Probability Methods in Civil Engineering

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Lecture No. # 22

Conditional Probability Distribution (Contd.)

Hello, and welcome to this lecture of this module.We are discussing multiple random variable in this module, and sofar what we havecovered that their joint distribution, marginal distribution. And fromlast lecture, we started that conditional probability distribution for the multiple random variables.And in last lecture, what we have seen we haveunderstood the concept, and particularly with respect to the discrete random variable. Andhow we canget their conditional probabilities, we have discussed througha problem which iswhere the random variables are in discrete nature.We will continue the sameconcepts,sameconditional probability discussion on this conditional probability density, and this we will continue for the continuous random variable from this lecture.

So, basicallywe are continuing that the conditional probability distribution which we will basically cover in this class for the continuous random variable. And we will discuss some problems, where we need to deal with thecontinuous random variable that we will see.And after that, we will go gradually to thediscussion and different measures of association of different random variables; this is very important,because when we are talking about that differentrandom variables, and we are consideringtheir joint occurrence, their joint probabilistic behavior. So, basicallywe want to know theirkind of association with each other orsometimes ifthe number of random variables is more than two, thenwe may interest to know that, what association istwo random variableswhen the other random variables are either partial doubt or taken care of.

But basically as you know from this last couple of lecture that we are basically starting with the two random variables which is the special case, which is known as the bivariate random variable.We will also discuss thosemeasures in terms of the two random variables, and the similar concept can betaken forward for the more than two random variables.So,that will be ourbasically I should not say that main goal, but one of the most important goal to understand the concept of multiple random variables.

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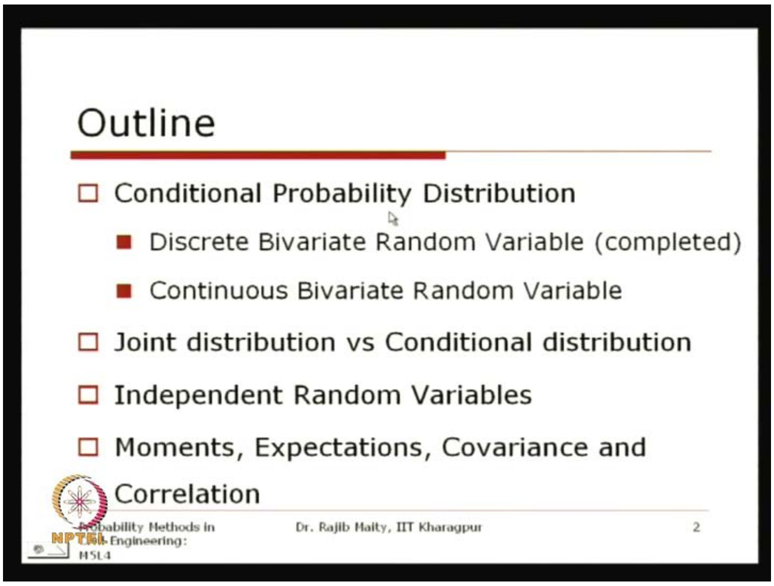

So, to start that we will firstsee that continuousrandomvariable in case of the continuous random variable, how we can get this conditional probability distribution,and as I told that thisdiscretebivariate random variable case means, in case of this conditional probability distribution, that we have completed in this last lecture. And today, we will continue with the same concept of conditional probability distribution, and we willknow how we can determine this in case of the continuous random variable.

Then also we will gowith the joint distribution versus conditional distribution; this is in case of the both the cases meansdiscrete as well as continuous, then the concept of independent random variable we will see in the light of this conditional probability distribution andthe sum of probabilistic condition. We will see, to declare two random variables to beindependent and after thatgradually we will move todifferent measures ofassociation among therandom variables that we are dealing with sofar, in this module through, their moments expectation covariance correlation may beall this property will not be covered in this lecture itself but gradually this will be the overall outline, how we will proceed one after another, so we will continue now with this continuous bivariate random variable for the conditional probability distribution.

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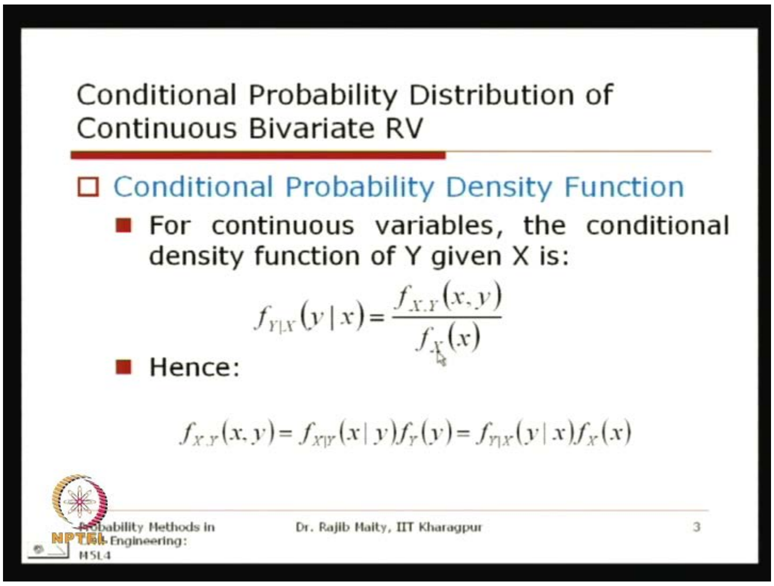

So, if we recall from ourlast lecture, that this conditional probability distribution iswe call it, when the information of the of one random variable is known to us.For example, this is,that distribution of Y given X and we have seen in the lastlecture, in case of the discrete random variable that; this is thejoint distribution in case of discrete; itispmf, that is jointpmf we have seen, and that divided by the marginal of the one random variable on whichthe distribution is conditioned on. So, hereas it is condition on the X we are using this f x (x); which is nothing but the marginal probability distribution forX.

