Probability Methods in Civil Engineering

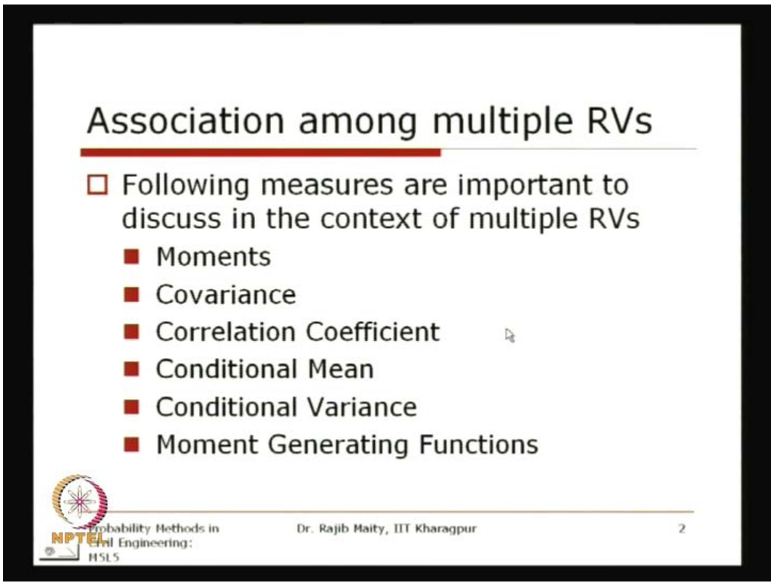
Dr: Rajib Maity
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture No. # 23

Properties of Multiple Random Variables

Hello, welcome to this lecture, as I indicated in the last lecture that today we will start different properties of multiple random variables. Particularly this calculation might be we came across earlier also in the context of the single random variable. Now, as in this module we are discussing the multiple random variable, we will just discuss the properties of this multiple random variable in the context of the multiple random variables.

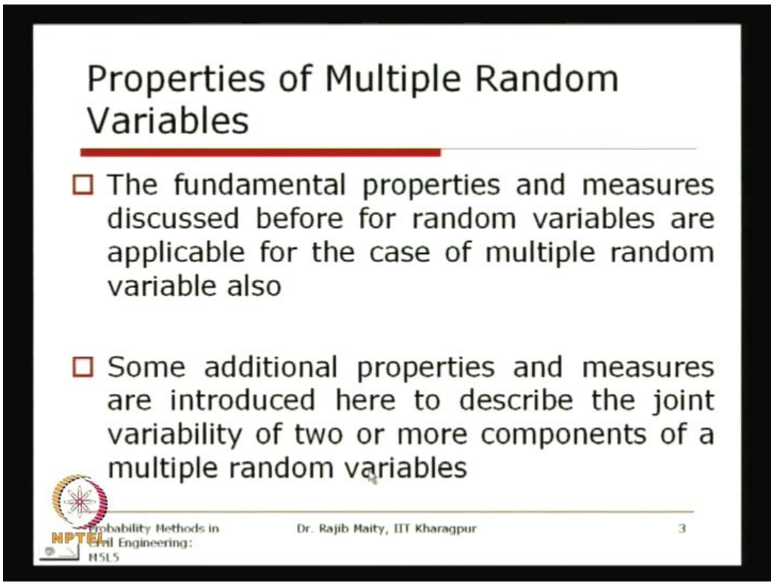
(Refer Slide Time: 00:52)



For example, that moment that we have seen that in the context of the area with respect to the origin, so far as the single random variable is concerned that we have that we have seen earlier. So, here now basically we are extending those ideas to the more than one random variables, where to start with as I am always doing that for the bivariate random variable case. And similarly the same concept can be extended to the more than two random variables as well.

So, we will be going through one after another their properties that moments then we will know, so moments one part will be that expectation where the expected values of different random variable associated with in the in the multiple random variable will be well be considered. Then the covariance covariance means, how the how the random variables are covariate each other, and from that it will be taken to a measure of linier association which is known as correlation coefficient that we will discuss, and gradually we will move through the conditional mean, conditional variance, and then moment generating functions. Maybe we will see we may cover the moment covariance, and correlation coefficient in this lecture, and gradually we will continue to the next lecture for the other properties of random variables.

(Refer Slide Time: 02:31)

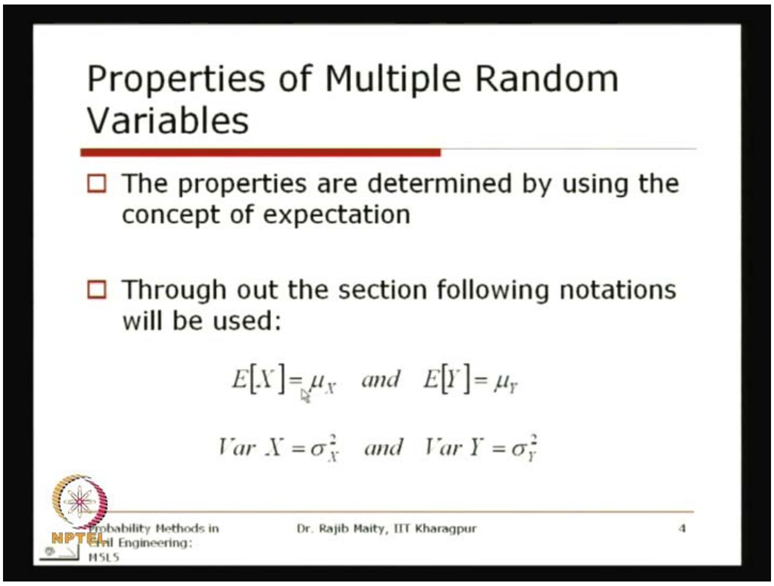


So, to start within the general discussion of this properties of multiple random variable, as I was telling, that the same concept what we discuss earlier for the single random variable will be used here also, only thing that dimension depending on the number of variable associated will will be increased, and for that one there may be means some more properties some more measures will be will be considered to describe their association with each other.

So, the fundamental properties and the measures discussed before for the random variables are applicable for applicable for the case of multiple random variable also some additional properties and measures are introduced here to describe their joint variability.

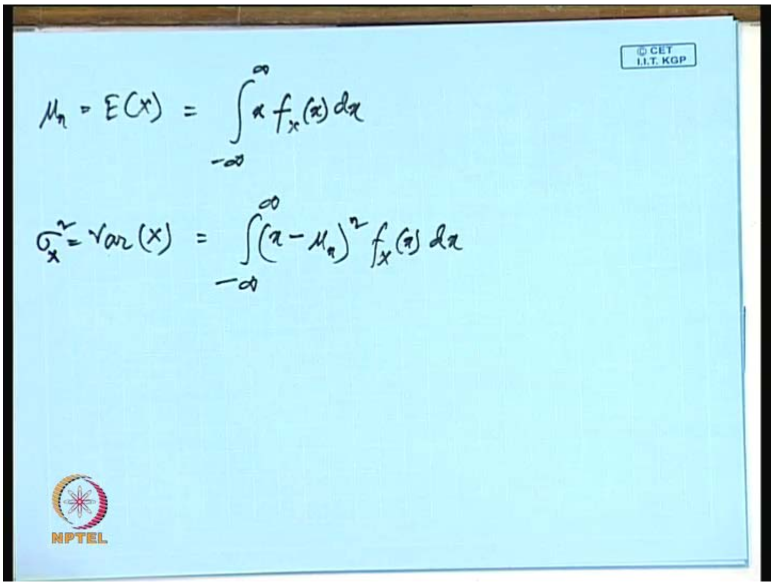
As, I was telling that it may be linear or nonlinear particularly when we are talking about the correlation coefficient; it is the linear association the joint variability of that two are more components of a multiple random variables will be discussed.

(Refer Slide Time: 03:45)



The properties are determined using the concept of the expectation, the expected value that we discuss earlier in the context of a single random variable here, we will discuss with respect to the first we will just take that same concept, with respect to the two random variable; that is bivariate case and same thing will be again extended to that more than two random variables. And in this discussion we will follow this notations where the the expectation that is expected value of the random variable X is denoted by mu x, and expected value of Y is denoted by mu y, variance of X is sigma x square and variance of Y is sigma y square. So, these are all in case of we discuss earlier; in case of single random variable and if you quickly recall that this expectation of X is that is that X multiplied by that its density.

(Refer Slide Time: 04:48)

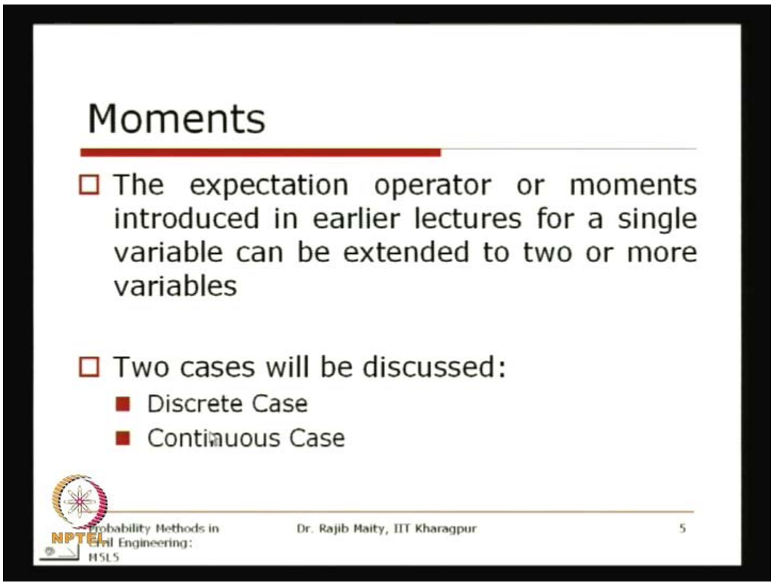


That is when we are talking about this expectation of X; that is, it is that over the entire support the integration for this x this multiplied by that x probability density and this integration will give you that expectation of this X, and when we are talking about the variance of X we are generally considering its its second moment with respect to its mean.

So, when we are talking the second moment with respect to the mean we are doing the the integration after with respect to its mean that is expected value, now we have denoted this one as your mu x. So, this is your mu x this square then that probability density this integration will give you that variance of X. So, these are these are for the single random variable.

