Probability Methods in Civil Engineering

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Lecture No. # 24

Properties of Multiple Random Variables

Welcome to this lecture.Basically, this todays lecture is the continuation of our previous lecture,where we have started the discussing properties of multiple random variables and in that, we started discussing some of them and today we will be continuing of the rest of the properties.

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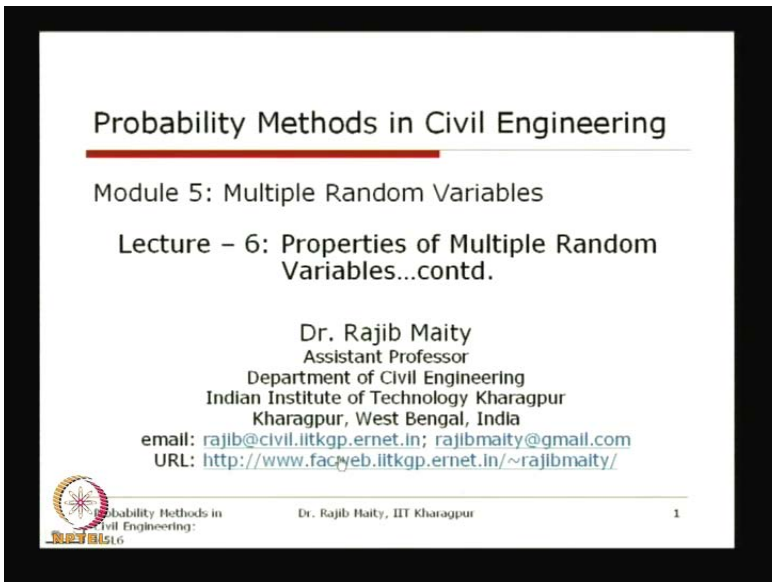

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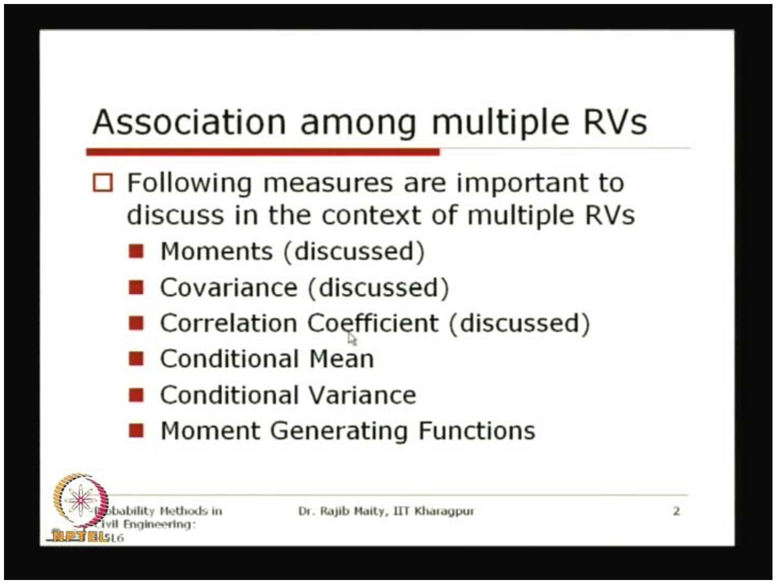

So, if you see that in the last class, we discuss about their moments and these all we are discussing in the context of the multiple random variable and this things we discuss earlier also with respect to the single random variable and you know that their same concept, we are using herewhenwe are discussing those basicproperties.

Say for example, when we discuss the moments also we have seen in the last class, that howthe concept is taken forward from the single to the bivariate case and from the bivariate to the more than two random variable cases.

Now, when we are means discussing about the multiple random variable that time, we have tothere are some more properties may come, which may not come in the with respect to the single random variable.

For example, thatthe conditional distribution we discussearlier now this conditional distributionsarein respect to the multiple random variables only minimum,we need two random variables where we can say that the distribution of one random variable, whenit is conditioned on the outcome of the other one.

So,those are the conditional distribution that we discuss, now related to those things there are some properties also we can we will come in this multiple random variable case.Andwe will bestarting ourtodays lecturein discussing of those theories and somerelated problems in civil engineering.

So,at a glance if you see what are thethings that we discussed in the last class? We discussed the moments,we discussed the covariance and we discussed correlation coefficient.

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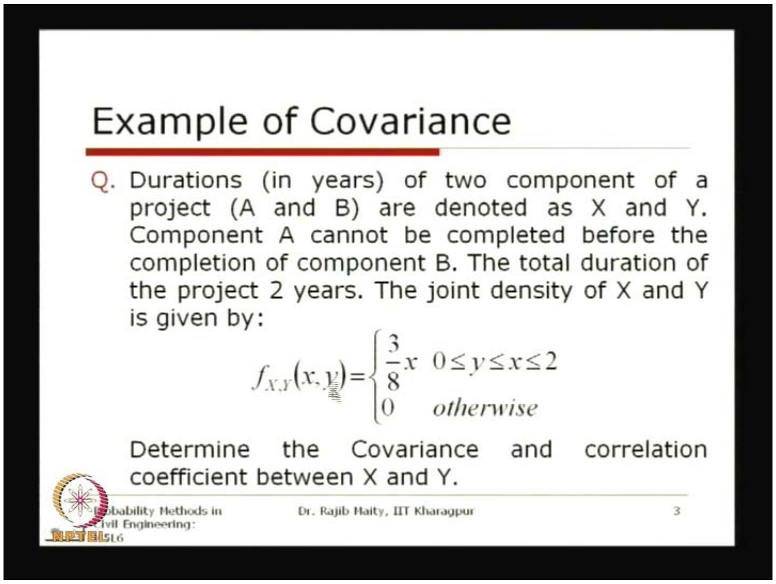

Now, todays classwe will start withoneexample problem, which to cover thiscovariance and correlation coefficient and after thatwe will discuss the conditional mean and conditional variance andgradually,we will alsogo the moment generating function.If this one if not todays class then may be next class.We will continue the moment generating function, but if time permits will cover this one also.

So, one thing what we arewhat I am trying to say here, is that this conditional mean and conditional variance when we are talking about this thingswas not there in the single random variable.But,this thing basically we are takingthe theory from the conditional distribution. The conditional distribution, in case of multiple random variables was discussed earlier.We will just see some of the properties, when the distribution is conditioned on the other random variable.

So, in the last class where wewhere we stop is that after discussing the theory of this covariance and correlation coefficient that we have seen and this is a basically,if you recall that correlation coefficient is the is the standardizedmeasure with respect to that covariance.Now, this correlation coefficient can varyfrom minus 1 to plus 1 that also you have seen.

So, todays class will start with oneexample problem where we will see that covariance and then correlation coefficient.Basically if you recall in the last class, we have taken some special problemwhere we havetaken the problems with theindependent random variable and I just giveone indication at the end that, we got the covariance equals tozero.So, there are could betworeasons. One reason is obviously whether therandom variable or independent or not and other reason thatthey do not covariate each other that is why it iszero.

So, means if the random variables are independent, then obviously the covariance will comezeroand we have seen one such problem in the last class, where the covariance iszero.So, if you checkthat whether the marginals, the product of the marginals are equal to the joint distribution which is theproof for the independencebetween the random variables.

So, thatwas thepoint that I indicated in the last class and the other problem also the waywe have taken we have started with the assumption of that independence and finally we have seen that covariance is coming to bezero.

Now,we will in this class that we are starting with a more general problem, where the covariance need not bezeroalways and from the covarianceusing whatever the theory we have discuss in the last class. We will also calculate, what is thecorrelation coefficient?Now, this correlation coefficient and the covariance that we are getting we are starting from theirdistributional propertiesor joint distributional properties.

So, these arebased on the based on the whatever the distribution it is,butin the practical application many times we generally have the data and based on the data how we can get the estimate of the correlation coefficient. Those are basically call the sample estimates, because we have the sample and from the sample we were estimating thismeasures and that is the statistical part.

We willtake up this issues is in module seven,here what we are discussing is the measures with respect to the population means the entire population of the randomvariable.So that means basically, we are starting from their joint distribution so that problemstates in this there aretwoprojects.There is one project in that project, there aretwocomponents andtwocomponentscan takedifferent times and thebasically in any project including the civil engineering projects. There are many interrelated components are there tosuccessfully execute thoseprojects.

Here, we have taken two componentsfor the simplicity sake,there could be more thantwocomponents in a project and here what we have taken thattheir time taken, there is a duration of each project that is x and y.

So, the duration in years oftwo components ofaproject a and b are denoted as x and y. So this theduration in years which is denoted as x, that is the component a takesx and component b takes y. Now, the component a cannot be completedbefore the completion of component b. So,as I told that components are always interrelated.So, it requiresbeing the complete the onecomponent first,until and unless you finish this one you cannot go for the secondoneso like that.

So, here as the component a takes the time x and b takes time y. That means the x should be greater thany and the total duration of the project istwoyears.The joint density is given by or it is assume like thisthat f x y is the three by eight x where, this y isy ranging from this 0 tox and x range from this y to 2.So, that total it is below less than 2 years and the y is less than x and otherwise it is 0.

Now,when we are declare that this is a joint distribution between x and y, then obviously the totalin a single random variable we call that total area below the curve.But here,when we are taking the two random variables you know that this volume now.Because it is a three dimensional now, so the total volume below thissurfaceshould be equals to one.So that you can check yourselfwhether thisis a validp d f or not first. So, what in this problem we are looking that we have to determine the covariance between the duration oftwocomponents that is between x and y and their correlation coefficient between this two random variables x and y.

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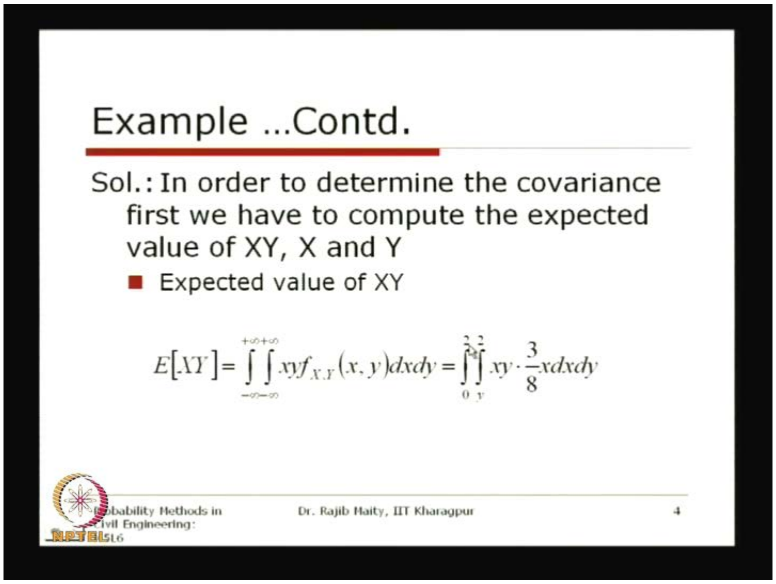

Now if you recall all the theory, that we discuss that covariance if we want to calculate it should be theexpectation of x y minus expectation of x multiplied by expectation of y. So, first of all we have to calculate theexpectation or expected value of x y x and y.Now the expectation of x y is you know this formula, we discuss earlier so x y multiplied by their joint densityand we have to integrate over the entire range.