So, in the last lecture, we haveseen that, why we need thisneed thismeans;this form actuallymixes;that mixes it suitable, first of all thisconditional density is itself,and is a probability density, sothe propertiesthat the commondensity function follows; if should be followed by this one also. So, theythe first one you know that this should be non-negative.

And second one you now that, this should bethat summation over the entire support should be equal to 1. So, here thesummation, if you want to know, then that you know that, we have to do the integration over the entire range of the random random variable concerned and that should be equal to 1, so basically that the pictorial representation that we give in the the last lecture.

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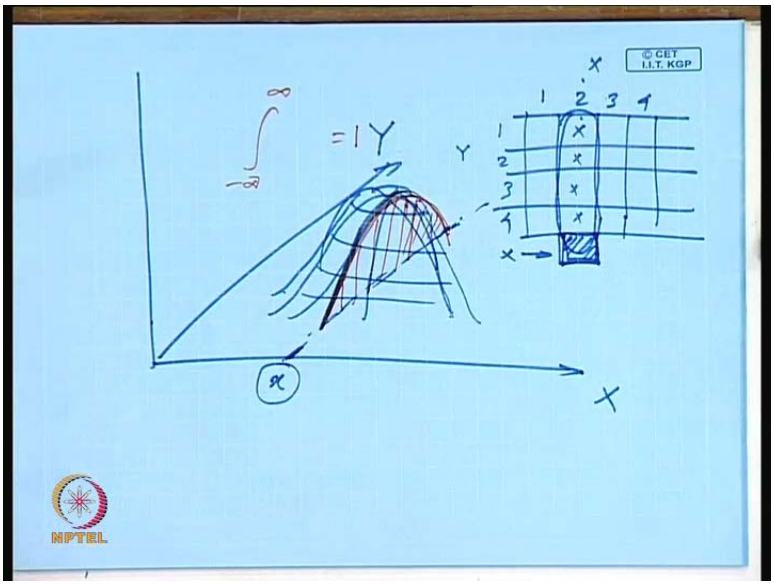

If you refer to that once again here, on this writing pad that;this one suppose that, if you take that;this is thatthis is that two axes or the two random variable axes representing two random variables, and if we are having some kind ofsurface here.

So, if this is the jointjointdistribution of thistwo random random variable; that means, when we are talking aboutthethis is condition on the X so; that means, this one this particular value of this X, I am just talking about. So, I amlooking through if this is; this has occurred. So, what is the distribution of Y,now when we are talking that on condition of this; basically you are taking, a taking you are looking at the cross section;that cross-section is passing through this valueX. Now, that in case of the discrete; also we haveseen in the last class; that there are saydepending onhow manyspecific outcomethat the X and Y can have and one example, we have shown that the X can take the 4 specific outcome 1, 2, 3, 4 and Y also can take 1, 2,3 and 4.

So, then for the condition on that X is equals to 2 then we have considered only in this column. So, this is for the discrete case and similarly, if I just take the same concept to the continuous random variable; that means, I am considering a section of this jointpdf through this line that, where this X is given to youwhich is equivalent to considering the one column.Now,you have alsoseen that, if this is the marginaldistribution of thisof the X then youknow this is the summation.

So, first entry, second, third and fourth, sothis probability that summed up to get the marginal distribution at x equals to2, soandthat one was taken and divided by this two sothat the summation of this all probability is equals to 1, soequivalent in this continuous case, what we willtake that sothis is the marginal marginal distribution that at this section.

So, whatever the section that, I see here in this section;through this section suppose that, this is a section that, I see through this one, through this X. Now, the total this one this whatever the total means; total that whatever the whatever the distribution through this x, if that is divided by this joint distribution distribution then the new, then the final distribution that we will get that will follow that the second property of a probabilitydistribution function is that;its integration over this range, over this entire range of this y should be equals toshould be equals to1.

So, this isthe fact that, we arewe are taking that this f (x) there is a marginaldistribution of thisof therandom variable on which the distribution is conditioned on. So, the expression for the expression for the conditional distribution says that that if f (x,y); this is the joint distribution that divided by its marginal of the random variable on which it is conditioned will get the conditional distribution.

So, this also we can represent if you want to know that, just after some algebraicjust take this one,this side then we get that the joint distribution of x and y is equals to probability of I think from this equation we will get this expression directly that is f,the distribution of y oncondition x; that is y given x y given x multiplied withthe marginal ofx.

Similarly, you can alsoif you if you if you see that, if you just change this one; that is f (x)f x given y thenthis will be the joint distribution; this will have no change because this is the joint distribution between X and Y, but this marginal will change of instead of x it will be y the marginal of y,you have to consider. So, there also if we take the joint distribution will equate then it will come that f (x) given y x given y multiplied by the marginal ofy the random variable Y. So, these two expressions are equal because both are equal to their joint distribution.

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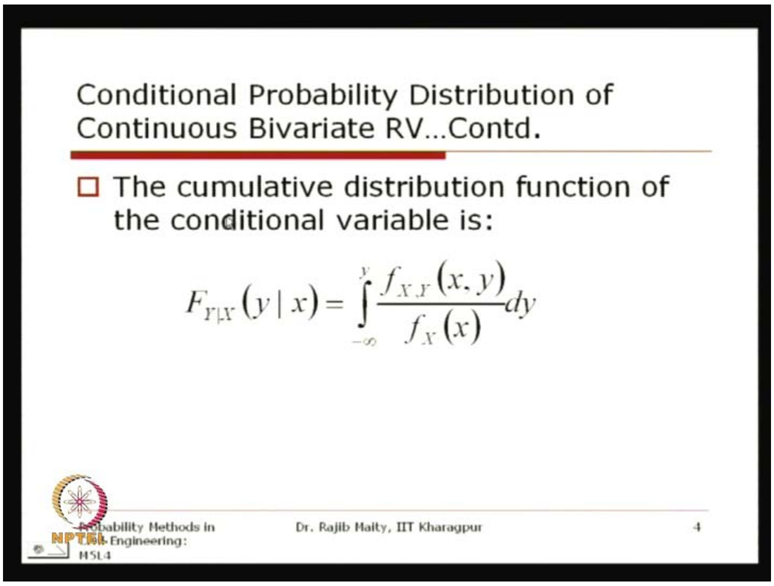

