Probability Methods in Civil Engineering

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Lecture No. #27
Functions of Multiple Random Variables (Contd.)

Hello and welcome to thisninth lecture of this module and you know that this module we are discussing about themultiple random variables, and currently we arediscussing; we started our discussion from the last lecture, that is, the functions of multiple random variable. In the last lecture, we have given the indications that what all the differentfunctions could be. Particularly the first step that we are doing is thatone function of two random variables and there are several functionsis possible.

Last class we have seen that summation of two random variables, what is theirproperties and all we have discussed.We will just start today's class with aproblem on that and then,we will gradually take up some othertype of functions of two random variables and we will see, and in the next class,we will go for the two functions of the tworandomvariables.

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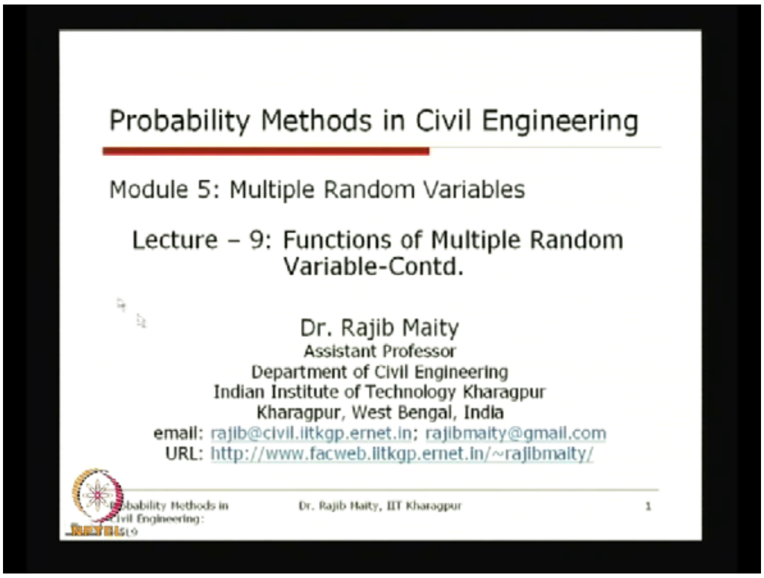

So, so, our, basically today’s lecture is a continuation of our last lecture that is the function ofmultiple random variables, andbasically we will becontinuingthe same thing again for the next lecture where we will discuss about the two functions.

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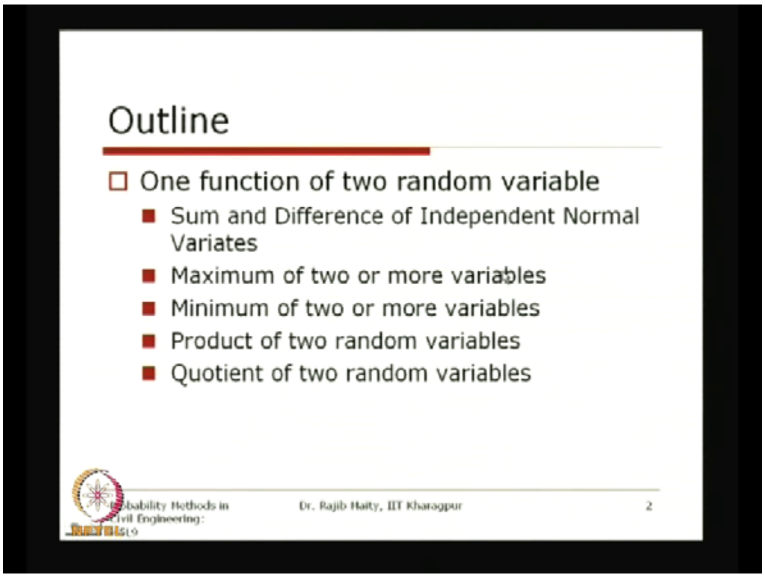

So, up to today's lecture, this a one function of tworandom variables, two or sometimes it is sometimes taking the more also. So, first,we will what we havedonein the last class is a, is the general discussion on this sum and difference of tworandom variables. So, here,we will specifically take up a specific case where the random variables arenormaldistribution, is having the normal distribution. So, how, so, the question is, if there are twonormal distribution, then if we add them orif we take a difference between them, then, the resultingfunction is follows what distribution that we will see first.

And after that,we will take the maximum oftwo or even more than two random variables and then minimum of two ormore random variables. Product of two random, random, variables, that is, if x and y is there, so what should be the distribution for x y and for the quotient of two random variable that is x y y. So, these things we will see along with their application some problems from the civil engineering for differentspecific case, that we will see foreach and every of type of this function.

So, this may not be the end for. So, the type of functions that we can consider out of two random variables, this may not be the complete list. There could be the other functions as well, where I can take this the summation of the square.For example, the x's square plusx's square plus y square.

So, that type of case also can be, can bethat their functions and properties of their that type of function can also be estimated, but the basic thing what I feel is that the basic idea if it is clear from whateverbeen taken up for thediscussion, if the basic concept is clear, then any kind any,any, type of functions can behandled. So, so that we will, what we will try through this few examples,we will just try to develop the concept so that this can be extended for the other type offunctions as well.

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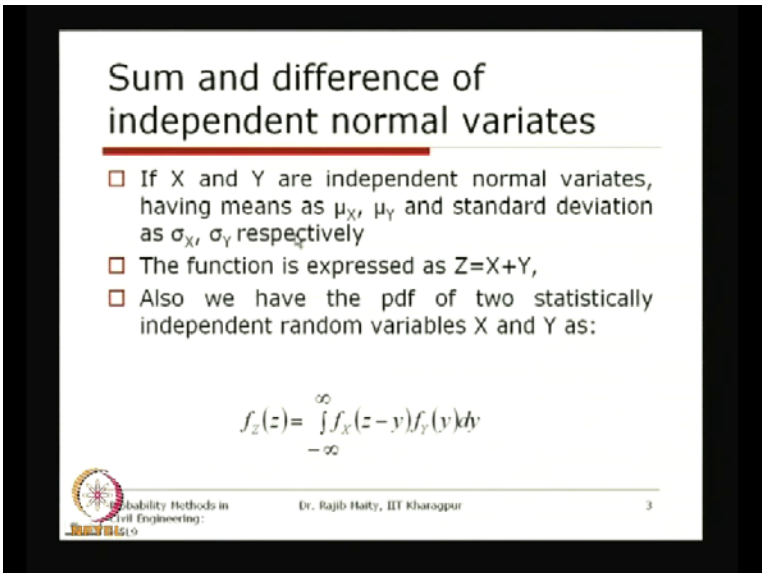

So, the first we will take the same thing again that we havediscussed, but it was discussed in the, in the, general concept for any type of function. Here, we are taking the specific case of the normal distribution becausethis type of situation is very commonparticularly in the civil engineering application that, when we are addingthe random variables and I can assume that thoserandom variables follows a normal distributions. So, we will just see thatin case of two random variables, what is their, what is the property of their summation and their difference and then we will try to generalize that thing from the two to more number of random variables.

So, here, thatif x and y areindependentnormalvariantshaving thenormal distributionfor their, and their parameters are mu x and mu y and standard deviations as sigma x and sigma y respectively. Then the function if we express is as that z is equals to x plus y, so we are taking. So, we are taking the summation of two random variables which are normally distributed having their own mean and, and, standard deviation. For x, it is mu x sigma x, and for y, it is mu y sigma y.

So, also we have the p d f of two statistically independent random variables x and y. This particular thing we discussed in the last class, that is, if two random variables are independent, then their, then the distribution of their summation is that, if that the z and this f z can be expressed as integration over the entire support, expressed in terms of anyone of the random variables.Here, it is expressed within terms of the y. So, this one is the z minus y because you know that this x can be expressed by that z minus y from here and f y y and that we,we, haveindication with respect to the y.

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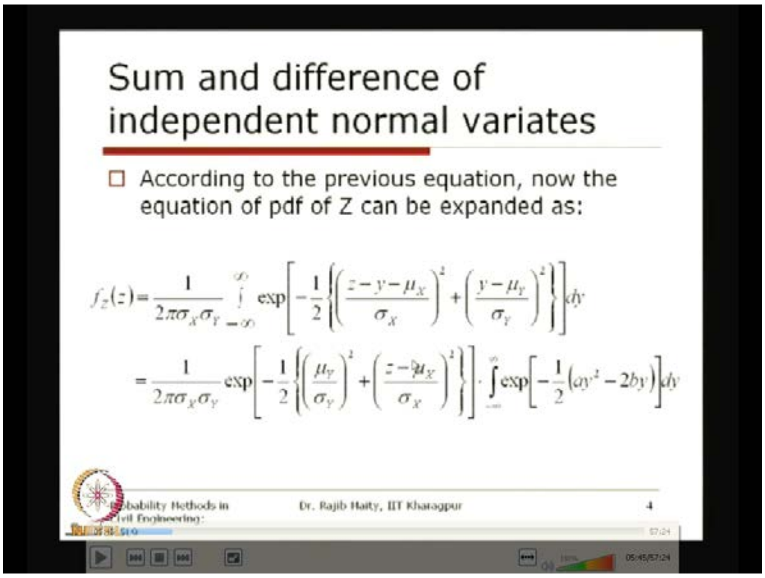

So, if we put this one that we discussed in the last class in this expression, if we put their form, that is, thatnormal distribution form, that also that also we we knowwhat is a normal distribution form, and then, we can see that this can be expressed in terms of this, that is, that 2 pi sigma x andthat 2 pi sigma y this can be taken out as a common, that is, come as two pi sigma x sigma y, and then, we are taking the integration over the entire support of this random variable y that is minus infinity to plus infinity and that, that, exponential of. So, this is the first component is comes from thatrandom variable x. So, how it comes that we are just replacing this z minus yin the place of the x. So, this is basically x minus mu x by sigma x whole square, whole square, that we know. So, that was a,a, replaced; that x is replaced by z minus y.

Andalso for the y, we do not have to make any change because we aretaking it to the random variable y. So, it is y minus mu x by sigma y whole square. So, this expression if we do thisintegration, then we can see that we can take out some of the constant term. So, this mu y sigma y and this z minus mu x sigmamu x by sigma whole square. So, this component we can take out from thisintegrationterm and remaining that aminus infinity to plus infinity exponential of 1 by 2 minus and ay square minus two b ydy.

Now, this a and b are the twoconstants in terms and that can be expressed in terms of the that z which areapart from the, apart from y, the other variables can be used to express this a andband which is herea equals to 1 by sigma x's square and plus 1 by sigma y square.

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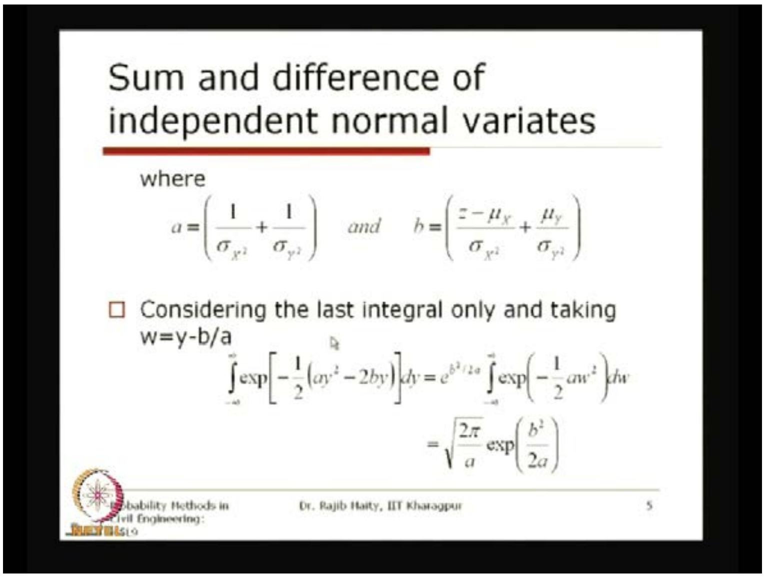

So, this square should be in the top, you know that this is the variants we are referring to. So, this one and b is equals to that z minus mu x by sigma x’s square plus mu y by sigma y square. So, these are the constants that a and b, and now, this, for this integration if you justdo some substitution like w is equals to 1 minus b by a and if we do this integration, we will,finally we will get this form, that is, square root of 2 pi by aexponential b square by 2 a. So, this expressionnow if we put in place of thisintegration that is here, then what is the form we will get? Total form for this distribution of this f z. That is ourinterest.

