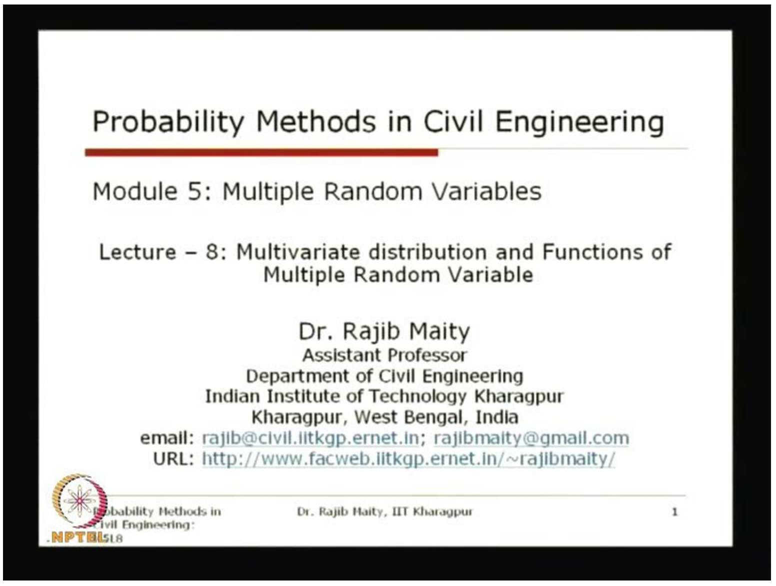
Probability Methods in Civil Engineering
Prof. Dr. Rajib Maity
Department of Civil engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 26
Multivariate Distribution and Functions of Multiple Random Variables

Welcome to this eighth lecture of this module on multiple random variables. And you know, we have started this multivariate distribution, and we will discuss about this functions of a multiple random variables.

(Refer Slide Time: 00:26)

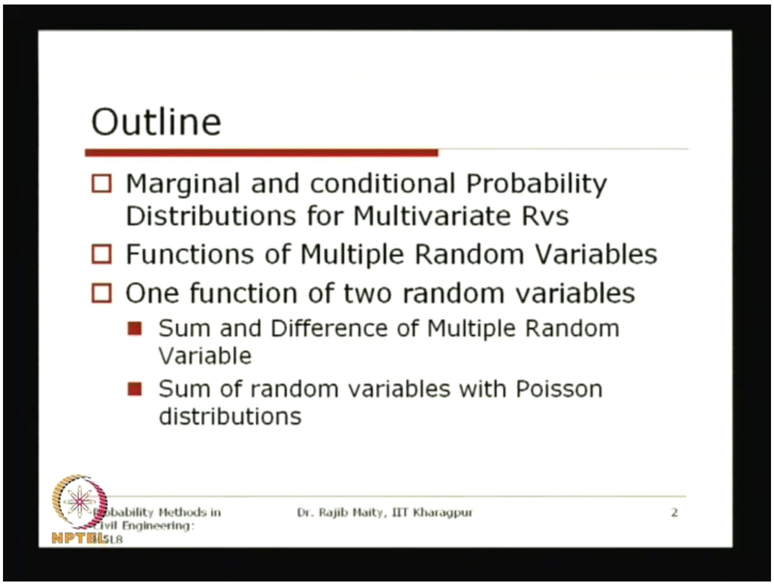


These multivariate distributions we started, and what we have seen is that the concept of this two random variables m, that is the bi variate cases can be extended if the number of variables involved in more than 2. And that we started the discussing in the last lecture, and there are some more part is pending.

So, we will take up that one first, basically the marginal distribution and the conditional distribution, these two things are pending. And we will also see one small problem on that, and after that what we will do, will just go to the functions of random variable, and we will take may be in this class, as well as in the next class. We will spend on that to discuss about the, what about the functions of the random variable.

So, you know that the functions of random variable, we have done it for the case of the single random variable earlier, but as a in this module we are discussing this multiple random variable. So, in this one, in case of the more than one random variable case, we will take up, and the functions, means the functions of both the random variables or all the random variables involved, that we will discuss.

(Refer Slide Time: 01:56)



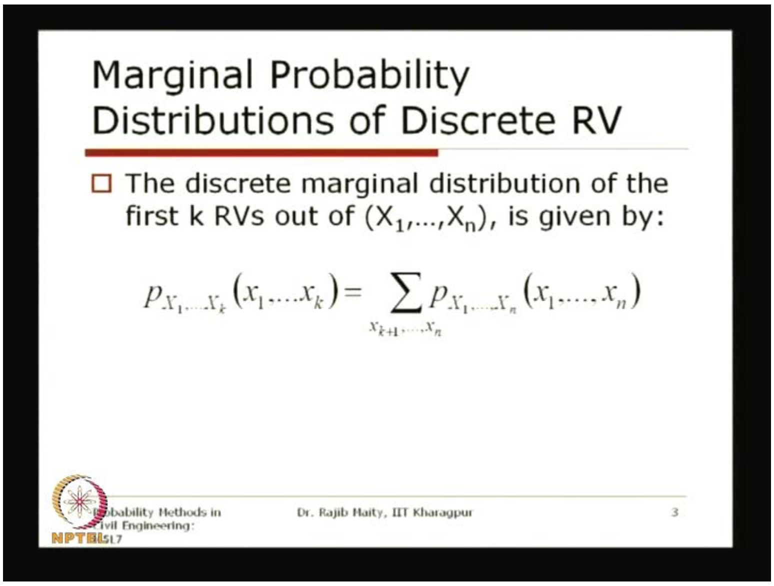
So, to start with today's lecture, we will start first with those marginal and conditional probability distributions for multivariate random variables. So, this two part was pending in the last class, we will start this one first, and after that we will go to this functions of multiple random variable, the concept from the single random variable will be taken forward through the Jacobi and all. And in this also, there will be someone function of two random variables, we will try to take up in today's class, and because there are other types is also possible, the other type is that two functions of two random variables.

So, it is not only that one function from two random variables, it is possible to have two functions as well from two random variables, but anyway today's class we will take up this one function of two random variables. Basically, we will put some put our discussion, will be oriented around the sum and difference of the multiple random variables, basically we will take up the bivariate case as we are doing for all the cases, you have, you know and that first the sum and their differences.

There are many applications you can imagine, for example the transportation engineering if you take, if towards a bridge from different source or from the different cities the highways are joining, so and the different roads are having different traffic volume. So, if you want to know the that probabilistic a behavior of the traffic volume at that particular point, so we have to go for the summation of the individual random variable, so we will take up that one. And there are some of the various specific cases, where you know it is possible to is possible to derive some simpler rules, so that easily we can do this, we can obtain the new pdf for the function.

For example, here may be we will try to cover to this class itself, that the Poisson distribution if we take up, this distribution different random variables, and if we add them up, then the resulting random variable what its nature and all, we will discuss through this today's lecture.

(Refer Slide Time: 04:21)



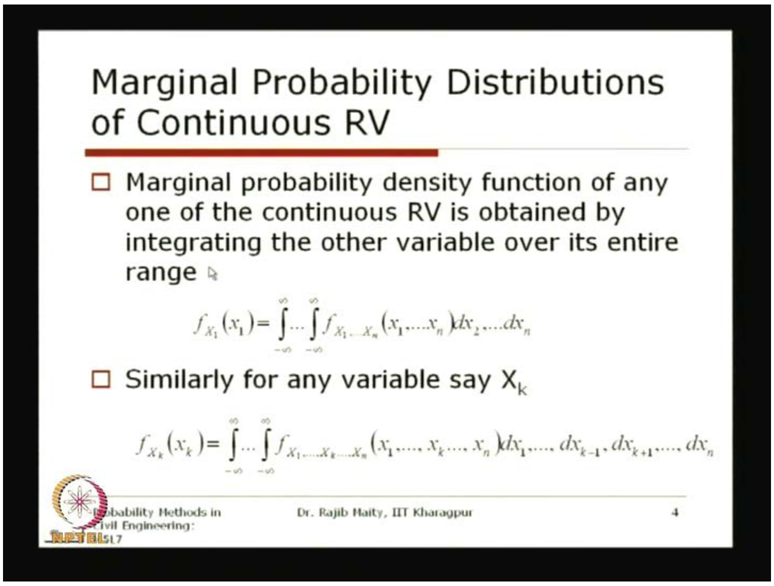
So, first we will take this one, these marginal probability distributions of the discrete random variables. Now, you know that we are discussing the general case, general case means when the number of random variables involved more than two, so suppose that there are n random variables, and these are discrete random variable, then we can get the marginal distribution of a subset of that total random variables.

Say that for example, here we are taking the first k, for example, so that discrete marginal distribution of the first k random variables out of X1 to Xn, so there are n random variables, obviously k is the subset of this n random variables. So then, in case of the discrete random variable, this marginal distribution is given by, so this P x1x2 up to xk will be equals to the summation of, the for all possible cases of the random variables from the xk plus 1 to xn.

So, if we do this one, do this full, this is a joint pmf, if you do this summation, then we will get the marginal distribution. So, basically you can see here that the concept is same, concept is same means that what are the random variables for which the marginal distribution we are looking for, so we are looking for the marginal distribution for the x1 x2 up to xk.

So, the remaining random variables is to be marginal out, in case of the bivariate distribution, bivariate case, we have seen that if we want to know the marginal distribution of one random variable, then we marginaled out the other one, so this marginaled out means that irrespective of the remaining random variable. What is a distribution of this, of the random variable for which the random, the marginal distribution we are looking for?

(Refer Slide Time: 06:39)

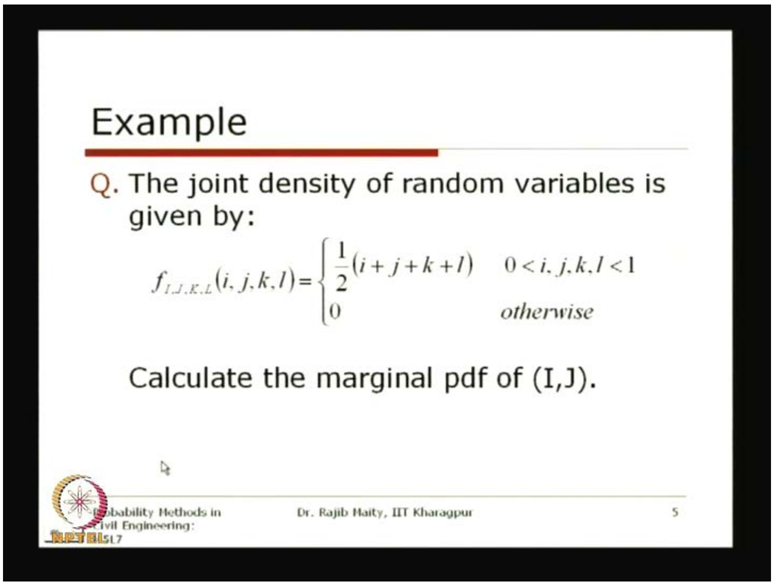


So, that means, that for this k plus 1 up to n, this has to be marginaled out, because we are looking for the marginal distribution for the first k random variable. So, in this way, we can get their marginal distributions as well, this is for the discrete case, we are going for this summation, and similarly the same thing we can go for the… if it is for the continuous random variable, then we can do the take the same approach. Now, we have to do the integration of the entire range of the of the remaining random variables.

So, here, one example is given here, for example that out of this n again, that x1, x2, x3, up to xn, so these are the n random variables are there, and we are interested to know the marginal distribution of the first one, that x1 itself the first random variable. Then what we have to do, we have to do the integrate for the entire range of the remaining random variable, so this remaining random variable means x2, x3, x4, up to xn.

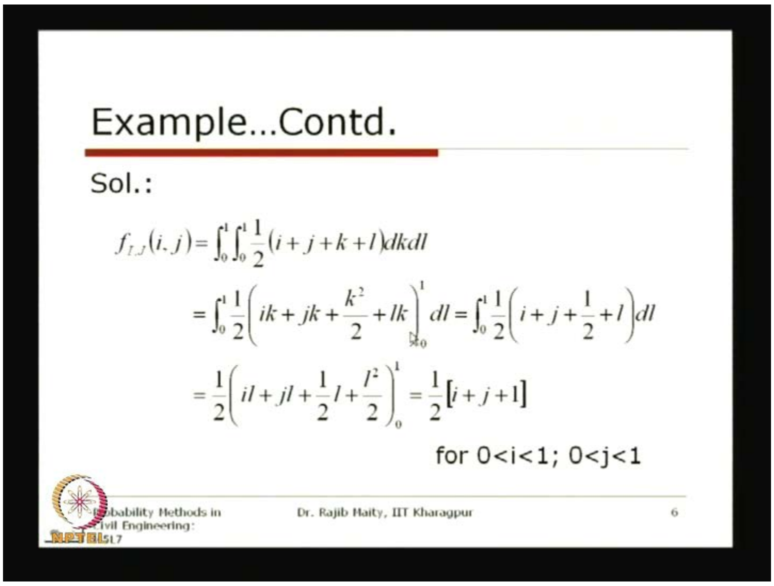
So, that is why we offer the entire range, we are just integrating out, that means we are marginaled out the other random variables to get the marginal probability distribution. Similarly, this is the first one, so in general we can get the marginal probability density function for the any general random variables, so that kth random variable, so we have to do this integration, that is the other random variables except that xk. The other random variable has to be marginaled out to get the marginal distribution for the random variable xk.