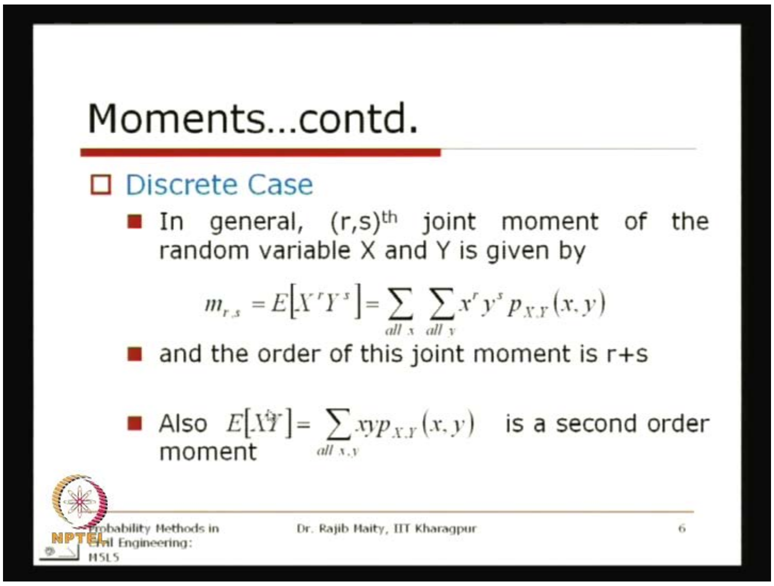
So, these notations we are just using. So, this this is noted as the sigma x square and similarly, for the variable y particular two two variables is considered in this bivariate case and that will be seen here with this notations. So, expectation of X is mu x, expectation of Y is mu y, variance is sigma x square and variance Y is sigma y square.

(Refer Slide Time: 06:11)



Now, the expectation operator or the moments introduced in the earlier lectures for a single variable can be extended to two or more variables, and two different cases has also discuss in case of this single random variable is that; one is the discrete case and another one is the continuous case. So, discrete case means when both the random variables considered is discrete random variable, and continuous means when we are considering it to be the continuous both the random variables are continuous.

(Refer Slide Time: 06:49)



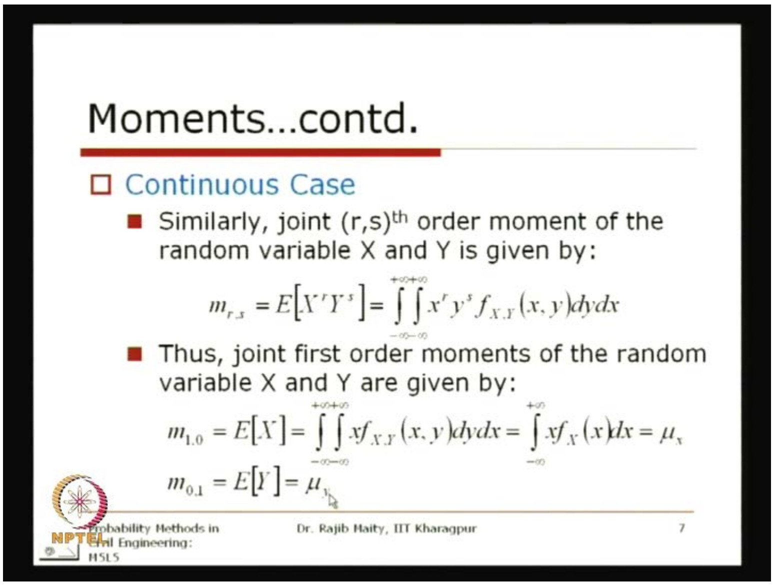
So, two cases, two different cases will be considered and so first we will take up this discrete case. So, in general when we talk that here, that (r, s) eth joint moment. So, now remember that in the single random variable, when we discuss we generally consider that single order means not order that which moment we are considering with the first, second and third we are just considering in this way.

But when we are considering more than one random variable; in this case we are taking the bivariate case first so that when we are describing some joint moment so we have to describe that both of them. So far, X it is r eth, for Y it is s eth, so this joint moment of the random variable x and y. So, the order of this r and s is respectively for the first and the second random variable. So, this is the way we describe this joint moment, and so when we are calling that it is (r, s) eth joint joint moment; that means, do we considering the expectation with respect to the origin that is, why it is x power or and y power s.

So, as these are the discrete case, that is why we are taking the summation for all possible X as well as all possible Y, x power r and y power s multiplied by their probability mass function joint probability mass function P x y (x, y) and for this case when we are we want to know, what is the order this joint moment this order of this joint moment is declare as to be the r plus s. So, what is the total of this r and s that is the order of this joint moment.

So, if we just now, we want to take that expectation of X, Y that is both the r equals to 1 and s equals to 1, then it is a it is that summation for all possible x and y basically this is double summation in the sense that two random variables are there, but here it is both are written in the same all x and y, x multiplied by y multiplied by their joint mass function. And so this expectation of X Y is a second order moment, because this is x is the r is equals to 1, and s is equals to 1 here, so this is a second order moment.

(Refer Slide Time: 09:27)



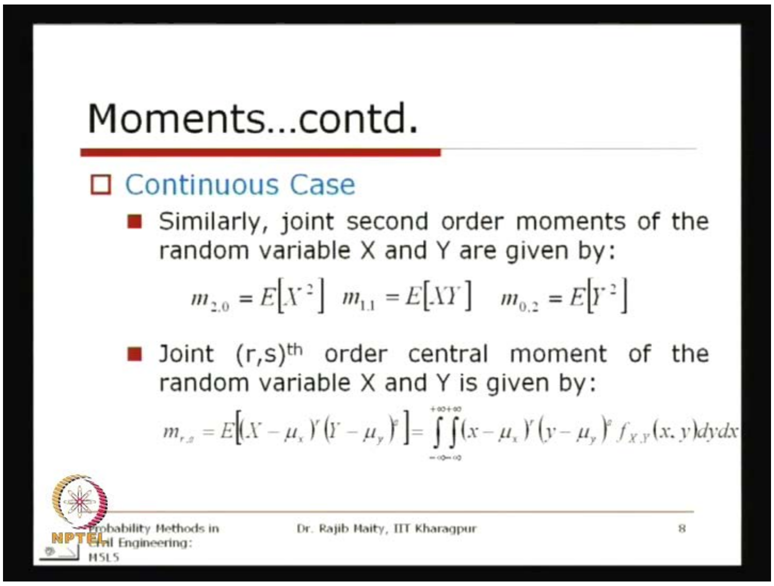
Similarly, if we consider the continuous case, then instead of that summation we have to do the integration over the entire support of the random variables. So, the joint (r, s) eth order (r, s) eth joint moment of the not order, this will be joint moment of the random variable X and Y is given by their expectation of x power r and y power s is the integration from minus infinity to plus infinity for both x and y. X power r y power s multiplied by their joint probability density function which is f x y(x, y), so this we will give that joint moment of order r plus s again.

So, now, if you just see that, the joint first order moment, if I just want to see the first order moments means one r, s either this 2 in to 1 and other one should be 0. So, make it the first order moment; so the first order moment, how it comes, if you just see for this x and y. So, that if you consider that m 1, 0 that is r equals to 1, and s equals to 0; that means, we are taking only x, y power 0. So, expectation of X so this will be; obviously, will go to that mu x, we now that; this expectation of X is mu x. So, in between steps that you know, that this simply following the general equation that is x power r, so, x power 1 and y power 0 which is one so, the then this joint density dy dx.

Now, you see when the first integration we are taking with respect to the d y so; that means, the X we can take out from this integration and if we just take this integration from minus infinity to plus infinity; that means, the entire support of y of this joint distribution then we know from our previous lecture that this is nothing but the marginal density of x, because over the entire support of the y; that y is taken out so we are doing the integration with respect to the y, so it is marginal out we can say so what is remaining is that marginal density of that x. So, now, this integration the first one with respect to y yields that it is the marginal density of x, so f x (x).

Now, if we take this integration you know this is just now we have discuss, this is nothing but here, the expectation of X which is that mu x. So, the first order moment when we take that, r equals to 1 and s equals to 0; which is yielding to be that mean of the one random one random variable x. Similarly, if we just take the other one that is r equals to 0 and s equals to 1; that means, we are taking the expectation of Y, which is obviously yield that mu y; that is the mean of the random variable y, which is shown here that is m 0, 1 expectation of Y which is equals to mu y.

(Refer Slide Time: 12:46)



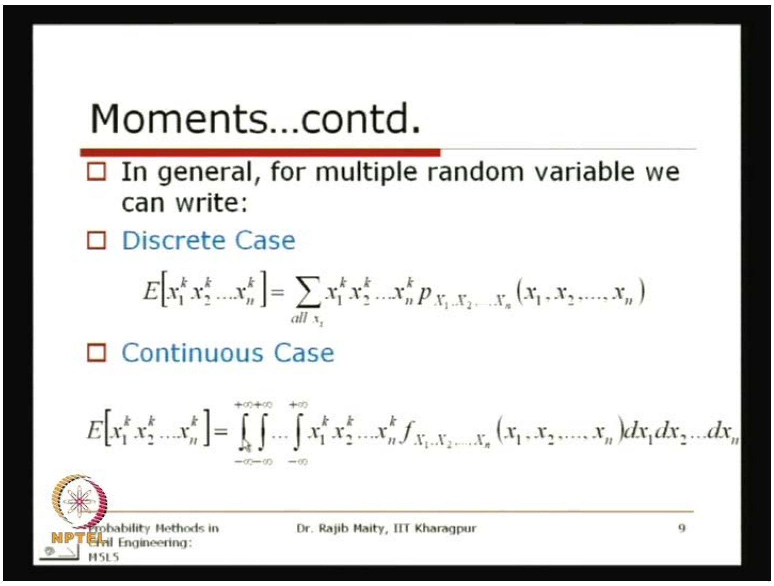
Now, if we go for the second order moments; now, second order moments there are three different cases; one is that r equals to 2, and x equals to 0 or r equals to 1, s equals to 1 or r equals to 0 and s equals to 2. So, both the cases as long as the summation is equals to 2. So, these are all second order joint moment; joint second order moment so this m 2, 0 is it is expectation of X square these are second moment with respect to the origin m 1, 1; that means r equals to 1 and s equals to 1; which is your expectation of X, Y and this one and this m 0, 2 is expectation of Y square second moment with respect to the origin of the random variable Y.