Nowwhen we are integrating over the entire range there is you know this integration over that area that you have totake thelimit. So first we are taking the integration with respect to x so it is y to 2,then this f x y is the 3 by 8 x and x y multiplied by and then y equals to 0 to 2.So the pictorial, if you want to see this one that is here.

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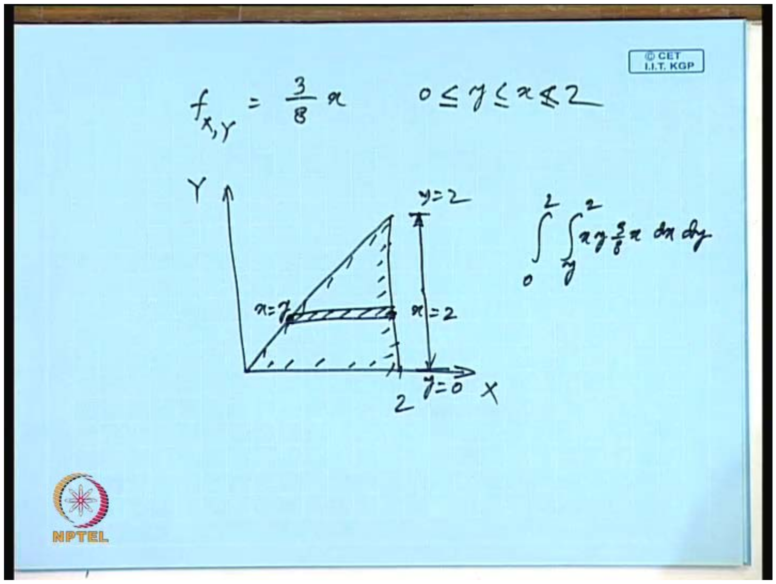

On the pad, if you see that f x y is equals to 3 by 8 x, for the range that is 0 less than equals to y less than equals to x less than equals to 2 was given here.So, if you want to see this one here basically if this is your x and if this is your y, thenbasically you are gettingarealike this, where it is a 2.So, the p d f is defined over this area where x is greater than y, so over this area it is defined.

Nowwhen you are taking thisrange for this entire support of this random variables, then you have to either consider a strip like this or a strip like this.So, if you here the where the limits are taken if you take a strip like this, that means yourthatrange of x.So,here you know that x equals to y in this point and here that your x is equals to 2.

Now, thisstrip first you move from this to this in the wide direction in the y direction.So,here the y is equals to 0 and here the y equals to 2 that is whywhen we were doing this integration over this area, that means you are taking thatthis one this x. So, that x y then f x y that means 3 by 8 x now, when we are takingthat integration limit x, so that meansyou are taking this y is y from y that is x equals to y to 2 and then when we are taking with respect to y you are taking 0 to 2.So, this isthe way how this integration limits are coming in so that iswhat is shown here so this integration from y to 2 and 0 to 2 over this range to cover this entire range of thissupport of the both the random variable.

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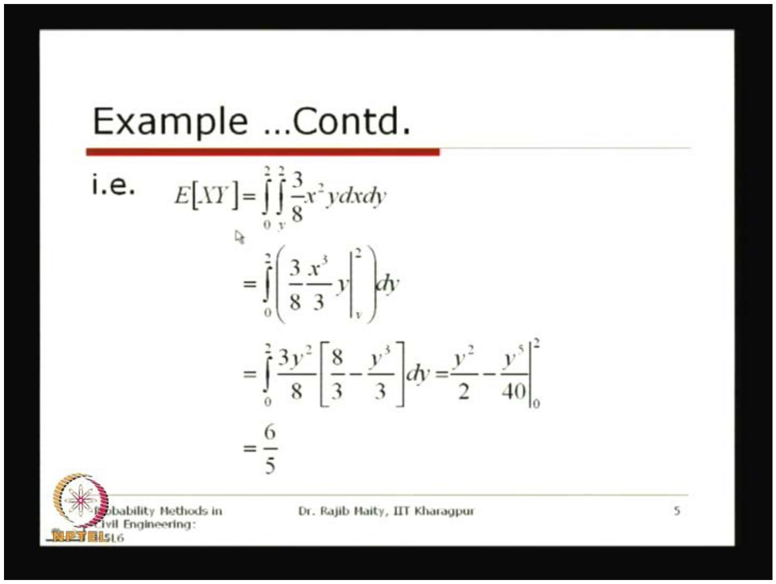

So, if you do this integration you will get that expectation ofx y is equals to6 by 5.

So, just we have to follow this few steps.

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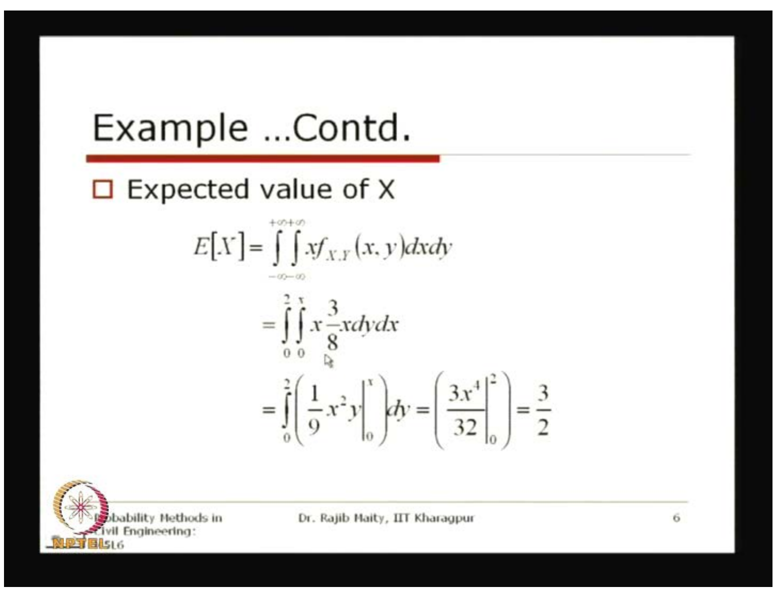

And thenif we want to know what expectation of x is, thenthis is easy.Because,this x multiplied bythis range so expectation of x means we have to integrate out the entire y and 0 to x should be the limit for theother one for this.So,this should be multiplied by this x and take that entire range again and if you do this one, then we will get thatexpectation of x equals to 3 by 2.

Sohere if you just see,this is also means taken the over the entire range,but here theway the integration is defined is a in a particular direction.So, first we are taking that while d y alimit so that is why this limit is 0 to x and then we are taking the x that is why it is 0 to 2.So, that means what I am taking is that this is also can be done if you say in this writing pad, theneither of this oryou can take this one also.

So, that means you are taking that integration with respect to y first y equals to 0to yis equals to x here and this one you are doing it from for the entire range of this x, so where the x equals to 0 here and x equals to 2 here.So in this way also we can do and this is exactly shown here for this limit here,where that y for y it is0to x and for x it is 0 to 2. So, if do this one then you will get theexpected value of the x is equals to 3 by 2.

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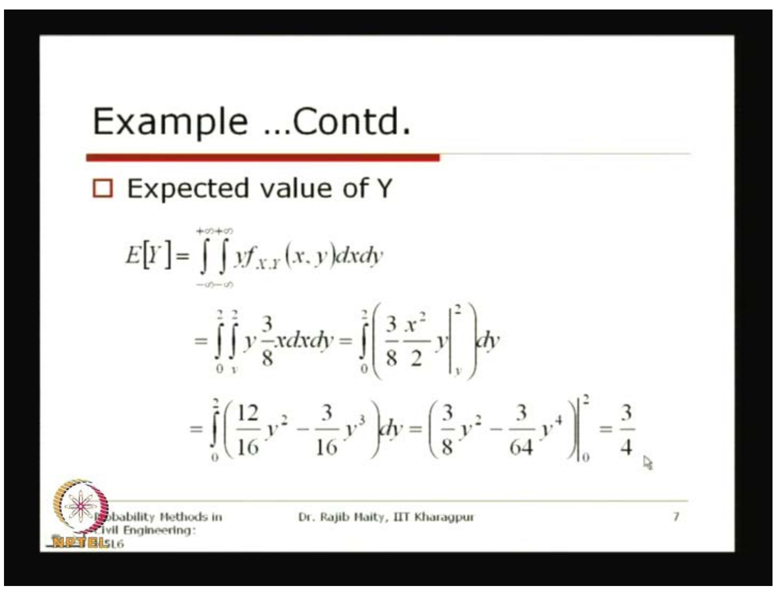

Similarly, the expected value of y also you cantake and if you take this one, then you will get the with the suitable limits and after doing this integration, we will get this expected value of y is equals to 3 by 4. So we gotexpectation of x y, we got expectation of x and y.

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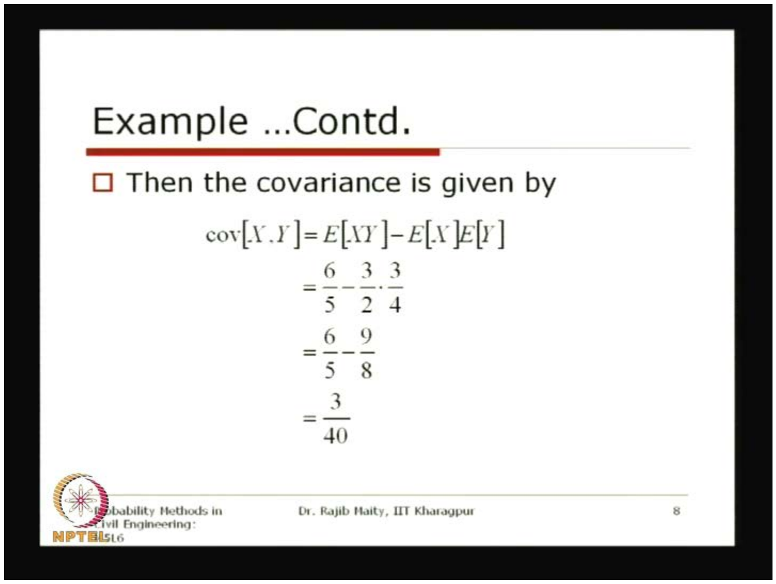

So, if you just fit in this covarianceformula covariance between x,so x and y is equals to expectation ofx y minus expectation of x minus expectation of y which 6 by 5 minus 3 by 2 multiplied by 3 by 4which is coming to be 3 by 40. So the covariance here, we have seen thatit isthe covariant to each other andtheircovariance is nonzero and which is 3 by 40.