(Refer Slide Time: 08:04)



So, one example is given here, suppose that the joint density, this was we take up in the last class as well for some other problem, but here we will be interested to know what is the marginal pdf of some subset of the random variables. So, here, there are four random variables, are there i, j, k, l, this four random variables are there, so there joint density is given by this, so this half into i plus j plus k plus l, where all these random variables have the range from 0 to 1. Now, if this is the thing, then we are interested to know what is the marginal pdf of i and j, so we have to just integrate out that k and l for their entire range, that entire range is that 0 to 1.

(Refer Slide Time: 09:01)

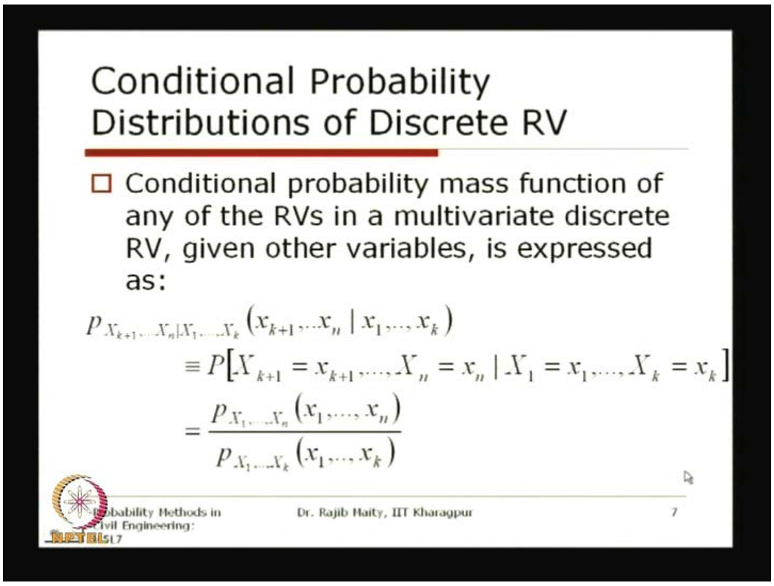


So, we have to go for the integration, and if we do this integration, that is f, i, j for this one, we have to integrate out that k and l, so here for all the random variables, this range is same, so you have to if it is not, if the ranges are if the support of that random variable is different, then you have to take care of that range of this integration. That is for example, the first one we are doing it with respect to that k, so as we are doing it with the respect to the k and this k range is 0 to 1. Similarly, then after we take if we integrate out, then we are doing the same function with respect to that l, for the range of the l, which is also here 0 to 1.

So, we are doing this integration, and by this you know this is simple one, so if we put this integration limit, we will get the new function which is half i plus j plus half plus l. And again for this one, we do this integration with respect to l, and we get this from and after that we get the final form as half into i plus j plus 1, and here also this range for this i and j are 0 to 1.

So, now you see in this function, which is obvious that it is irrespective of whatever the value of this k and l, so here in this function k and l should not appear, because this is also mathematically also obvious, we have integrated out that one. So, this is the marginal distribution for this i and j, which is half into i plus j plus 1for the range of i from 0 to 1 and for j also from 0 to 1.

(Refer Slide Time: 10:46)



Now, the conditional probability, this also we have discussed in case of the bivariate one. Here, we are taking up that for this n numbers of these random variables, and first if we take that discrete random variable case, so conditional probability mass function for the pma, that conditional pmf of any of the random variables in multivariate discrete random variables, given the other variables is expressed as this. So, here, you see that the conditional distribution of the random variable k plus 1, k plus 2, up to n.

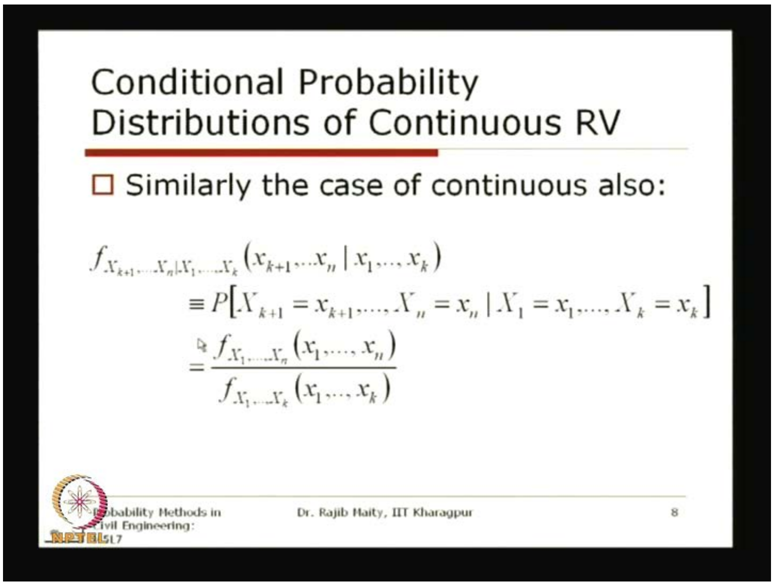
So, for this random variables, we are looking for the conditional pmf, in the condition is given, the condition is that, we know that x1 to xk, so for this subset, that is first to kth random variable is known, so based on that condition, what is the distribution for the remaining random variables, which is k plus 1 to xn. So, this is, now you know that the probability of this xk plus 1 equals to xk plus 1 in this way up to xn equals to xn on condition x1 is equals to x1, x2 is equals to x2 and there is some specific values for this random variable starting from the first to kx random variables.

Now, you know, for the bivariate case, if we just want to relate how to get this probability, it is thus directly what we can write, is that first of all this is the joint pmf for this all the random variables, starting from x1 to xn, so that should be, that the normalizing fact has to be with the marginal distribution, on which this distribution is conditioned on.

So, now, this distribution is conditioned on the subset of this x1, x2, x3 up to xk, so we have to get their marginal distribution, so that marginal distribution should be used as a normalizing for this joint distribution, joint pmf to get the conditional pmf for this xk plus 1 to xn, so this denominative here. You can see that this is the pmf for the x1to xk, which is the marginal joint probability distribution between x1, x2, x3, up to xk, so in this, we will get the conditional probability, it is must function.

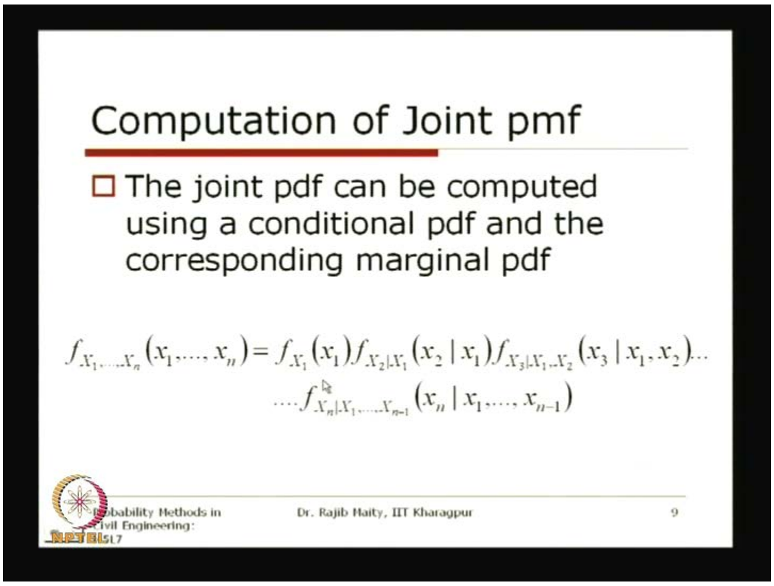
So, it is not only that, so here is one example is shown, means one specific case not example, the specific case is shown, where we are taking that k plus 1 to axn on condition of x1 to xk, so it is not necessarily that the random variables that we are considering, it should be one after another, it is not that case, you can pick up any random variable of your interest, and of condition on whatever. So, this k is a general notation, so you can even pick up that x1, then x4, then x9, so it should be conditioned on the remaining random variable, and accordingly, we have to say that which marginal distribution you should have before you can get this conditional distribution.

(Refer Slide Time: 14:21)



Similarly, if this is a continuous thing, then you have continuous random variable, then also for the k plus 1 to xn random variable, condition on that to x1 to xk, you have to get that joint distribution divided by the marginal distribution of the random variables from x1 to xk, that is on which this conditional probability distribution is conditioned on, so this is the marginal joint distribution of the remaining random variables.

(Refer Slide Time: 14:56)

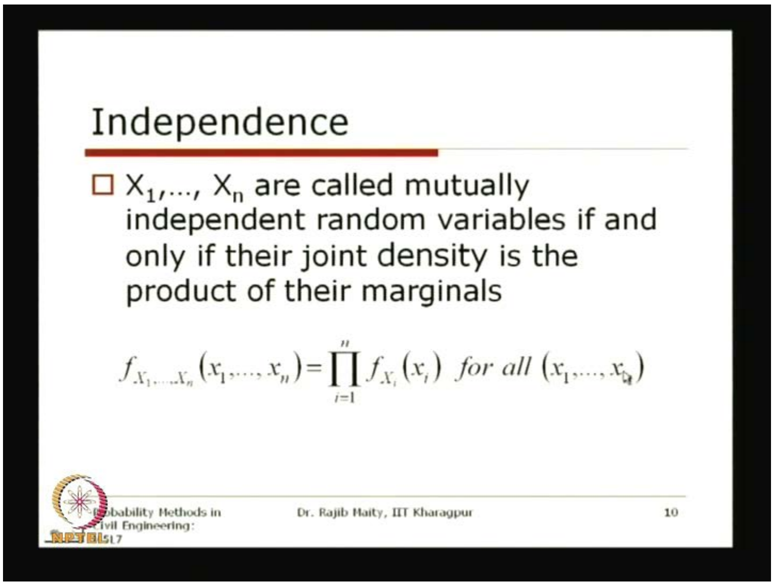


So, with this, one more thing that we have also discussed in case of the bivariate one, that we can get the joint distribution, if we know that conditional distribution and the marginal distribution, if we know then we can get the joint distribution. So, here is that one, is shown here, that is, this is the joint distribution, now this is the joint distribution between the random variable x1, x2, up to xn.

This one can be expressed in a general term of this that marginal of the x1 multiplied by the conditional of x2 given x1, that multiplied by the marginal of that x3, given x1 and x2, and in this way it will go on, so the general term of this series will be that, conditional distribution of xk given that x1, x2, x3, up to xk, and finally, at the last, the marginal distribution of xn, given that the random variable starting from x1 to xn minus 1.

So, this is the general term to get that joint distribution between this x1up to xn. You can recall that in case of the bivariate one, that is when the n equals to 2, so only two random variables are there x1 and x2. So, this expression, basically the first one if we just take up, that was the expression that we discussed, in case of the, when we considered only two random variables x1 and x2. So, when we are extending for this n numbers of random variables, this is the way the expressions get expanded.

