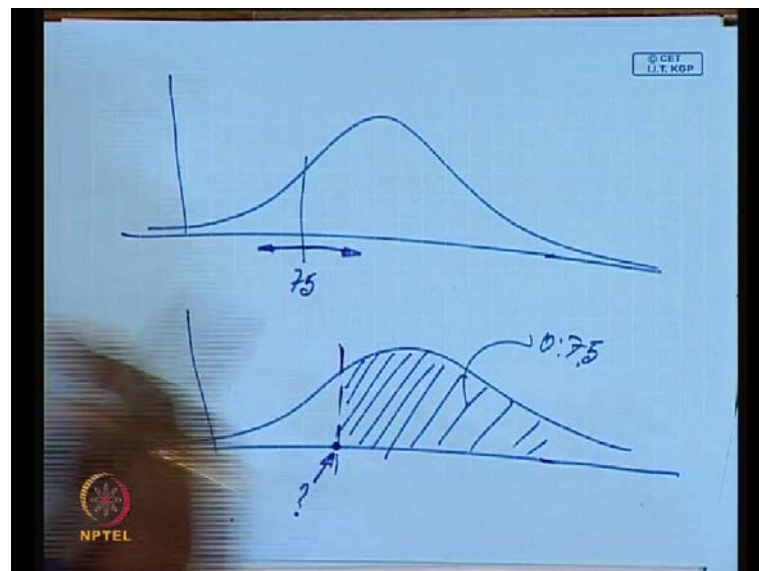
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So, the table that we shown here we should know here that the table that we shown here is continuing from this minus infinity to plus infinity, you can use, you can directly use this table also to know that what is the probability for this minus 1.32, that also you can use directly or if for some textbook only the positive side is given, using that the property of this symmetry, then we can just convert it to this positive number, with the consideration of the fact that the total area is 1. So, then you can use this, that particular table as well.

The second question is, what is the probability that the annual rainfall in the certain year is at least 75 centimeter? So, before I start that one, so, the concept here is, that if you understand that, how we are we are obtaining the probability for the normal distribution after reducing it to this standard normal distribution, then all of this type of problem will be straight forward, may be for the rest of the problem that we are discussing, we will just discuss their inside, but how we are referring to this table and all we will not be discussing in so details. So, I hope that you understood how we are getting the probability by using the standard normal table.

So, here the second question is, that the probability, that the annual rainfall in the certain year is at least 75 centimeters. So, what does it mean, at least 75 centimeters means, what we are looking for is the probability x is greater than equals to 75. Now, this probability x greater than equal to 75 can be, we are first converting it to the thing that 1 minus probability of x less than equals to 75. So, why we are doing this 1, you know that now the probability of x greater than equal to 75 plus probability of x less than equal to 75, basically includes everything, all these all the area under the normal curve.

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So, what we are talking here is that, if this normal distribution and if this is the mark for this 75, then the then the probability x greater than equals to 75 plus probability x less than equals to 75 is covering the entire area below this curve, that is which is 1. So, that is why the standard normal distribution you know that is available for the less than equal

to values. So, as it is available for this less than equal to values, then that is why we are interested to transfer it from this greater than equal to sign to this less than equals to sign.

Now, you know one thing is also important here, just to say you that, whether we will include the greater than equal to sign will be included in the both the side, this question may come to your mind, basically in the mathematical concept, what we should not, we cannot include both the equality sign in both the side, either this should be equal to or that should be equal to.

But, remember that, so far as the continuous random variable is concerned, this is of no harm, because for the continuous random variable, if we want to know the probability of it is specific value, which is equal to the specific value is 0, that we discuss at the beginning of this course. And this is what and this normal distribution you know, this is a continuous random distribution, continuous probability distribution, So, for this one, if I want to know, what is the probability that x equals to 75. That means, that probability x equals to 75 is 0. So, what we can, what we can think in this way, that probability x greater than equal to 75 is equals to 1 minus probability less than 75 plus 0 and that 0 is nothing, but probability x equals to 75. So, whether you are including or excluding the equality sign, it does not matter mathematically, so far as the distribution is continuous.