Now, suppose thatwe areconsidering that the Y given X. Now, if I want to know the cumulative distribution of that distribution of that conditional distribution Y given X, then we have to do the integration of, because this is now the thisfull expression is now the distribution for that conditionalprobability density.Andso we have to integrate it from the left extremeto a specific value Y, with respect to y only, because we aretalking about the Y random variable Y here to get the cumulative distribution function.

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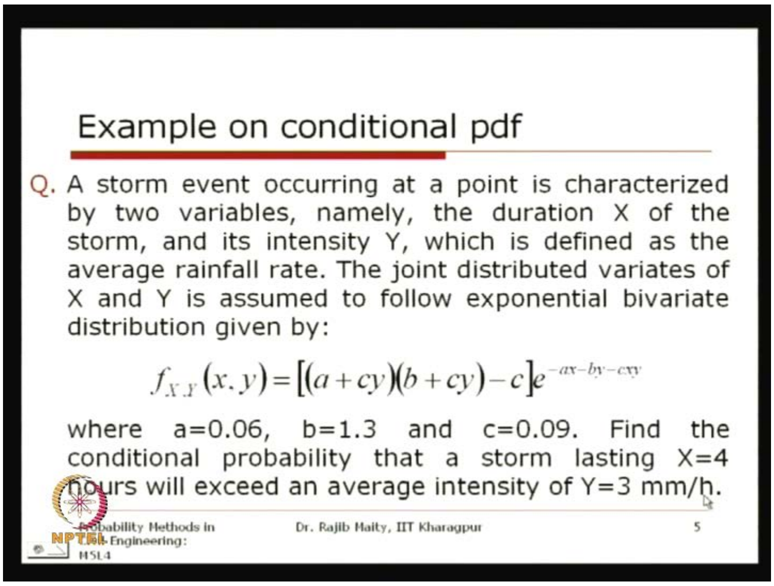

We will take oftake of one problem here,which is on this storm event the similar problem was taken earlier; but here we will specifically discuss about the conditionaldistribution and we will see that, how we can getthatconditional and how it is,how we can apply that whatever the conditional distribution we have we have obtainedfrom thisjust now what we discuss.

So, this problem is on astorm event; which is occurring at a point and it is characterized by two variables and these two variablesare;one is that theduration of the storm event and second one is the intensity of thestorm event or we can say that; this is the maximum intensity. So, what happens in general that if we see the behavior of the of the storm event thengenerally, if the duration increases the maximum intensity decreases soand vice versa sothat there could be some probabilistic relationship between this two; this two variables basicallythis storm events are characterized characterized by3variables along with this duration maximum intensity as well as the total depth of rainfall.

So, for example, if we see that once storm event which iswhich is lasting for 1 hour then the maximum intensity, and on the other hand, if we take the durationwhich is; which for 6 hoursthen the maximum intensity; this two for the 6 hours maximum intensity will be lesser then the storm eventof this 1 hour one hour duration.As, I told that this storm events are generally characterized by with also another random variable; which is the total depth of rainfall, but here as we are discussing the bivariate case now,we are considering the two random variables, sofor a storm event we have consideronlytwo random variable to attributes;one is that its duration another oneis maximum intensity.

So, there are two random variables now; one is X and other one is Y, and their joint distribution of this X and Y is assumed to followan exponential bivariate distribution, which is given by this expression f (x, y) this is joint distribution; which is equals to a plus cy multiplied by b plus cy minus c exponential minus a x minus by minus c x y; this a, b, c are different constant and that generally depends on thespecific location and that and this has to be determined.

Where it is supplied that; this a equals to 0.06, b equals to1.3 and c equals to 0.09, sowith theseparameters with the values of this constant. So, our joint distribution is completely defined, nowwhich is look for is the find the conditional probability that a storm event lasting X equals to 4 hours like sothe duration is 4 hours will exceed an average intensity of 3 millimeter per hour. So, here the unit of the intensity is millimeter per hour and that X isyour in hours. So, which is actually this constants are are of course, the related to the units of this random variable; that is being beingconsidered. So, now, for the as I just told that if the duration increases then thethen the intensity will decrease, soherethe question is that.

After giving this joint distribution question is sofar, the storm duration is X. So, now, the conditional probability we should obtained on condition X; that is distribution of Y on condition X. Now, condition of X is that; X is equals to 4 hours.Now then we have to determine, what is the average intensity? So, thetheprobability that the average intensity will exceed3 millimeter per hour; this isthis is what we have to determine.