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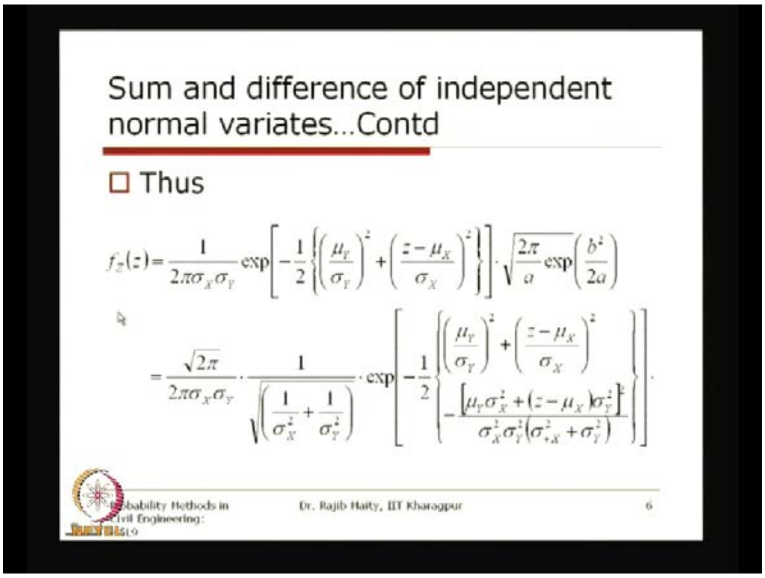

So, here it is that, so, we are just placingthis form in place ofthis integration, and after somealgebraicarrangement, rearrangement of this whole expression, we canwe can arrange it in this formwhich evenfurther can be rearrange in thisfinal form which is that 1 by square root of 2 pi in bracket sigma x's square plus sigma y square exponential of minus half z minus mu x plus mu y divided by square root of sigma x's square plus sigma a squarewhole square.

Now, if we see this form, that is, for thefor the function z which is the summation of x and y, so this form is also one normal distribution with some different parameter. What are those parameters? So, this is the, this is the mean which is now the summation of the individual means of this x and y and this is the standard deviation which is the sigma x's square plus sigma a square,square, root.

So, if wetake this one, of course, this will be the square root of 2pi and this part will b outside of this square root or there will be a square. You know that this is a normal distribution form and this is coming from, this is a form of this that random variable z. So, the sum of two independent Gaussianrandomvariables is also a normal density function, but this normal density function having the parameters are as we have justnow seen thatmu z of this resultingfunction is equals to mu x plus mu y and sigma z square is equals to the summation of the individual variants.

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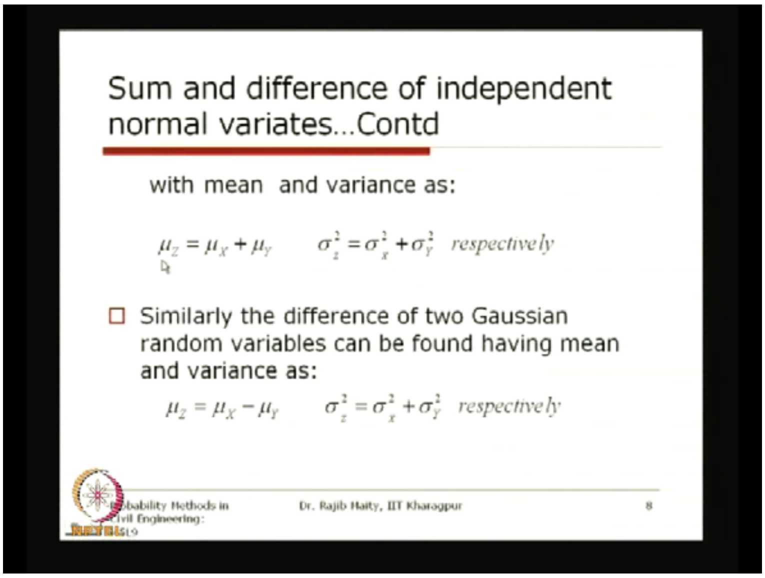

So, this can be now, so, this is veryinteresting that if when there we are taking two normal distribution, we are adding them up, then the resulting variable is also normal, also a normal distribution having their mean is the summation of their individual means and the variance is the summation of,of, their individual variance.

Similarly, we can also do the same exercise to know what is the distribution if we take the difference; means that x minus y if we take, what is the distribution of this function their,their, difference. If we do the same exercise, it will come as that, that, difference also will be a having a Gaussianrandomvariable and theirparameters that is the mean and variance will bethe first we take the mean. The mean will be the difference of their individual means.

So, here, you remember that the function we have taken, it is the x minus y.That is why it is mu x minus mu y. If we take the function as the y minus x, then it could have been mu y by mu, sorry, mu y minus mu x, but if you see the variance,variance is not that difference in case of the difference in the random variable as well. It will be always be additive.

So, the, for in case of the difference also, the variance will be the summation of the, their individualvariances. So, irrespective of whether it is x minus y or y minus x or even x plus y. For all this cases, the variance will be the same, that is, the summation of the individual variance, but this mean will be changing as per thefunction is.

So, now, these things as you can see thathere we have taken the two random variables.Now, this can be easily extended for the more than two random variables. So, if we take say that n numbers of random variable, all are independent andthat their distribution is normal with their individual means and variances. Then the resulting function, if the function if I take that their summation, then the resulting function will also be a,a, Gaussiandistribution having their mean is equals to their summation of theirindividual means and the variance is the summation of their individual variance.

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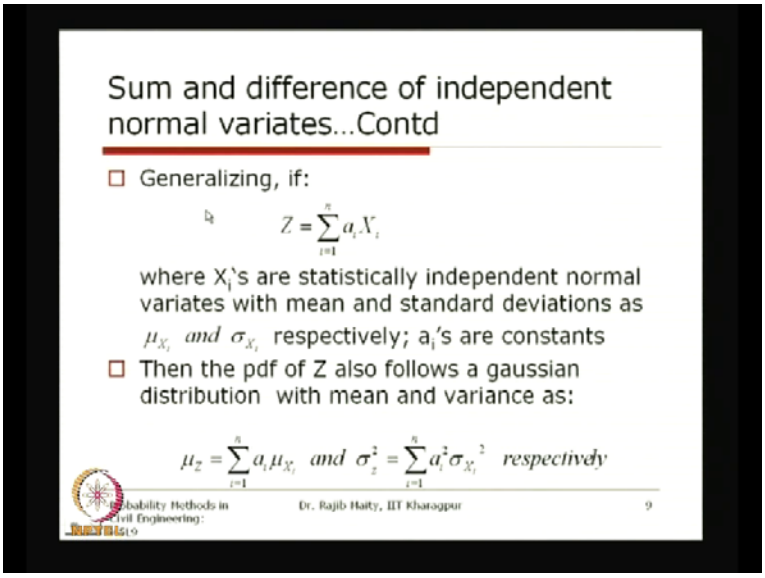

This is what is explained here, that is, if you just want to generalize it, that is the z is equals toeven that if we,we, can even we can multiply with a constant, that is,a i x i. So, each random variable is multiplied with the constant that is a i. So, these are statistically independent, normal variance with mean and standard deviations as mu x i and sigma x i respectively and a’s are, a i’s are constants.

So, in that case, that z will have, z will also follow a Gaussian distribution with mean and variance. As the mean will be the summation of their individual means. Now, you know that if we just multiply with the scalar component with normal distribution, their meancan be just multiplication of their, with their constant that we have seen when we were discussing that expectation.

So, here, so, that mean also will be, that is the summation of that a i mu x i for i varies from 1 to n for all the random variables, all the random variables concerned, and the variance will bethat a i square. So, this also we explained when we are taking the second moment, that is, the variance of the random variable when it is multiplied by a constant, that constant should be squaredin case of the variance. This we, this we also discussed earlier.

So, in this case now, if there are n summation of this n suchrandomvariables, their variance will be the summation of that a i square sigma x i square for i varies from 1 to n.

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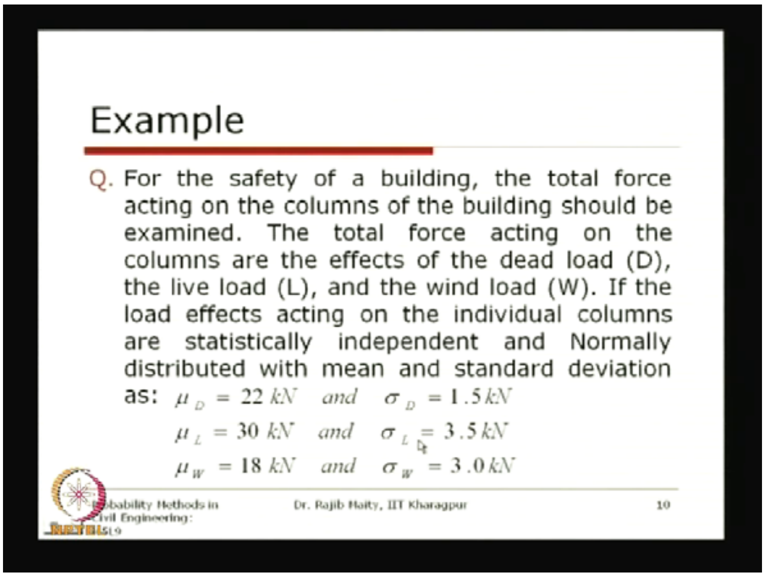

So, if we take one example now on this, the summation of where all are normal distribution, we have taken aa structural safety problem. For the safety of a building, the total force acting on the columns of the buildings should be examined. The total force acting on the columns are the effects of the dead load, live load and wind load. There are other loads as well, but here we have taken three different types of load. These are, these are the primary and most important load and there are different combinations we generallyconsider for this.

So, if the load effects acting on the individual columns are statistically independent and normally distributed with their respective mean and standard deviation, as this for the dead load, the mean is 22 Kilo Newton and the,the, it is it is standard deviation is 1.5. For the live load, the mean is 30 kilo Newton andthis, their standard deviation is 3.5, and for the wind load, it is 18 kilo Newton andtheir standard deviation is 3 Kilo Newton. So, if this is thesituation of the loading system, then, we have to determine the mean and standard deviation of the load acting on the column. So, it is a total load which we have to take the combination of this all this three loads.

Here, we can just take, for the simplicity sake, we can just add them, individually we can add them and find out what is the distribution, but you know that in sometimes due to the factor of safety, we have to multiplythis force with someconstants. So, those are separate issue, but you know that if there are some constant is multiplied also, thenthe,the, last thing that we have discussed that ai; a i is the constant that is multiplied with the individual random variable that also we can do. But here, what we are looking for is that just the simple summationof the all three forces what is their,their, distribution.That is what is, is, asked for.