Now, joint (r, s) eth order central moment of the random variable X and Y when were talking the central moment with respect to that random variable X and Y; that means we are taking the moments with respect to their respective mean. So, respective mean further for X it is mu x and for Y it is mu y.

So, that we have to we have to consider we we are basically shifting that origin to that to the coordinate mu x, mu y, so with respect to that the point mu x comma mu y we are taking that moment. So, as we are calling that (r,s) eth moment r, s this joint central moment then; that means, the X minus mu x power r, and Y minus mu y power s and as you know that these are the functions of those random variables simply we have to replace that X minus mu x power r and Y minus mu y power s multiplied by their joint probability density, and take the integration over the entire support of the random variable X.

So, maybe we can see here, that for the continuous case we have discussed some more of this central moment as well as the different order of this moment and for the discrete case we have just started with with the, what is the general expansion of the moment so whatever we discuss for both that continuous. And the discrete both are applicable for the both the types of this random variable only thing that, as for the discrete case it is the probability mass function we can go for the summation. And here for the continuous case we are just going for the integration for the entire support. So, just this is the difference otherwise both the both the equations, all the equations that we have discuss is applicable for both the cases.

(Refer Slide Time: 15:42)



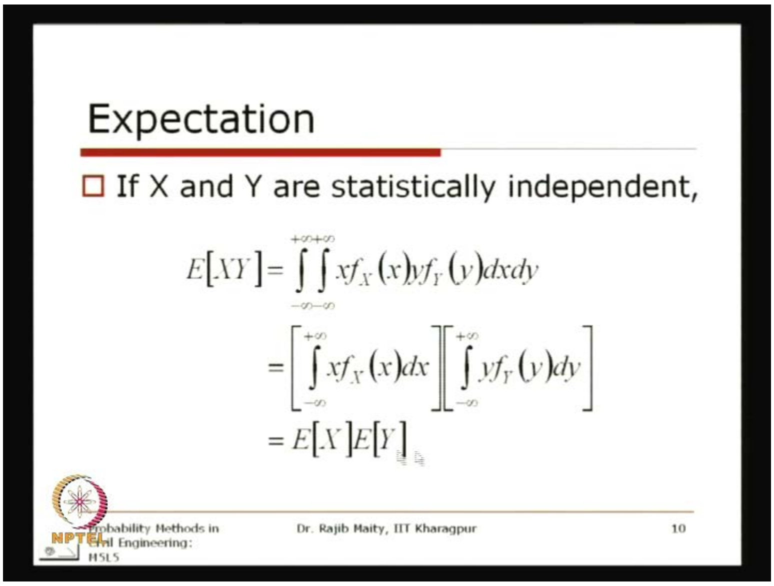
So, next if we just want to increase the number of random random variable we have considered here, if we just want to extend it to that to the n numbers of random variables. So, here if it is the discrete case so far, as our notation is concerned these things should be capital that is the is the random variable x 1. So x 1, x 2 up to x n; so, there are n random variables are concerned and for all the random variables we are just considering that power k; that is that k eth you can just take the k comma k comma k up to n numbers of k eth order moment for the n numbers of random variables. If it is a discrete case then we have to do the summation summation for all x i.

So, they are x 1 power k x 2 power k up to in this way up to that x n power k; this is multiplied with the joint probability mass function of x 1, x 2, x n. So, here this is the random variable X 1 so this will be the capital, so if you do this one then; that means, the same concept is extended for the n numbers of random variables, where all are discrete random variables. Now, we are getting that this k eth order for all the random variables k eth order joint moment is express by this so far as the random variables are discrete.

Similarly, if the random variables are continuous, then instead of this summation we have to consider the integration over the entire support for all the random variables from x 1, x 2 up to x n their power, all powers are k and multiplied with their joint density, now this X should be capital as you know that; this notation for the random variable is the capital X.

One thing can be mentioned here, it is not necessary that all the power will be k; you can have the different powers. So, different when we are talking about that power; that means that, which moment we are considering so far x 1 we can consider some some order for x 2, we can consider some some other it is not necessary that all this power should be should be same, so in that way we can make the expression more general at the cost of the computational ah difficulties.

(Refer Slide Time: 18:10)



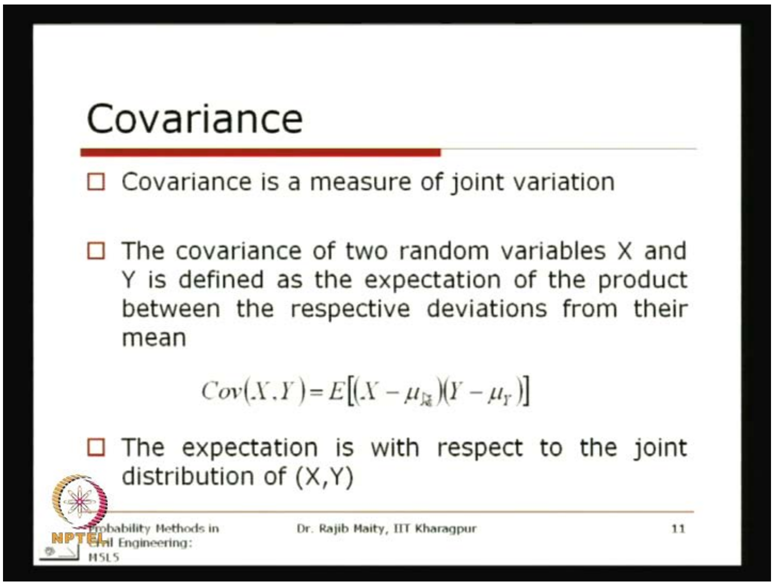
So, now if we come back to that bivariate case again, and we will see that some more properties, how these are related with the with the some basic notation that we have explained earlier. For example, that if the X and Y are statistically independent so this statistically independent we have discuss in the earlier lecture, that when we can say that there, so that occurrence of one does not have any influence on the occurrence of the other random variable.

So, and that can be verified through the fact that their joint density is equal to the multiplication of their their their marginal density so that f (x,y) is equals to f (x) and f (y). So, the joint is the product of their marginal, so if that the case is satisfied then those are called the statistically independent random variable and its in such cases; so that this joint moment that is expectation of X Y can be shown to be equal to their multiplication of their individual mean; that is expectation of X multiplied by expectation of Y.

Now, as we know that, when we are writing that f (x, y) basically, before I write this one there are some steps you know that this is basically, that X Y multiplied by just now the expression that we have shown that X Y multiplied by that joint density is this; this is for the discrete one, and is simply we have to just put that integration sign as we are considering the continuous one. So, here also may be that r equals to 1, and s equals to 1, if you put then x y multiplied by their joint density.

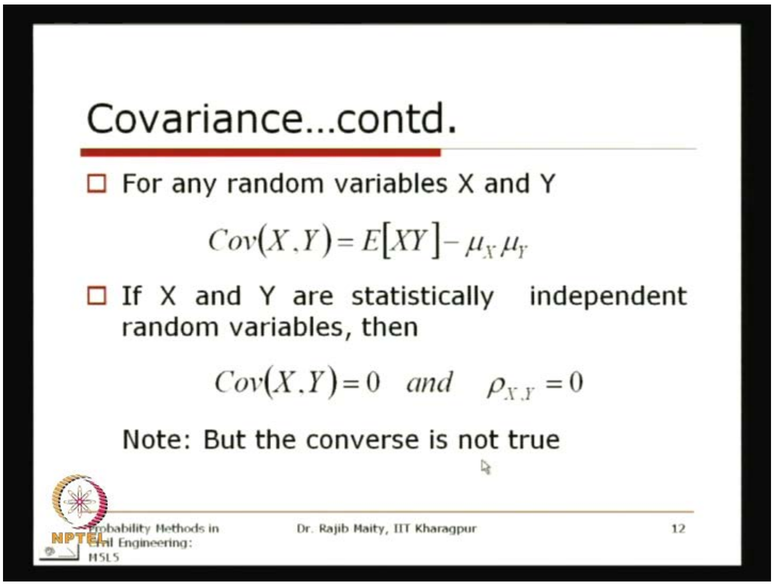
So, now as we are telling that this joint they are statistically independent; that means, their joint density is the product of their marginal marginal density, so we are separating out that x multiplied by the marginal of x and y multiplied by the marginal of y. So, if I do that one then we are coming to this form; that is x marginal of x multiplied by y its marginal of y dxdy. Now, we can separate out this integration because this is entirely for this variable x and this entirely for the variable y, we can separate out and do this integration separately and this one you know that; this is nothing but the expectation of X and this is nothing but the expectation of Y. So, if they are statistically independent with this condition expectation of X Y is equals to expectation of X multiplied by expectation of Y.

(Refer Slide Time: 20:56)



Next one important concept that we will discuss is the covariance; the covariance is the measure of the joint variation. So, how the both the random variables vary with each each other; that means, if I that linguistically, if I explain that whether, if one random variable increases what is the condition of the random the other random variable. So, the covariance of two random random variables X and Y is defined as the expectation of the product between the respective deviations from their mean. So, respective deviations from their mean when we are talking about; that means, we are considering that X minus mu x there is a mean of that X and Y minus mu y mean of the Y. So, this is basically the central second order moment when we are considering that for the X minus mu x this power so this r is equals to 1 and s is equals to 1, so this one this expectation is known as the covariance between X and Y. So, the expectation is with the respect to the joint distribution of this X and Y. Now, if we if we explain this one; this this this expression we have already seen that, how how we can explain this this expression

(Refer Slide Time: 22:20)



Then there two things can come out that is for any random variable if we just take that X and Y, then we can show that the covariance between X and Y is equals to the expectation of X Y, minus mu x mu y. So, basically if you just take this expression and we if we come through their their respective expansion of this variable, that is this one that is taking that r equals to 1 and s equals to 1. And then if we take, if we expand this one and then we can show show that this one is equals to, this this expectation is equals to their joint expectation; that is expectation of X Y minus the the product of the mean of the x and product of mean of the x and mean of y.