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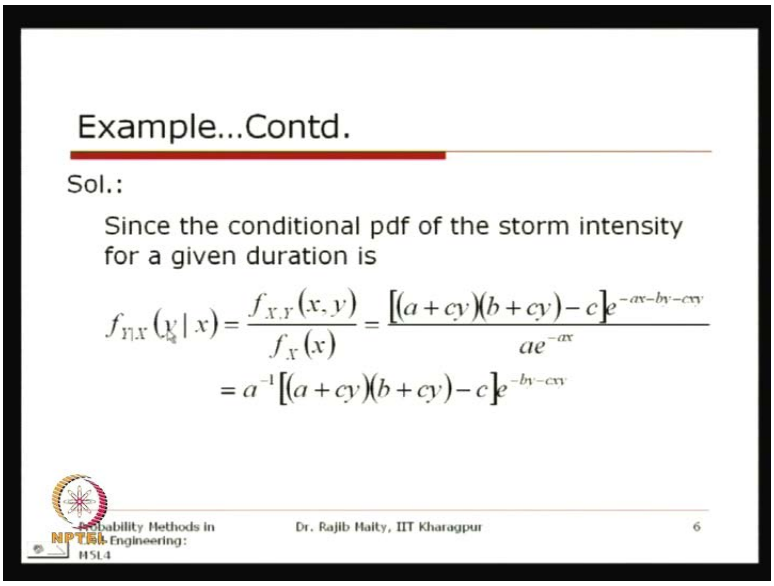

To solve this one;first of all we have to we have toobtain thisconditional distribution and you know thatfor to obtain this conditional distribution, we have to know its joint distribution as well as the marginal distribution of the random variable; on which this conditioned on, as we have seen that; this is conditioned on the x, here x is the duration of this storm, sowe need to know the marginaldensity of x, and in the earlier lecture, if you refer to them from there you can see that; for this type of distribution the marginal distribution of the x is on is on exponential distribution and itswith the with the parameter a.

So, this f x is the the marginal distribution of x is,a exponential minus a x sothis is the exponential distribution for the x. Now, once we know this this marginal distribution; that means, it isstraightforwardto get that their conditional distribution it is just the division of this joint distributionwhich we know, and for its marginaldistribution which is a e power minus x. So, we get the form of thisform of this distribution that is conditional distribution and of course, herewhatit is not mention here is that, the both x and y you know that for the exponential distribution support from 0 to infinity. So, here as we are this both; this x and y are there are. So, this support for this x and y both are from 0 to infinity. So, this is the complete description of this joint distribution of y given x.

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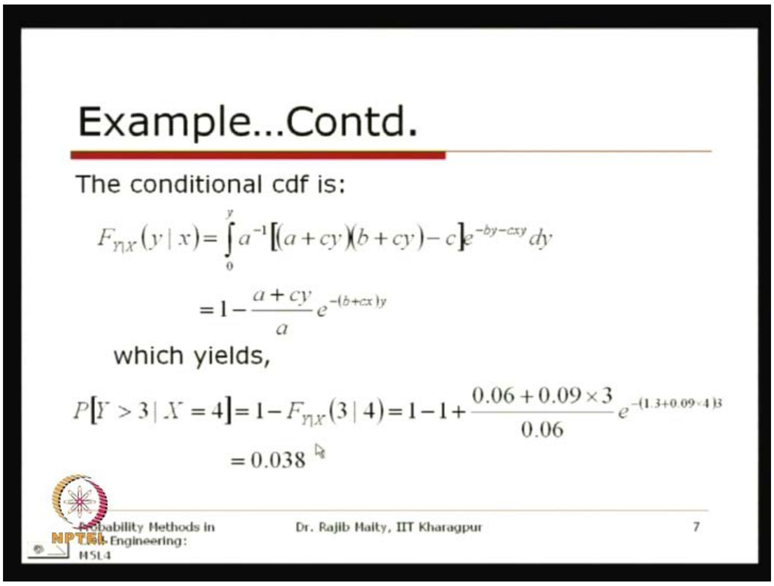

Now, we have towe have toget itsitsitsprobability sowe can directly do that integration or what we can do,we can also get that cumulative density first and cumulative density, we have seenjust frompreviousslidethat; as it is thaton the y on condition x. So, we have to do the integration from the left extreme to the specific value ofy to get itsget itscdf; that is the conditionalcdf, conditional cumulativedistribution function. So, at the left extreme is here 0, so0 to y and this is the conditional distributionfunction; this we have to integrate with respect to yand after this integration and we have to followsome integration by parts also and you will see that, it comes to the form of 1 minus a plus cydivided by a exponential minus b plus cxy, sothis is the cumulative;this is the conditional cumulative distribution function for y given x.

So, now, I think just we have to put those values herewe want to know that what is the exceedance probability of this Y greater than 3given; that Xis equals to 4 and you know that, as it is given that Y greater than 3. So, if we justdetect from the total probability which is equals to 1, then it will be that Y less than equals to 3, given X equals to 4 so that means, that is the f (y, x)3 given 4 means Y less than equals to 3, given X equals to 4.

So, we just put this value of Y equals to 3, and X equals to 4here with the other parameter that was defined and we get the probability that; it will exceed at 3 millimeter per hour for a 4 hours duration storm is equals to 0.038.

So, you see from this example that, if we know the joint distribution and the corresponding marginal distribution then this type of anyanyany such question can be answered. So, one question we have seen that what is the probability that it will exceed three millimeter per hour for a 4 hour storms and we have seen that the probability is very less. So, it will help to a take a decision find as the probability is very less; that is 0.038 then we can say that sogetting more than 3 millimeter per hourmay be very less probable. So, like like this any of this type ofanswer we can get.

But one thing that, I wasmentioned earlier also here is that to, if we know the marginal distribution for two random variables thenit is not that easy to know, what is that joint distribution,only one thing we cansay,from based on the theory is that, if the joint distribution is yours is joint normal distributionor joint Gaussian distribution then, what we can saythat their marginalalso will be anormal distribution, but again that that the reverse that is, itwe cannot say it is vice versa.

Sothat if I say that two random variables X and Y; both are having the marginal density of normal distribution X and Y we cannot say that the joint distribution,what is that joint distribution soeven though we are starting this type of problem that this joint distribution is given to us. So, getting the joint distribution from themarginal'sneed not bevery easy task and in this module towards the end we will discuss abouta recently or relatively newer concept in the civil engineering; which is known as copula and this copula is generally one of thelatest thing that is there to gettheir joint distributionthat,we will discuss later for the time being we arestarting of whatever the problems we are describing we are describing that theirjoint distribution is known.So,this is thequestion ask for and we got this probabilityfor this particular condition.