(Refer Slide Time: 16:39)



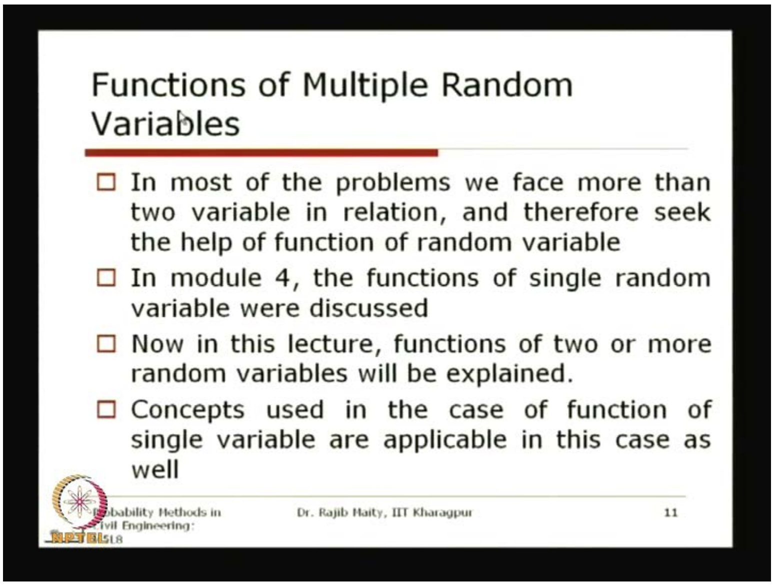
And finally, the independence between the n numbers of random variables, earlier we discuss the independence in case of two random variables x1 and x2, and there we have discussed that if and only if, of course that joint distribution between two random variablea can be, is equal to their product of their individual marginals, then we can say that this two random variables are independent.

Now, in case, now, if we just extend the same thing to the n numbers of random variables, then again it will be, the concept will remain the same, that is if the joint distribution of those n numbers of random variable can be expressed as their product of their individual marginal distribution, then we can say that this n random variables are independent to each other.

This is explained here, so if x1, x2, up to xn are called the mutually independent random variables, and this is possible if and only if their joint density is the product of their marginals, so this is the joint density from x1 to xn, and from this x1, x2, x3, which is equals to their product of their individual marginal, so i x, fxi xi, i valid from 1 to n, that means for all x1, x2, x3, up to xn.

So, with this, the properties that we wanted to know for the in general case, that is the for the n numbers of random variables that is over, and next we will take you through the different functions of this multiple random variables, first we will start with the two random variable case, and then again following the similar way, we will extend this for the more than two random variables as well.

(Refer Slide Time: 18:43)

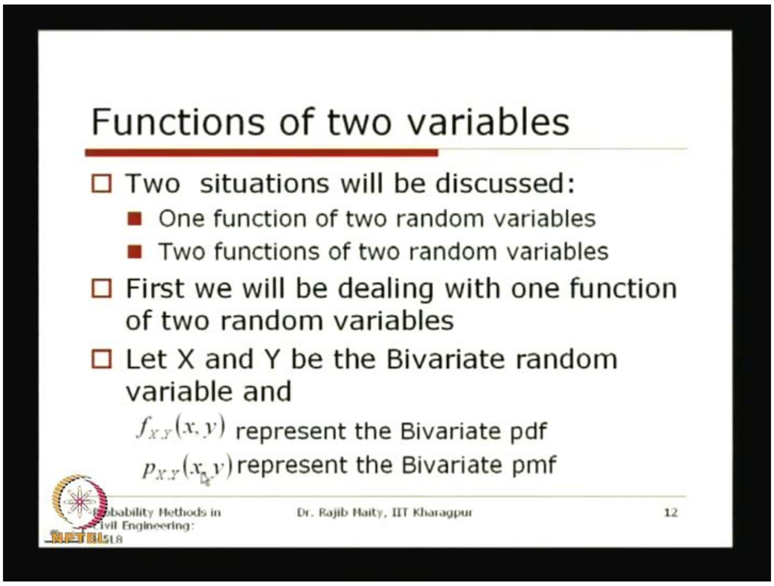


So, to start with the functions of multiple random variables, in most of the problems, we generally face more than two random variables are in relation, and therefore we need to know the functions of the random variables, so that this once this functions of this random variables are known, then how we can get the pdf or of their after that cdf of the functions of the random variable.

So, in earlier module, in module 4 particularly, the functions of the single random variable have discussed, so there also we understood the basic concept, that is if the x is a random variable, and we generate a function of that ax, that is if y equals to gx, then the function y that is also another random variable.

So, if that is also a random variable, then we know that how to know that different properties? That is different properties means, if we know the pdf of that function, then everything is known, so that we discussed. Now, here, now, in this one, in this module, we will be take up that two or more random variables, this will be explained. And the concept that we used earlier in case of this functions of single variable, is are also applicable in this case as well and that we will take you through one after another.

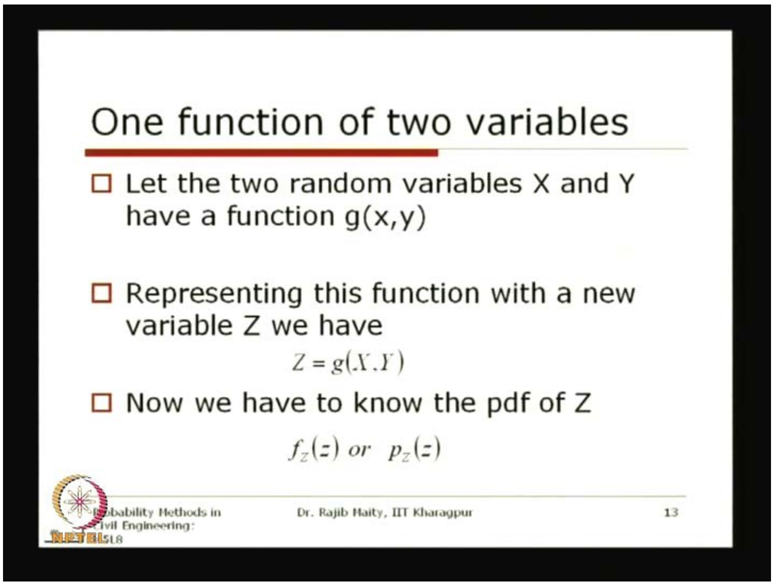
(Refer Slide Time: 20:19)



Basically, here the two situations can happen, two situations means that as you know that we are first of all we are taking the bivariate case, so for the two random variables, we can have one function or we also may have two functions of the two random variables. So, first of all we will take up this one that is one function of two random variables. So, one functions of two random variables means here, suppose that two random variables are there x and y, then the x plus y is one function, so which is using both the random variables, so x plus 1 x minus 1 like this.

So, for this discussion, you know that this fxy is the represent of bivariate pdf, and the pxy represents the bivariate pmf, we will use this one to know that what is the pdf or pmf as the case, may be for their functions.

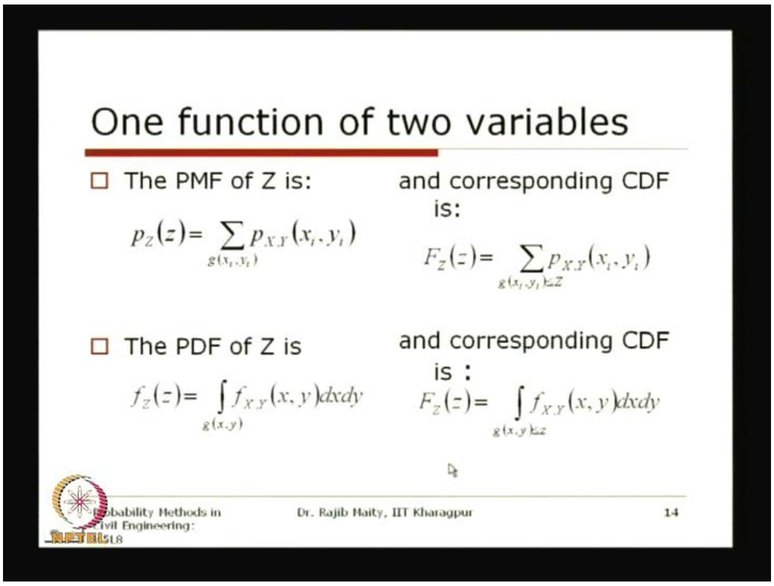
(Refer Slide Time: 21:17)



So, let us consider that there are two random variables, which are that x and y, and this x and y is having a function, which is represented by the gxy, so g is the functional form, and which is basically giving another that function, that gxy itself is a random variable. So, suppose that gxy is equals to z, now we are interested to know the properties of this z.

So, if we know that pdf, that is that fz of z or if it is discrete, then it is pz of z, that is pmf. So, these two things we have to know, and what we know is that we know that joint distribution that is fxy, joint pdf or joint pmf that we know, and we have to, we are interested to know what is this fz of z or fpz of z.

(Refer Slide Time: 22:12)



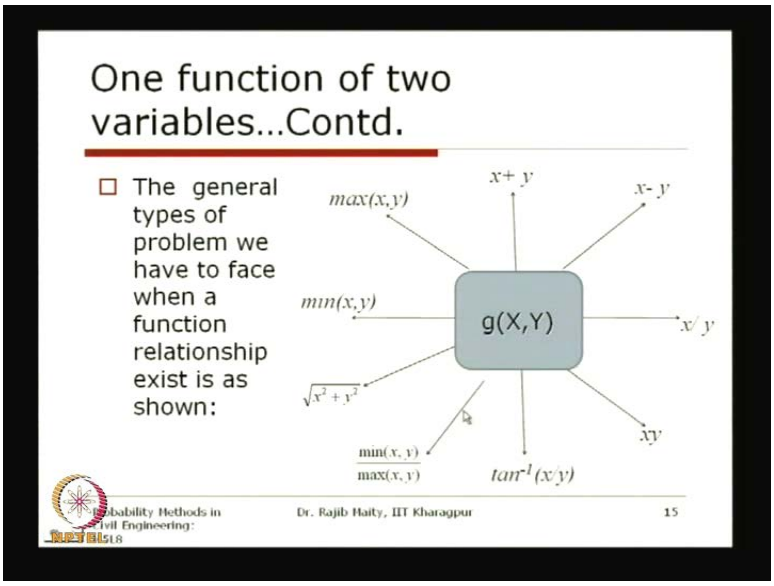
Now, if we want to know that this pmf of the z, then this pmf of z is explained, is expressed as this, that pz of z is equals to summation of this, they are joint pmf or all the possible values of its function that is g xi yi. Now, this i can vary, this xi yi is the combination for this all visual combination. And all visual combination, this is we are talking about the increase of the discrete, because only few specific points where the probabilities is defined.

Now, once we know this pmf, then that pdf is again just that for a particular value, it could be less than equals to that z, this will be obviously small z, because this is the small z, so if it is less than equals to this one, then if we sum it up, then we will get that their corresponding cdf.

Similarly, for the continuous case to get the pdf of this z, we have to integrate for the visual range of this function, that is g aa xy in this way with the respect to that dx and dy, so double integration I have to do as the number of random variables involved at 2, is the bivariate case that we are talking about now.

Similarly, once we know this one, if we want to know that cdf, then cdf should be, this should be less than equals that specific value of z, and over that range if we just integrate both this random variables x and y, so then we will get what is this corresponding cdf.

(Refer Slide Time: 23:59)

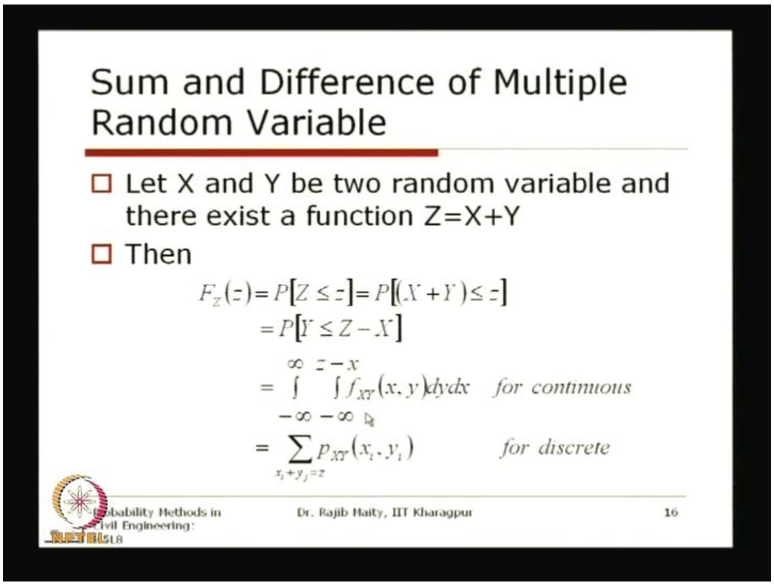


We will just see it in case of a particular application in a minute, that is we will just take the sum and difference first, which is also at, here are many applications in civil engineering, particularly in the transportation engineering, that we will talk, but before we go to that specific functions, we can just have a look that what are the different possibilities of the single function out of two random variables involved that is possible.