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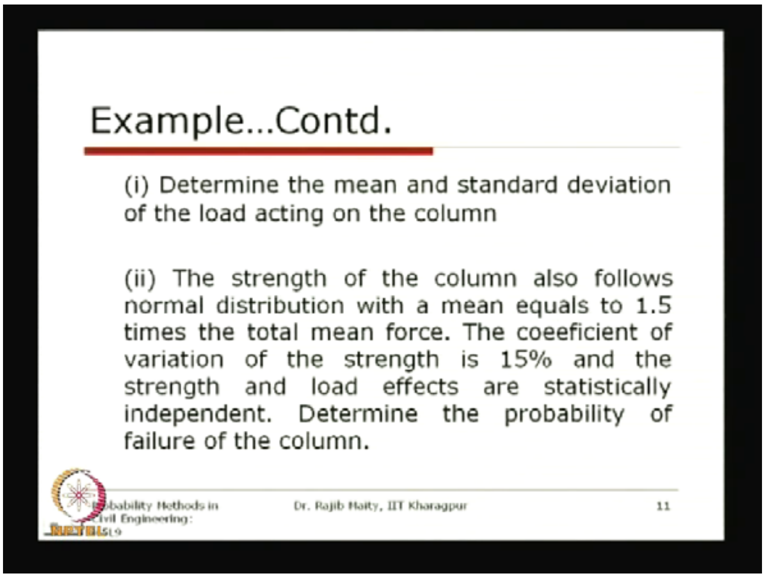

Now, the, and the second question is that the strength of the column also follow a normal distribution with the mean equals to the 1.5 times ofthe total mean force and the coefficient of variation on the strength is 15 percent, and the strength and the load effects are statistically independent determine the probability of the failure of the column.

Now, to agive a little bit background of this, the, what strength that is being used here for the column is that how much force it can resist. So, that is also is having the same as that your loadingunit, so that now what to determine the failure or to,to, declare that this is failed.What we can say is that, if the difference is becoming negative, that is the, if the load is exceeding what isthe strength of the column, that time we are calling that this column isfailed. So, you can see that the first problem basically we have to go for a summation and the second problem we have to go to a difference,difference, between the strength and thetotal load on the column.

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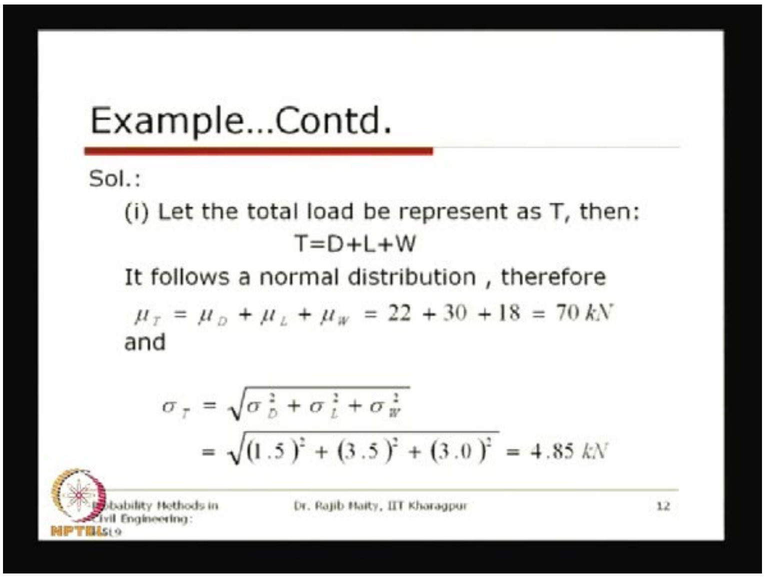

So, to solve the first partthe total loadto be, total load be, represent. If it is represented as t, then this tis equals to thatd plus l plusw.Now, as I was telling that there could be some, some, factors ofmargin factors or factors of safety that we can multiply with this individual loading system and that need not be same for all the loading pattern. So, depending on that, this final answer may change, but here, if we just consider that total load is the dead load plus live load plus wind load, then it follows a, then we know that if all this three are thenormal distribution, then it follows a normal, normal, distribution as well. So, this mu of this t, that is, total load it is. Mean will be the summation of their individual means, that is, which is 70 Kilo Newton and its standard deviation is nothing but the positive square root of their summation of their individual variances.

Now, this 1.5 square root which is thestandard deviation for the dead load to 1.5 square,3.5 square and 3 square, and after that, we are taking the positive square root which is the 4.85. So, the total load is also a normal distribution having its mean is 70 Kilo Newton and standard deviation is 4.85Kilo Newton.

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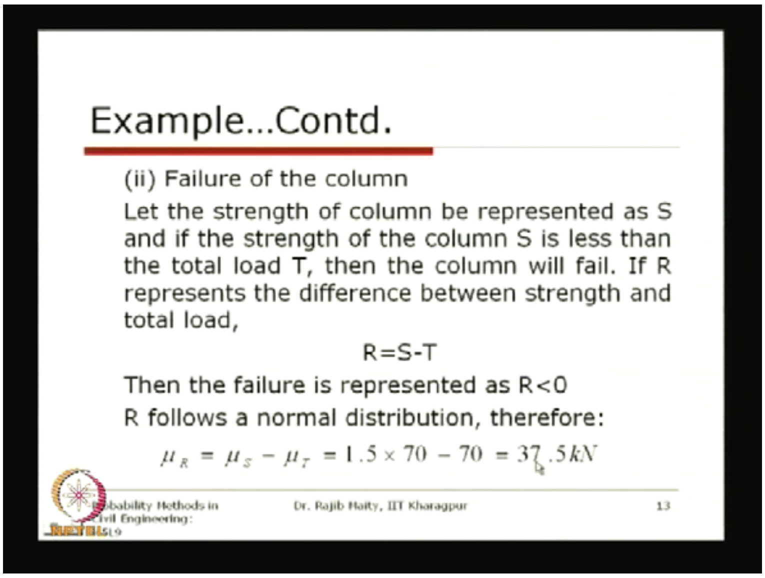

Now, that failure of the column, so let the strength of the column be represented at s, and if the strength of the column s is less than the total load t. So, here, now from thisto calculate the failure, you have to consider the total load. So, if it is less than the total load, then the column will fail, and if the r represents the difference between the strength and the total load, so we are justintroducing a new random variable which is r and which is the difference between that s minus t. Now, it can be easily sent from this expression that, if is s is less than t, that time obviously the column will fail; so, that time r will becomenegative.

So, if the, then the, the, failure is represented as r less than 0. So, r follows a normal distribution.Therefore, you knowthat as s and t both are normal distribution. So, it should be the mean of this r will be that mean, difference ofthis their means, that is, mean of this strength and means of thistotal load, and mean of this one as expressesis 1.5 times of the total load. So, 5.5 multiplied by 70, that is the mean for the strength and minus mean of the total load is 70 thatwhat we have just seen in the previouspart of the problem. So, the difference is 37.5 Kilo Newton.

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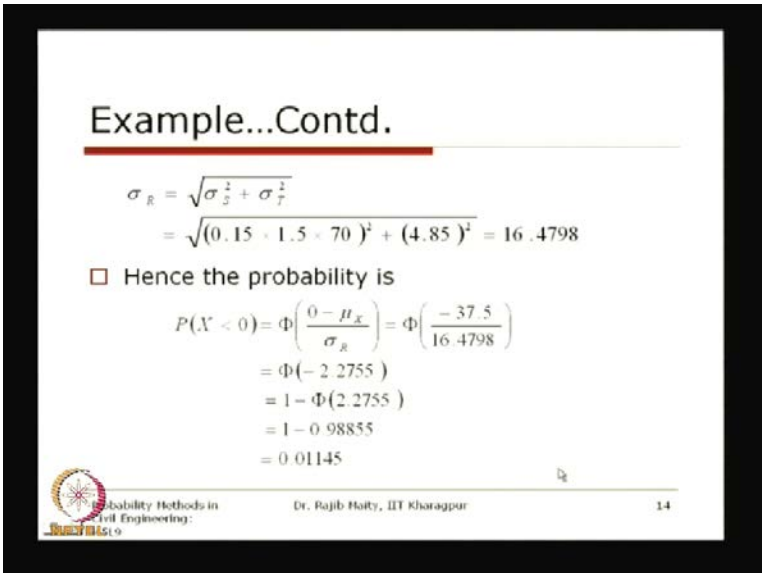

So, so, the r is having a mean of this value and we have to see it is standard deviation as well. So, this standard deviation is you know that. So, we have to find out what is thestandard deviation on the strength first and standard deviation strength. It is the coefficient ofvariations that, that was given that c v wasgiven as 15 percent. So, now, this 15 percent multiplied by 1.5 multiplied by their mean. So, it will give you thestandarddeviation of the strength and that square. So, variance plus the variance of thetotal load which is 4.85 square.

If you take this one, so the variance comes as 16.4798 Kilo Newton this one. So, this should be the Kilo Newton. So, so that r is also that difference of their strength andtotal load is also a normal distribution having it is mean 37.5 Kilo Newton andstandard deviation is 16.4798 Kilo Newton.

Now, if we want to know the probability of their failures, so probability of the failure is equals to probability of, we have denote it as r, so r less than equals to 0. So, r less than 0of course, here to be carried to the field, which is equals to now you know that we have to reduce, to get the reduced variance that is. So, this is the value that x minus orr minus r is here is 0, so 0 minus mu r divided by sigma r. So, this is the mu r. What is, this will be r; this is not x. So, mu rwhat we have calculated is this 37.5. So, minus 37.5 divided by standard deviation, which is16.4798. So, this, the, phi of minus 2.2755.

So, this now we can get it from the standard normal distribution table that, you know thatin the earlier class, we have discussed how to refer to that standard normal distribution table, and from there, we can gets its value at theprobability. So, now, so, it can also be writ10 that 1 minus phi of 2.2755 due to the symmetry of this standardnormal distribution curves and this value is 0.98855 point which is giving the probability of the failure of the column is 0.1145.

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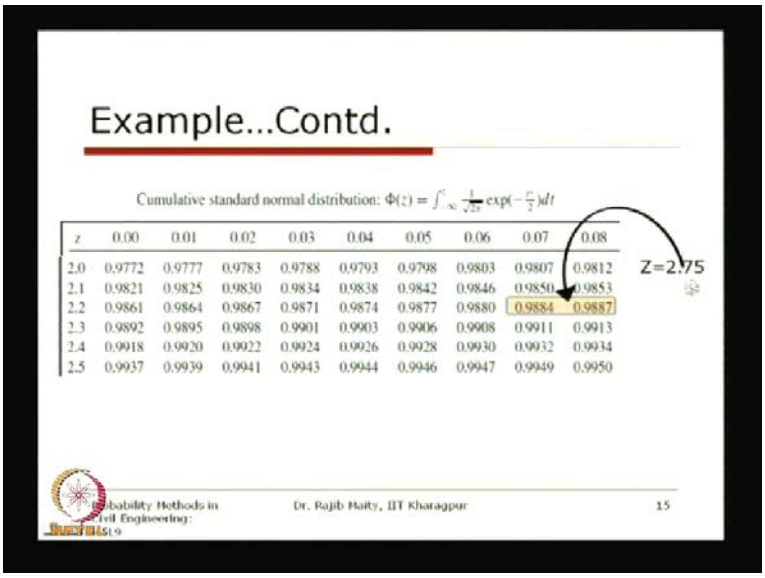

Now, to get this one, you know that. You have to refer to this standardnormal distribution, and for this 2.275 was, yes,2.2755 will be there this is a 2.27 is this and 2.28 is this. We have to get an interpolation between these two values which is 0.984, 98, 98845. So, what is this value that 98855 that value you got at the interpolation between these twoextreme that is two 2.27 and 2.28. This will be 2.275,that is, we have seen that what we are referring to here 2.275.