So, with this, now we will follow again the similar procedure, we will first we have to transfer it to get that reduce variate, which is your 75 minus the mean 150 divided by the standard deviation 38, which is equals to minus 1.97, this z less than equals to minus 1.97 can be directly obtained from the table or as I told, if only the positive side is available, then also you can further transfer it to that 1 minus probability of z less than equals to that plus 1.97. So, using this one, if you now again if you get that the value of this z for 1.97 and you do this calculation basically, this probability less than equals to this one will be equals to your 0.976.

So, the probability that the annual rainfall in a certain year is at least 75 centimeter is your 0.976. So, for the catchment, that we are talking about, there is a high probability, that 75 centimeter of rainfall. So, getting 75 centimeter of annual rainfall is highly probable that is what we can infer.

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Problems on Probability Calculation...contd.

(c) Let x_{75} be the 75% dependable annual rainfall.

So, $P(X \geq x_{75}) = 0.75$

or, $P\left(Z \geq \frac{x_{75} - 150}{38}\right) = 0.75$

or, $1 - P\left(Z \leq \frac{x_{75} - 150}{38}\right) = 0.75$

or, $P\left(Z \leq \frac{x_{75} - 150}{38}\right) = 1 - 0.75 = 0.25$

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And the third thing, that the last question that we have seen that let that 75 percent dependable annual rainfall. Now, the 75 percent dependable annual rainfall, what we mean is that, let that x_{75} , this is value is denoted that 75 percent dependable annual rainfall. As I told that, this dependable rainfall means, I am assured that this much rainfall can be obtained at this station with the probability level 0.75.

So, if that is the question then what I am looking from this one, so, if this is the distribution of this annual rainfall, then I want to know a specific value for which, it is located in at some point. So, this value I am looking for, what is this value, in such a way that this is 75 percent dependable. This is 75 percent dependable means, whatever that probability on the higher side of that value is equals to your 0.75.

Now, from the c d f point of view, if you want to know, that means, what is the value up to which the total area covered under the p d f is equals to 0.25. This is what is the concept and here now you can see that as we have denoted that x_{75} is that particular value, so that probability of x greater than equal to that value is equals to 0.75. So, probability of z , when we are doing that transformation, when we are getting that reduced variate; that means, that x_{75} minus, that mean that 150 divided by the standard deviation 38, that is the your reduced variate which is equals to 0.75.

The difference from this earlier 2 things, that we are discussed is that, there this probability we are looking for and this specific value was known here it is reversed; that

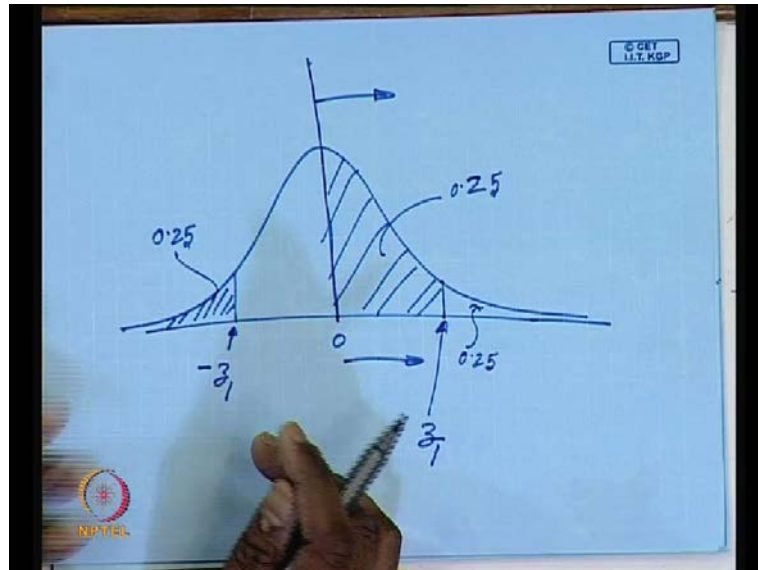
means, I know the probability, I want to know what is the specific value both the things can be done. Now, from that, so, if we just, so, this is the greater than equal to sign, if you just refer to this one. That means, this area I am just talking about the 0.75.

Now, from the c d f we always know that this is the less than equals to, that means, if we can convert into this side, then we can we can make use of that table. So, that is the interest. So, for that what we are doing is that, instead of using the greater than equal to sign, I am interested to know, what is the z less than equals to this quantity, this reduced to variate, in that, what is happening to make this one this change, that we have to subtract it from the total area below this curve. So, total area or the total probability is 1. So, 1 minus, so, this is replaced by 1 minus probability z less than equals to this value, which is equals to 0.75. Now, we are just doing this algebraic thing. So, this will be equals to 0.25. So, this probability should be equals to 0.25.