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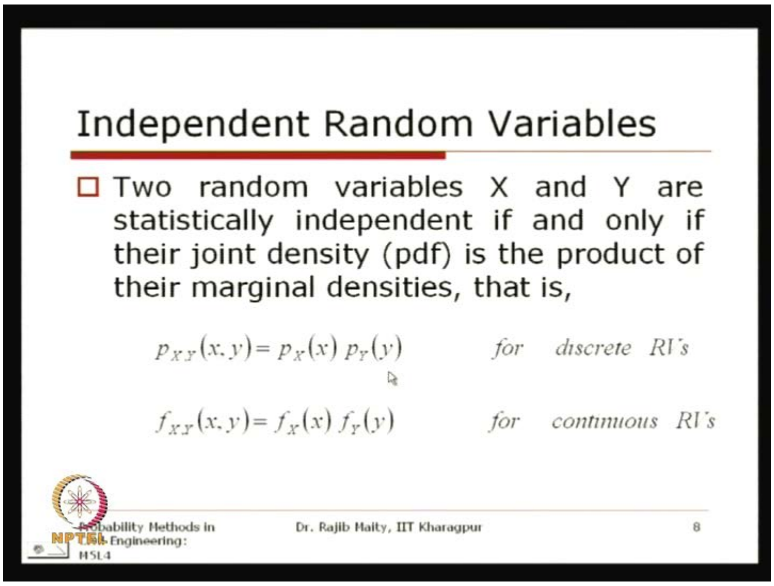

Now, we will discuss about another important concept is called the independence and in the light of this conditional distribution. So, first of all if we just directlystate, what ishow we can decide thatwhether two random variables are independent or not mathematically. So, there are two straightforward expressions are there; one for discrete and other one for the continuous, but before I got to that that mathematical expression.If I just want to know, what does this independence means simplyfor two random variables, if we if we declare that the outcome of onerandom experimentdoes not have any influence on the outcome of the other one then we can say that this two are independent.

Now, to test this one mathematically we have towe have to take the help of that joint distribution as well as their marginal distribution that is two random variables.If it is available it is not necessary that two random variables only we can say even more than two; that is, if it is more than two or n numbers or random variables are available then, if you know theirtheir marginal distribution of this n random variables. And if we know theirjoint distributionas well then the check is that, whether the product of the marginal distribution is equal to theirjoint distribution if this is true; then we can declare that this isindependent.

So, theSo, thestatementstates that two random variables X and Y are statistically independent,if and only if their joint density pdforpmfof course, pmf for the discretetheir joint density is the product of their marginal densities, that is p x (y) jointpmf between x and y is equals to that p x (x)marginal for x multiplied by p y (y) marginal for the y. And this is for the discrete random variable and for the continuousjoint the densitypdf jointpdfbetween x and y is equals to marginal density of x multiplied by marginal density of y; this is for the continuous random variable.

Now,in the light of theirthat conditional distributionthere we will just see in a minute, how we can get this expressionbutin this light,what I like to stress is that,this is to declare it is statistically independent; this isthe condition should be if and only if. So, this one this price I want to stress that is; this is thethis is the condition if it is satisfied then only we can say that X and Y are independent.

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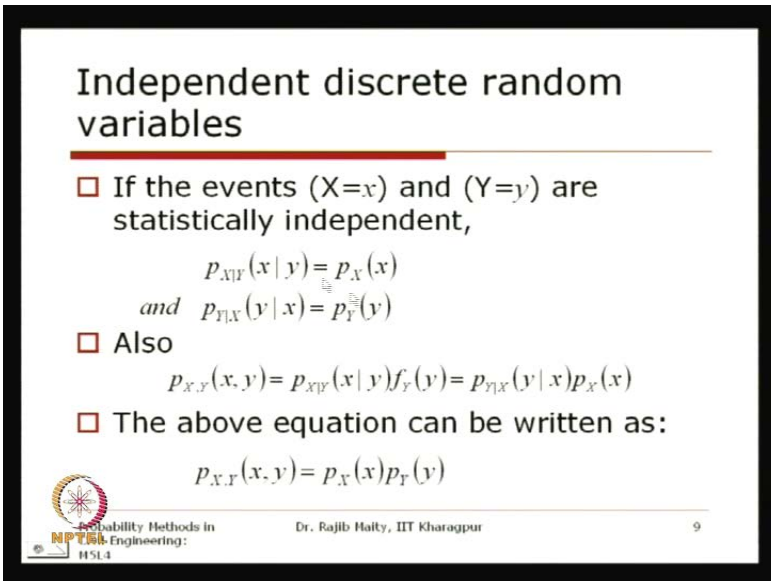

Now, in the light of theconditionaldistributionconditionaldistribution that we discussed, sofar if you see that is we havewe have told that the outcome of one random variable does not have any influence on the other.So that means, here that there are suppose this is for the discrete case there are some two events; one is thatX equals to xspecific outcome, andY equals to y; this two are statistical independent, thenobviouslythat probability that x given yshould be equals to probability of that x; that is marginal of the x, because just nowwe know that this the outcome of one had no influence on the outcome of the other.

So, thewhenever we are telling that this x given y, sothis given y have noinfluence on thisjoint distribution, that means;that the probability of this x given y is equals to the probability of x, because this is have no influence of the outcome of other andvice versa. The probability ofy given x is equals to the probability of y; that is the marginal of marginal density for y, and also from the conditionaldistribution, what we have seen thatjustfew minutes back that this joint density is equal to that conditional multiplied by themultiplied by its marginal sothis will be p, not fp means we are using the notion p for the discrete random variable.

So, this condition multiplied by its marginal, soit is condition on y somultiplied by the marginal of y orthe conditional density condition of x multiplied by its marginaldensity. Now, if you just replace this conditional probability with this what wewhat we have discuss just now that is p x on condition y isequals to p x or the p y on condition of xwhich is equals to p y, thenall this expressions leads to that this joint density; that is p (x, y) is equals to their product of their marginal; so, this isin case of the discrete random variable.