So, this is what is shown here, that is covariance of X and Y is equals to expectation of X Y minus mu x into mu y, if this X and Y are statistically independent random variable, then this covariance is equals to 0 and this rho x y; this correlation coefficient will come to this one in a minute, this will also we can show that this is also 0.

So, before that if they are statistically independent, if we say then just now we have seen that this expression of X Y is equals to expectation of X multiplied by expectation of Y, this is what we have seen just now, if they are statistically independent. So, this is now your mu x and this is mu y. So, if you put this one in this form so far the statistically independent random variable X and Y, so this one is becoming 0 so that is why, the covariance between two random variable if there statistically independent, then the covariance equals to 0.

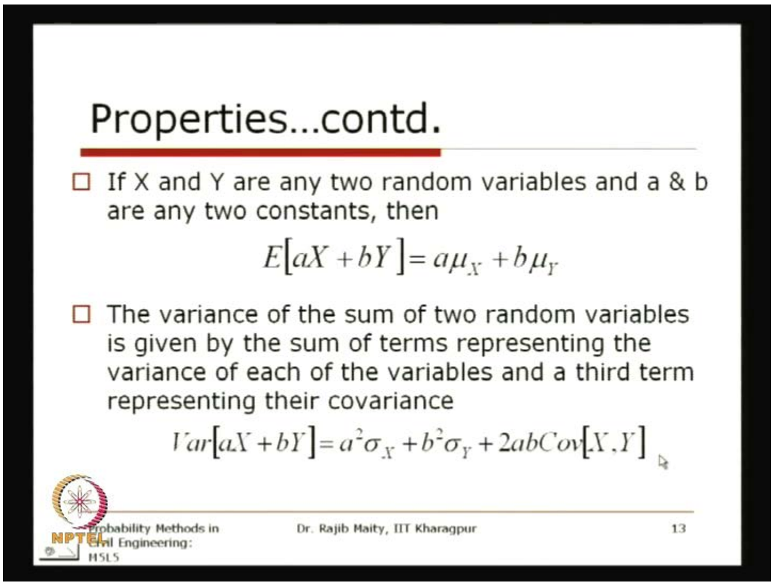
Now, up to this point whatever we have discuss, we have got three important information and we will see some problems on this this information, if we can state that the joint density of two random variable is the product of their individual marginal then those two random variables are independent. Now, if the random variables are independent then their covariance is equals to 0.

So, we can take up some some problems, where if you just show that their marginal densities are the product of the marginal density is equal to their joint density and then from the joint density, if you want to calculate the covariance then the covariance should come equal to 0. So, have we have generally some this type of some problem and we will just discuss after this after this discussion, when will just start with that assumption that this is independent. And we will just show also that whether they are really the product of the marginal are equal to their joint distribution or not and after that from the joint distribution we will calculate the covariance and we will see that whether that covariance is coming to 0 or not.

So, these properties will be cross-checked through those problems. So, after this one one thing is also important after seeing all this thing, but the converse is not true the converse in not true means, that if sometimes we see that the covariance is 0, but that does not indicate that those are statistically independent.

So, if you recall our earlier lecture that we told that the two random variables are statistically independent if and only if that condition, that their joint is equal to the multiplication of the and product of their individual marginal. So, from that one we are starting and we are ending up to the condition that they are they are independent, then the covariance will be equals to 0, but if I somehow, I show that together covariance is equals to 0 we cannot say that the reverse is true, that is that they are independent we cannot say that one; this will also will be showing in terms of some pictorial view in after few slides.

(Refer Slide Time: 26:57)



Now, some more some more equations, these equations we also discuss earlier with respect to the single random variables. Now as the number of random variables are more than one in this multiple random variables so we will also see this properties as as well, that if X and Y are any two random variables and, a and b are any two constants, then expectation of aX plus bY is equals to a mu x plus b mu y, recall that in the earlier lecture in with respect to the single random variable, we have shown that expectation of E X is equals to a mu x, that is whatever the constant is multiplied with the random variables, it can be can be taken out from this expectation operator, and so that it will remain as this expectation of X which is nothing but mu x.

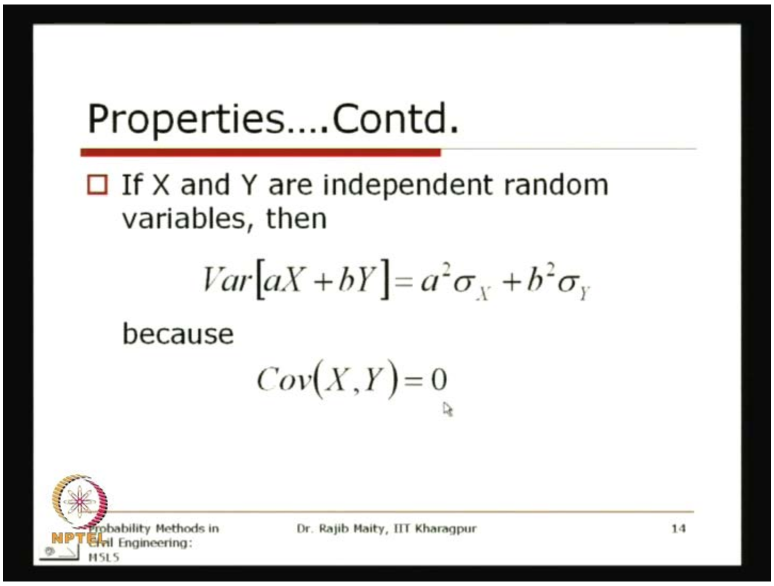
So, this is now additive also if there are more than one random variable and they are added to each other with some other constants. So, this can be again applicable and we can just write that expectation of aX plus bY equals to a expectation of X which is mu X and b expectation of y which is mu Y earlier in the context of the single random variable when we are considering the variance then also we have we we have seen that whatever the constant that is multiplied when we are taking out of this variance operator those will be in terms of the square we can take it out and then the variance of the random variable.

But when we are considering more than one random variable then the new term that is a covariance also will come in to the in to the expression that is the the variance of the sum of two random variables is given by the sum of terms representing the variance of each of the variable and a third term representing their covariance. So, variance of aX plus bY when we are when we are taking, so a can be taken out of this variance operator so that this will be a a square so a square sigma X which is the variance of X plus.

Similarly, for the second one the b square multiplied with the variance of Y which is sigma Y plus another term we will come which is 2 ab multiplied by the covariance between X and Y; so, this is the full full expression when that covariance is considered.

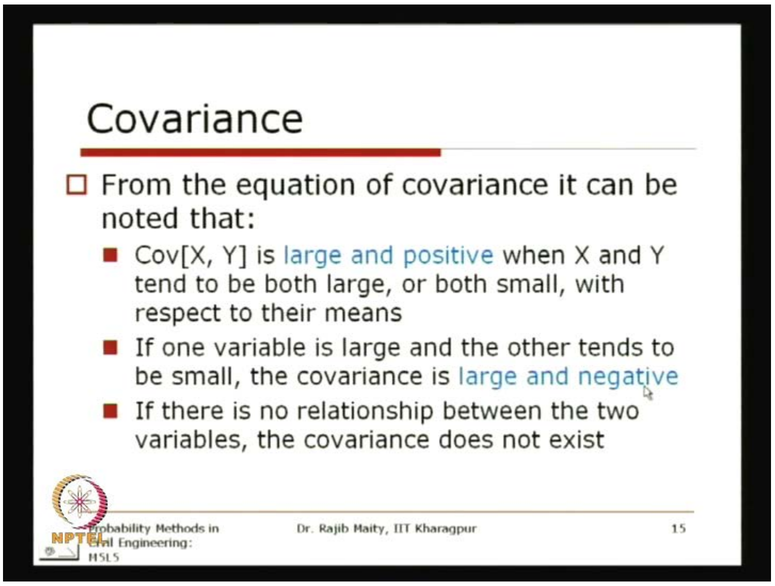
So, easily you can just draw one more step that if the random variables are independent then we have just now we have seen the covariance will be equals to 0. So, in that cases the variance of aX plus bY will be equal to a square sigma X plus b square sigma y. No third term because the covariance will be 0 in case of the independent random variables.

(Refer Slide Time: 30:02)



This is what is explained here also again that if the X Y are independent random variables then the variance of aX plus bY equals to a square sigma X plus b square sigma Y because the covariance between the X and Y is equals to 0.

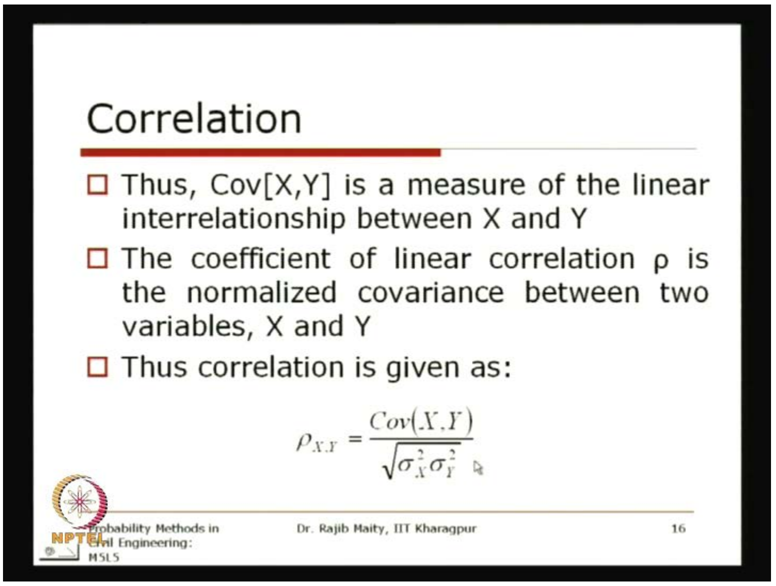
(Refer Slide Time: 30:20)



Now from the equation that whatever the expression of this covariance what we have seen it can be noted that this covariance X, Y is large and positive when X and Y tend to be both large or both small with respect to their means. Similarly, or in contrast to this if one variable is large and the other one tends to be more and vice versa of course. The covariance is is is large and negative and if there is no relationship between two two random variables instead of no relationship we can say that if there are if they are independent then the covariance does not exist.