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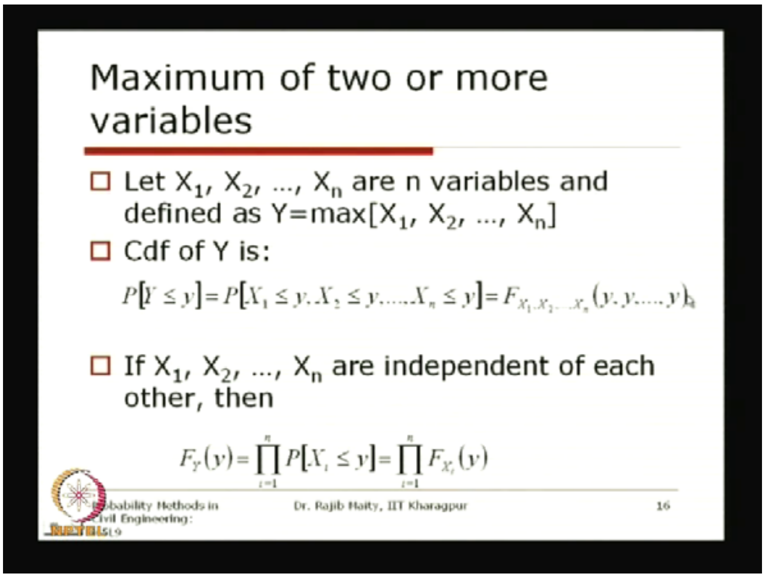

Well, so, ournext discussion is on this maximum and minimum oftwo or more randomvariables. So, if sometimes this is also important is that we have the severalrandom variables and we are interested to know what is that, what is the maximumof that, of the individual random variables. Suppose that we know that suppose at a place the earthquake's intensity we know and what is the maximum intensity of individual earthquake is having some. So, we,we, are interested to know that in what should be the maximum intensity, what are the properties of themaximum intensity.

So, in a particularstretch or period, if we are, if we are interested to know that this manyevents has occurred or this many events are likely to occur, then what is the, what is the distribution of its maximum intensity of the earthquake like that. Similarly, suppose that we are just picking upthe annual maximum flows. So, so, we are taking thatthemaximum flows from each month and what is their distribution, and we want to know that out of this n years, what is the maximum flowthat can, that can, occur or similarly what is the minimum flow that, that, can occur. Sometimes that is also important forsomemanagement purpose.

So, to address that type of question, so we need to know that distribution of the maximum of two or morevariables. So, let that x 1x 2 x 3 up to x n are the n variables and these are defined, and the function which we are defined as the y is equals to maximum of this nvariables, so maximum of x 1x 2 up to x n.

So, now if we want to know what is the c d f of they, then c d f of the y is that y less than equals to the specific value y, is equals to that it should be (( )). So, that x 1 is also less than equals to y; then x 2 is also less than equals to y like that. All the random variables should be less than equals to y, which is nothing but the joint c d fof all this random variables x 1 to x n for their, for the specific value y for which we are looking for the c d f.

So, if this now,now, the one more assumption if we put that. If this x 1x 2 are independent to each other, then their joint distribution can be expressed as the product of their individual marginals. So, now, so, this is the marginals of this distributions, and so that we can this sign is for the multiplication of individual, individual, c d ffor the, for the x i. So, i varies from the 1to n, that is, it is the product of all the product ofthe marginal c d f 4 4 all the random variables.

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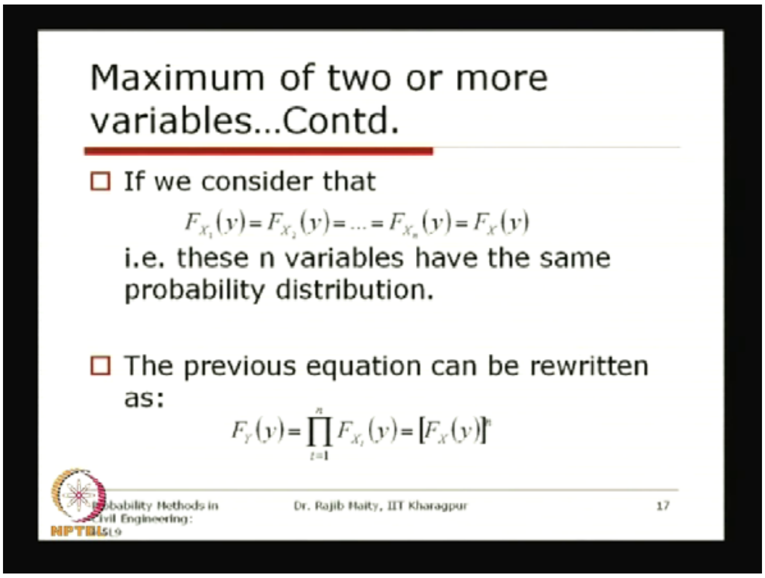

Now, again, one moreassumption if we just say that all are, all are, the distribution of all of them are similar, that is, f x 1 in terms express in terms of y, f x 2 express in terms of y. All this distribution the distribution of all this random variables x 1x 2 up to x n are having a similar distribution which is can be written as f x y. Then this nvariableshave the same probability distributions. So, if that is the case, thenf y. So, the product will be just their power to the n.

So, you see that here to reach from this, this, expression and what we have started with, so, this,this, expression this c d f. From this original expression, if we want to go to this expression, that is, which is the product of the say f x y power n, there are two assumptions involved - one is that they are independent and identically distributed. If that is the case, then we can say thatthis f y y is equals to f x express in terms ofy of course, power n.

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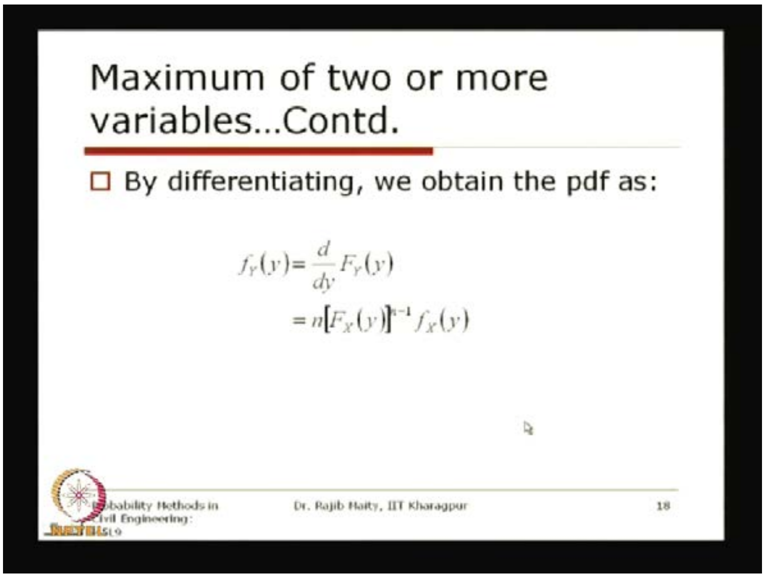

Now, if we want to know what is its c d, p d f, that is, we have to do thatdifferentiation with respect to y and this is the final form, that is, n f x in express in terms of y power n minus 1 and this differentiation you know, that is, their p d f that is f x of x that is p d f of their individual random variableexpress in terms of y.

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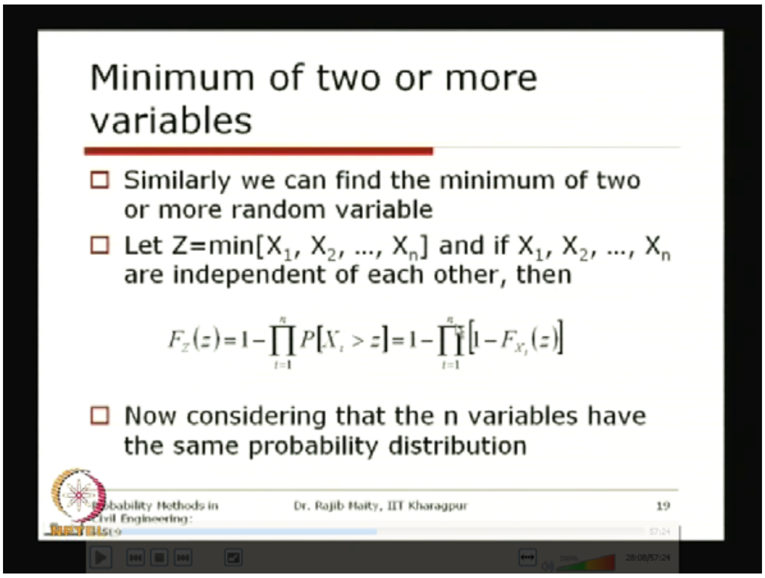

And similarly, if we take thatthat minimum of two or more random variables, that is, when we are talking that this z is equal to their minimum of x 1x 2 up to x n, and if that x 1x 2 x n are independent to each other, then this f z is equals to 1 minus ofthis product of their, this probability, that is, x igreater than z. So, we have taken that 1 minus.

So, now this one can be expressed as 1 minus the product of their 1 minus theirc d fof the individualrandom variables. Now, considering that this n variables have the same probability distribution, that is, that is, if they are having the identical distribution, then this product can bereplaced as a power of ntothis 1.

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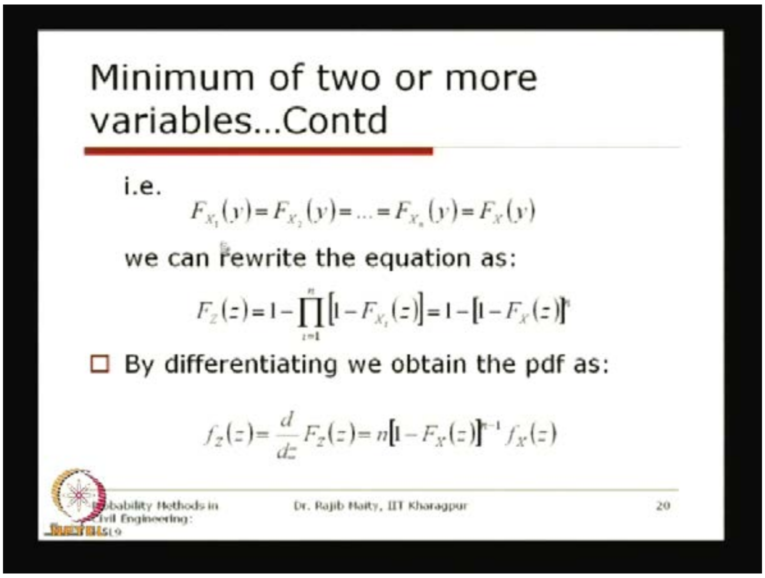

So, that, so, this is their, what is their identical distribution, that is, f x for all theindividual random variables x 1x 2 up to x n. So, this one what we can write a, so, finally, what we can write that this c d f of the, of the, function z which is the minimum of all thisrandom variable x i is equals to 1 minus,1 minus, f x expressed in terms of z power n. Now, if we differentiate this oneto getto, get the, p d f of z is equals to you have to differentiate with respect to z. This function which is n multiplied n times of this 1 minus f x expressed in terms of z power n minus 1 and multiplied by that f z of z.

So, this is the expression that we aresent is for when we are interested to know the minimum of n random variables. So, which is n multiplied by 1 minus f x expressed in terms of z power n minus 1 f xp d f expressed in terms of z, and if we are interested to know what is there, if what is the distribution of theirmaximum one, the maximum of the n random variables, then that expression isthat n times of their c d ff x is expressed in terms of y power n minus 1 multiplied by theirp d fexpressed in terms of y.