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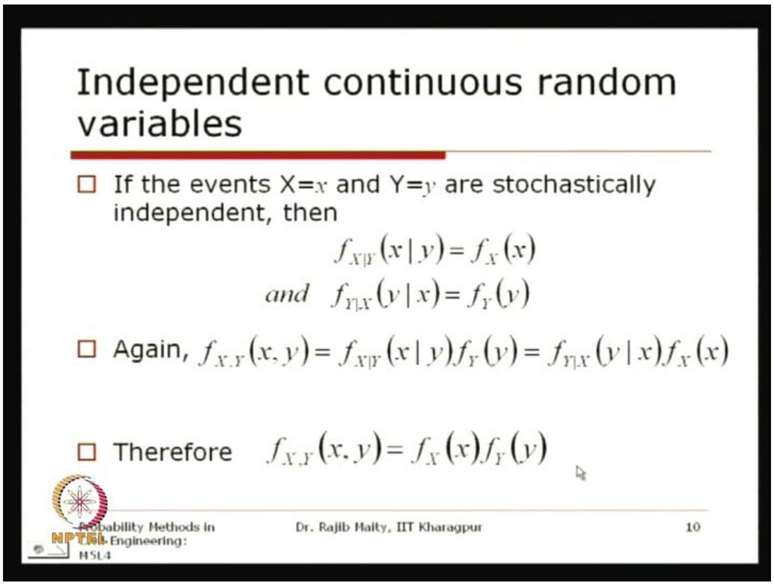

And similarly,if we see that for thecontinuous case also the similar thing;only in case here we have to consider thepdfnot thepmf, sothatpdf of this; that is x on condition y it should be equals to its marginal, that is f x of x and similarly f y given x that ispdf of y given x should be equals topdf of y alone, because the outcome of this y does not of anyinfluence from the outcome of x. So, following again the similar concept here, also we can justwrite that; this joint distribution we know that is equals to theconditional distribution multiplied by its marginal and it is condition y. So, marginal is y here and conditional x if it is condition on x that is given x then it should be multiplied with the marginal of x.

Now, this conditional probability, if we replace from this their marginal itself thenboth this expressionwill lead to that joint density jointpdf is equals to the product of their marginaldistribution; that is f x (x) multiplied by f y (y). So, these are the two conditions; one for the continuous another for the discrete one which should be satisfied and if this is satisfied then only we can say that this X and Y are statistically independent.

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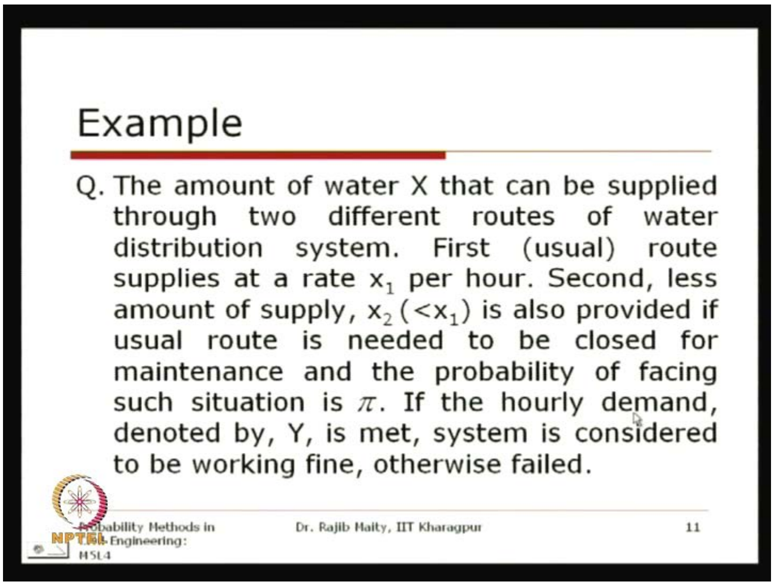

Now, we will take another problem on this continuous random variable, where we willwe willwe will say discuss again the similar thing just the previous problem was based on thisdiscrete random variable. And here we will take anotherproblem on this water distribution system, and wherethat when we will see that thatits the probability of its failure.

Just little background to this problem is that some time for this water distribution systemweuse alternate path andfrom the ifeverything is usual, then we determine thatthat peak demand, and just on that this supply is determined and of course, the supply is determinewith the various factors; that is its socioeconomic factor water availability and solot of things is there. So, we determinethis much water is the available and also that demand is also variable you know that in during the peak peakhours the demand is more, and during the lean hours the demand is less. Now,when we can say that this system is working fine is that whenever the supply of thewater is sufficient or sufficientin the sense; that it is the more than, what is demand Then we say that system is working fine, but if thesupply is less than whateverwhatever the demand then the system is declare as failure.

Now, the failure of a system is,what additional distribution system is the also can have manyfold reasons;onereason here that we have taken in this problem is that thesudden requirement of this closing of the path. So, sometimes there are some alternate routes are available, but this alternate routes generally havingdue to thedue to the various factor, ifit is possible to supply only very less amount of water through the other otherotherpath.

So, this is also now that the less amount of water, if it is during the peakhours there is chance thatthe therequired demand may not be supplied during the peak hours sothat time the system generally fails. So, based on that of this now,there are two different supplies where we can say that whether,which supply is available nowand again depending on the hourly demand also varies. So, for example, that fromthe peak hours also says that morning7 am to 8 am or8 am to 9 am.

So, what is that, what is the demand and in this demand times there are we can now imagine that this, if we consider these are the random variables then the this random variables are also related to each other and dependent on each other before we can determine the probability offailure. So, one such example problem is taken here in the context of that on the conditional probabilityfor the continuous random variable.