Now, how we will make use of the table. So, now, how we will make the table is that, we do not know what is z, what we know, we know what is the probability covered. So, from the table, we have to find out the probability equals to 0.25 and corresponding to that what is the z quintile that we can get. So that means, if we just go back to that original table, so, instead of looking for this z past, we will look for this, which is the, where is this probability is there. So, where is this probability there means, that for example, here the probability is 0.8186. So, if I am interested for this probability, then I should say that this is for the z value which is 0.91, similarly I have to look for the probability which is equals to 0.25.

So, this is; obviously, as this is the symmetrical you know that, from z is equals to 0.5 this is for the probability is equals to 0.5 the z value is 0, So, obviously, the z value will be negative in that standard normal distribution to get the probability is equals to 0.25, because the 0.25 is less than 0.5. Now, what we can do, if both the tables are available to us like this, then we will, so, from the negative side, we will just find out where from this 0.25 is there and from there we will get what is this z value. If it is not available, then what should I do, how can I use that only the positive side to know what is that quintile, let us explain it first graphically.

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That means, this is the standard normal distribution, this is your 0 and what is available to us only this side, only the positive side from 0 onwards is available to us and what we are interested to know is that, we are interested to know this value up to which the total area is equals to 0.25. So, how we will get this one is that, we will find out for the value where, so, if only the positive side is available, where this area is equals to 0.25. Why 0.5, because I know from this value, total area below this curve towards the positive side is 0.5. Now, if I can say that this is equals to 0.25, that means, this area this, white area is also 0.25. Now, suppose that I get this value equals to say z_1 , at z_1 I have seen that this from 0 to that z_1 value this probability is 0.25, that means, from z_1 to plus infinity is the area is also 0.25.

Now, using the symmetry of this normal curve, what we can say that, this value will be minus z_1 . The value will be same only it will be on the other side of this 0. That means, towards the negative side of this 0. So, to make this 1 equals to 0.25, this will be a minus z_1 . This can be done, in case from some textbook you get only the positive side values of the normal distribution.


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Problems on Probability Calculation...contd.

From the standard normal table, the value of the reduced variate Z corresponding to the non exceedence probability of 0.75 is 0.675. So, the value of Z corresponding to the non exceedence probability of 0.25 is -0.675.

$$\frac{x_{75} - 150}{38} = -0.675 \quad \therefore x_{75} = 124.35 \text{ cm}$$

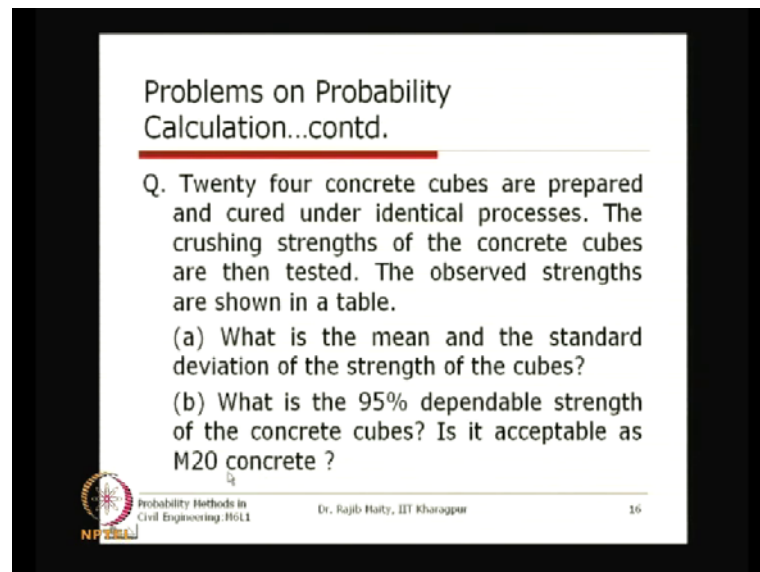
So, the 75% dependable annual rainfall is 124.35 cm.