So, using these one if we now when we are talking that this large and positive and large and negative if we just want to make it standardize, then we can have some some base over which we can we can just conclude the conclude or some we can quantify the nature of their association. Now, to standardize this we have we are using their individual standard deviation.

(Refer Slide Time: 31:39)



This is what how we will show that thus the covariance X, Y is a measure of their linear interrelationship between X and Y, the coefficient of linear correlation that is rho that is correlation coefficient rho is normalized covariance between two random variables X and Y.

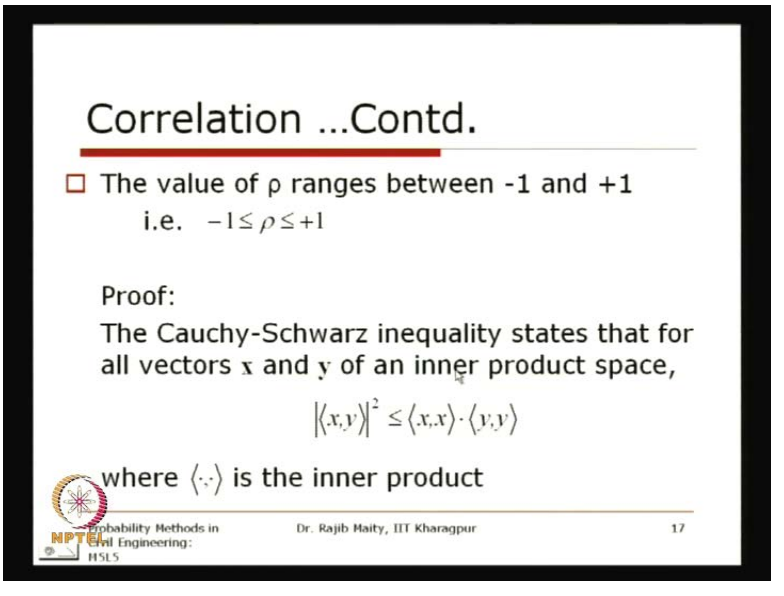
Now, how this normalization is done is this that rho x y it is the correlation coefficient between X and Y is equals to covariance between X and Y divided by product of their standard deviation which is square root of sigma x square sigma y square you know these are the variance and their positive root is your standard deviation. So, product of the standard standard standard deviation so this is the normalizing factor if we normalize this then this this total quantity is the is the correlation coefficient.

Now, why we have use this one for the normalizing constant is that this covariance X Y is having a maximum upper limit is equals to their product of the individual the square root of the variance product of the variance.

Now if this is the maximum limit of this one this will also will show in terms of some inequality theorem, so if this is the maximum one then we can say that so this total quantity can go up to the maximum of the plus 1; obviously, there will be we have to take the square root of this total term so that it will we can also show that it is limit of this quantity is basically from the minus 1 to plus 1, and if they are independent to each other then we know that this covariance is 0. So, this correlation coefficient also will be 0.

So that is what actually was mentioned in this one as I told that we will discuss in a minute that if this covariance is 0 then this rho X Y that is correlation coefficient between this two are also equals to also equals to 0 which is now clear from this expression that if covariance is 0 then this correlation coefficient between X and Y is also equals to 0.

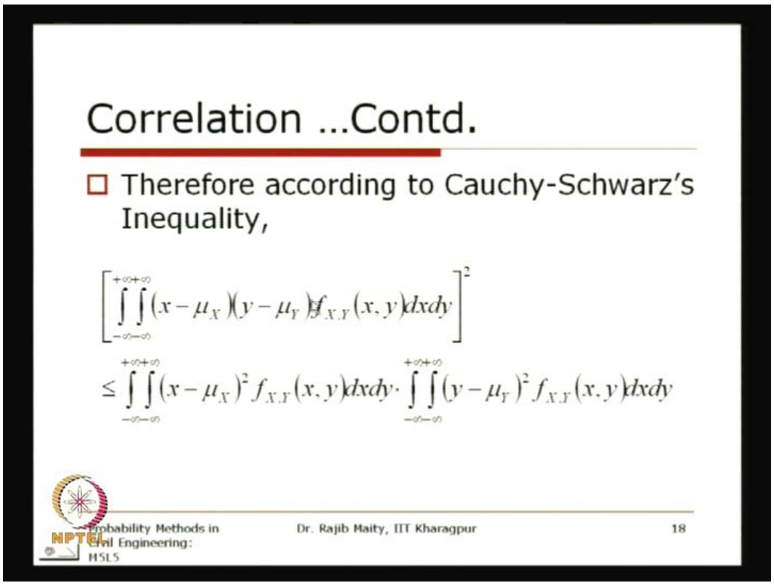
(Refer Slide Time: 34:06)



Now, as I was telling that this this correlation coefficient is range between 0 and sorry range between minus 1 to plus 1, so this rho x y is from minus 1 to plus 1 so this plus one basically indicates that there is a positive dependence. And a perfect linear relationship if if it is exactly equal to one perfect positive relationship and if it is equals to the minus 1 then it is perfect negative linear relationship; that means, one increase of one is ensures that the decrease of the other one similarly for the plus 1 increase of 1 ensure that the increase of other one also.

Now, to prove this one there is a inequality that is Cauchy Schwarz inequality it states that for the vectors x and y of an inner product space that if we take this inner product their mode and square which is less than equal to their individual inner product.

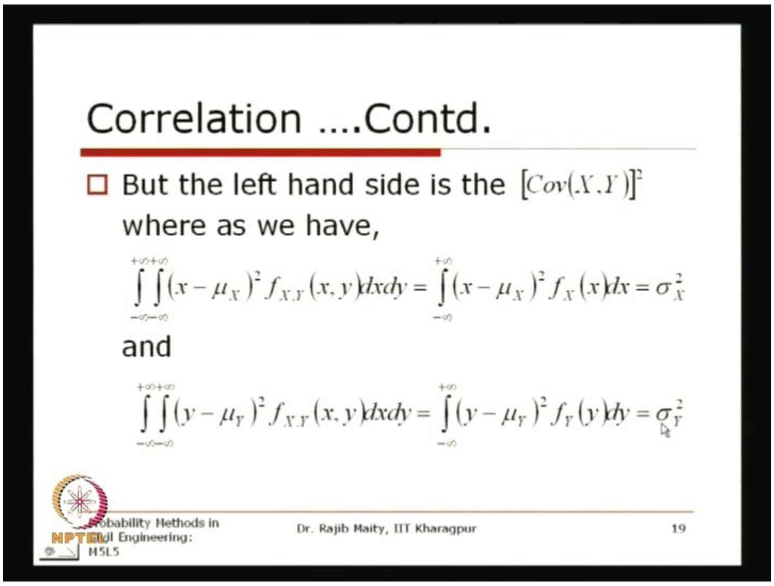
(Refer Slide Time: 35:20)



Now, using this one if we just fit into that expression of the that expectation of their their covariance with the central moment. You know that this one we we describe earlier, this is the covariance with respective to their that is central second order moment. Taking r equals to 1, and s equals to 1; that is X minus mu x Y minus mu y multiplied by by their joint density and integrate it over the entire support of the random variable X and Y.

Now, if we take this one as a square now which is here the the so that X minus mu x square one this joint density multiplied by their this the other one one that is Y minus mu y square f (x) that the one density because there is there is square. So, one we are taking here and other one we taking here. So, this one if we just take this integration, so with this is satisfied with this inequality so maximum of this quantity should be the product of this one multiplied by the product of the product of this and this.

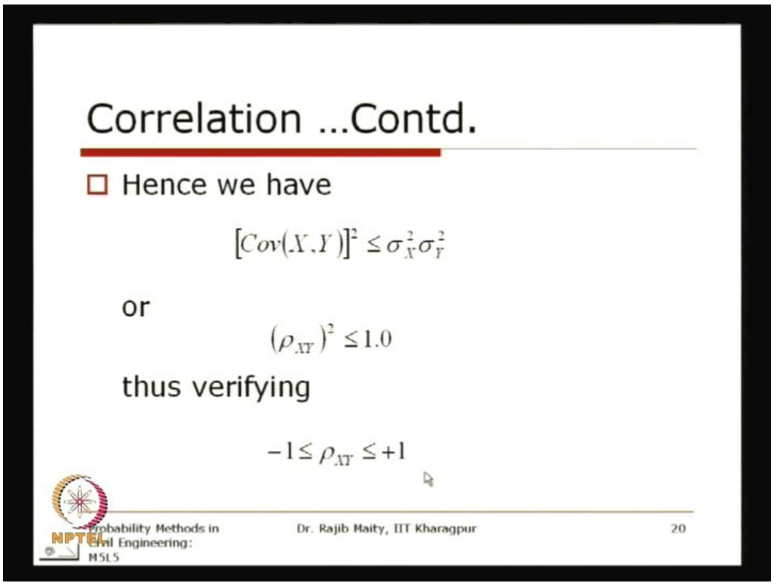
(Refer Slide Time: 36:36)



So, as this is the maximum upper limit now if we just see what are this two are individually means that is X minus mu x square their joint density dxdy which is nothing, but if we just first take the because this is with respect to the x now if we take this first integration from this the entire support of the dxdy and. So, then with respect to the y this is this can be taken out from this integration so what is remaining with in this integration is that minus infinity to plus infinity their joint density with respect to y; that means, we are marginal the y is marginal out what is left is that marginal density of x.

So, that is why this marginal density of x multiplied by this X minus mu x square and you know this expression is nothing, but the variance of X and which is the central moment that is with respect to the mean the second order moment of the random variable X only, so, this is sigma square similarly the other one is the is equals to the sigma y square.