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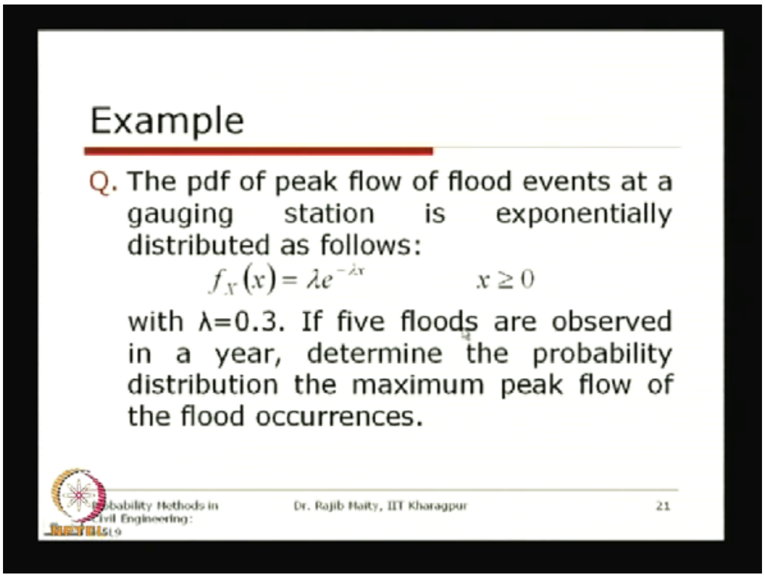

So, now if we take oneproblem based on this one, suppose this is the problem on the peak flow, so there are different flood events occurs and we are, we want to know. So, to characterize their flood, there are different attributescan be considered - one is the duration of the flood, total volume of the flood, peak flow of the flood. So, there are different characteristic.

So, in this example, we have taken the peak flow, and here, what we are interested? We know what is the distribution of the peak flow. Now, if there are certain numbers of flood occurs, and to prevent thatto do a proper flood management, we want to know what is the maximum peak flow that can, that can, occur for different designpurposes. So, a that type of problem is taken here thep d f of the peak flow of the flood events at a gauging station is exponentially distributed as follow, so f x x lambda e power minus lambda x, where x is greater than equals to 0 with the lambda parameter as 0.3.

Now, if there are so n flood events, so, here, we have taken the 5 flood events. Ifthe 5 floods are observed in a year, determine the probability distribution of the maximum peak flow of the flood occurrences. So, so, here, what is that intuitive is thatthose assumption that we are, that we havediscussed about during the theory. So, for the all the flood, it is having the same distribution. So, all the floods means in case of all the floods, their distribution of their peak flow are similar, and obviously, that for this one, for the, for the one flood to an another flood that their distributions are also independent. So, these two assumptions are there when we are declaring that, that the distributions,that p d f of the peak flow of, of, the flood events at that gauging station is having this distribution with the lambda parameters is this. So, so, there are 5 such flood events. What is the, what is the probability distribution for themaximum peak flowout of this 5 flood events.

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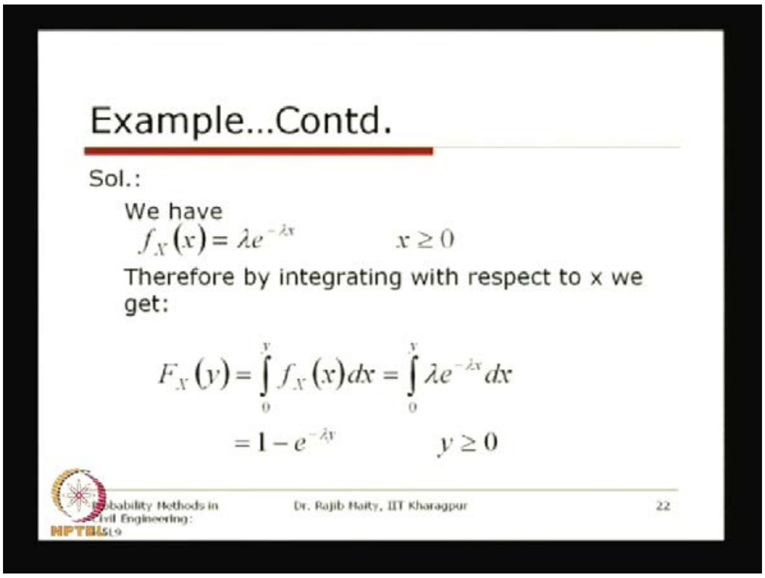

So, to solvethis one as we are interested to know there, the maximum, maximum, the peak flows. Then this is their p d f, that is, given that is lambda e powerminus lambda x, and this is an exponential distribution, and you know that if for the c d f of the exponential distribution is 1 minus e power lambdax. So, here, we are expressing in terms of they as we have just,just, keep parity with the theory. So, this is expressed in terms of the y and which is thec d f of the individualpeak flows for the different floods.

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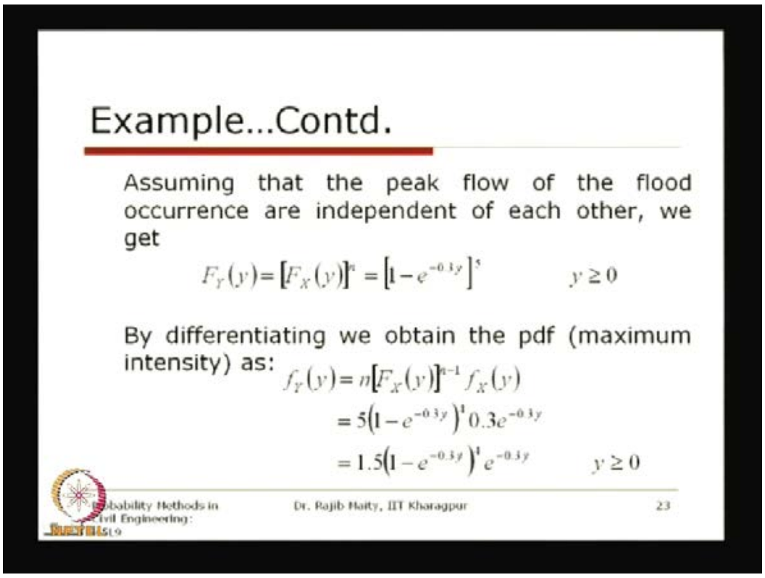

Now, if we assume that the peak flood, the peak flow of the flood occurrences are independent from each other, then we get that this f y, that is, y is themaximum of that 5 maximum peak flow out of this 5 flood occurrences. Then this f y should be the multiplication of their individual c d f. So, so f x express in terms of y power n. So, 1 minus e power minus 0.5y power 5, so, where this yis greater than 0 equal to 0. So, now, by differentiating, we obtain the p d fof this maximum, maximum, peak flow as this f y y is equals to n f x ypower n minus 1 of their individualp d f.

So, this is now n is equals to 5 f x y is that c d f is 1 minus c power lambda e power minus lambdaminus lambda y. So, which is lambda is 0.3. So, this is replaced by this one and expressed in terms of y power n minus 1 which is 4 and this distribution this p d fis 0.3 powerminus 0.2 y that is lambda equals minus lambda y. So, the expressing this 1 point51 minus t power minus 0.3 y power 4 e power minus 0.2 y for y greater then equal to 0. So, this is for, this is a distribution for the maximum peak flow.

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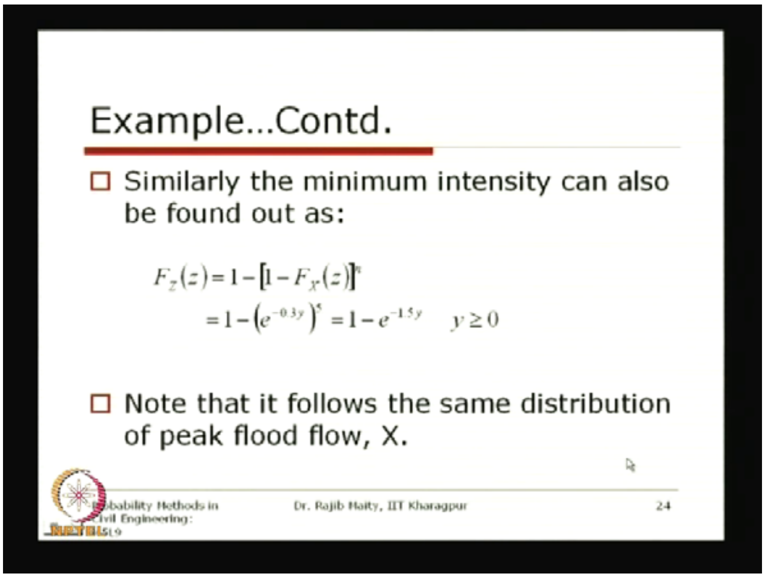

Now, if we are alsowant to know what is the minimum of this peak flow, then this expression we canrefer to that is f z z equals to 1 minus of 1 minus f z power. This also expression we havediscussed earlier which is then 1 minus. This is now e power minus point t3 y power 5 and this is can be expressed also 1 minus e power minus 0.5 y. So, why this is have been discussed here is that just to give a special note that, when we are interested for the exponential distribution if theindividual random variables are distributed.Then the, if we are interested to know what is thatmaximumout of this n individuals random variables, then we have seen what is theirexpression.

But if we are interested to know what is the minimum of the different random variables in case of the exponential,I am just we are talking. So, that also we will have the,we will have the similar form of this distribution, that is, that also will be exponential distribution, which you can see it from here also, that is, if we, if we are talking about the minimum 1 and if this is expression, if we just shown, which is coming as 1 minus e power minus 1.5 y. So, this is the form of the exponential distribution with obviously is some different lambda parameter. So, so the note is that it follows the same distribution of the peak flood flowx in case of the minimum only not for the maximum.

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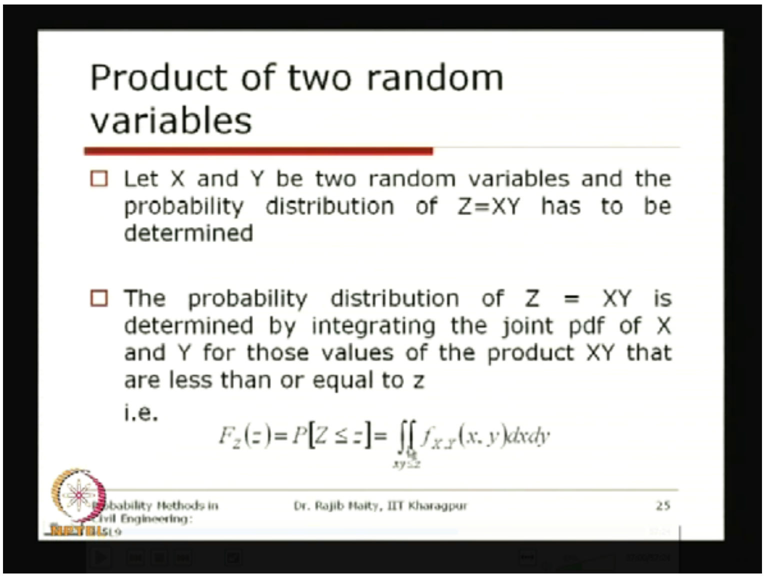

So, other twothings, the other twotype of variables, type of the function is that,one is the product that we will discuss; another one is the quotient that we will discuss, and with reference to thatsum that integration a area and all, so that this,this, concept. So, last time, in the previous class also we have explained that how we should take the integration limit in case of we have discuss it for the summation anddifference x plus y andx minus y. On what area and what should be that indication limits that we have we have discussed. So, here also we will take up.Similarly, the one is the,the, first one is that x y and other one is the by y.