So, here are someone glimpse of that one, so that that the first what we have expressed is that the summation that is x plus y, this gx can be the their summation of the random variables involved, or it can be the difference, that is x minus y. Similarly, it can be the ratio, that is x by y or their product x y, some trigonometric functions of that one that is the tan inverse x minus ax by y, or it can be the ratio of the minimum of both the random variable divided by the maximum of both the random variable, or there could be some other algebraic operators square root of x square plus y square or simply the minimum of both the random variables or maximum of random variables.

We are not saying that this is the only possibility, there could be some other, but just to show you that some of the things that we may be interested, we will pick up this summation and difference first, to explain it in details, in the sense that just to show a pictorial view how or what should be the integration limit, on what we should go for the integration to get their behavior of the function, that is based on these two random variables, so that will take up first, and similarly the same concept can be used for the other cases as well, what we have shown just now.

(Refer Slide Time: 26:00)



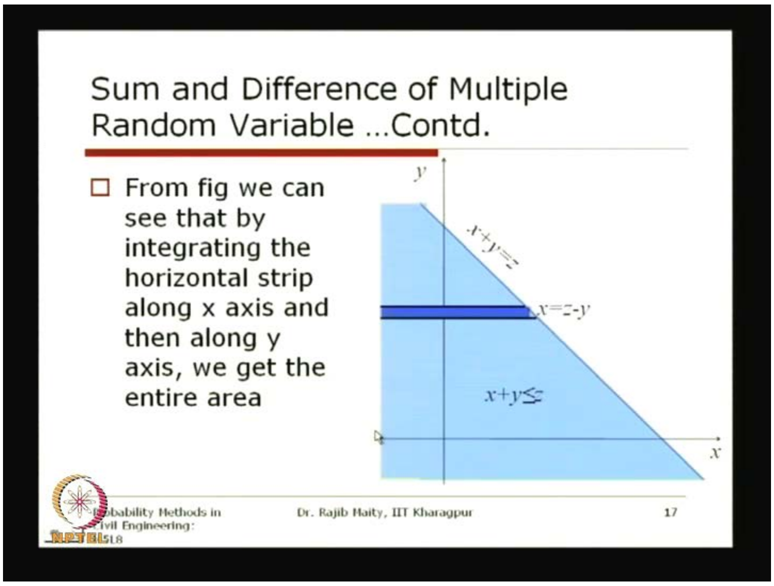
So, to start with, let us take that this sum and difference of the multiple random variables, so that let that x and y be two random variables, two random variables there will be 1 x and there exist a function, which is that z equals to x plus y. Now, if this is the function, then that Fz is equals, that you know from the cdf, that this is the probability, that the random variable z should be less than equal to the specific value of this z, now this z as we have already denoted that z equals to x plus y.

So, this means that this probability x plus y should be less than equals to the specific value, remember that as we told earlier also, this lower case letters are the specific values of the random variables, and the upper case letters are the random variable itself. So, this one we can just, so you can see this should not be the capital z, this should be the small z, basically we are just taking from this equation to here, so this will be the small z, which is the specific value of this random variable z.

So, the probability of the y less than equals to small z minus this x, and if you want to know this probability, then we have to do the integration over some range of the plane xy, that will explain in a minute. So, that range, now will show that how it comes from this minus infinity to this z minus x, and this is with respect to the y, and that should be integrated with respect to the minus infinity to plus infinity.

This is of course in case of this continuous, and for the discrete, for whatever the possible case is, where that xi plus yi is equals to that z or is less than equals to z, for that all those visual combination of this xi and yi we have to add up this pmf to get there that cumulative probability distribution.

(Refer Slide Time: 28:17)

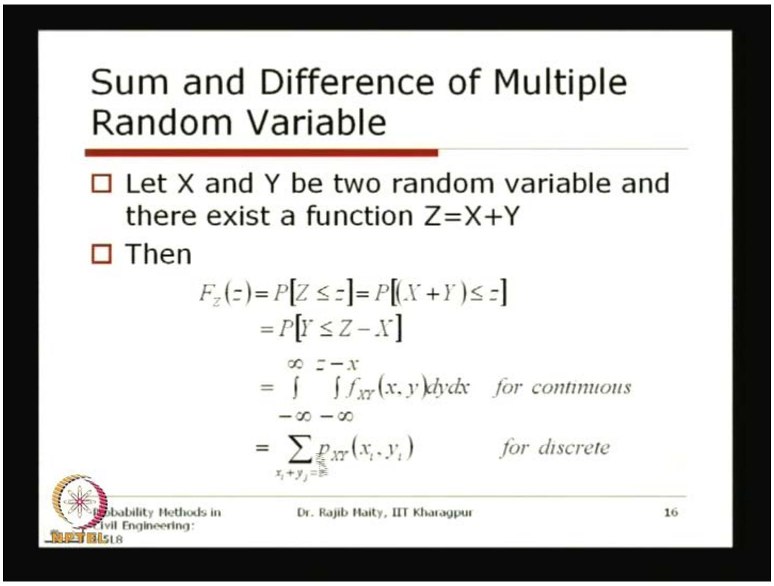


So, now, to see this range here, one some pictorial representation is shown here. That from the figure, you can see that by integrating the horizontal strip, that is horizontal strip that we have taken out, so this line that you see, this is basically showing you that x plus y equals to z. Now, this shaded area, which is light blue including this strip as well, this area is nothing but, which is the x plus y is less than equals to z.

Now, to get this cumulative distribution function, we are interested in this area, so as I told that x plus y should be less than equals to z, it is that area we are talking about. So, probability over this one should be integrated, now this obviously we are talking with respect to the continuous random variables, and if it is a discrete then, what will happen? In this shaded area, whatever the points are there, points means the combination of this xi and yi that we have to add up.

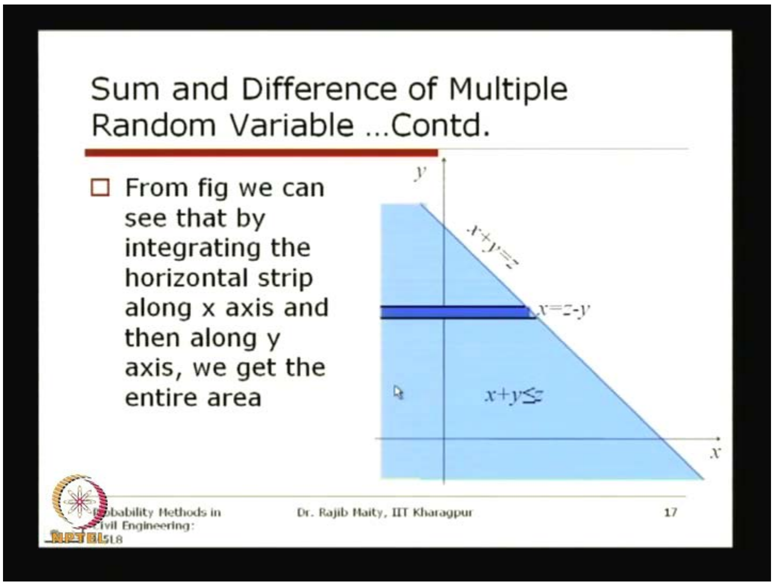
So, now, there are two possibilities, you can take an a strip like this, so for this strip, basically this area going up to the minus infinity, so this strip is basically from the minus infinity to that x equals to z minus y. Now, this strip if you just indicate out with respect to the y, if you integrate from this minus infinity to plus infinity, so that is eventually covering the full shaded area to get this integration.

(Refer Slide Time: 29:59)



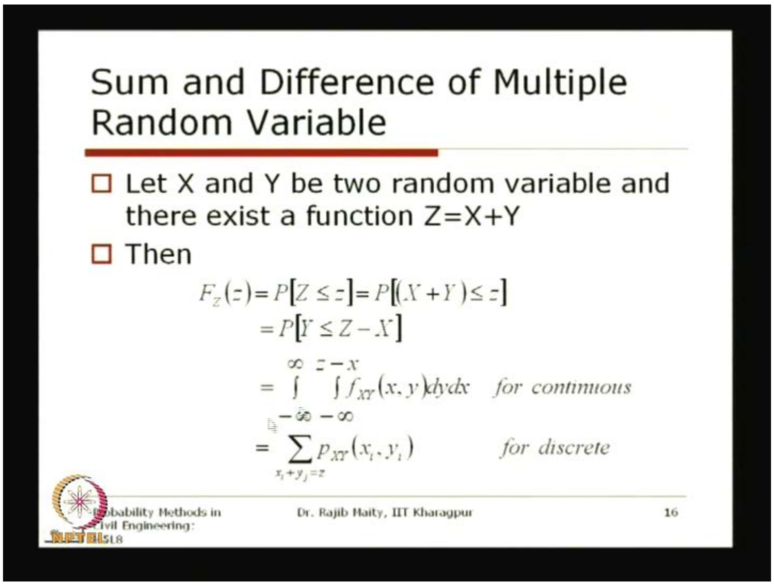
So, for the x, the limit is from the minus infinity to z minus y, and for the y, the limit is from the minus infinity to plus infinity. So, this is what you have just shown, here as the indication limit for this x, so this will be dx dy, so for this x, so this should be dx dy, for this dx, the limit is from the minus infinity to z minus x, and for the dy, the limit is from this minus infinity to plus infinity.

(Refer Slide Time: 30:22)

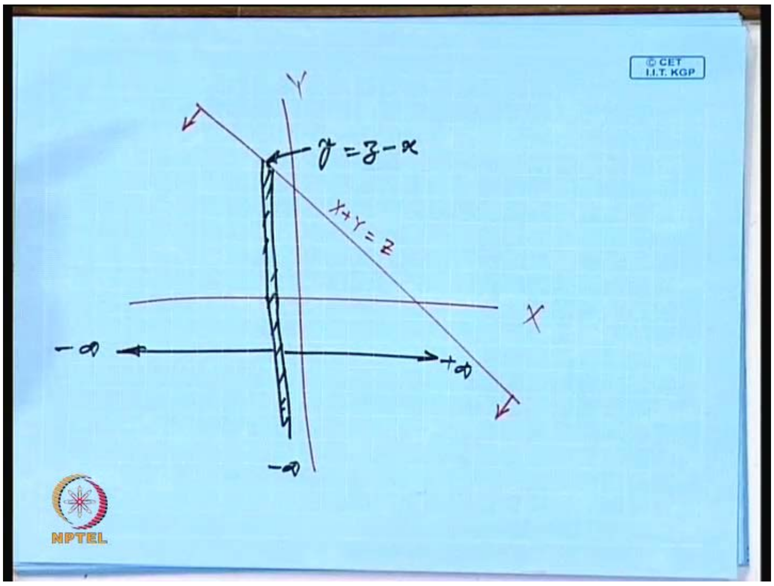


So, this is only one possible case that we have shown, the indication could have been also done taking vertical strip, so in this vertical strip, then you have to do the integration with respect to the y first, and in that case, the range will be from the minus infinity to that y equals to z minus x, so for that range, and for the ax that you can see that from the minus infinity to plus infinity.

(Refer Slide Time: 30:59)



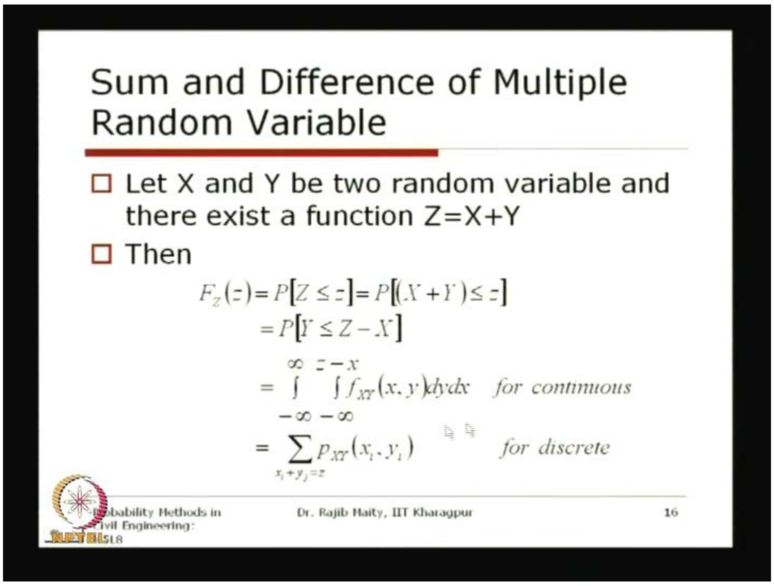
(Refer Slide Time: 31:06)



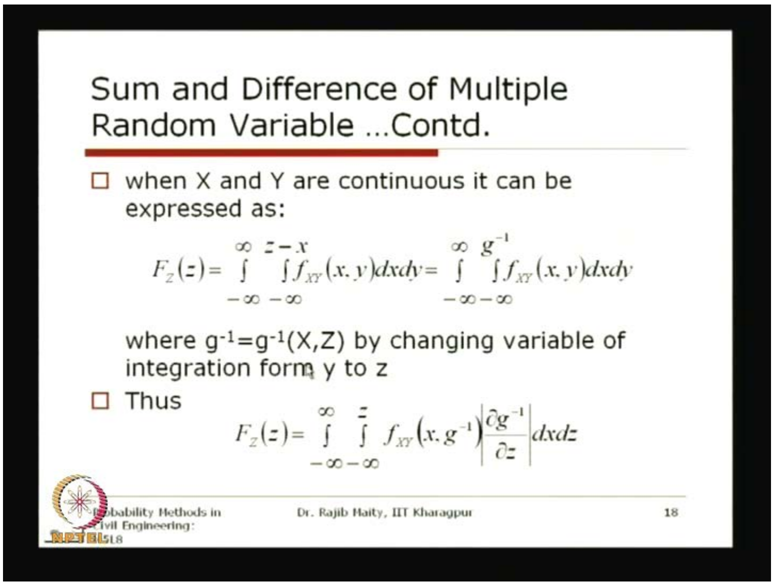
Basically, the diagram should have been shown taking the strip like this, because here the limit that you can see, it is written that z minus x, that so if you just see it here, what I am talking about, suppose that these are the two axis, one is this x and the other is the y, and what we are looking for is this line, where this x plus y is equals to your z, so x plus y less than equals to z, means we are just talking about this area.