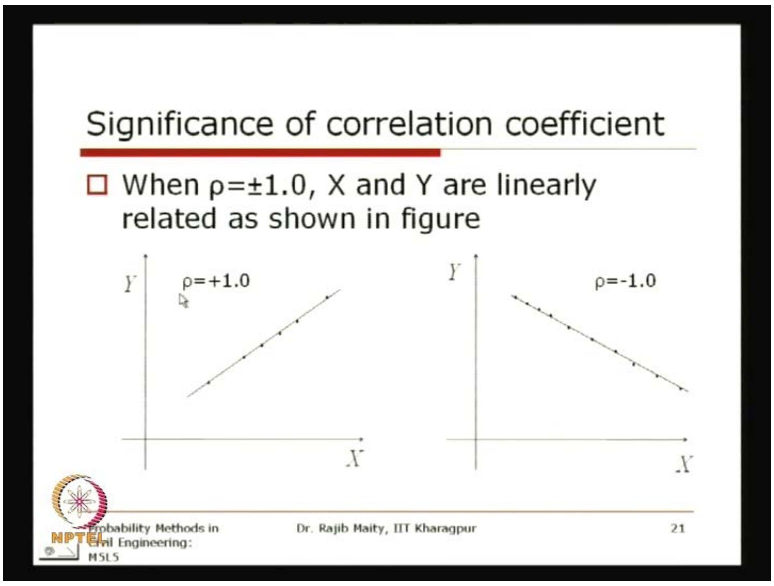
(Refer Slide Time: 37:47)



Now, if we just put this one in that that that expression that is the expression of this one what we started with this one. So this is your that covariance the covariance square is less than is equals to this is your that sigma x square and this is your sigma y square.

Now, this is here is shown here the covariance square less than equals to sigma x square sigma y square, which is now if we just take this one this one to the left side then this will become as a rho xy square; so rho xy square, because we have just now we have seen that rho xy is the covariance of X Y divided by square root of sigma x square sigma y square. So, this is your correlation coefficient square which is less than equals to 1 this is verifying that this correlation coefficient should be between minus 1 to plus 1.

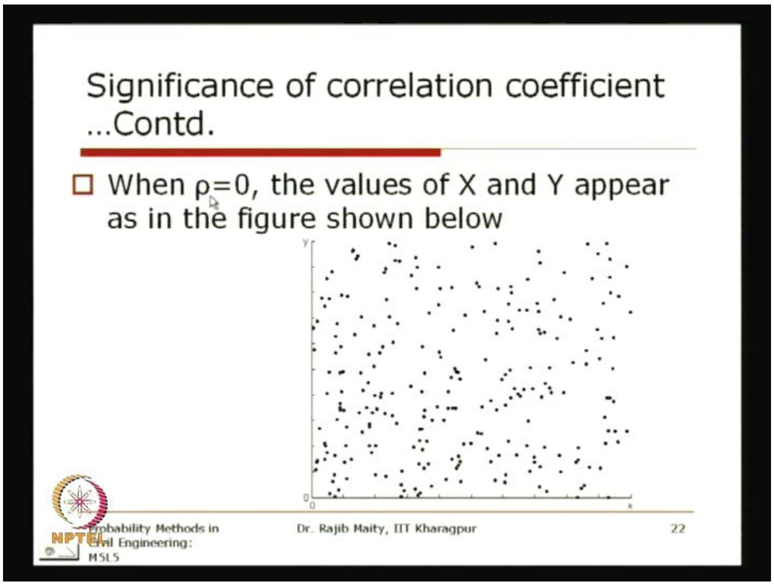
(Refer Slide Time: 38:43)



Now, if we say that what does these things means that when this when it is exactly equal to the plus 1 or minus 1, so when it is equals to exactly equals to the plus 1; that means, there is a perfect linear relationship and which ensures that the increase of one random variable ensures the increase of the other other one. And also the opposite when it is exactly equals to minus 1 then the increase of one is ensuring the decrease of the other one and vice versa.

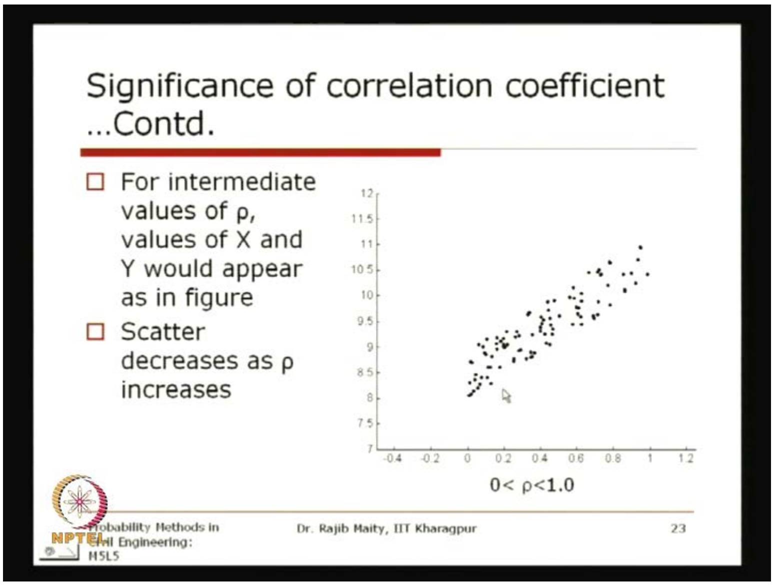
So, these are all on a perfect straight line as long as these are on the perfect straight line if it is having a decreasing trend towards the positive X then this is minus 1 and otherwise it is plus 1.

(Refer Slide Time: 39:32)



Now, in between if we just see if we see this kind of relationship that is this is the X this is the variable X this is variable Y where we are not seen any kind of trend. So, there it is a we can get a exactly equals to or very close to 0, the correlation coefficient will be very close to 0 or exactly equal to 0.

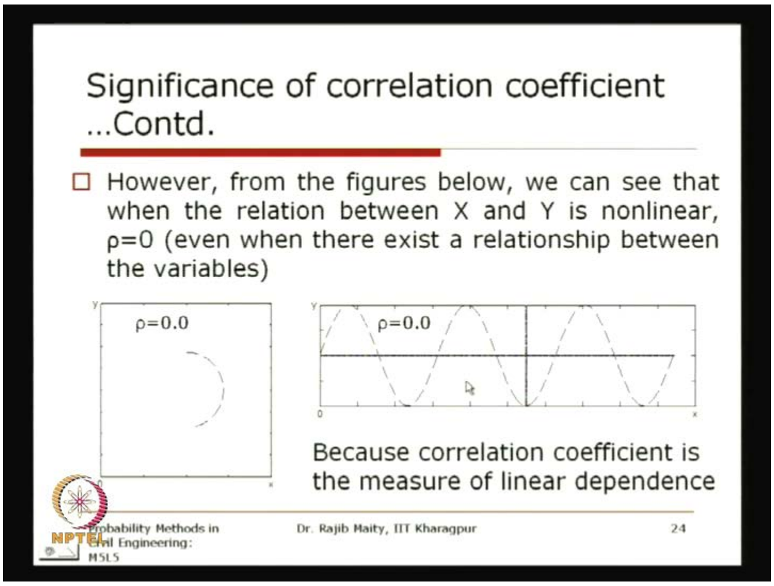
(Refer Slide Time: 39:56)



Now, in between this one so this type of relationship we can say that this is you are having a positive relationship, so this will be the rho will be equals to between 0 and 1, and for this case as we have the way we have generated this correlation coefficient is coming around 0.92 or so this type of, so this is also positive relationship the increase of one is ensuring the increase of the other one and the correlation coefficient r is equal to approximately equal to 0.92.

So, here you can see that this this axis limits so this is that for the X it is generally varying between 0 to, so one and for Y it is varying between 8 to 11 kind of thing so this range does not have anything to say that how their relationship will be. So, only thing that what we are saying that how what is their linear dependence or linear association between two two variables which is reflected to through the correlation coefficient.

(Refer Slide Time: 41:08)

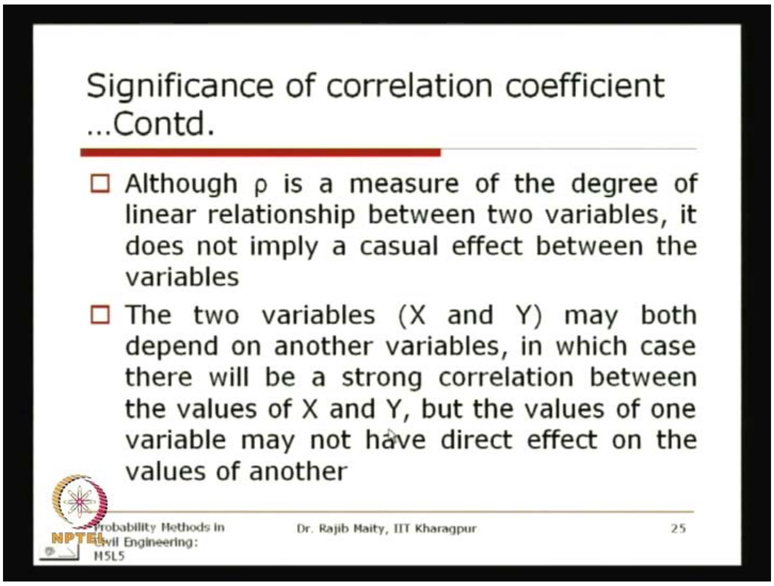


So, however one thing is also important here to note that from the figures that we have shown below we can see that when the relation between X and Y is non-linear for example, there we're here we are talking about a semicircular shape that is there is a. So, X and Y are related through the equation of this circle, so which is a non-linear equation here there is a relationship with a with a sine curve so there are the relationship between X and Y is perfect, but need not be linear.

So, as long as the relationship is not linear so this rho is equals to 0 this can be mathematically this also can be proved even for the for the expression for example, that X equals to Y square so which is perfect relationship. So, the first order the quadratic relationship, but if we calculate their correlation coefficient between X and Y s X and X square say then also it will become 0.

So, this is basically a point of caution that that correlation coefficient should if it comes near 0 that should not be concluded that there is these are these are independent, which actually we are trying to mention when we are telling that the reverse is not true when the covariant between two variables are 0 that does not mean that they are independent this is what is reflected here because the. So, why we will get here is 0 is the correlation coefficient is the measure of the linear dependence or linear association. So, linear here is important as long as the relationship is linear or how much linear relationship is there there we can get a get some correlation coefficient, but for that non-linear relationship this is equals to 0.