Now the second thing that was asked in the question is that, we have to calculate thecorrelation coefficient. Now to get the correlation coefficient,you know we have to find out the their individual variance means the variance of x and variance of y and that we have to take this square root and with thatquantity, we have to standardize thiscovariancethen we will get thatcorrelation coefficient.

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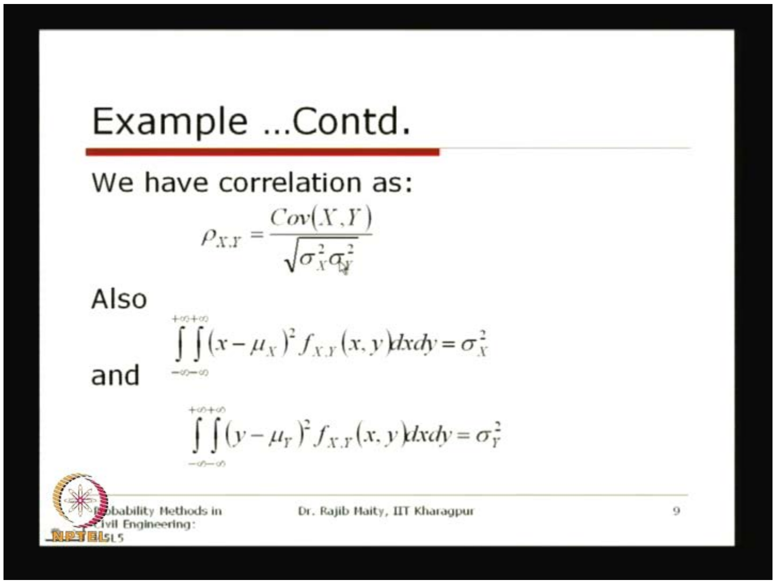

Through, the formula is this that correlation coefficient equals to the covariance between x and y divided by the square root of the variance of x and variance of y.

And now to calculate the variance of x thiswas discussearlier.Sothis x this is the variance,you it is the central moment with respect to the mean of that random variable and that we have to second moment.So that is why, we are taking the square that multiplied by their joint density and taking this integration with over the entire range of this x and yas it is thebivariatecase.

So we will get, what is the variance of x. Similarly we will get the variance of y,if you take thisif you follow this integration.So we have to solve thistwo,first along the covariance, we know the what is covariance so we have to solve thesetwoto get, what is their quantity and then we can fit in this equation to get their correlation coefficient.

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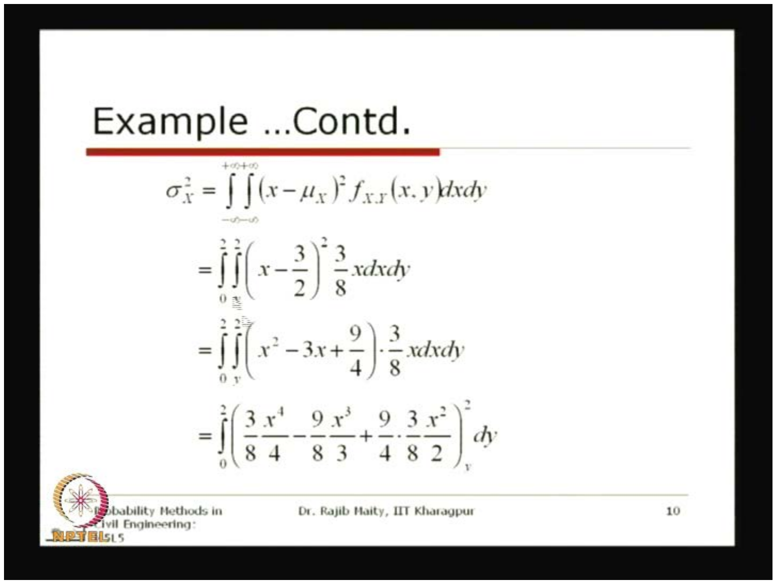

So thissigma x square, when we are takingthis one and the mu x, we have alreadydetermine that is expectation of x which is 3 by 2.We have seen it earlier, now we are placing this one and with the suitable limits, for the entire range of this support of random variable and if you do this integrations step, we see that the sigma x square is coming to be 3 by 20.

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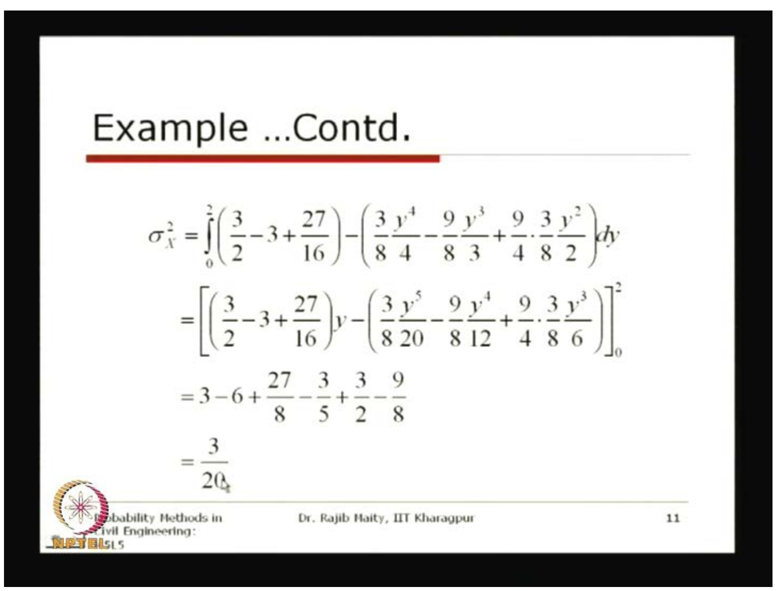

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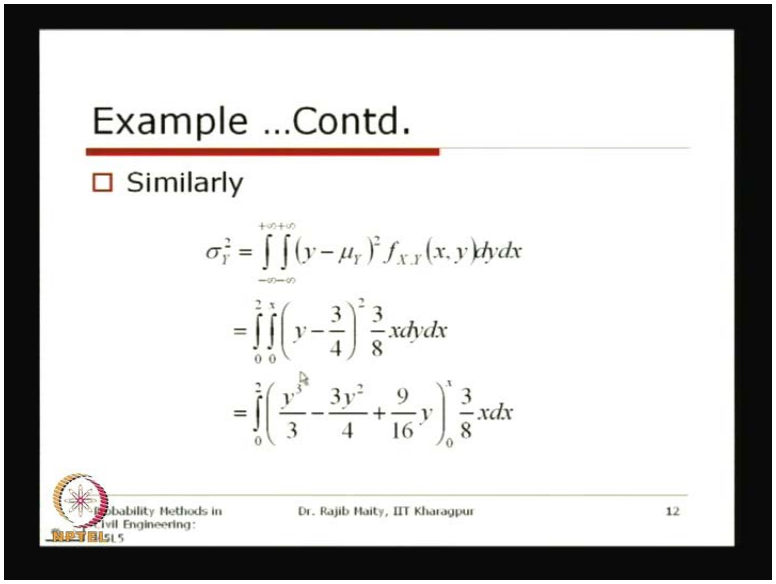

So, sigma x square is 3 by 20 and similarly if we get that sigma y square,where we are taking the y minus mu y and this mu y.You know, this is theexpectation of y and this expectation of y is equals to3 by 4 and if you take this 3 by 4 and with this suitable limit of thisentire range of the random variable.

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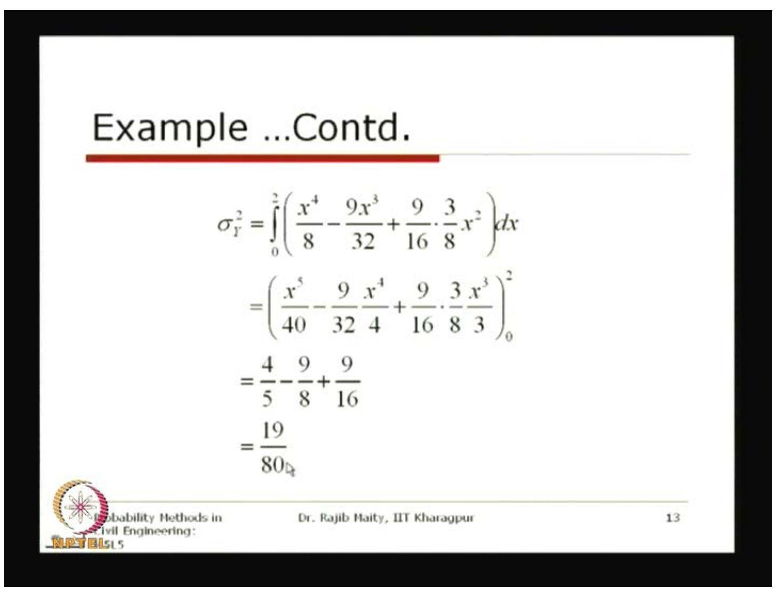

Then, what we will get thesigma y square is equals to 19 by 80.So, this is the sigma y square.

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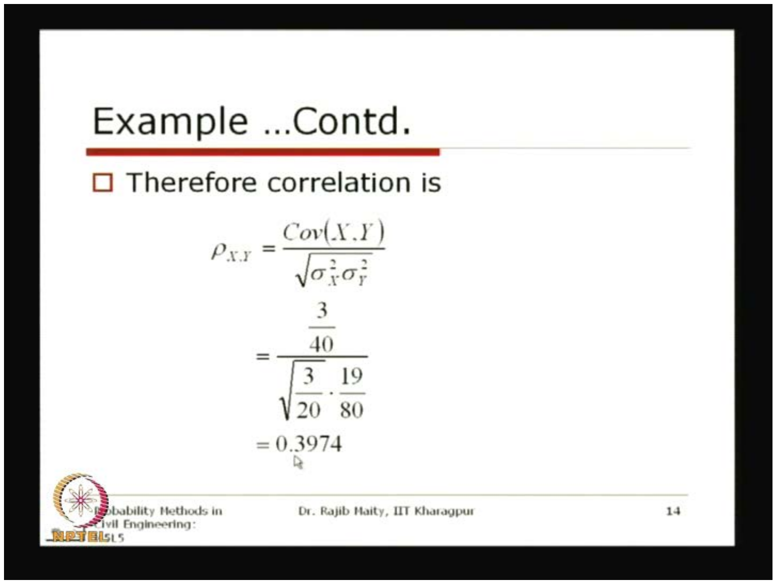

Now, if you put thistwoin thisequation of thiscorrelation coefficient, then we get that.So, this will be the entiresquare root, because this root should be going up to here.So,3 by 20 multiplied by 19 by 80 take the wholesquare root of this one and then in the numerator it is 3 by 40. If you do this one, we will get the correlation coefficientbetween x and y is equals to0.3974.