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Now, here you can see that from this standard normal table, the value of the reduced variate z corresponding to the non exceedence probability of 0.75 is 0.675. So, the value of the z corresponding to the non exceedence probability of 0.25 is minus 0.675. So, the value that the z 1 here, that we are referring to, is your, if you see the table, you will see that it 0.675. So, that means, here that we are value, we are referring to it is minus 0.675.

Now, if this is the thing, what we are doing is that. So, this is your now the reduced variate z , what we have seen it earlier, that is $x_{75} - 150$ by 38, that this probability is equals to 0.25, that means, this value which is nothing, but the z is equals to minus 0.675. This is what is explained here. So, this is equals to minus 0.675 and if I doing this 1×75 is 124.35, that means, the 75 percent dependable annual rainfall for that catchment is 124.35 centimeter.

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Problems on Probability Calculation...contd.

Q. Twenty four concrete cubes are prepared and cured under identical processes. The crushing strengths of the concrete cubes are then tested. The observed strengths are shown in a table.

(a) What is the mean and the standard deviation of the strength of the cubes?

(b) What is the 95% dependable strength of the concrete cubes? Is it acceptable as M20 concrete ?

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We will go to the next problem, where also the similar the normal distribution is used and this problem is on the testing that strength of concrete and you know that the characteristics strength of the concrete is determined based on this 95 percent dependability. Now, what is done you know that there are some several concrete cubes of specific dimension 150 millimeter by 150 millimeter by 150 millimeter is casted and they are cured in some identical condition and for 28 days at the end of 28 days, we generally get the value of the crushing strength in the laboratory and different crushing strength are obtained.

Now, from there, we have to see that, what is the strength we have to draw, because obviously, if we go for the different concrete cube, many concrete cube for all the values. So, as many concrete cube, those many values we will get. So, we have to conclude something probabilistically. That type of problem is taken as the second problem here that 24 concrete cubes are prepared and cured under the identical process the crushing strengths of the concrete cube are then tested, the observed strength are shown in a table. The question is, what is the mean and standard deviation of the strength of the cube and what is the 95 percent dependable strength of the concrete cubes. Is it acceptable as M 20 grade concrete.

Now, you have to know that, this M 20 great concrete means, it is characteristics strength is 20 Newton per millimeter square. So, this 24 cubes are tested for their crushing strength and this values are shown here. So, what we are assuming that this follows a

normal distribution. First we are calculating their mean and their standard deviation. So, first the mean is that 24.96, you know that will summation divided by 24 is giving that 24.9676 Newton per millimeter square. And second one is that the standard deviation, the sample estimate you know, that $\frac{1}{n-1} \sum (x_i - \bar{x})^2$ from i equals to 1 to n , here n equals to 24.


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Problems on Probability Calculation...contd.

Crushing strength of concrete cubes

Obs No.	Crushing Strength (N/mm ²)	Obs No.	Crushing Strength (N/mm ²)	Obs No.	Crushing Strength (N/mm ²)	Obs No.	Crushing Strength (N/mm ²)
1	19.3465	7	20.0212	13	24.3619	19	22.0908
2	26.3294	8	28.8753	14	20.3917	20	28.0456
3	25.0496	9	27.5152	15	26.4099	21	26.3734
4	33.2465	10	22.8869	16	22.6434	22	23.2713
5	22.8927	11	32.0763	17	24.5022	23	28.1044
6	18.3680	12	27.8196	18	18.0644	24	30.4283

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Problems on Probability Calculation...contd.

Soln.:


(a) Let the crushing strength of the cubes be a normally distributed RV.

Mean strength for the sample is

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} = \frac{999227}{24} = 24.9676 \text{ N/mm}^2$$

Standard Deviation for the sample is

$$s = \left[\frac{1}{(n-1)} \sum_{i=1}^n (X_i - \bar{X})^2 \right]^{\frac{1}{2}} = \left[\frac{1}{(24-1)} \sum_{i=1}^{24} (X_i - 24.9676)^2 \right]^{\frac{1}{2}} = 4.1594 \text{ N/mm}^2$$



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Problems on Probability Calculation...contd.

(b) Let x_{95} be the 95% dependable crushing strength.