So, first, we will take the product of two random variables. Let that x and y be two random variables, and the probability distributionof z is equalsto x y has to be determined. The probability distribution of z equals to x y is determined by instigation the joint p d f of x and y for those values of the product x y that are less than or equal to z. So, this f z express in terms of this; z is equals to the probability of z less thenthat specific values.

If want to know this one, then we have to integrate that joint distribution of this x and y for the area where the product of x y is less than z. So, this is the basic equation that you know which we want to, we are interested to know that what is the c d f of the z. Now, now, properly we have to identify that on what area this integration should be done for which variable.

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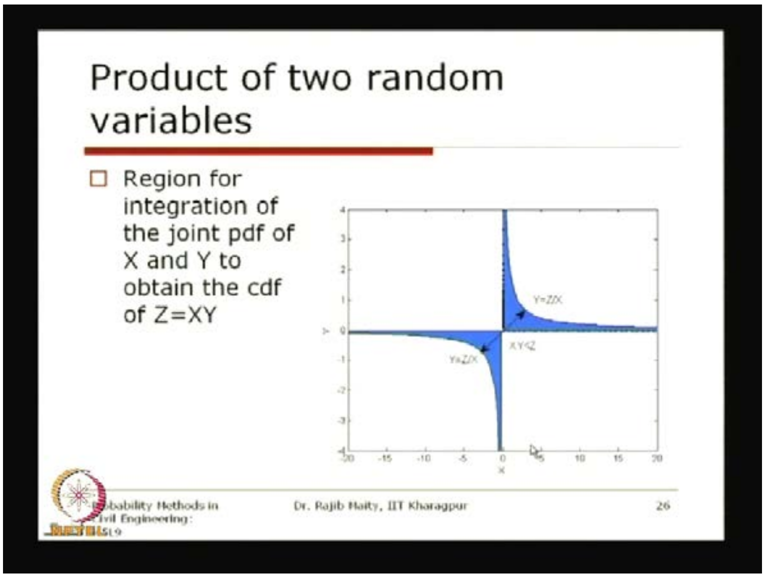

So, if you not take this one, that is, first that, that, we are discussing the z equals to x, z equals to x y. So, there are two possibilities are there - one is this. So, this is the line where if we just take a specific value of z, then this is the, these are the lines where this y is equals to z by x and this is one and this is another; this is the first quadrant and this is the third,third, quadrant. So, and these are going to this infinity plus infinity. This is also going to the plus infinity; this is going to the minus infinity and this is coming to the minus infinity.

So, if, so, if this is the, the,the, shaded areas is that x yah x x y less than z. So, now, this when we are talking about x y less than equal to z and we have shown this one this shaded area. Now, depending on the what is the support of this individual random variable x and y, this, this, area whether we will input this coordinate or this coordinate or not, that is, the separate issue, but this should be obviously with bounded by these two curves. So, in this first quadrant, this is the curve, and in the,the, third quadrant, this is the, this is the curve.

So, if you want to now integrate it, now you can easily see that we have to do the integration in two different parts - one is that for this if we, if I just take this is with respect to this axis, so, one is that, we have to take it from this side, and the other one, we have to take it from this side.

Now, if you want to, if you want to do this integration and if we are, if we are taking athis small strip that we are discussed in the previous class,class, also that if you take small strip from hereto this say infinity, depending on the case whether this area will be included or not that is the separate one, but it should be lower bounded by here; it should be lower bounded, but for this side, thiswill be upper bounded. This two are the thingsthat we have to consider here.

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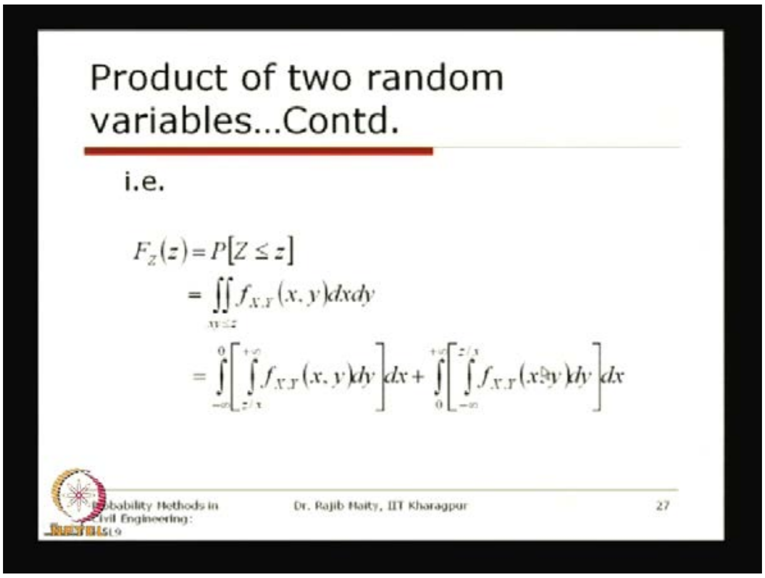

So, now, if you want to do this find out their integration limits, then you see that for this f z that the expression we have explained is first what we are taking is that, for thisz by x for the y, for that y we are we are taking first. So, this,this, is now we are for the left hand side v left hand side; that means, in this area.

So, this is now the lower bounded by this z by x. So, y is equals to z by x and going up to plus infinity. This is also a general term whether it will go to the plus infinity or it should be bounded by, bounded at 0. That depends on this what is the range of that, that particular random variable. So, this is the thing. So, that strip if we take, and after that, after that, for the xaxis the indicationminusinfinity to 0.

So, minus infinity 0 means it is going to this minus infinity and we are doing into two parts. So, first, we are coming up to this axis which is that x equals to 0.Now, for the right hand side, if you want to express for the other one, other expression which is, we are taking the first for this with respect to the y and this should be theupper bounded. Now, which is upper bound is that z by x. So, this upper bound means here.

So, from this minus infinity, it will go to that, that up upper bound. Now, again the similar point whether this area will be included or the lower limit will be 0 to that z by x, or from the minus infinity to z by x, that depends on the individual randomvariable. So, and after that, when we are taking thatlimit for the x, so this limit for the x will be from this axis, that is,is from y axis, that is, x equals to 0 to the plus infinity.

You see here the limit is from the 0 to infinity. So, the first one in case so far as the x factor is concerned, we are taking it the from minus infinity to 0, and for the other, for the, for the positive axis, we are taking from the 0 to infinity, and similarly, this inner layer limit, that is, for the y is also changing accordingly.

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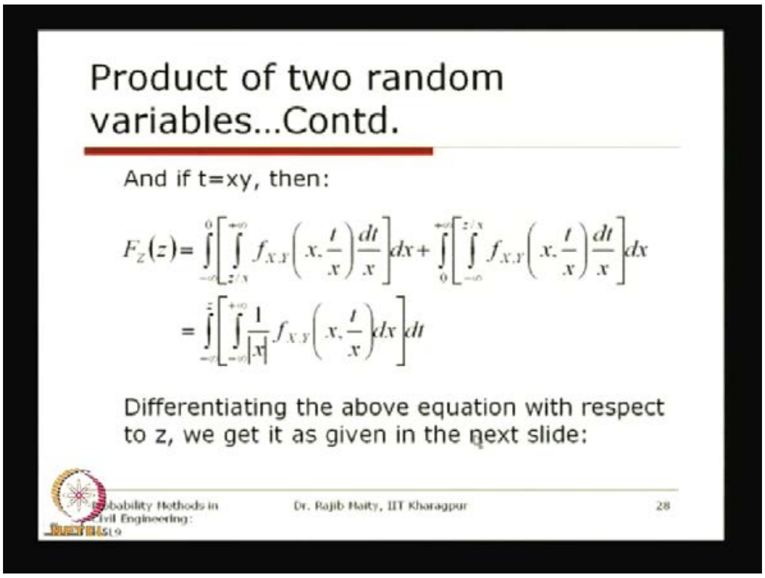

So, now, there are some certainsubstitutions. If that t is equals to x y, then if we want to express in terms of this t and this integration if we do, and finally, we get thetotal that final form as that from minus infinity to z and then integration from minus infinity to plus infinity 1 by mod x f x y, that is, the join distribution expressed in terms of x and t by x with respect to that x, integration with respect to x and final integration with respect to that t.

So, that it will be expressed in term of z only to get there what is their distribution function for the random variable z, which is actually equals tox y, that is, product of x and y. So, differentiating now again. So, this is a c d f. So, differentiating the above equation with respect to the z, we get as it is a given here.

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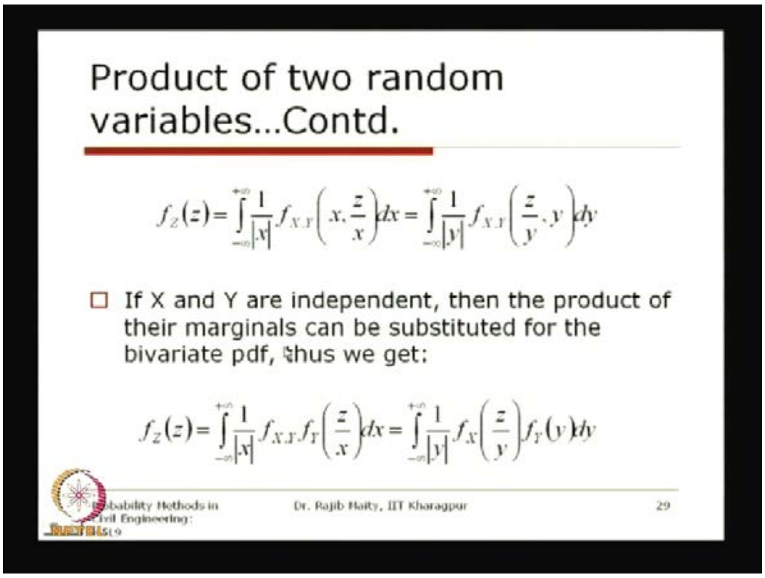

So, if we just do this differentiation, we get its p d f. So, we will get the final form as this 1 by mod y f x y z by y y with respect to, with respect to y or you can also do it with respect to the x, that is,1 by mod x f x y x by z by y, sorry, z by x with respect to x. So, depending on which variable you are just substituting, repeating on that, you are getting what is itsform of this p d f.

Now, if the x and y are independent, then the product of the marginals can be substituted from their bivariate p d f and thus we get that. So, this joint actually you know that, if it is independent, then this joint can be, can be, express at the product of their individual marginal. So, this, this expression just we have taken that with respect to the x. So, this expression can be expressed in terms of this f x of z by y and f y of y with respect to y. And similarly also is this can be also expressed in terms of the x as well, that is, f x of x and f y of z by x with respect to, with respect to, dx and 1 by mod x from minus infinity to plus infinity both the expression are possible.

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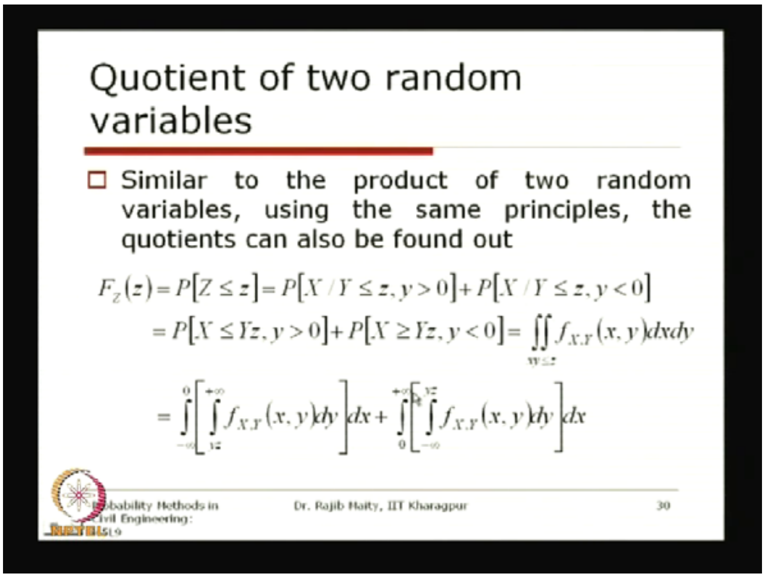

Now, similar to theproductof two random variables using the same principle, the, the, quotients also can be found out. So, the, to find out the,the, quotient that is f z of z, that is, z is less than equal to this z which we can write that x by y is less than z and y greater than 0.