Now, if you just take a strip like this, so this is basically going to this minus infinity and going up to the area. If we just take this is y, y equals to minus infinity to this one, to this point, this point is y, is equals to z minus x from this line. Now, this strip here basically taking it from the minus infinity to the plus infinity, so this is the range for this x.

(Refer Slide Time: 32:00)



(Refer Slide Time: 32:14)

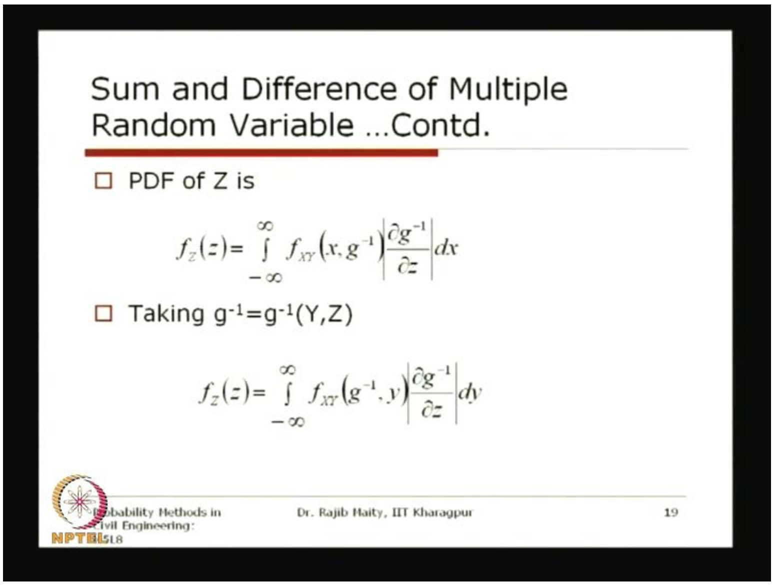


Basically, with respect to this diagram, if you see this equation, then it will be more clear, it is minus infinity to z minus x, and for the x, it is minus infinity to plus infinity. So, this way if you do this one, then you will get, when the x and y are continuous, it can be expressed as from this minus infinity to infinity fxy dx dy, and this one when you are talking about this z minus x, which is refreshed by this y, that means the g inverse, the function inverse that we are talking about, where this g inverse is the g inverse of xz.

By changing the variable of integration from y to z, what we can get also is that this fzz from minus infinity to plus infinity, from this minus infinity to the z, this function with respect to this joint density, taking this x and g inverse multiplied by their this partial derivative with respect to z of that function dx dz. Now, may be it may not be that straight forward to understand at this stage, because basically this is a Jacobian that we talking about, and this will be, and why we are just talking about this, as you know that the way the integration that we are taking, and integration limits we have discussed, which will be more easy to do indication in this way.

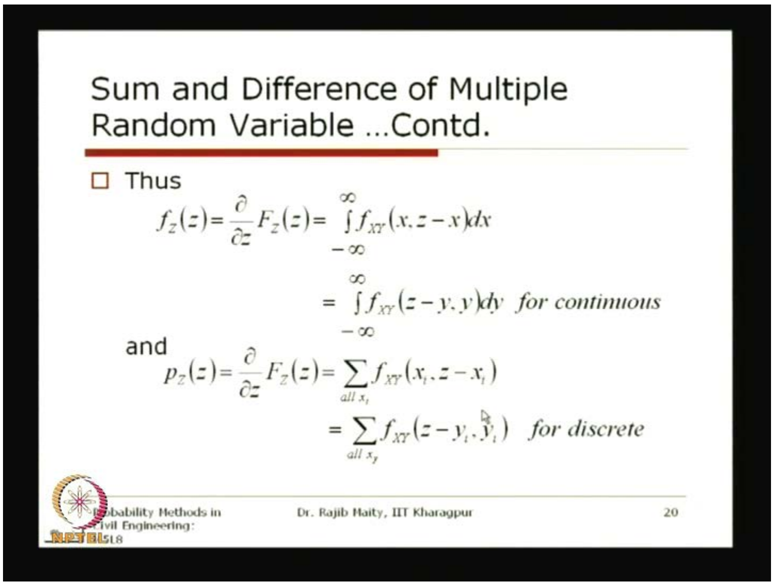
But, you know that this is a single function that we are talking about, next when you will go to this two functions or even more, then what will happen, we will see that this taking this jacobian will be more easier, and this one also we have discussed earlier when we are talking about this single random variable. And in the single random variable, we have seen that how the Jacobian can be can be useful and it will be even more useful when you are talking about the two or more function. So, that is the, what it will be more clear when we will be take up possibly in the next lecture the two functions out of this two random variables, that time we will just see it again.

(Refer Slide Time: 34:04)



So, for the time being, so this can also be expressed, there is a pdf, can also be expressed through this integration over the entire range with respect to the x by taking this Jacobian, that is dpartial derivative of g inverse z. And if you take this inverse equals to this, again you can do it with to respect to the y as well.

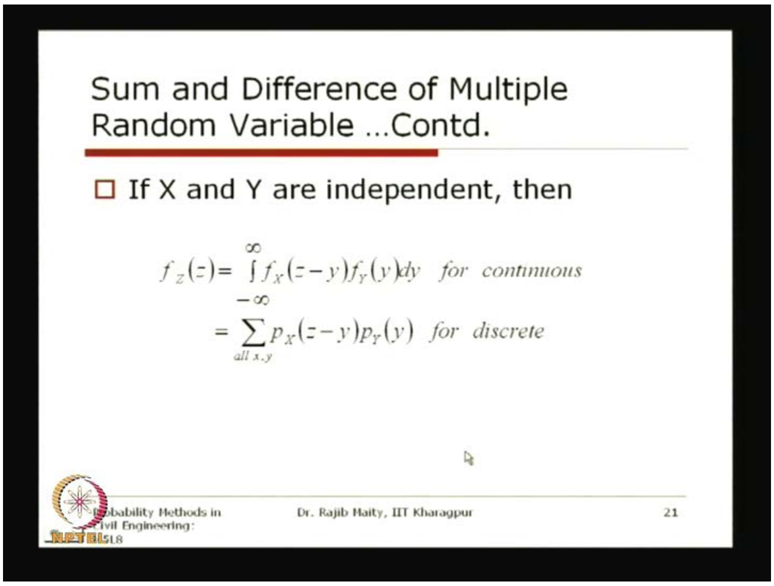
(Refer Slide Time: 34:29)



So, thus this fz can finally be, so within the pdf can finally be shown for this entire range of this dy, you can just replace this x as this z minus y, and also the opposite.

Also possible that is you can replace this y also that z minus x with respect to dx, also you can do similarly if it is that discrete random variable and then this will be the summation for all possible xi of this of their joint pmf. This should be P joint pmf and this is by replacing that yi, that is for all possible yi and this will be that pmf for that x and y, between x and y.

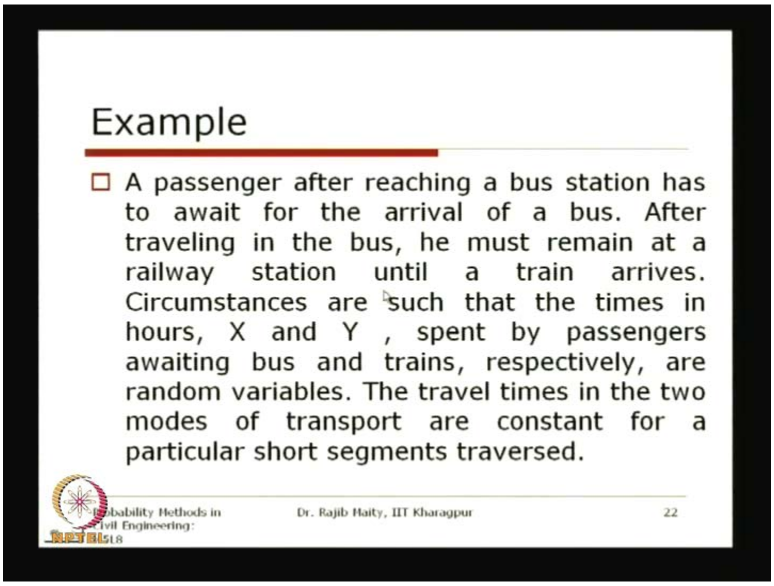
(Refer Slide Time: 35:23)



Now, if x and y are independent, and if you are you are interested for the summation, the distribution of their summation, then this can explained to be minus infinity to plus infinity fx z minus y multiplied by fy z minus y dy, because you know that their joint is now the product of their individual marginal.

So, fxx and the x is replaced by their z minus y. Similarly, for in case of the discrete random variable, this will be for all possible xy, we can just write that for this Px of this z minus y, and Py for this y. For both this cases, that is for that continuous and the discrete, you can replace that y and do this instigation with referred to the x also, that you can understand from the previous slide.

(Refer Slide Time: 36:21)



So, now, we will take of one example, and this example is based on the traffic control. Traffic control means here that there is some waiting times, and for between two destinations the passengers are travelling from one point to another point, where two mode of transport. Say the first segment they are travelling by one mode of transport, say by bus, and the next one they are travelling by train.

So, if you just have to model their waiting time for the full sector, for the full stretch, in considering the time for the bus or the waiting time for the bus, as well as for the waiting time for the train, and a then there is a physic application, that the total waiting time should be the summation of this two random variable. So, this two random variables means, we are considering the waiting time for a particular passenger, for both the mode of transport or the random variables.

So, what is the total time that a passenger has to wait? So, this type of problem if you want to model, then the theory that we have discussed just now, that summation of two random variables is important. Similarly, so why we have taken up this one is that there is a waiting time, and this waiting time can be modeled through that exponential distribution, and so that can be easily shown here through some calculation.

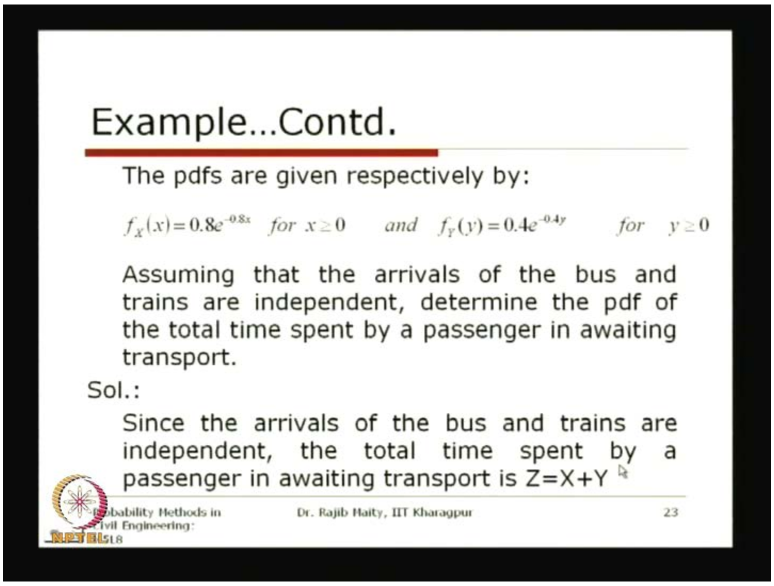
This can also be shown in case of the, suppose that two river streams are margin at a point, and we know the nature of the flows for both the tributaries and so what is after joining so what should be the nature of their distribution. Again coming back to the transportation engineering, so if suppose that there are two routes are connecting to two cities and both the cities, so the roads are coming and joining to a point, where it might be critical with respect to the traffic volume.