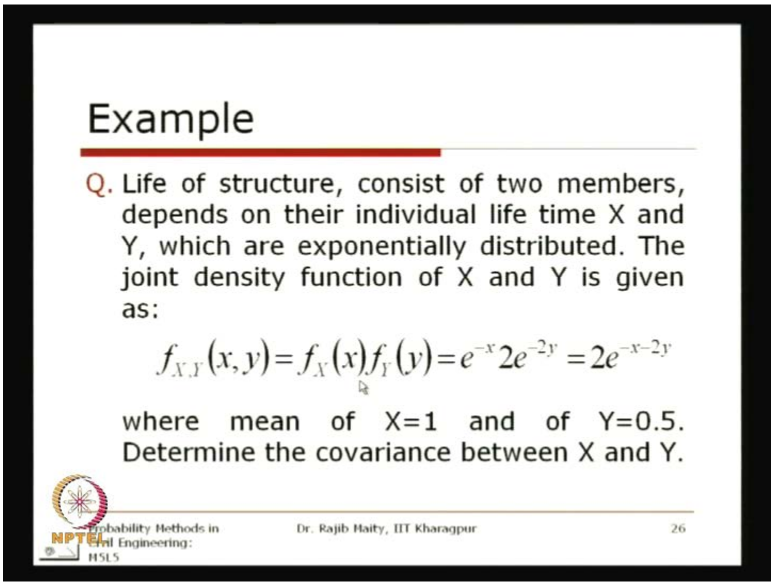
(Refer Slide Time: 43:04)



So, the significance of this correlation coefficient although the rho is a measure of the degree of linear relationship between two variables it does not imply that a casual effect between the variables this is one thing which is also important to note that the correlation in causality should not be should not be mixed up, because the positive correlation coefficient also is not a not a indication that the that the one variable is having some casual effect on the other one, so but however, in many application particularly in the civil engineering application we to identify some what is the what is the casual factors we use various statistical measures along among which the correlation coefficient is one of the most important out of them. But this thing should be kept in mind that even though you get some very significant correlation coefficient that may not always indicate that there is a casual relationship so that is what is mentioned here that although the rho is a measure of degree of linear relationship between two variables, it does not imply a casual effect between the variables the two variable X and Y may both depend on the another variable.

So, this is one one special case that we are giving as an example, but not the only case where this type of relationship may occur only one special case we are just mentioning here that two variables X and Y may both depend on the another variables. So, in which case there will be a strong correlation between the values X and Y, but the values of one variable may not have the direct effect on the values of the other. So, even though we are getting a we may get a strong correlation coefficient that does not indicate that they are having any casual effect on each other.

(Refer Slide Time: 45:08)



We will take one example particularly that I was discussing that when we say that they are independent, and their covariance should be equals to 0 so that type of one example is taken here this is a structural ah engineering problem if the one structure is made up of two structural element. And failure of the life time of the each member is we can say that this is exponentially distributed depending on their quality of the material, and both can have the exponential distribution and both can be independent to each other.

In such cases one such problem is taken here so the life of the structure consist of two members depends on their individual life time X and Y. So, here the random variables which are consider is the X and Y is the individual life time of the two members of a structure and both the X and Y are exponentially distributed and their joint density joint density function that is X and Y is given as f (x,y) is equals to that product of their marginal distribution and the marginal distribution of x that is the first random variable is that e power minus x and second one is the 2 e power minus 2 x.

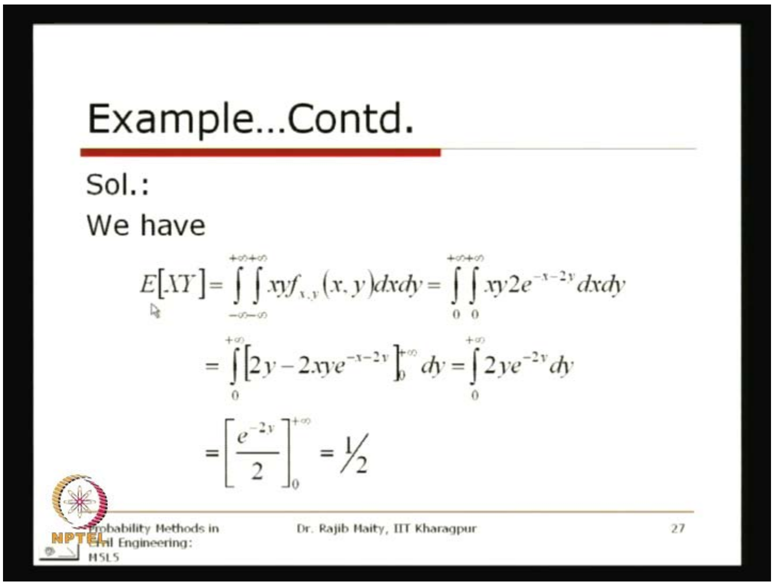
So, we can see that both are exponentially distributed f x (x) is that e power minus x and other one is 2 e power minus y so and we are taking their multiplication; that means, that X and Y are independent to each other or I should say it is the other way as they are independent we are assuming they are independent that is why we are getting their joint density is equals to the product of the marginal's so that e power minus x multiplied by 2 e power minus 2 y which is giving you this this expression as their joint density which is equals to that x and y the limit of this x and y should be from 0 to infinity for both x and y.

One thing is also important here when we are taking this type of relationship we are supposed to basically fine let me finish this one so we have to determine the covariance between X and Y. So, when we are when we are interested to know their covariance of X and Y, if it is not this type of known distribution that is exponential distribution is known to us and we know their properties. And by this time in the earlier lectures also we have describe different standard distribution for which we know the different properties.

So, they are exponential distribution which is having a form of this lambda e power minus lambda x you know that for their mean is 1 by lambda so otherwise what could happen. If the joint distribution is given for some other problem we are supposed to obtain their marginal distribution from the marginal distribution we are supposed to get their mean. And after that we can use that one in the in the equation of the covariance that type of example also we will take up after this, but as in this particular case it is the exponential distribution and we and we know their what are the properties l is the first moment we know that its mean is 1 by lambda so we know what is the mean of X we know what is the mean of Y.

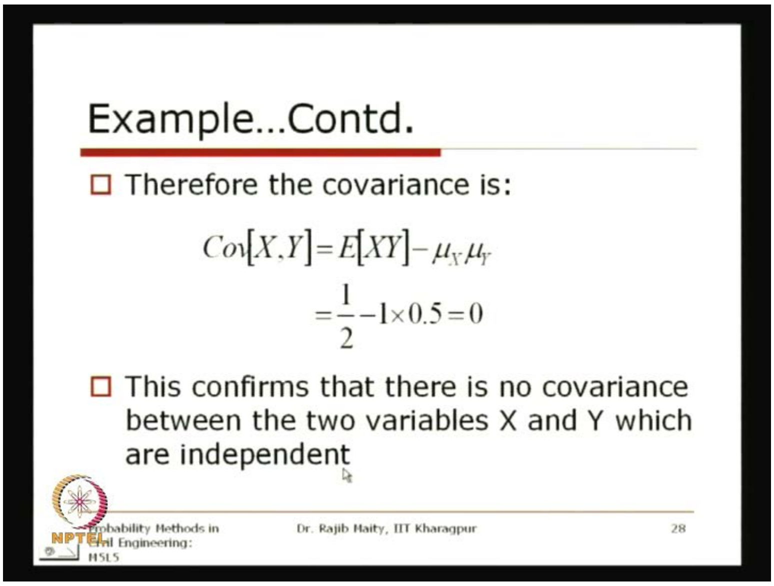
So, here so we can see that as this is the exponential distribution is e power x so the mean of the X is equals to 1 and mean of the y equals to 0.5 that is 1 by 2. So, this two information is already known to us only thing we have to calculate that covariance that is the expectation of e x we have to calculate and from there we can calculate their covariance.

(Refer Slide Time: 49:01)



So, we have to first get that this is only unknown that we have to determine the expectation of X Y, so which we know that this is from the over the entire support that X Y multiplied by their joint density which is equals to here from 0 to infinity 0 to infinity these are the support of this X and Y, and we have to do this integration so after doing this a few step of this integration we can we see that this expectation of X Y is equals to half that is 1 by 2.

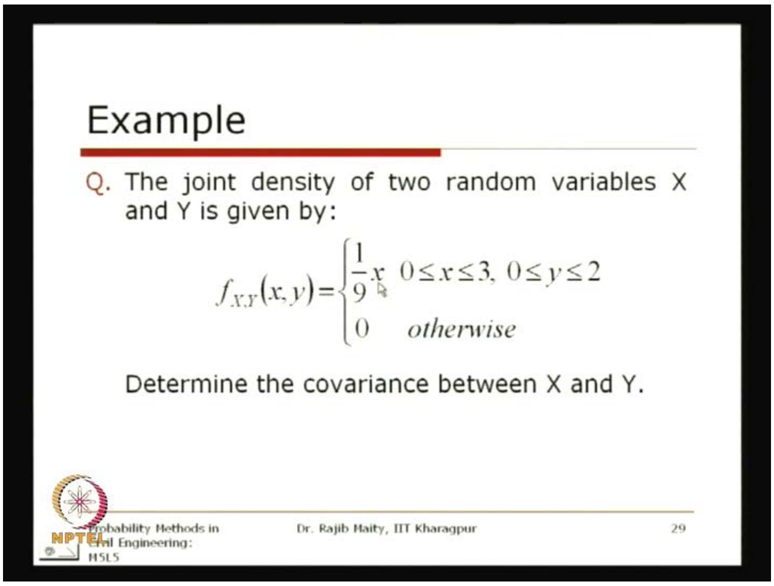
(Refer Slide Time: 49:38)



Now, if we take this one in the expression of this covariance covariance that is covariance of X, Y that is expectation of X Y minus mu x mu y now we already know what is mu x and we already know what is mu y, so the this is equals to half minus 1 multiplied by 0.5 which is equals to 0.