So here, where we have seen that oneproblem, we have taken and in that problem, we have seen thatthere is some covariance and from that covariance, we calculate the correlation coefficient as well.Next,we will move to the conditional mean and conditional variance and you know that these,we are not discuss with respect to the single random variable. This is discussing only with the respect of the multiple random variableandthe minimum two random variable you need.Basically, we will start our discussionwith the two random variables only and we will see thatwhat is this conditional mean and conditional variance,which is our next discussion.

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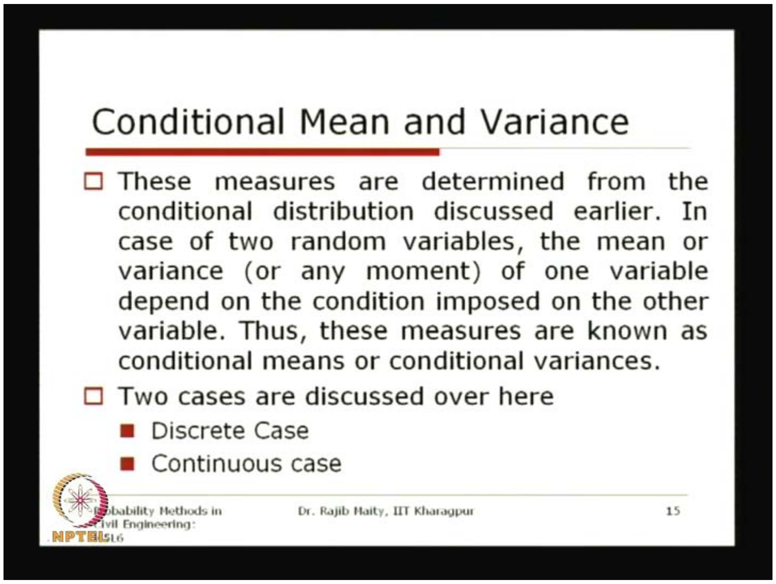

So here,if you see that this conditional mean and conditional variance that means that these measures are determinedfrom the conditional distribution discussearlier.So, conditional distribution we have seen that from the joint distribution, now if the one the outcome of the or therange or the outcome of the one random variable is known.Nowwhat should be the distribution of the other random variable that was the theme of thisconditional distribution and that theory we have discuss earlier.

From that conditional distribution, so once we have decided that this is the conditional distribution, then that distribution function also follows thebasicaxioms of this p d f that,so it should be greater than this is a nonnegative number greater than equal to 0 andthetotalprobability, if you integrate it over the entirerange then it should be equals to 1.

So, that too should be followed now simply means, if you just extend the concept thensay sothat we should treat as a distribution function and from the distribution function, we can calculate whatever the statistics or whatever the measure towant to calculate. So, similarly the conditional mean when we are referring to that means from the conditional distribution, we can calculatefollowing the equation of this, how to calculate the mean following that equation, we can calculate the mean and that mean we will be called as the conditional mean.Because that distribution is already conditioned on the outcome of the other random variable

So,in case of two random variables because we are always starting with this two random variables first, so in case of two random variables the mean or the variance or I should say that this is of any moment of any order.Because, mean you know that firstmoment with respect to theorigin variance is the second moment with respect to the mean.So, basically when we are talking about this measurewith respect to this measure with respect to the conditional distribution.

Basically, there is no need that we have to stop only in this mean and variance.It can be go on for thehigher moments as well.Butanywhere to start with the discussion, we are taking this meanand variance and thattoofor the two random variables.So, thismean or the variance of one variable depends on the condition imposed on the other variable.

So,now if we recall that our discussion on this joint distribution and from the joint distribution to the conditional distribution, that means we are considering the distribution for a particular value of the other random variable and that we have seen that how to normalize that one to make sure that this total in case of the single random variable of the two random variables.

Then the conditional is for conditional distribution is a single random variable,now how tomake that one standardize tomake it the total area, below that random variable equals to one that we have seen. For the distribution we are calculating this measure.So, that is why the condition so whatever the condition that we have imposed on the other variable dependingon that means of that original random variable say x can change.

So, that is thus these measures are known as the conditionals.So that is why the word conditional comes, before that measure that is conditional mean or conditional variance. May if you take up thetwodifferent cases that is one is discrete case and other one is the continuous case, if you take upthesetwocases separately.

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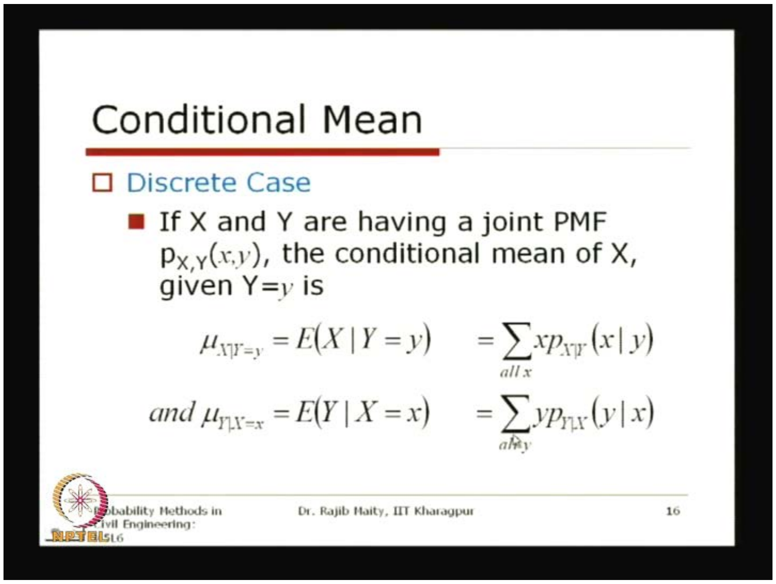

Then, inwhen we just discuss that these are thediscrete random variable.So, two random variable x and y are discrete and their joint PMF is that your p x y x y. So, this is if this is the joint PMF, then the conditional mean of x given y equals to a specific value.Now, that y equals to specific value, when we are telling that means that x on condition y. Now, this mean is the expectation of this quantity that is x given y, y is equals to y which is a specific value of the y.

Now this one we already know that, what is their distribution function from the discussion of the conditional distribution?Nowthe conditional distribution, when we are taking.So we know that conditional distribution just conditionalPMF that is x given y this, we have seen earlier how to obtain thisparticular distributional form.

Now, as I amtrying to say that this is andthis is aPMF or probability mass function for the x and which is should follow all the properties thatPMF should follows.So, this we can treat as an as aPMF and from thatPMF, if you want to calculate the mean.You know that this is astandard formula that we have to multiply with that which expectation that we are talking about the expectation of the x. So that variable that we have to multiply and we have to sum this one for the all possible values of x.Now if we do this one then we will get the mean and that mean is for the condition and the condition is that y is equals to y.

So this is the conditional mean, in case of the joint PMF and this is for the x and similarly if you just reverse it for the y, then it will be like this. That expectation of y oncondition x is equals to x, that is given x should be so in this case we have to multiply it with y and this y and this PMF should be the conditional PMF of y given x and this we should get for the so, thistwoare for the discrete case.Because,weas this random variables are discrete,so we are taking thissummation of all feasible values of x or y.What the case may be.

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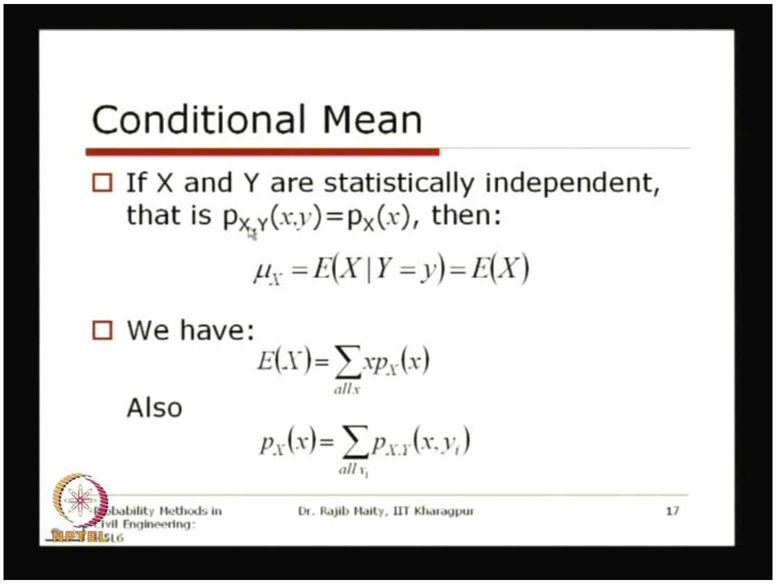

Nowifso one special case we always discuss, that if they are statistically independent.So, this independent has beendiscussed in the previous lectures.So, two random variables when we say that they are they are independent.So, that conditional distribution, that is the probability of x given y or y given x should be equals to the distribution of x only.Because, they are independent they are statistically independent.

So, in such cases the conditional mean and variance will be the equal to theirown mean. So,that is what is explain here that is if x and y are statistically independent,that is the probability of x.So, this should be the probability of x given y not x comma y. This should be probability of x given y should be equals to it is that probability of x make the distribution of the x.

So, we know this one so this x given y, if you just replace with respect to that p x in the previous equation, this is what we have seen so that x probability of x given y. So, this we are replacing with respect to the p x of x, because these are statistically independent.Now this x multiplied by p x of x for all feasible x this is nothing but the expectation of x.

So, that is why I here is written that mu x that is the expectation of x given y equals to this is equals to the expectation of x only.This is quite obvious, because they are independent that x and y are independent.So, that whether what condition is imposed for the other random variables should not influence theproperties of theproperties of this random variable, what is discuss here is that x. So, that whatever the condition is givenfor the y should not be,it should not have any influence on theother one that is x. So this should be equals to x.