So, this range we have to, we have, we have to, consider and plus the probability of x y z for the case when y is less than equals to, when is y is less than 0. So, for this can be expressed that x is less than equal to y z and y is greater than 0. So, two different cases, two,two, different areas again - one is that y is greater than 0; other one is the y less than 0. So, this can be expressed that x yless than z, no, this is, that this is that x by y less, then z this is an wrong. So, x by y less then z for the joint distributionthat integration if we do.

Similarly, again you can justexpressthisx by y or that x equals tothat y z that, that expression we just take and that limit if you consider, then the integration limit in case of the y if you take the strip and then the y, then this integration limit will befrom y z to,to, the plus infinity, that is, x equals to y zbecause of our function is z equals to x by y.

So, from the, so, forthe lower limit, for the y will be y z 2 plus infinity and for the xit will be minus infinity to 0 first. When we are considering and that y that the one zone, that is, out of this, there are two separate zone. So, it is for,for, the negative side of the x axis that is from a minus infinity to 0 and the other one is for the positive side of the x axis from this 0 to infinity and this limit will be from the minus infinity to y z of this limit to instigation. If we do, we get the c d f of z.

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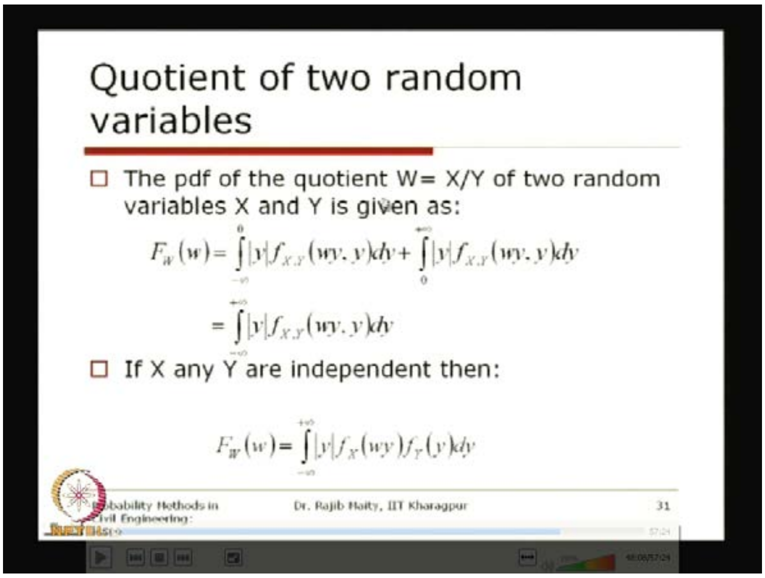

Now, the p d f of the quotient, that is, w is equals to x by y of two random variable section given as. So, finally, we are getting that minus infinity to 0 mod y f x y, that is, joint distributions express in term of w y, y d y plus for this 0 to plus infinity mode y joint distribution express in terms of w y and y and we are getting the final from this minus infinity to the plus infinity mod of y joint distribution expressed in term of w y and y dy.

Similarly, this expression can also be expressed as you know in terms of the x aa. In terms of the x also, there the accordingly you have to change the substitution which one u, that y should be substituted and it should beexpressed in terms of the x, and then, the integration should be takenwith respect to the,with respect to, x.

Now, againif the x and y are independent to each other, thenwe can say that this joint distribution of this, of their quotient is equals to this minus infinity to the plus infinity mod of y. Now, this one, this joint distribution is basically expressed in terms of their product of their individual marginal. So, this is the marginal of this x express in term of w y and this is the marginal y expressed in terms of y and doing the integration with respect to y, and similarly also you can express the same thing with respect to the x as well by proper substitution. So, instead of y, there will be express it in terms of x first and then we will do the instigation with respect to x.

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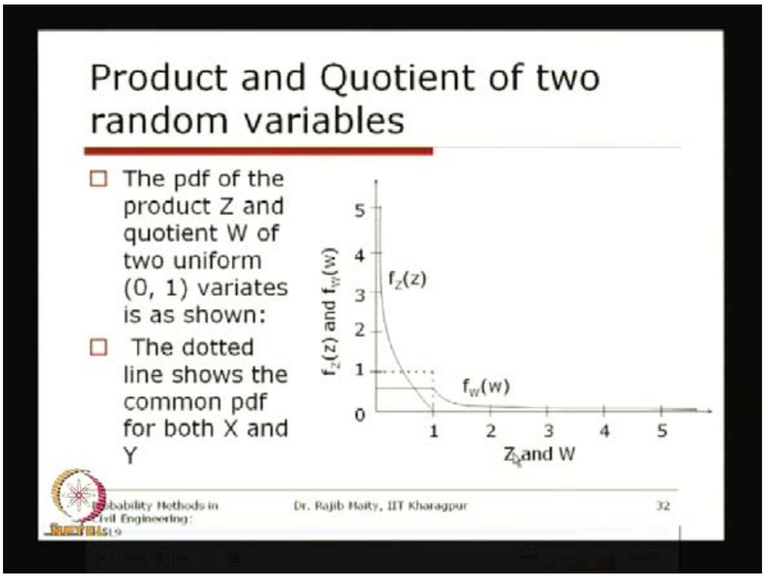

Here, one example is shown. Suppose that there are two randomvariables - one is thatproduct of z and the quotient is w a here, that is, so, their individual. These are the two random variables is consideredwhere both the random variables are uniform. So, this dotted line as you can see here, this is the uniform distribution for both the random variables and it is uniform between 0 and 1.

Now, if we want to know what is the, what is the, distribution of their, of theirproduct and this product is said z. Then this line you can saythat this is their p d f for the, for the, product, and this line that you can see this is there, the p d f for that quotient. So, this can be directly, can be obtained from the expression just now which we have explained just now.

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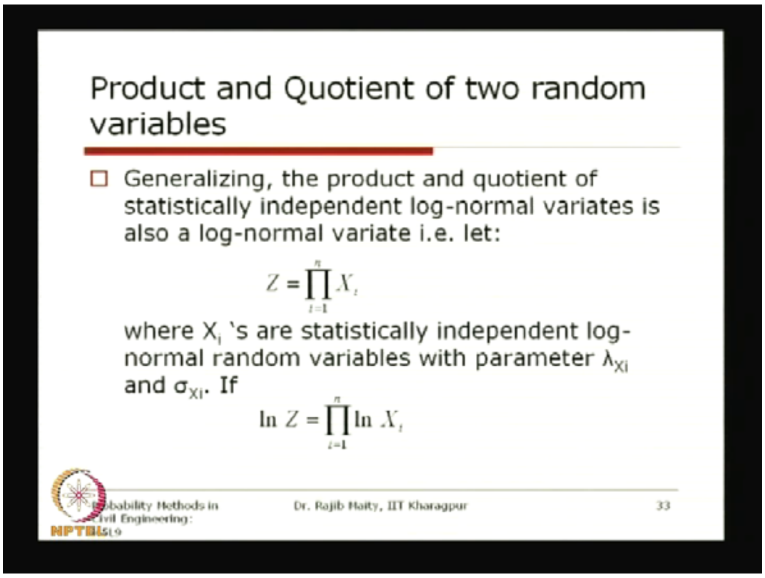

So, now, so, if we just want to generalizedsomething, that the product and the quotient ofquotient statistically independent; lognormal variate is also a log normal. So, here, we are taking a specific distribution again that, we are specifying that all these normal, all this random variables are log normally distributed with theirand their product as we are writing that, this is z; z is the multiplication of this, all these random variables from this x 1x 2 up to, up to, x n, and this x i are statistically independent log normal random variables with parameter lambda x i and sigma x i. Lambda x i is basically they are mean that is thelog of that z is equals to the product of the log of there all the a random variables.

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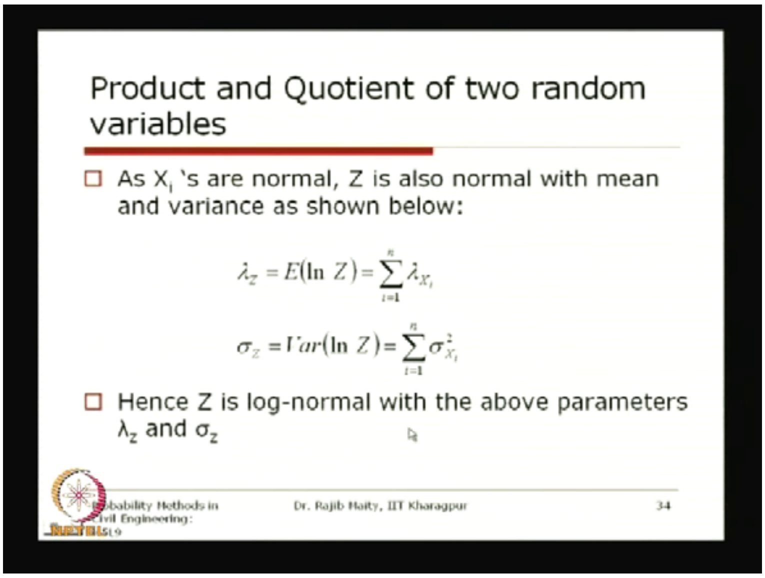

If we take that product, then as this x i's are normal and this z is also will be normal with the mean and variants as soon below. This is if they are normal, because we have taken the log. So, once you take the log of the log normal distribution, then it becomes normal that you know we have discussed earlier.

So, this now the final a parameter,that is, lambda z is equals to expectation of log of log natural of z which is the summation of the individual. Individual means that parameter the lambda x i and this is the standard deviation of this that their individual variances. So, this is the summation. This will be a square, this will be a square, the sigma z square. Otherwise, you have to take the positive square root for both this expression.

So, hence this z is also a log normal distribution with the above parameters, that is, lambda z and sigma zmeanthis lambda z is theparameter when this is the mean of this log term fromrandom variables and this is their standard deviation. This is the, this is the standard deviation of the log transformed random variables.

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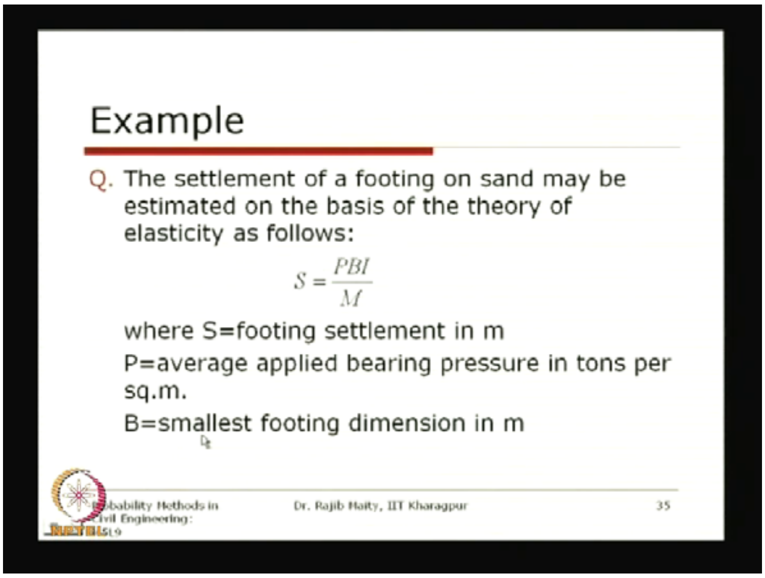

One small example we will take - the settlement of a footing on the sand may be estimated on the basis of the theory of elasticity as follows s equals to p b i by m - where s is the footing settlement in meter. P is the average applied bearing pressure in tons per square meter and b is the smallest footing dimension in meter.