So, if I know that the nature of the traffic volume for both the roads, and so we can from there we can access what should be the critical condition at the junction by knowing the nature of their individual traffic volume for both the roads. So, these are the type of application that you can see in the different areas of civil engineering.

So, one such example is taken here for the passenger waiting time, so a passenger after reaching a bus station has to wait for the arrival of a bus, after travelling in the bus, he must remain at a railway station until a train arrives, so you can say now there are two mode of transport, one first one is the bus and second one is the train.

Circumstances are such that the times in hours, so times what is explained in terms of hours, this are the x and y, that has to be spent by the passenger awaiting for the bus and the trains respectively, so the waiting time for the bus is denoted as x, and waiting time for the trains is denoted by y, and these are obviously considered to be the random variable.

(Refer Slide Time: 39:48)

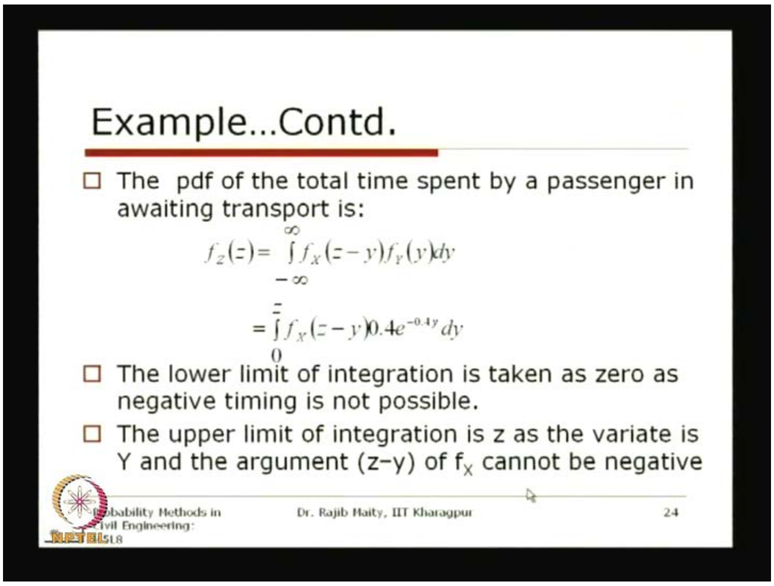


So, the travel time in two modes of transport are constant for a particular short segment traversed, and the pdf of their both this random variables x and y are given by this exponential distribution, which is that 0.8 exponential minus 0.8x for this x greater than equals to 0. So, this you know that lambda is equals to 1 by, so lambda is equals to 0.8, so the mean waiting time if you see for this the first mode of transport, that is for the bus, it is 1 by 0.8.

You know that for the exponential distribution the mean is 1 by lambda, and similarly for the train that the pdf is shown as the 0.4 exponential minus 0.4 y for the support, for this random variable y is also again greater than equals to 0, so the waiting time cannot be negative.

So, assuming that the arrival of the bus and the trains are independent, determine the pdf of the total time spent by a passenger in awaiting the transport. Since the arrival of the bus and trains are independent, the total time spent by the passenger in the waiting transport is z equals to x plus y, so this is the new random variable z, that the nature that we are looking for, so what should be the pdf of this z?

(Refer Slide Time: 41:19)

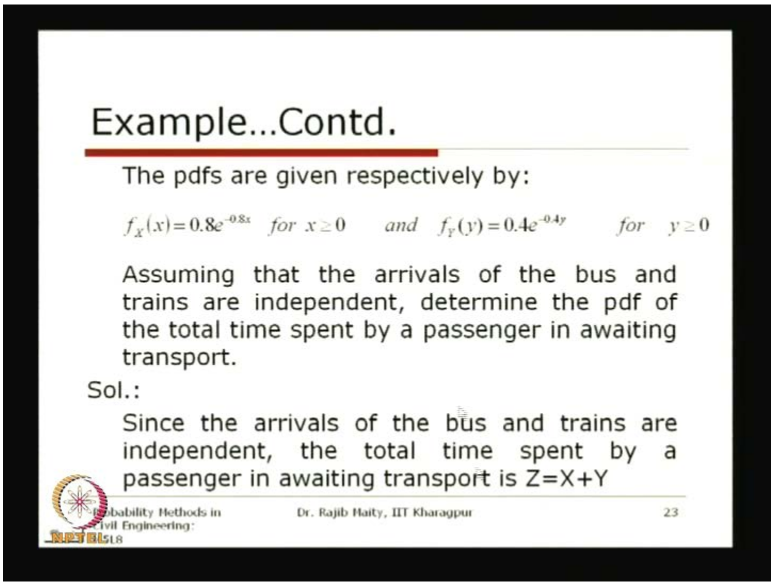


Now, as we have told that, now this is a little bit more even more simpler, because we have considered that these are independent, so as they these are independent, now their joint distribution, because I need to know that their joint distribution, the from, the theory whatever we have seen just now, that we know that we need to know what is their joint distribution for the two cases of the joint bivariate pdf, so that pdf we know, and from there we will calculate for the specified range, we should do that integration to know what is their nature, that is pdf or cdf.

So, similarly, here just what you have seen that, if they are independent, then we can do for this one, that is their joint will be the product of their individual marginal, that is fx and fy, so fx now is replaced by this y, that is z minus y, and fy is the. Now, we will just put that now this integration limit, so as we are talking that over the entire support of this random variables, so the entire support here is this, that is so when we are talking about this z the total time, so that the lower limit should be 0.

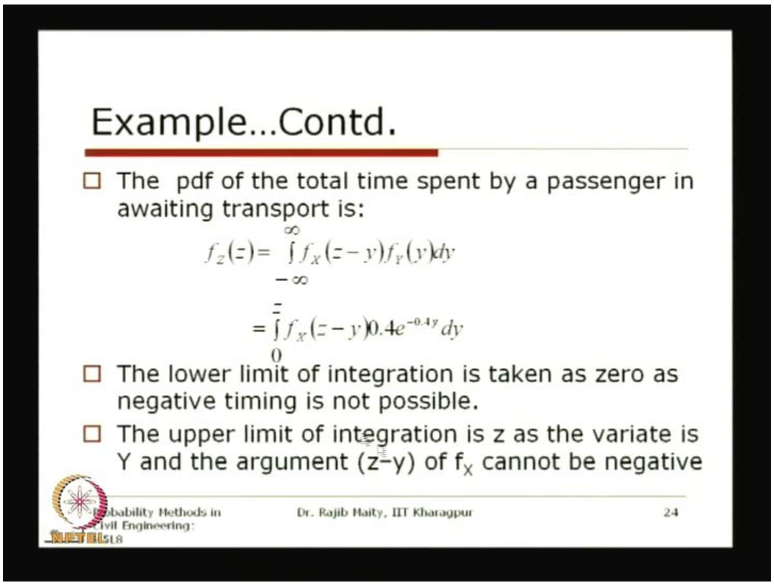
So, x plus y is giving you the z, so x plus y as it is giving you the z, then the obviously the lower limit is 0, because the waiting time for the passenger even it is for the both the transport, it will always start from 0, so that is lower limit is 0, as the lower limit for both the random variables are 0. Now, we have to see that what is the upper limit for this y against which you are doing the integration, so this upper limit of the y can never be more than z, because that this z is nothing but is equals to x plus y.

(Refer Slide Time: 43:30)

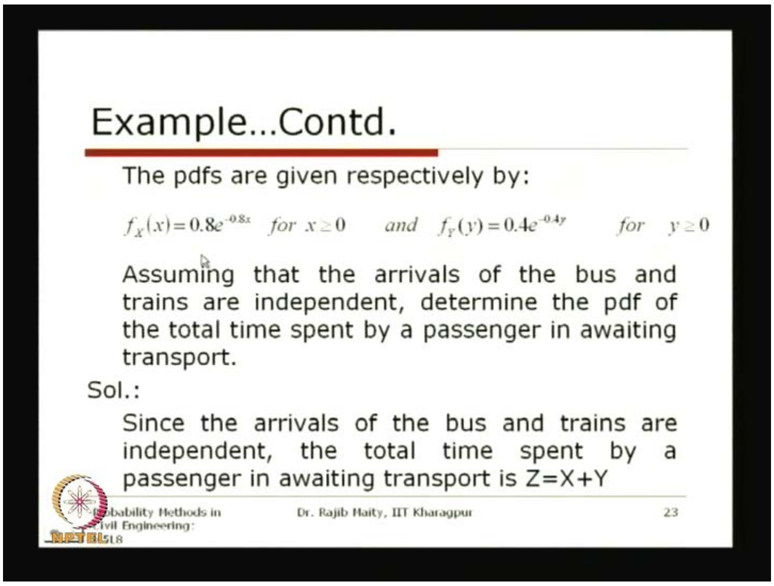


Now, even so minimum value that x can take is 0, so that means the maximum value that the z, that if it is the x, that is for that equation, that is the x plus y, so that possible range for this y, the upper maximum upper limit that the y can take is should be equals to z.

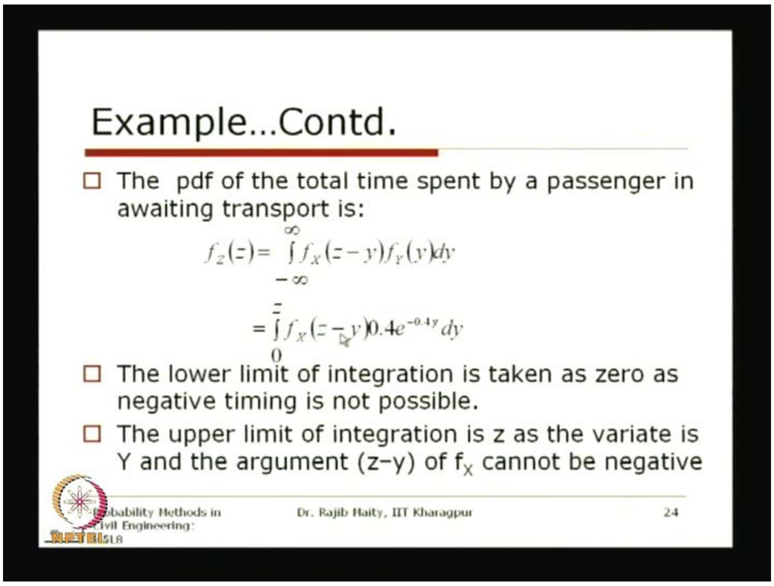
(Refer Slide Time: 43:44)



(Refer Slide Time: 43:53)

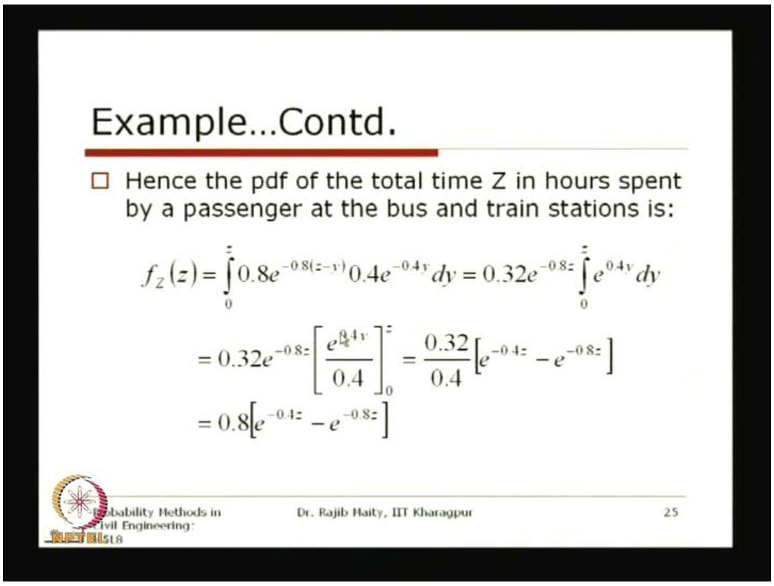


(Refer Slide Time: 44:05)



So, this is what is shown at the integration limit between this 0 to z, that and now the remaining part is straight forward, you have to just put this individual pdf for this x, that is 0.8 e power minus 8 in place of x. You have to write that z minus y and this is for 0.4e power minus 0.4y, and for this range you have to do the integration.