Similarlythat we can write with respect to the y also, that is expectation of y given x should be equals to expectation of y alone.So, these are thetwocases when they are statistically independent.

Now we will just see another case, wherewe will just find out that toget there,how the expectation we can just this we can see in the other context.So, this starts a like this means the we know that this equation that is the expectation of x is equals to these are all,we are discussing in case of the discrete random variable.That is why we are taking the summation.So, the expectation of x is equals to their x multiplied by it isthe densitythat probability mass function to get its expectation.

Now this p x x what we can write is that is equals to, if you just take this is basically marginal of the x.So,how we will get the marginal the marginal if you just sum upif you if the other variable that is y I that is y. If we if you marginal out this y then what we will get is thatmarginal distribution of the x. So this also we have discuss earlier, when we are discussing the marginal distribution.So, this p x can be replace with this equation, where we are introducing the joint distribution.

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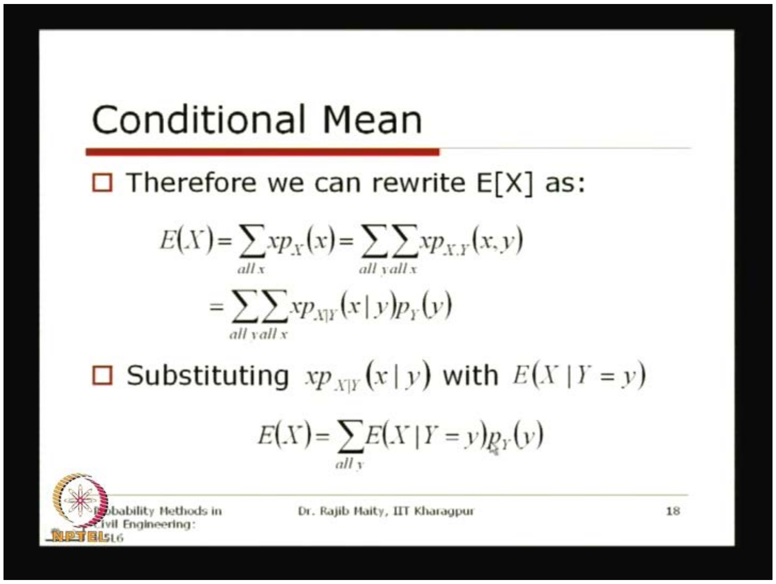

Now, so this expectation of x is this one that, we have shown and this one is now replaced with thejoint distribution for when we are taking it the summation with respect to the all y. Now this joint distribution also can be replaced with respect to their conditional.Now how is it done is that this is so this p x y from the conditional distribution discussion in the earlier lecture is covered that thisone is equals to that probability of x given y multiplied by the marginal of y.

Now, if we justif we take now this quantity that conditional distribution multiplied with the x.This can be replaced with the expectation of x given y, because this we have seen it seen earlier where it is shown it here.So this is the expectation is the conditional expectation which is this expectation, so what exactly this one we are using here to replace this quantity.

So, this one can be replaced with this expectation of this.So,basically this one in the sense that including the summation of all x, so what we can write thus expectation of x is equals to the this expected the conditional expectation multiply with the marginal of the y, when we are taking the summation for all possible y.

So, this oneso before I go to this continuous case, so this is basically what you can say that it is the expectation of the conditional expectations.So, this is the expectation multiplied by theirmarginal, which is a if you take it for this all over this the summation, then we are getting the expectation of the other random variable which is x.

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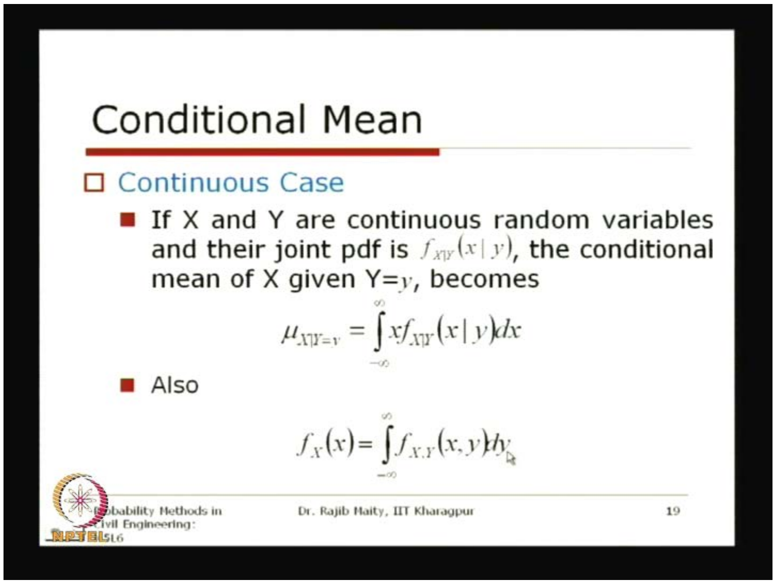

This that will also be discuss in the light of the continuous random variable also.Now, we will again start the same thing, in case of the continuous random variable. Now here if the x and y are the continuous random variable and their joint PMF is that probability of x and y, that this is the joint distribution isnot that x this, is this is a mistake this is a comma that is x comma y, that is x comma y.

The conditional mean of the x given y equals to y, becomes that mu x on condition y equals to this y, which is equals to you know that we have to multiply with the x for this conditional distribution and the over the entire range if we do the integration then we will get that their conditional mean.

Similarly,we also know actually we are taking this discussion with the same line, that we have discuss just now with respect to the discrete random variable.

So, we are also taking this same line just to conclude, whatever we have concludedin case of the discrete random variable within the light of the continuous random variable also, So,that the marginal distribution for the continuous case that is f x x is equals to if we take that,if d y is marginal and out that means, if we integrate over the entire support with respect to the y. Then, we will get the marginal distribution.

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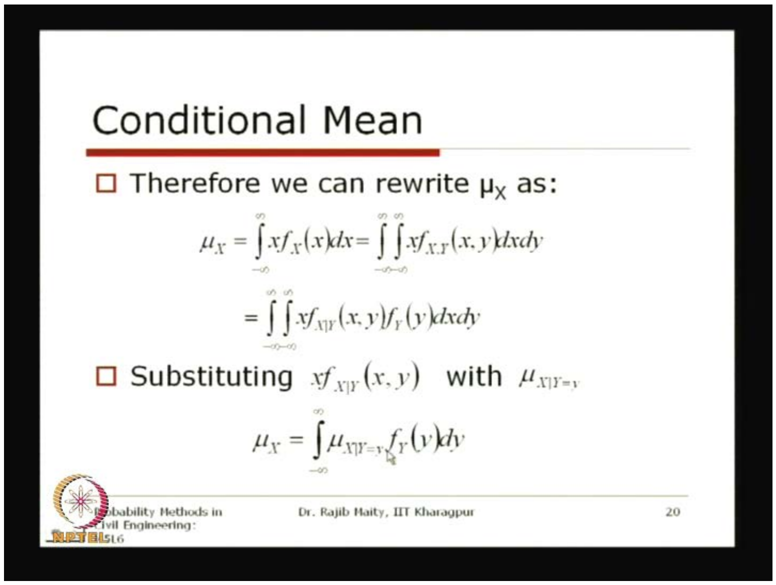

Now,we this one if you just take this mu x equals to this one and this f x we can just again replaced with the joint density and this joint density is integrated with respect to the y.So, that we will get this marginal one, so this f x is replaced with the integration f x y d y for the entire range.

So, again this joint one can also be replaced with respect to theirconditional distribution and this conditional distribution is x given ymultiplied bytheir marginal distribution.So, this we will get what is that e x, now if you substitute this one with respect to substitute this one means along with their integration with respect to ax. Ifwe integrate this one with respect to that conditional mean,which is the mu xgiven y.

So, then we will get the mu x is equals tomux given there is conditional mean multiplied by their marginal density, over the entire range of therandom variable for is the marginal density is taken to get the expectation of the other random variable.

So, this is exactly what we discuss in case of this discrete random variable and here we are putting the same thing in the light of the continuous random variable.

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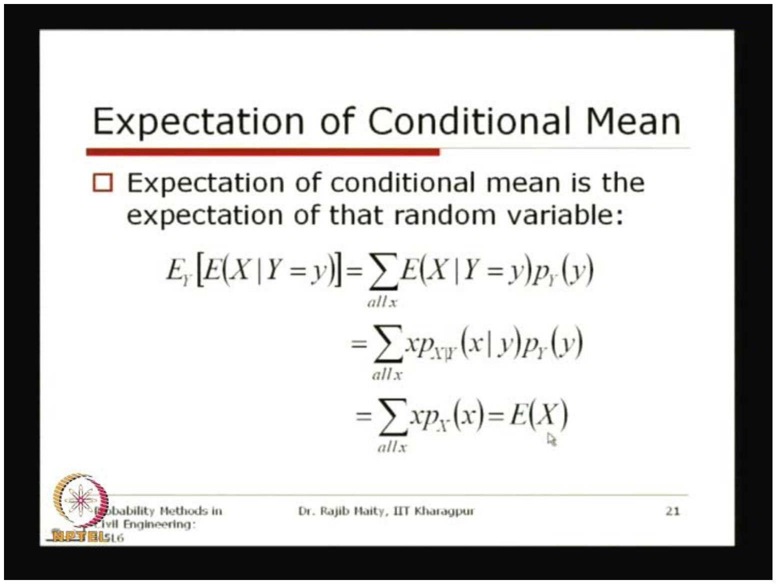

Now the expectation of the conditional mean, nowwhat happens that this measures when we are taking with respect to thepopulation, then this conditional mean this itself is a isone random variable.So, as this conditional mean are the whatever the conditional measure that we are taking those are also the also can be considered as a random variable.As these are also the random variable this will be clearer when we will be discussing about the sample statistics, because those are also the sample statistics,when we are discussingthose are also is considered to bea random variable.

Here also what we is telling that the conditional mean or conditional variance are this kind of measure, these are also the random variable.So, as these are random variable we can take whatever we can whatever the properties,we are we have inspected for the normal for a general random variable we can also do that one.

So,here we are taking the expectation of the conditional mean.So this expectation of the conditional mean is the expectation of that random variable.So how it comes that we will see now, so what we are taking that this is the conditional mean that is x given y. Now, what we are doing that we are taking the expectation of this full quantity which is written here e.

Now this subscript y why this is written here is that, now this subscripty indicates that this one whatever the inside quantity is the conditional mean and the condition inconditionwith respect to the y.So that iswhy this y subscript is given just to differentiatebetween the expectations of the conditional mean.