So, the problem states the amount of water that can be supplied through two different routes of the water distribution system. So, the among this two different routes the first one is the usual route and this through thisfirst route the water can be supplied at a rate ofsayX 1 unit per hour, and secondthrough the second route the less amount ofsupply is possible;which is denoted as X 2 and X 2 is less thanless thanX 1;which is also also provided if that is the second route is usedif the usual route is needed to be closed for themaintenance or any other reason.

And the probability of facing such situation; such situation means here, that I am using the alternate pathnot the usual one. So, this say situation the probability of this situation is pi, sowhether thesupply of the water is supplied to the community through the usual path or in or through some alternate path. So, the probability of supplying the water through the alternate path is pi and obviously, this will be less.Now, if the hourly demanddenoted by Y is met then the system is consideredto be working fine otherwiseit is failed, now this now once we are declaring that is the hourly demand, if we consider that two different hours.

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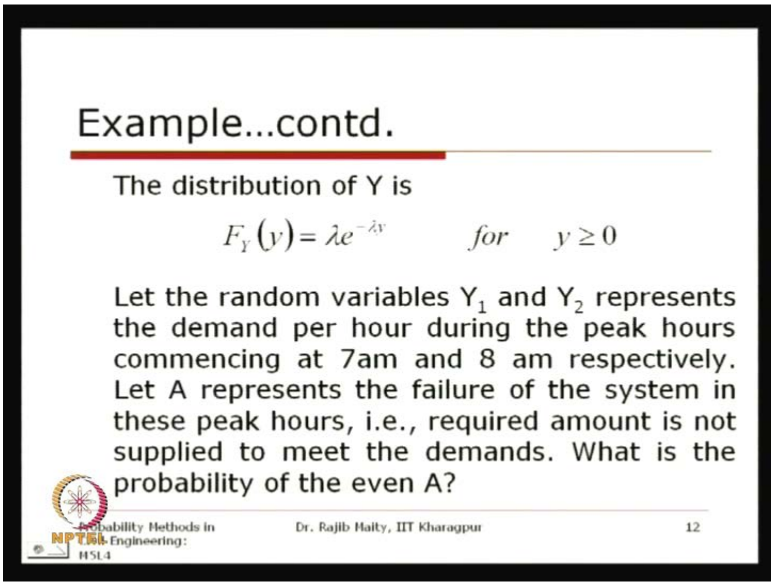

For example, herethat two different times; one is the 7 am and 8 am which are considered to be the peak hours is isisis considered for two different random variablesY1 and Y2,before that this demandcanfollow a specificdistribution andit is declare that this distribution of this y; that is the demand is hourly hourly demand, it follow a distribution that is a lambdaexponential lambda y, forfor ygreater than equal to 2,you know that, this supportfor this exponential distribution is0 to infinity.

Now, if we consider the two random variables here that is Y1 and Y2, which which represents the demand per hour during the peak hours commencing at 8 am and commencing at 7 am and 8 am; that means, are7 to 8 and 8 to 9respectively this two are the requirement that that demand is also also vary.

But any way we have consideredthat both the thingsare following for the simplicity sake we have to assume that both the demands following the samedistributional form; one is thatlambda e power minusminus lambda y which is a exponential we could have evenconsiderthat twovariables here is following twodifferent type of distribution;means the type of distribution can be kept same that is the exponential distribution, but this parameter can be changethat is for the Y1 it is lambda 1 and for the Y2 it could have been lambda 2, but just for the demonstration purpose here it is consider that both are following the same distribution.

Now, let A represents that the failure of the system in this peak hours, that is a required amount of water is not supplied to meet the demands.Now, the question is what is the probability of the event Aeventat thist is,this is a spelling mistake event A sothat during this peak hours; that means7to 8 and 8 to 9 this two time whatever the demand is that is Y1 and Y2 thisrequired supply is not there.; that means, that Y1 is greater thangreater than either x1 or x2 whatever is being supplied or that Y2 is greater thanthat x1 or x 2whatever the supply during that time, so, have to determine the failure of the system through this.

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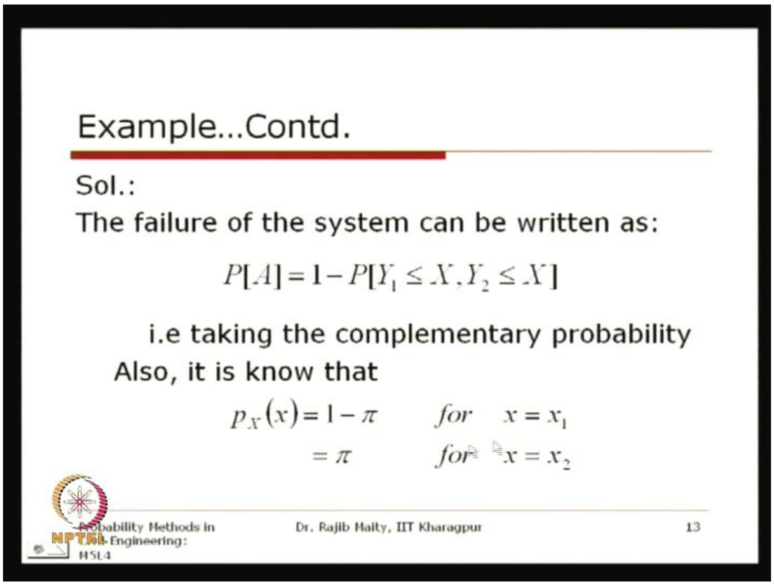

To solve this one thefailure of the system can be written as like this, sothis is the a that event we have declare andso that means, as I told that Y1 should be greater thanthat X that is the supplywhich is you know that this supplyhere isthis type of random variable is call thatdichotomous dichotomousmeans it can take only two possible values. So, here we have seen that it can be it can take onlythe two values; one is x1other one is x2. So, x1 is the usual supply and X2 is the supply throughthe alternateroute sothe failure means Y 1 should begreater than, what is the supply that, Y 1 should be greater than Xor that Y 2 is greater thangreater than X.