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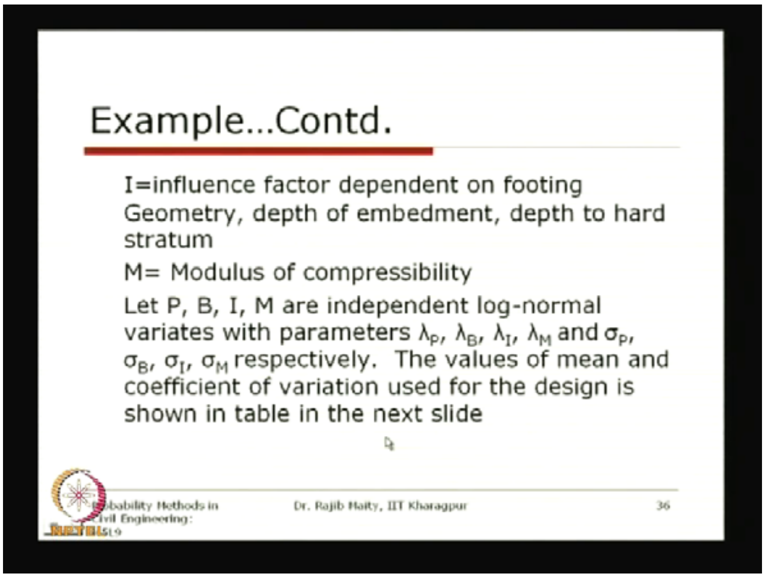

I is the influence factordepending on the footing geometry depth of an embedment, depth of, depth of embedment means how,how, deep it is from the surface depth to the hard stratum. This m is the modulus of compressibility.Let p b i m are independent log normalvariates with parameter lambda p, lambda b, lambda i and lambda m and their and this sigma p sigma b sigma i and sigma m respectively. The value of the mean coefficient of variation used for the design is shown in this next slidewhich here, that is, the, this isthe mean and the coefficient of variation for individual this parameters.

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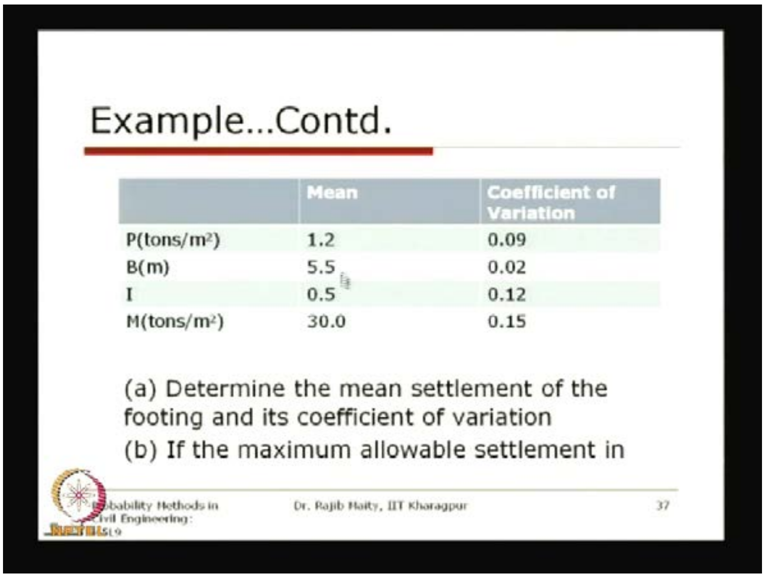

To determine the mean settlement of the footing and its coefficient of variation if the maximum allowable settlementin is, is, 15 centimeter. What is the reliability excessive,excessive, settlement? There is probability of no settlement.

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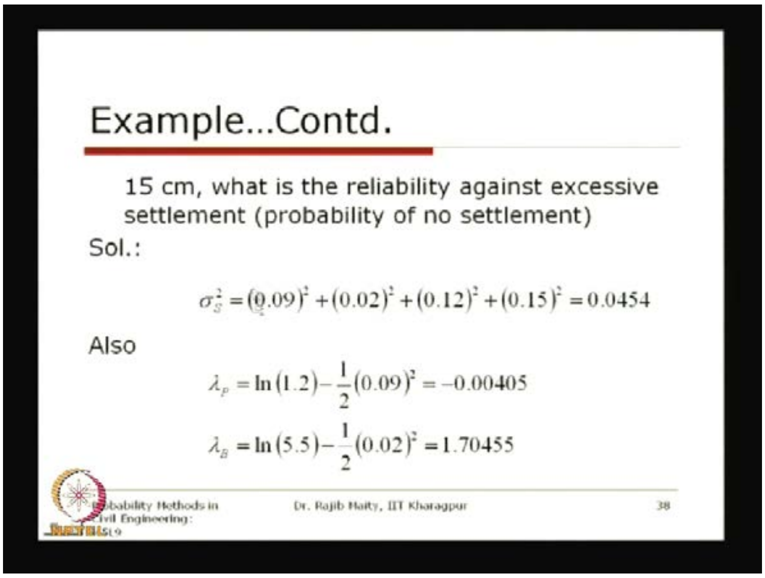

So, the as we have seen for this log normal variates, that is, the sigma s's square which is the both it is considering both that is. So, sigma s square is equals to the summation of their individual squares. So, this 0.0454 lambda p is equals to by from this expression.We will get that minus 0.00405.

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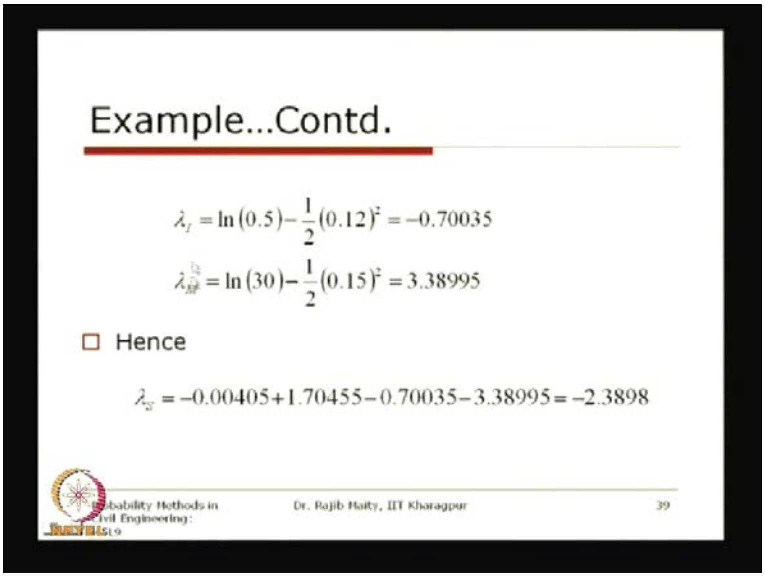

Similarly, lambda d and lambda i and lambda m of this parameters we will get, and then, so the, for that resulting random variables which is s shouldbe thesummation of all those lambda parameters. So, this is the, these are the lambda parameters for the individual random variables. So, this is the, this is the lambda for the finalrandom variable which is s.

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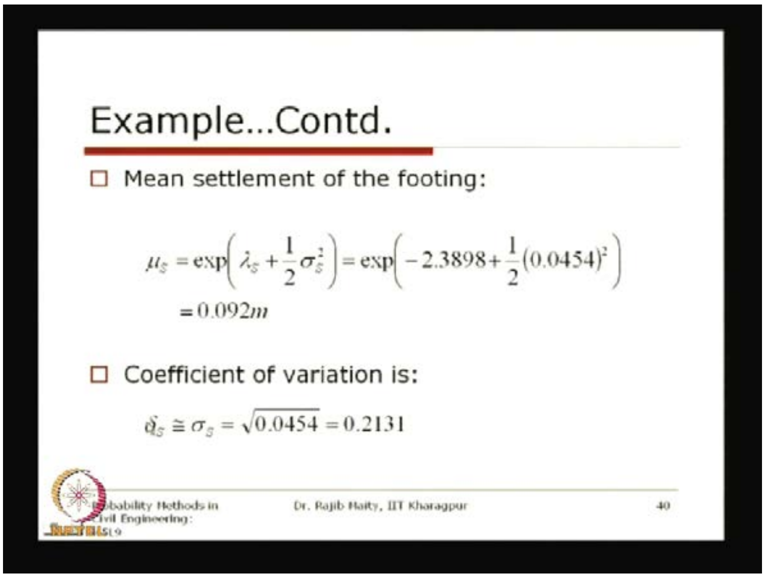

Andthe mu of this means settlement of the fore footing as you know from this log normal distribution is from this expression, we just refresh this lambda and sigma s this. So, this will the mean settle will be the 0.0, 0.092 meter, and the coefficient of the variation which is approximately equal to the sigma s and which is 0.2131.

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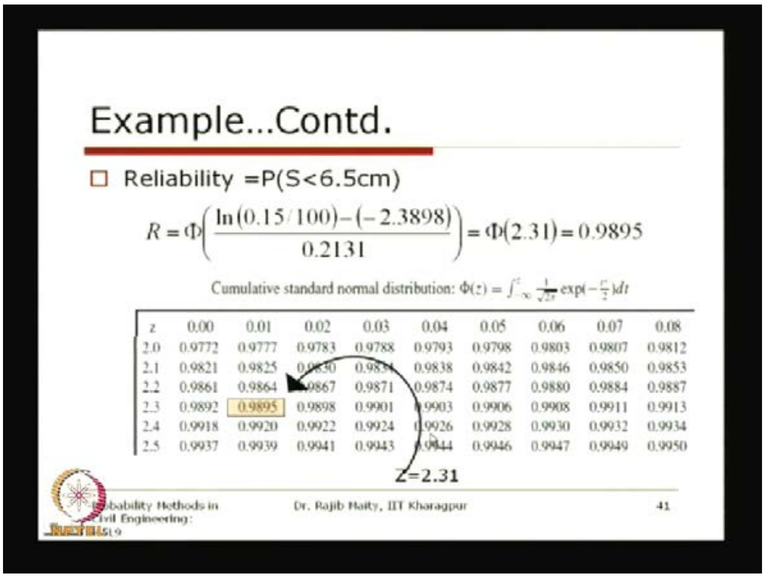

If these two parameters we get, then the reliability that, that what we are expressing that it should be, the settlement should be less than 6.5 centimeter. Then the reliability, so, we can just express, we can just find out the reduce variate by taking the log of their, this 0.15 by hundred this much meter.

So, this is 0.15. I think in the problem, it was for the,the,15 centimeter. So, this is 15 centimeter. So, this will be the 1.5 centimeter. So, if we just take this one, this is the correction 1.5 centimeter. So, this log of this 1 minus their mean divided by their standard deviation we get the two point three one and which is equals tothat 0.9895, which we can get easily from thecumulative standard normal distribution table.

So, the reliability is equals to 0.9895 for the problem that we have discussed. So, in this class onincluding the last class, both the classes we have, we have taken two, two, that functions of the multiple random variables particularly we have discussed one function from two random variables, and in the next class, we will continue and we will take up the two functions of two random variables. So, means what are the joint distributions of those two resulting random variables that will be discussing in the next class. Thank you.