Nowwhat we can write again that this one multiplied by theirdensity, that is the density of this with respect to the y.So, this p y y we can write, so these are the following the sameformula that iswe are using thiswhatever the random variable multiplied by their distribution and sumup for the all possiblevalues of the random variable.

So, this one again now this one we can write that x multiplied by probability ofx oncondition y,x on condition y multiplied by this p y y and this one again sowhen we are writing that this multiplied by this one this can be written as their marginal of thatx.

So,the p x of x, now this x p x of x we know that this is equals to the expectation of x. So, that is why the expectation of the conditional mean is the expectation of the random variable.So, the expectation of conditional this is the conditional mean for the x if I take the expectation of this quantity then this is nothing but the expectation of x alone.

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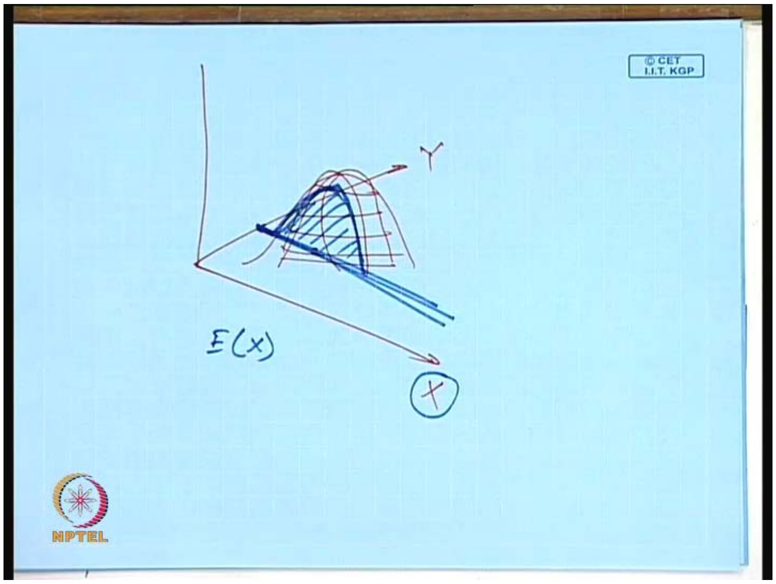

Conceptually, if you want to understand this one, what thus this mean suppose that we haveconsider that,say let us refer to this figure suppose that there are two random variable this figure.I havedrawn earlier also,so what we are so supposethat there is a surface kind of thing. So, which yourcan be considered as a joint distribution between the two random variable, this is the x and this is y.

Now when we are talking about the conditional that this conditional means, suppose that this is what is the distribution of the x condition on certain value of y. So, suppose that this is the value of the y that we are talking about and so we are just we are just taking a section through this and whatever the cross section that we can see here.

So, this is the distribution when we are taking when we are normalizing with this marginaldistribution of this y. We are getting that value so for this one we are calculating one mean and that mean is your nothing but you are the conditional mean. Now so this can vary over the entire range of this y. So, this basically thissurface can go then take any value when we are taking the expectation of those conditional means, basically we are searching over the entire range of this y.

Now when we are taking that expectation for the conditional mean over the entire range which is nothing, but what is the overall expectation of this x. So, that overall expectation of this x is nothing but the x.So, this is the exponential background with respect to the two random variable. Of course,is thattwobehind the statement that the expectation of the conditional means is the expectation of that random variable itself that is, this expectation is equals to expectation of x.

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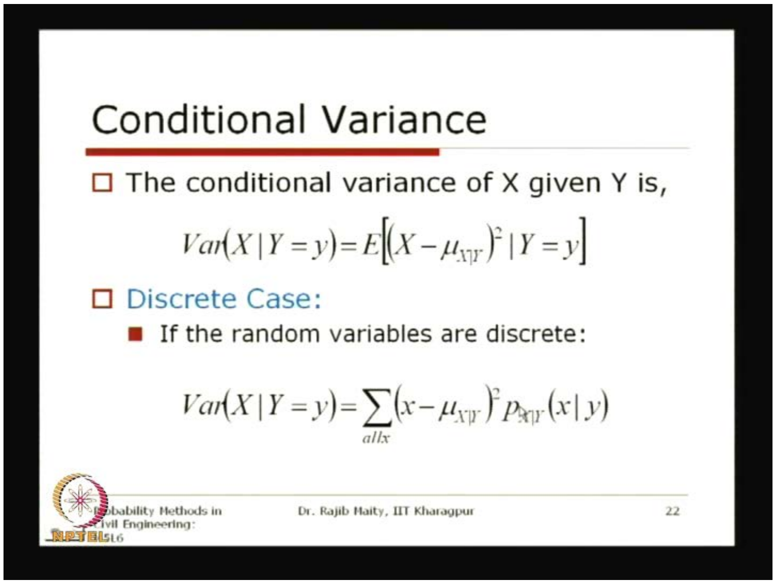

Now we will take that conditional variance, now again following the same formulae for the variance.Here, also we are takingthe conditional variance that means the variance of x given y. So, variance of x given y equals to small y that is specific value of yis equals to the expectation of x minus that conditional mean.So, this is if you are considering this is as the random variable that means, we are taking that conditional mean that square obviously this is condition on this y. So, this expectation is your the conditional mean.

So,remember that when we are taking this conditional measure this both this mean and variance,we are taking for thatfor that random variable which is condition on the other one.So, if you want to see it here in this figure, suppose that for this is the conditional distribution, now this distribution is having one mean which is not necessarily to be equal to this expectation of x.

Now when we are calculating the variance of this one for this condition that is, y equals that y is equals to y in this location.So, that means we are considering what is the mean for this condition which is denoted as say that mu x oncondition y equals to small y. So, with respect to this one we are taking the second moment to get the conditional variance this is what is explain in this equation.

Now again there will be similarlytwocases one is that, if the random variables are discrete, then the variance will be the summation for all possible x,where the x minus mu x square multiplied by their density and the density again is the conditional density that is probability of xgiven y.If you do this one we will get theconditional variance for a discrete random for the discrete random variables x and y.

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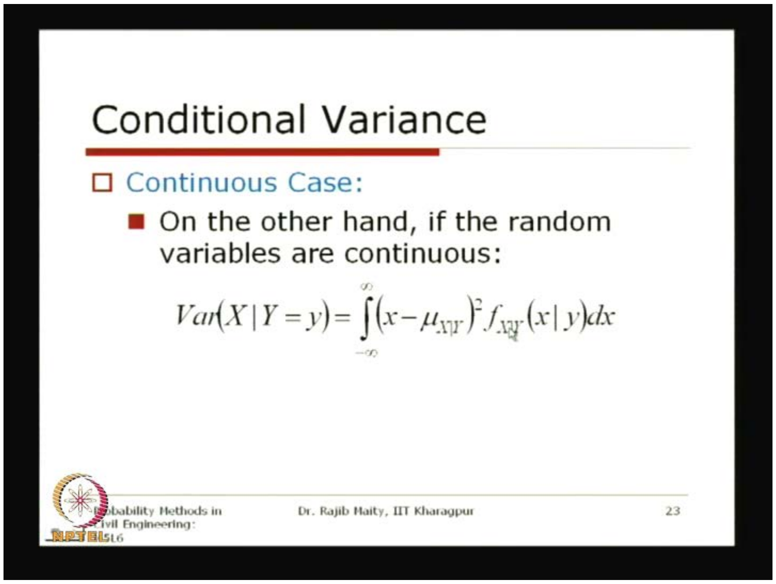

Similarly for thecontinuous case also that, if the random variableare continuousthen the variance will be equal to from the minus infinity to the plus infinity x minus mu x given y. Again this is the conditional mean square multiplied by their conditional density and it is integrated over the entire rangefor thatconditional random variable for that x. So,if you take this one we will get the conditional variance in case of the continuous random variable.

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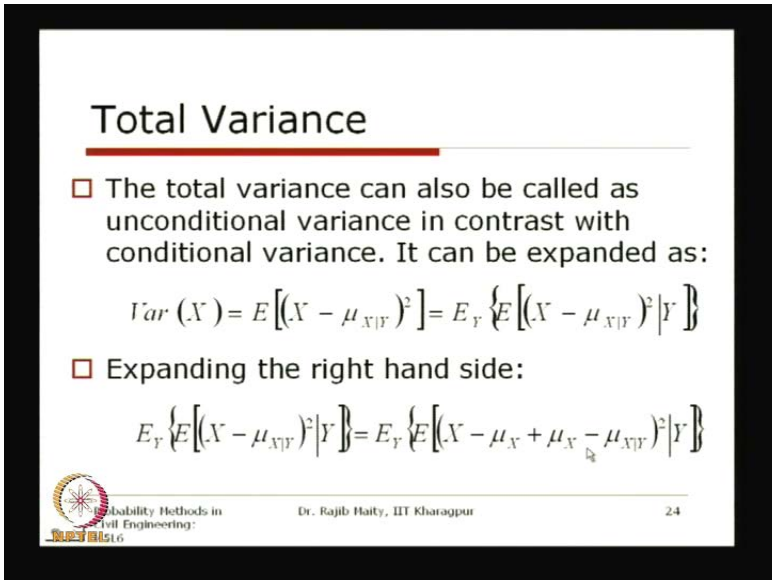

Nowthe so conditional variance when we are talking about, nowhere what we are trying to say that its relationship with the total variance.So, that total variancewe have not used it earlier,but here what we are referring to this total this is known.In the sense,what we are telling that this when we are talking the total variance we are basically talking with respect to the conditional variance.So, the conditional variance means for a specific value of the y now when it is notconditioned on any specific value of the y, that means the overallvariance that we areconsidering so that we are terming as thatthat total variance.

So, total variance in other words we can also say that unconditional variance.So, that variance of x so just why total word is used is the just make it contrast with respect to the conditional variance, conditional variance we can also call it as the unconditionalvariance of a random variable.So, this total variance that is why it is stated the total variance can also be called as theunconditional variance in contrast with conditional variance and it can be express as this sothis variance of x is equals to x minus mu oncondition of y whole square andthis can also be express that expectation of y oncondition that now this is what we are taking that the conditional expectation.

Now, from this expression the earlier that we have discussed.So, from there the conditional that the conditional expectation is the expected value x minus mu x oncondition y square given y.

Now, if you just explain this right hand sideof this expression, where itcan be it can be expanded as this one that is this square if you just expand inside this square term.There is one mu x is added and other one is just subtracted.So, basically keeping this full quantity same and then we will take this square.So, taking this one as the first term and this is as the second term, we will just expand itusing thesimple equation of this square quadratic equation.