(Refer Slide Time: 44:07)

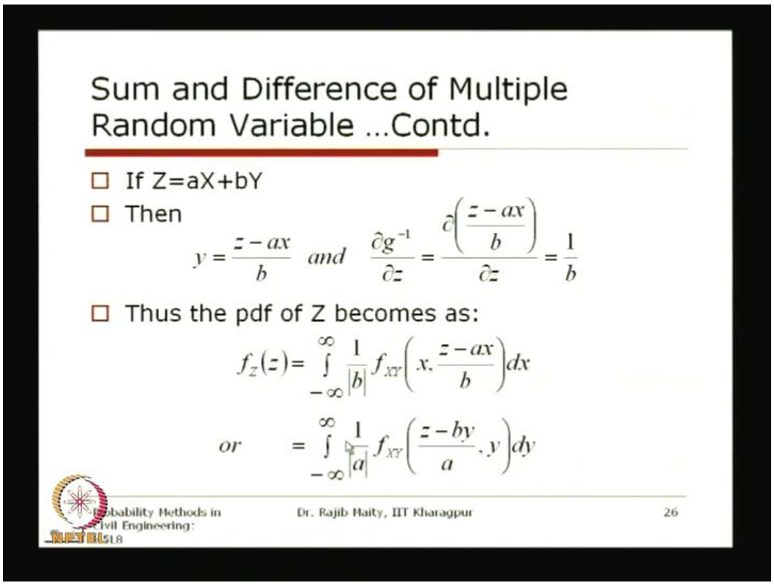


This is what is done here, that is for this 0 to z 0.8 e power minus 0.8 z minus y, which is multiplied by the other marginal pdf, that is 0.4 e power minus 0.4 y. And if we take this one, just after following few steps of this integration, that is you can take out this 0.32 e power minus 0.8 z, and integration over this 0 to z e power 0.4 y, so this y you are just taking care the 0.8 y as well as this minus 0.4 y, so it gives us 0.4 y. And if you do this integration for this range, so it will come like this, and if you just put this one, that is 0.4 minus 0.4 e power exponential minus 0.8 z.

So, this 0.4 is taken out, so finally it comes at 0.8 e power minus 0.4 z minus e power minus 0.8 z, and the range of this z is of course from 0 to infinity. So, this is a complete definition of this z, of course this range is also important. That is z is greater than equal to zero, that you can understand, so as this is the pdf, and if everything this is alright, then you can do once this one as a integration from zero to infinity, and check whether the integration is coming out to be 1.

So, that is the basic requirement for this function to be a valid pdf. And as you know that, for whatever may be the value of this z, as the nature of this function is this, is always greater than equal to 0. So, first condition is has satisfied, second condition you can check whether integration from 0 to infinity is becoming 1 or not.

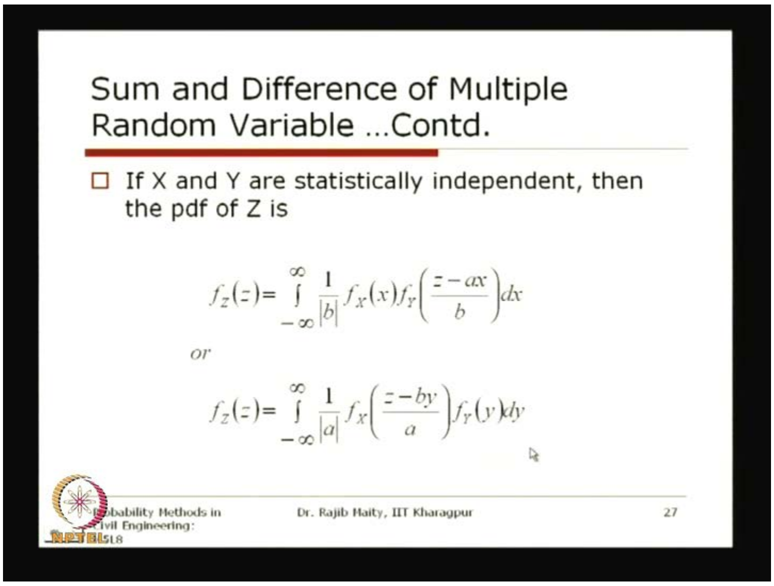
(Refer Slide Time: 46:11)



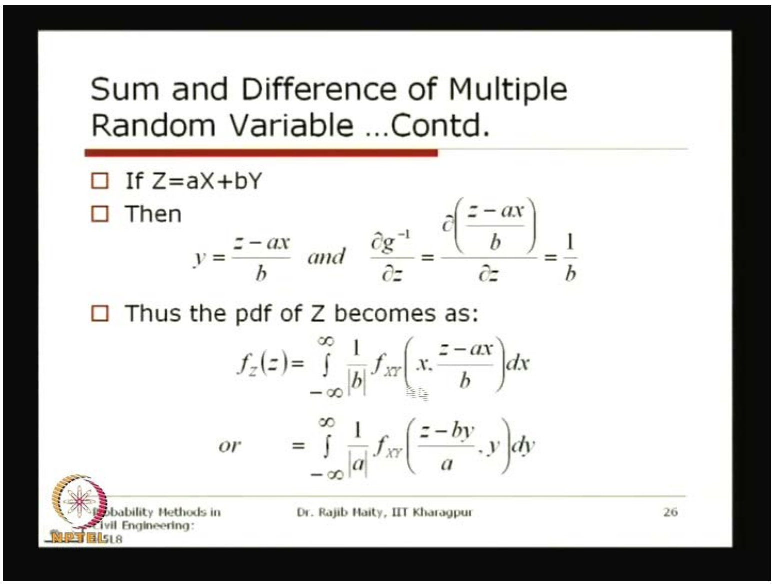
Now, if we just take it through this more general one, that is in case of this z is equals to aX plus bY, so there are, instead of simply we are adding up, we are just multiplying it by some coefficient, so aX by the coefficient a, and y by the coefficient b, then what we get that y is equals to z minus aX by b, so this one, this derivative that we get is equals to 1 by b.

So, from this again, from thus say that basic equation that is fz should be equals to that mode of this derivative, from this minus infinity to plus infinity fx of this z minus ax by b, which is dx or we can even replace that other one that is x equals to your z minus by by a, so in that this derivative will be equals to 1 by a, so this is then the integration will be with respect to y, so this also we can do.

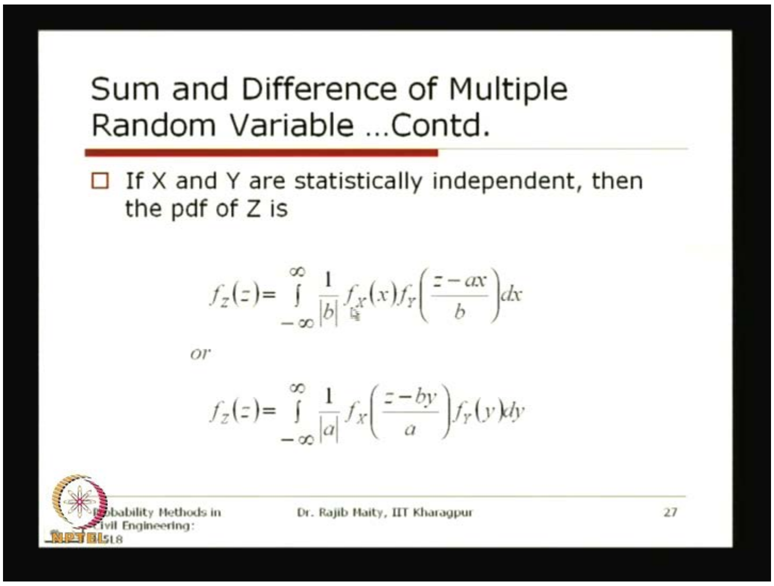
(Refer Slide Time: 47:15)



(Refer Slide Time: 47:28)

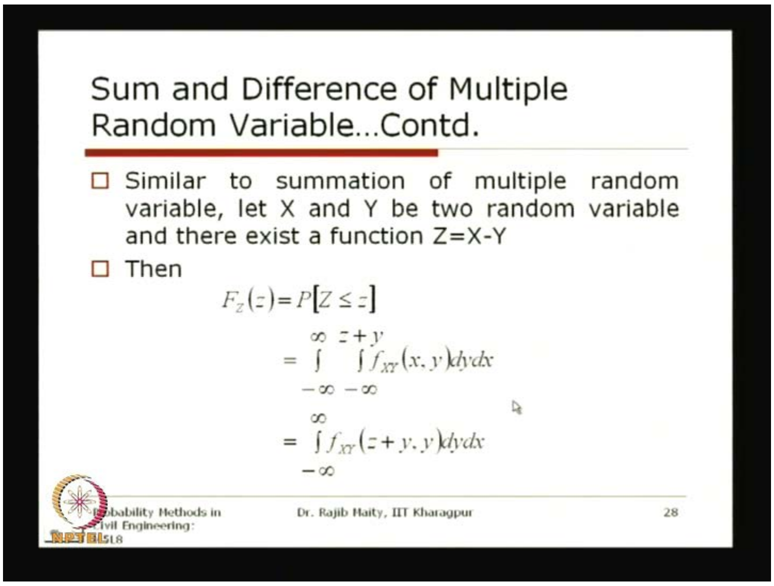


(Refer Slide Time: 47:32)



Then, if the x and y are statistically independent, then the pdf of z is again you know, that whatever the joint distribution that we are writing here in this form, you know that this joint distribution will be the product of their individual marginal, so we have to just write that fxx multiplied by fy of y, and y is nothing but z minus ax by b. Similarly, instead of replacing the y, we can replace the x also, then we have to do this integration with respect to y.

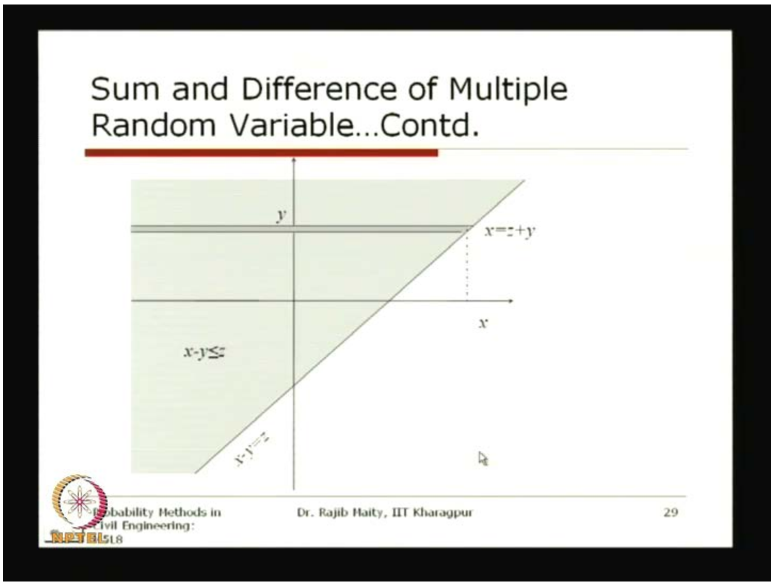
(Refer Slide Time: 47:55)



So, now, similar to the summation of the multiple random variable, let x and y are the two random variables, and there exist a function, which is that x minus y, so this is the difference between two random variables here. Now, of course, the x is greater than or whatever, this x minus y in general case that is z, is the difference between two random variables.

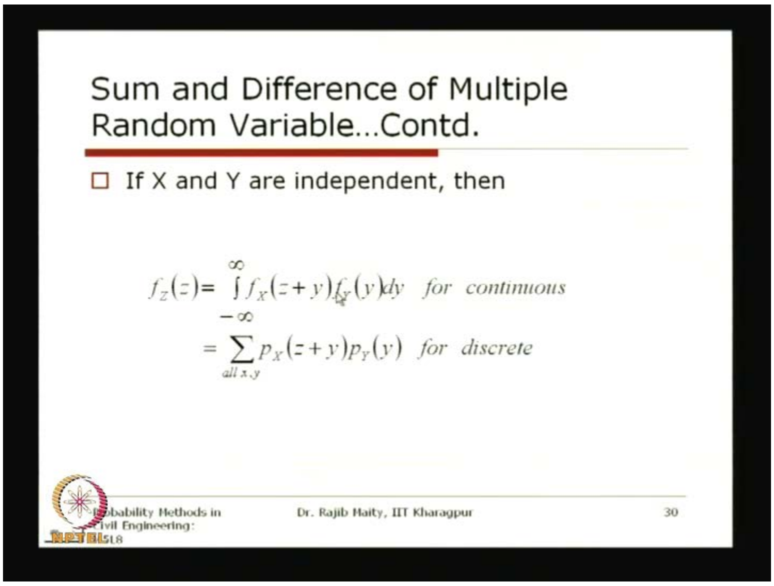
So, here also following the same procedure, that you have done for this summation is that, now here the limit that is that upper one for this, first with respect to dy, this will be minus infinity to z plus y. And after doing this one, we can get that from this over the entire range of this x, that is you are getting this joint distribution for this z plus y, so you are replacing that x and this is a double integration for their over that entire range of this x and y.