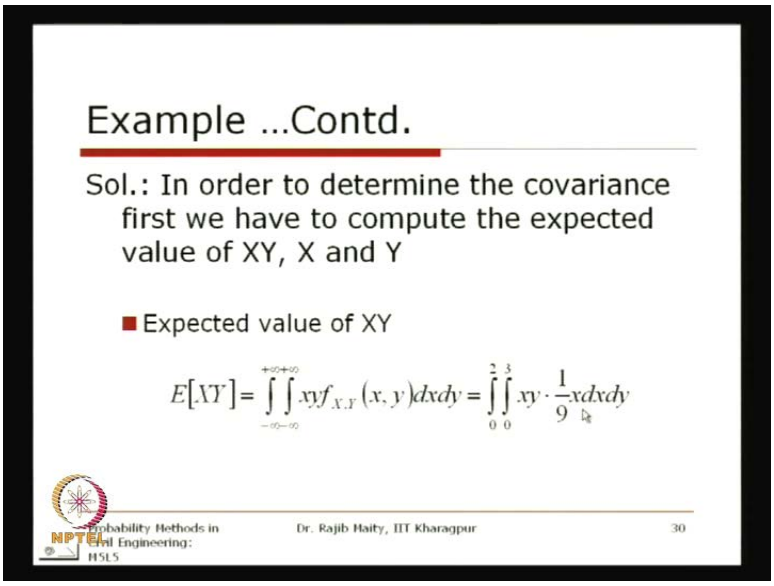
So, this answer basically was expected because we started with that thing that these are these are independent independents. So, as they are independent their covariance should be equals to 0 that is why we can note here that this confirms that there is no covariance between two random variables which are independent. We'll take up other problems similar, but for those cases we do not know what is their individual individual mean mean.

(Refer Slide Time: 50:33)



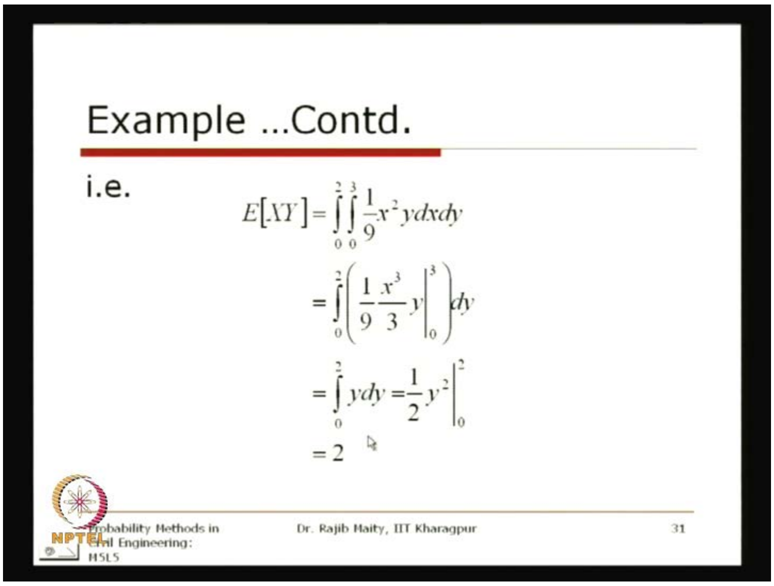
So, we have to calculate their mean as well as their joint moment that is X Y we have to calculate and there we can see that what is their covariance is. So, the joint density of two random variables X and Y is given by this f (x, y) is equals to 1 by 9 x, and the support is that from 0 to 3 and y is from 0 to 2. So, two different range of this support this is the joint pdf form there is only one variable x so you know that when we are declaring that one joint density is there we are ensuring that this is a valid pdf joint pdf. And you know how to verify that we have discuss earlier that is integration over the entire support of the random variable should be equals to 1, and this should be greater than equal to 0 for the entire support. So, with both the things are satisfied. So, what we have to do here in this problem that we have to determine the covariance between X and Y. Now, to do that one we do not know their marginal's and we do not know their means we have to obtain that one first.

(Refer Slide Time: 51:51)



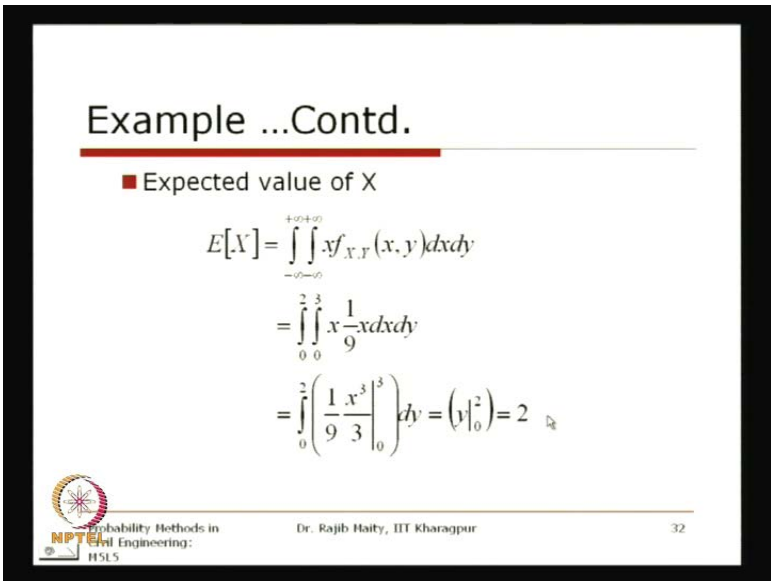
So, in order to determine the covariance so you first we have to compute the expected values of the X Y, X and Y all 3. So, expected value of this X Y is equals to their entire support from this minus infinity plus infinity X Y f (x, y) from 2 to from 0 to 2, and from 0 to 3 that is x y dot 1 by so this is your that joint joint density, so if we do this one this integration occur full state we can check that this expectation of X Y is becoming plus 2.

(Refer Slide Time: 52:26)



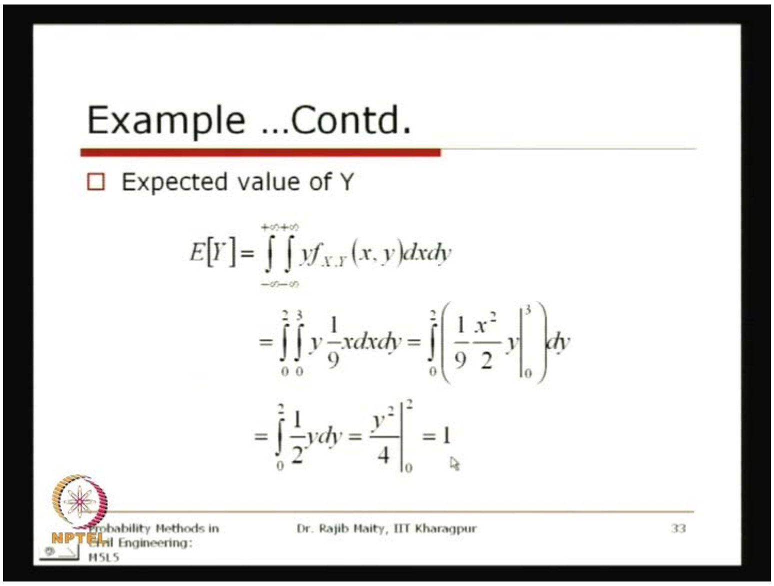
Now, similarly we have to get that expectation of X as well as we have to get the expectation of Y and from there we can get what are this things we now the expectation of this X is equals to their X multiplied by the joint density of dxdy.

(Refer Slide Time: 52:46)



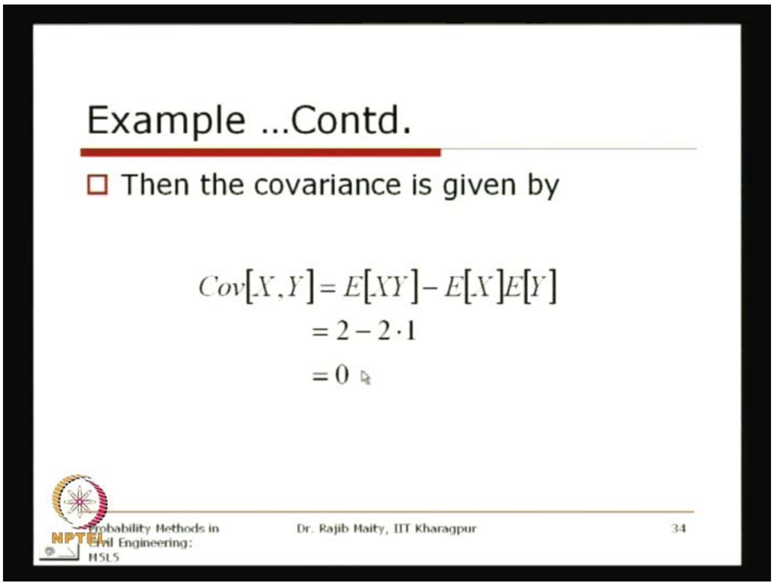
So, this is the entire range and if we do this integration with respect to the with respect to that X and then Y then we will get that this range which is we will get ultimately we will get that is equals to 2 expectation of X is equals to 2.

(Refer Slide Time: 53:16)



Similarly, if we take that expectation of Y which we do following the same expression we will get the expectation of Y is equals to 1, so we got expectation of X expectation of Y and expectation X Y.

(Refer Slide Time: 53:30)



So, the covariance is equals to you know that expectation of X Y minus expectation X into Y which is equals to 0 now this is interesting. So, we started with a problem where we have seen that the covariance is becoming to be equals to 0. So, this indicate or this may indicate I should not say that this this indicate that the covariance this simply indicate the covariance between the X and Y equals to 0 this may be a result of that that this that X and Y might be independent, if they are independent; obviously, the covariance will become 0, but we cannot we cannot ensure from this result that they are they are independent because of we have got that covariance is equals to 0.

So, covariance equals to 0 is not the proof to declare that this X and Y are equals to 0, but further to receive the shape we can just check whether really the X and Y are independent that is why we got the covariance equals to 0 or not. So, for that what we have to do we are having that joint density joint density we discuss. Now, we can get their individual marginals and if we get their individual marginals after that we can do one check whether they are independent or not.

So, our marginal density what is given here is this thus 1 by 9 x for this range. So, we can get their marginal's one marginal of x and marginal of y and we can multiply this two marginal and we can see whether this coming equal to this are not if it is so then they are independent we got the covariance equals to 0, if we does not come also that does not mean it will always come as I was as I was telling. So, you can just take up is as your cross check we can just take it once yourself to see that why the covariance has come to 0 for this particular problem. So, there are other properties as we are starting this class for this multiplied random variable and those properties we will take up in the next lecture. Thank you.