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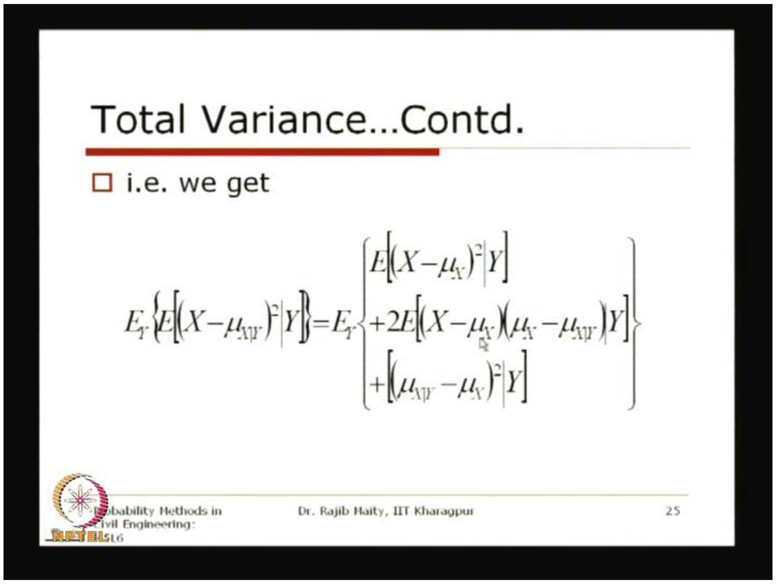

So, after expanding what we are getting is that, this squaresee again the condition on y, then that x minus twice of the expectation of that x minus mu x and mu x minus mu y of y and then mu conditional mean minus this mu x square condition on the y.Now if you see that this is, basically what this x minus mu x is the first moment with respect to the mean.So,you know that first moment with respect to the mean iszero,so thissecond term iszero.

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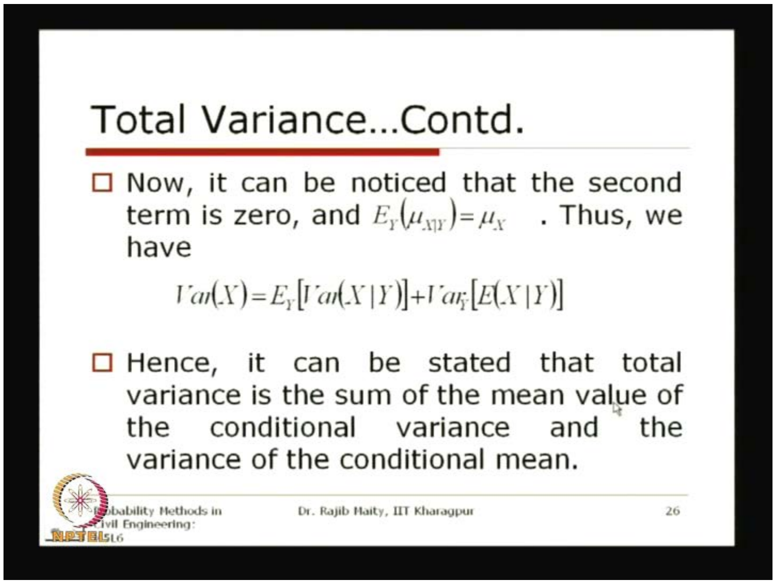

And the first one that this one this expectation of x on condition y,that is if you take the expectation of the conditional mean that, we have just now discussed.The expectation of the conditional mean is equals to the mean of the random variable itself so which is your mu x.

So, thus so thistwoequation if we justput there then what we get is that the variance of the x, which isthat expectation of y variance ofx given y plus variance y. So, again here the y when we are talking it is the condition and condition on thus condition on the y of the expectation of x given y. So, there is a kind of symmetrical nature you can see,though when we are taking that expectation of the variance first and expectation is taken oncondition of the y and then we are taking the variance of the expectation the variance is taken with respect to the y.

So, hence it is it is stated thatthe total variance again the total variance that we are talking about there is a variance of x, of that is the unconditional variance the total variance is the sum of the mean value of the conditional variance.This is the mean value of the conditional variance and the variance of the conditional mean.So the variance of the conditional mean this is what is explain here.

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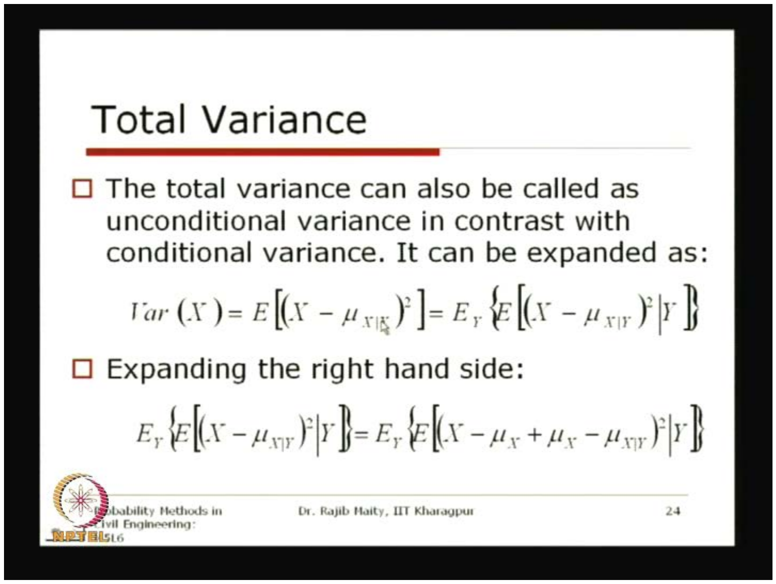

What we have referred here, so this is the variance of x this is might be this is a mistake.So, this will be the x minus mu x whole Square.So, there is no condition will not come here.

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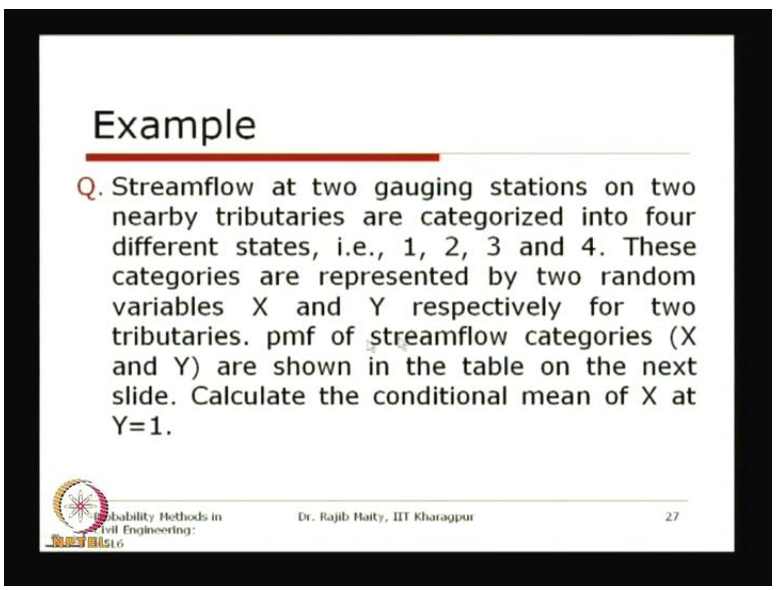

So now we willtake up oneproblem on this,conditional mean and basically to start with,we are starting with the discrete random variable where we can seethe all the quantities of this PMF and we can understand that how the conditional mean is calculated and we are taking that our same problem that, we have discuss earlier for the discrete case.In this case earlier we have calculated may be different thing that marginal distribution, conditional distribution and all.Here, we will this here we will see how to calculate and what is the conditional measure there is a conditional mean and conditional variance with respect to thesame problem.

So, that problem is that the stream flow attwogauging stations ontwonearby tributaries are categorized into four different states that isthe 1 2 3 and 4.These categories are represented by two random variables, one is that x and other 1 is the y. So fortwodifferent tributaries and two random variablesfor their category of the stream flow.

This p m f of this, so the joint PMF of the stream flow categories x and y are shown in the table on the next slide.Calculate the conditional mean of x at y equals to 1.So, means that the conditional mean of the x given that y equals to 1.

So we have to calculate first, what is the conditional distribution from the conditional distribution?We have to calculate their mean as the equation we have just now seen.

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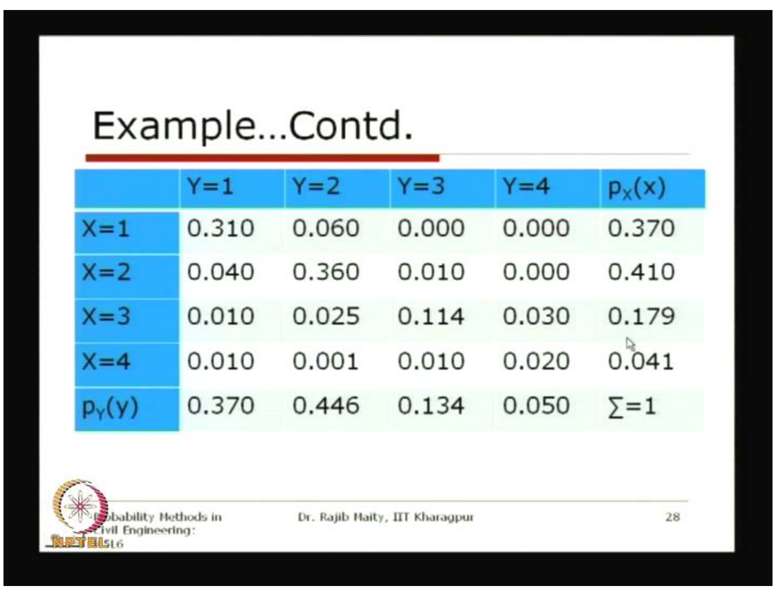

	Y=1	Y=2	Y=3	Y=4	$p_X(x)$
X=1	0.310	0.060	0.000	0.000	0.370
X=2	0.040	0.360	0.010	0.000	0.410
X=3	0.010	0.025	0.114	0.030	0.179
X=4	0.010	0.001	0.010	0.020	0.041
$p_Y(y)$	0.370	0.446	0.134	0.050	$\sum=1$

So, this is therethe joint distribution where this x equals to 1 2 34 four different states and y equals to 1 2 34four different states and these are their marginal distributionsfor this is for the marginal distribution of x which is the summation of eachcolumn and this is the marginal distribution of y, which is the summation of each column this is the summation of each is row and this is the summation of each column.So,this weknow earlier this,we have discussin the earlier problems also. Now what this problem we are taking is that, what is the conditionalmean of x given y is equals to 1.