(Refer Slide Time: 48:53)



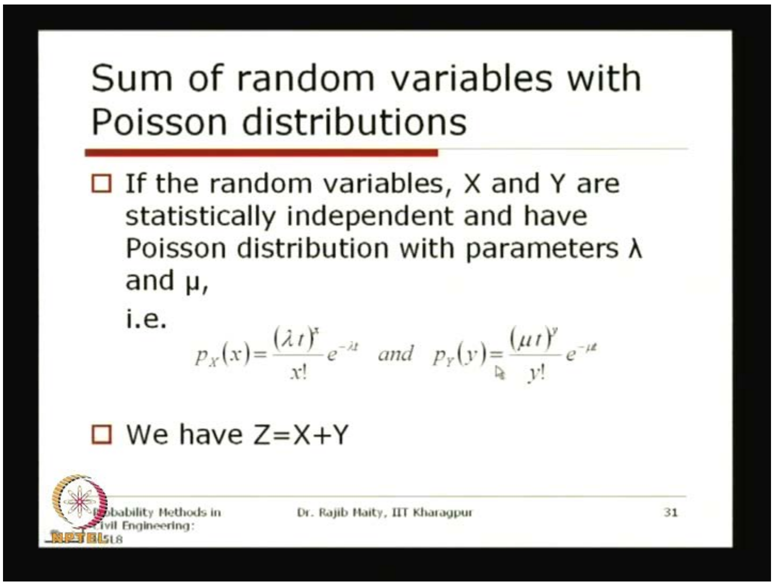
Here also means the pictorial representation, that is x plus y, so this line is basically showing that x minus y equals to z, and this side is equals to your that x minus y less than equals to z, so up to this you have to do the integrations similar to the earlier discussion. You can take a strip either in this direction or in this direction, and for both the cases suitable with the suitable integration limits, we can do the integration to get their pdf.

(Refer Slide Time: 49:23)



And again if they are independent, then we know that their joint distribution will be the product of their marginal, and when we are replacing this one, obviously it will be the z plus y to get that fx of x, and this fy of y, and then we will do the integration over the entire range of y in case of the continuous. And for the discrete also, then this for the individual pmf, we can multiply for all possible sets of this x and y to get that if the random variables x and y are discrete.

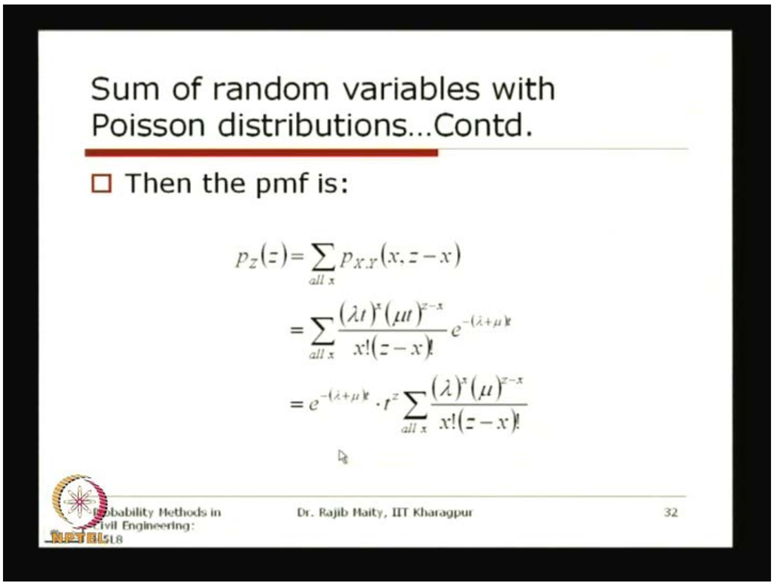
(Refer Slide Time: 50:03)



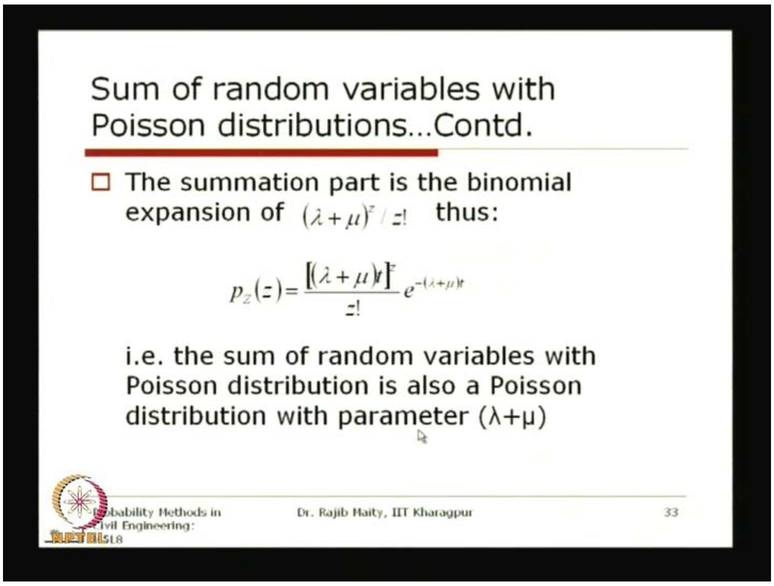
Now, as I started with that, if the random variables, in case of some specific random variables, if we just add them, then there are some specific results may come out. So, there are many such or few such distribution, where this summations are straight forward, first of all we are taking if the distributions are Poisson distribution, that is both the random variable follow the same Poisson distribution with of course it might have the different parameter, but the distribution nature is same.

So, if the random variables, which are x and y are statistically independent, and have the poisson distribution with the parameters lambda, and second one is the mu, then so that, you know that if it is the Poisson distribution, which is a discreet distribution and this Pxx is equals to lambda t power x by x factorial of e power minus lambda t, and for this Pyy equals to mu t power y by y factorial e power minus mu t.

(Refer Slide Time: 51:26)



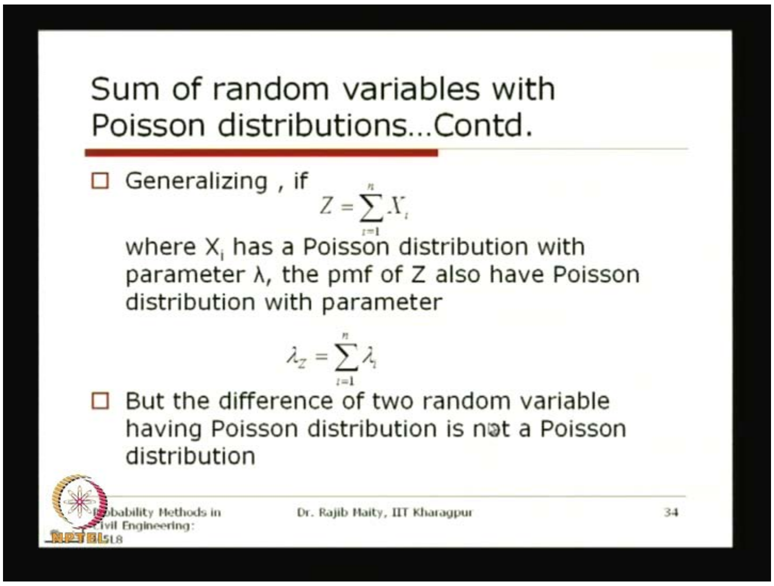
(Refer Slide Time: 51:46)



So, if these are the distribution, then if we have the functions like x plus y, then following the same equation that we have shown just now, for the in case of the discrete, obviously this will be the discreet distribution. So, we will just put this functions and just we get them for all visual x, we will be get the summation, then by this few step, we will get another distribution, which takes the form like this, that is lambda plus mu t power z by z factorial e power minus lambda plus mu t.

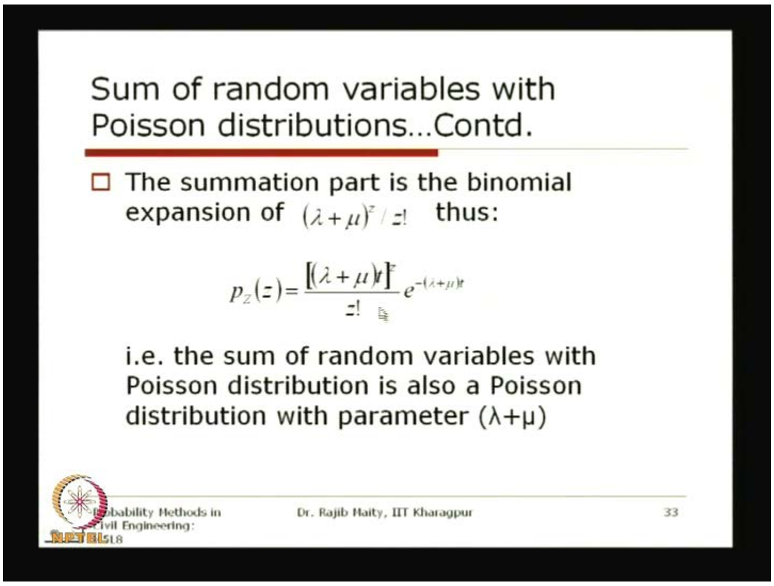
Now, if you see this one, this is also takes the similar form of the Poisson distribution, and of course that parameter has changed now; the parameter what is here is this lambda plus mu. So, the sum of the random variables with the Poisson distribution is also a Poisson distribution with parameter lambda plus mu.

(Refer Slide Time: 52:31)

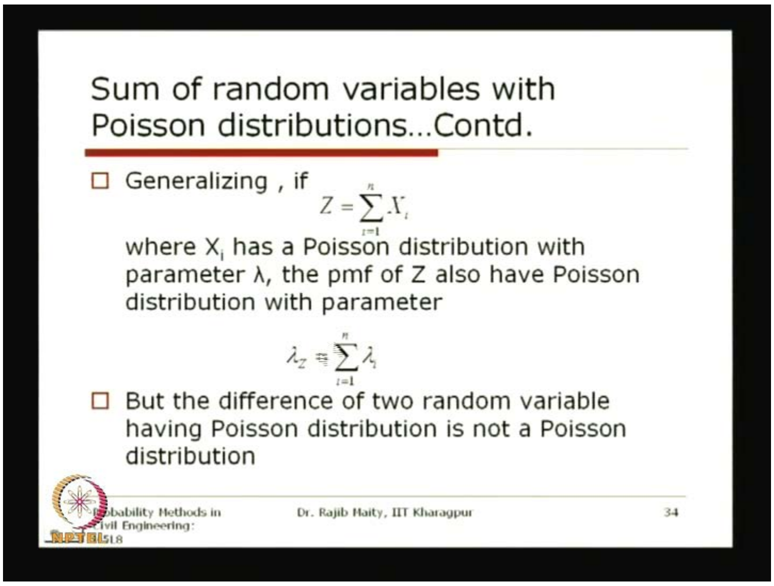


So, similarly, you can extend this one, so in case of this is two random variable, which are Poisson distribution, if there are more, say that n numbers of this Poisson distributions are there and z is nothing but the summation of all of them, then the this summation, this z will also have the Poisson distribution with the parameter and that parameter lambda z should be equals to the summation of all the parameters of this Poisson distribution.

(Refer Slide Time: 53:03)



(Refer Slide Time: 53:08)



But the difference of two random variables having the Poisson distribution is not a Poisson distribution, that also you can check whether the same so whether this type of form is coming or not, that you can check, so the difference is not a Poisson distribution. But, so far as the summation is concerned, we get the similar distribution, where the parameter is also the summation of the individual parameters.

So, in this lecture, we have discussed about the, we have started with this discussion for the multivariate random variable, first for their marginal distribution and conditional distribution and after that we have started with the functions of multiple random variable. And in this lecture, we have covered mostly the one function of the two random variables, and in the next lecture, we will take up the two functions from two random variables.