So,

$$P(X \geq x_{95}) = 0.95$$

or, $P\left(Z \geq \frac{x_{95} - 24.9676}{4.1594}\right) = 0.95$

or, $1 - P\left(Z \leq \frac{x_{95} - 24.9676}{4.1594}\right) = 0.95$

or, $P\left(Z \leq \frac{x_{95} - 24.9676}{4.1594}\right) = 1 - 0.95 = 0.05$

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Problems on Probability Calculation...contd.

From the standard normal table, the value of Z corresponding to 0.95 is 1.645. So, the value of Z corresponding to 0.05 is -1.645.

So,

$$\left(\frac{x_{95} - 24.9676}{4.1594}\right) = -1.645$$

or, $x_{95} = 18.125 \text{ N/mm}^2$

As x_{95} is less than 20 N/mm^2 , it is not acceptable as M20 concrete.

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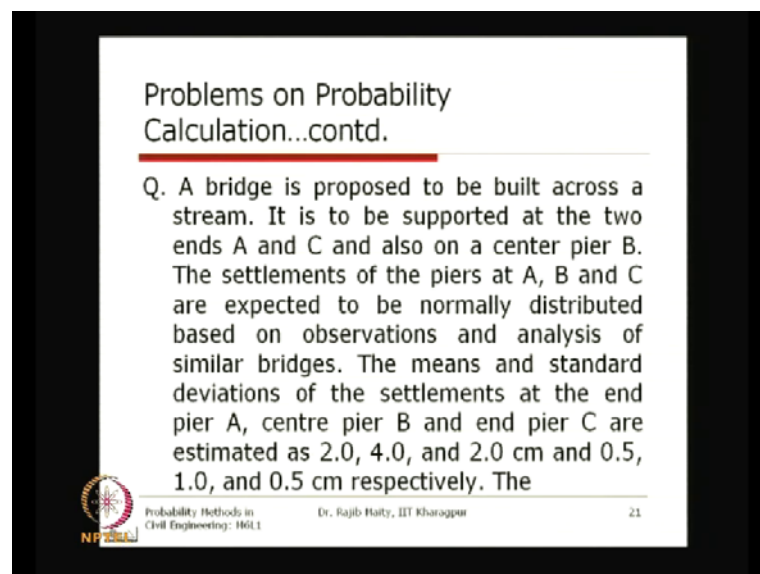
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So, using this one we get the standard deviation is equals to 4.1594. So, we got this two values now, the 96 percent dependable crushing strength is similar to the same that in the earlier problem, we called that 75 percent dependable rainfall. So, that means, the probability of x greater than equals to x_{95} , it is denoted as x_{95} , following the same

principle in the earlier problem. This minus that mean divided by the standard deviation we are getting their reduce variate.

We are getting it the less than equal to sign, that is 1 minus probability of this and we are getting that this equals to 0.05. Now, we have to find out the probability equals to 0.05 for which reduce variate. So, here you can see that, from the standard normal table, the value of the z corresponding to 0.95 is 1.645. So, the value of z corresponding to 0.05 is minus 1.645. So, this x 95 point that minus 24.9676 divided by their standard deviation. This is the reduced variate, which should be equals to minus 1.645. So, x 95 is 18.125 from this experiment that we got. So, the x 95 that is 95 percent dependable that strength is less than 20 Newton per millimeter square which is the requirement for the M 20 grade concrete. So, we can conclude that it is not acceptable as M 20 concrete.

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Problems on Probability Calculation...contd.

Q. A bridge is proposed to be built across a stream. It is to be supported at the two ends A and C and also on a center pier B. The settlements of the piers at A, B and C are expected to be normally distributed based on observations and analysis of similar bridges. The means and standard deviations of the settlements at the end pier A, centre pier B and end pier C are estimated as 2.0, 4.0, and 2.0 cm and 0.5, 1.0, and 0.5 cm respectively. The

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So, we can similarly we can go for the other problems, for the other civil engineering aspects, for example, we have we can, there is another problem on this settlement of this bridge p r, which is also important so far as the practical application is concerned. So, we will take up that problem in the next class and that using. So, there also we will be using the same concept and the same standard normal distribution table to get some answers based on the probability models.

And, gradually also that, what we will take up that problems also. So, from that data, we have to conclude that what distribution it is following through this probability plotting,

that we will take up and then also we will take up the central limit theorem, that we started with in this class. So, we will continue again in this same topic in the next lecture.

Thank You.