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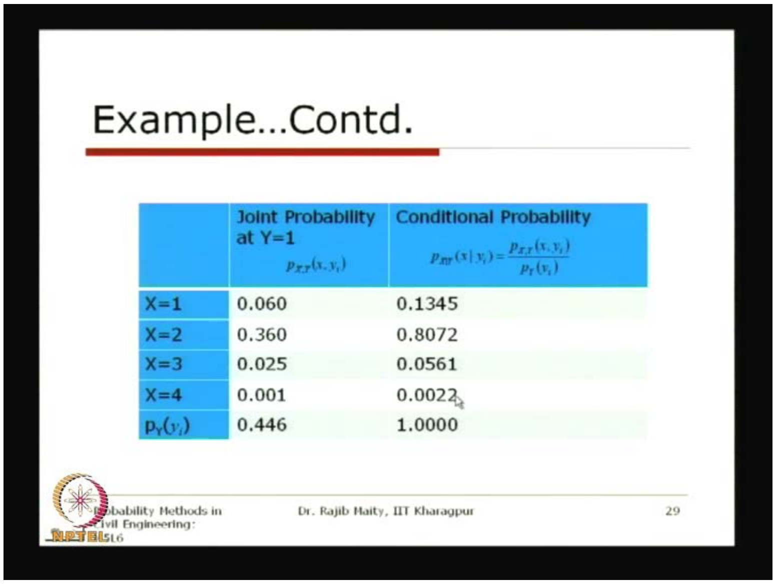

So,first of all we have to calculate their conditional probability.So, joint probability at ones that is given y equals to 1 this is this value we can get it from that joint PMF that is y equals to1. No there is a mistake this should be y equals 2.So, that at 2 so basically we arejust taking this columnout from here.So, the problem also the mistake should be corrected as the calculate the conditional mean of x aty equals to 2 not 1.

So, this we are taking and we are just you know that how to get the conditionalprobability that also we have discussearlier that is this distribution should be divided by their marginal distribution at that value of y that is y is equals to 2 here.So, that is the marginal distribution withwe can also see from this table that is the 0.446.

So, if you just divide we can get their conditional distribution as that is x equals to 1 it is 0.1345, x equals to 2 0.8072,x equals to 30.0561 and x equals to 40.0022.

So, this is the conditionalprobability now if you want to know their conditional mean, which means we have to multiply with their outcomerespective outcome and we have to sum it up.

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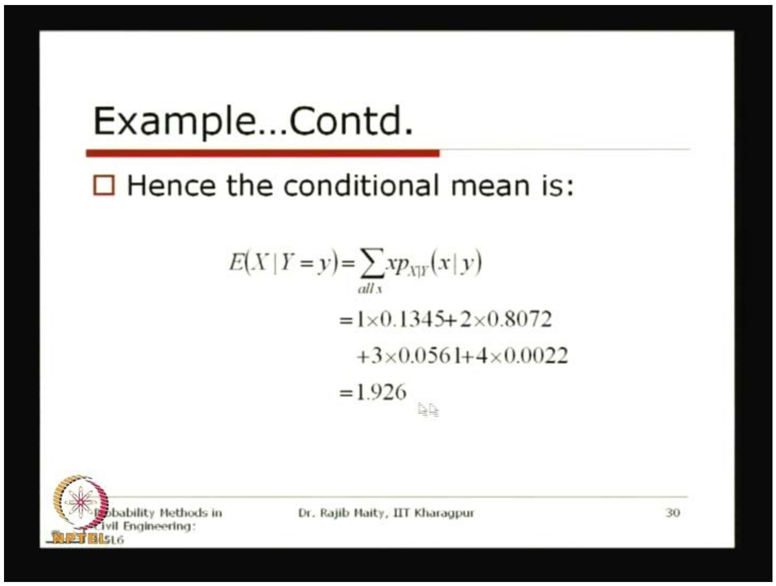

As thus a straightforward equation that is the expectation, the conditionalmean that x giveny is equals tothis should be your equals to xprobability x given y is equals to 1 multiplied by this probability and 2 multiplied by their anotherprobability and all like this all the outcome we should multiply and we should get what is the mean of that x given y is equals to 2 here so that is 1.926.

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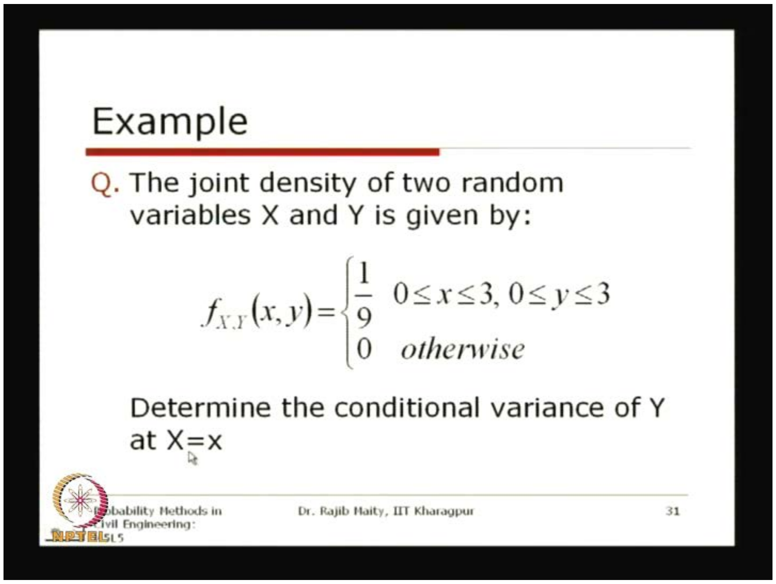

Nowwe will take anotherproblem,this is a joint distribution and this joint distribution is basically a uniform distribution that, we have taken here that is that x and y. So, in many cases we cancome across to this one that, they are related to their joint distribution is onesome joint distribution,but that distribution that joint distribution can also be uniform. The uniformdensity for a single random variable, we have discussed and that their we have seen thatfor the any value of that random variable that, probability density is constantprobability density same for the entire range of the random variable.

So, similarly if we just extend this onetwothat multiple random variable then it looks like this.This is for the suppose that this is youra and this is your b and this is your that distribution of this f x of x this is your x. So,basicallythis so this is one that is one by b minus a. Now, if you take it to thetwodimensional case, so there we will get that suppose that this is x and this is y, basically what we aretaking is this is that, so this is that surface, where theheight I see inrespect to thisjoint distribution x comma y x y.

This one is constant for the entire range say that this is your, so it is notnecessary to start with the origin, this can start from anywhere. Say that for the x range is from say, For example the way the figure is drawn that is 0 to say a and this is 0 to say b. So, over this entirerectangle one side is a and other side is b so thus surface is uniform.

So, this type of one problem is taken and in many cases we can comeacross of this type of situation,where the distribution isuniform.So, f x y y is equals to 1 by 9 for the range 0 to 3 and 0 to 3for both x y. Determine the conditional variance of the y at x equals to x.

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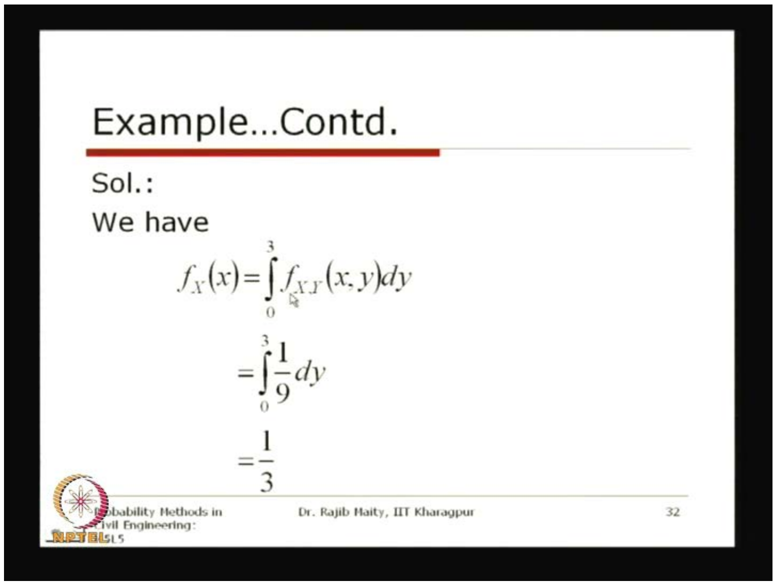

So,similarly we have to so first of all, we have to calculate theirmarginals and f x of x equals to if you do this integration we will get 1 by 3.

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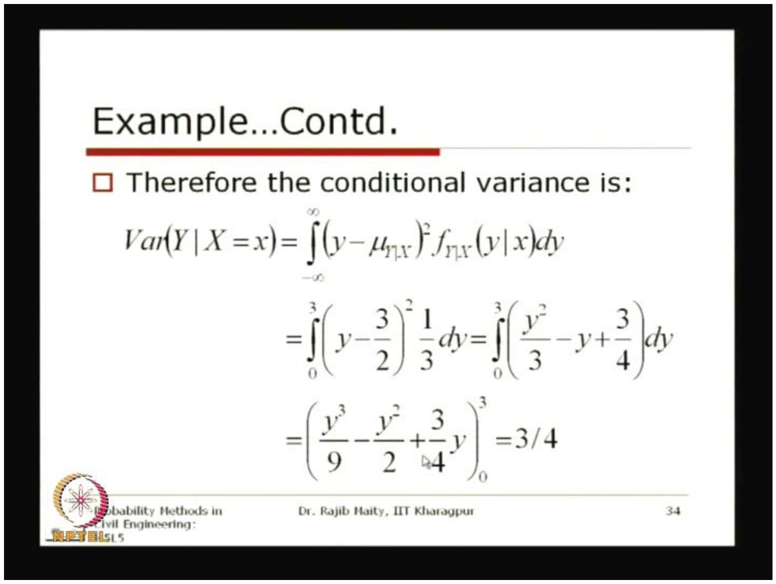

And f xso f y given x is equals to again 1 by 3 for the conditional distribution and conditional mean y given x is equals to 0 to 3 and this is, if you follow this integration then we will come that conditional mean of y given x is equals to 3 by 2.Therefore the variance the conditional variance when we are talking about this should be, equals to variance of y given x equals to x is equals tofrom the minus infinity to plus infinity.This is basically 0 to 3 y minus conditional means square their conditional distribution with respect to the d y.

If you do the integration, sowe just fit it in this equation y minus 3 by 2 square multiplied by 1 by 3 and if you do this indication, we will get the variance is equals to 3 by 4.

Sotodays class, we have coveredup to that conditional meanand conditionalvariance andwe have seen some example problems also on this and in the next class one thing is pending for this multiple random variable, which is the moment generating function and this we will discuss in the next class thank you.