Now, the failure is also here it is represented as A1 minus this Y1 less than X and Y2less than equals toless than equals to X, because of thisthe help from the cumulative density the concept of the cumulative density you know that we need to know that less than equal to. So, we have justconsidered that; this is 1 minus of thiseither that Y1 less than equal to X and Y2 less than equals to that supply X.

Now, it also known that sofar, as the distribution of this x is concerned that is that I told that it is a dichotomous variable, soit can take only two possible value value;one is that x 1and other one is the x 2 and in the problem it is declared that probability of facing such situation that; such situation means, when we supplying thewater through the alternate path that is x 2 is thex 2 is the supply sothat probabilityis your piand. So, the probability of supplying that x 1 amount; that is when x equals to x 1; obviously, this should be 1 minuspi, because this two are the mutually exclusive events. So, either of this two should occur, soif the probability of one event is pi it should be that for the probability of the other one should be 1 minus pi.

So, now this is the marginal; this is basically the marginal distribution of that of that x and which you are can say that for this two differentcase.

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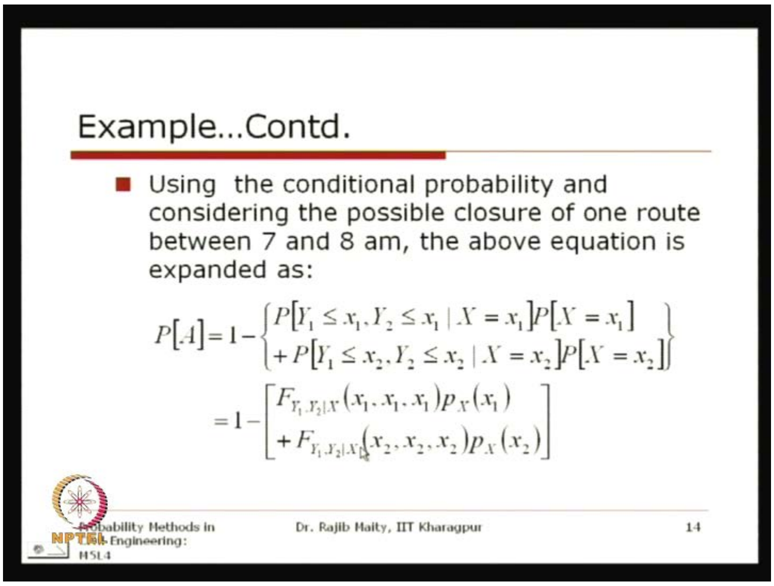

Now, using the conditionalprobability andconsidering the possible closure of one route between that7 and 8 am, the above equation is can be expanded that is the probability of failure this thing can be expanded that. So, 1 minus as I just letonce again refer to this one, so1 minus Y 1 less than X and Y 2 less than X. So, we are now expanding this this term that is Y 1less than X and we are also expanding that Y 2 less than X.

How we are doing that is that, the probabilityof Y 1 that is between 7 to 8 is less than x 1 comma probability of Y 2 less than x 1, because we are assuming both the times supply is usual on condition that X is equals to x 1 which is multiplied by its marginal. So, this is the conditional multiplied by its marginal that is X is equals to x 1 this and the other situation is the supply is x 2; that is probability that Y 1 less than equals to x 2 and Y2 less than equals to x 2on condition that supply is x 2this multiplied this full quantity multiplied by the what is the probability of x 2; that means, this is the cumulative conditional distribution for this Y1, Y2 when this variable when the x,now in the supply is x 1 and this is the cumulativedistribution when the supply is x 2.

So, which is represented through this expression though thiscumulative distribution multiplied by the marginalof that X for the value x 1 and this cumulative multiplied by the marginal for thefor the value X equals to x 2.

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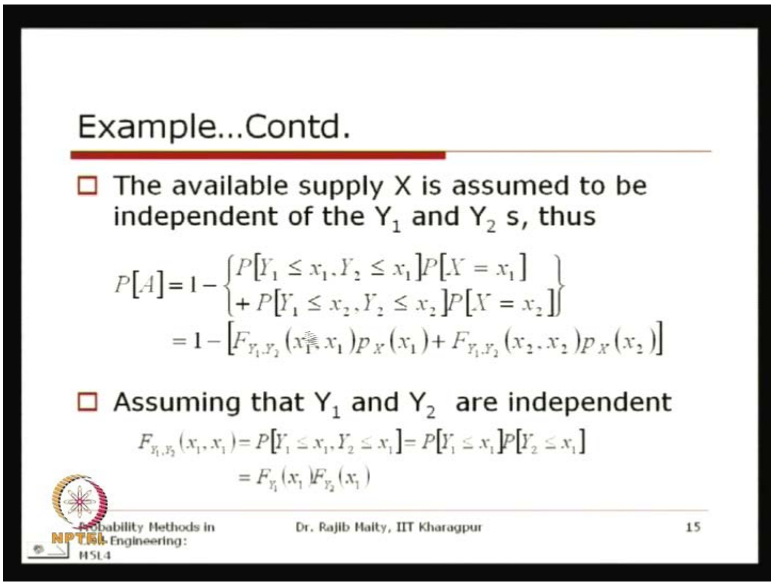

So, this two are obtained toand after that, if the available supply x is assumed to be independent of the Y 1 and Y 2 here, one thing I want to mention that even though this is this iswritten here it is notfor the practicalwhen we decide that, what should be thesupply for this one there lot of analysis is been is done to determine that what should be the supply, but once we have decided that, now the what is meant here for the independent; the independent means the demand and supply are independent what we mean that.

Now, the variable demand and the supply these twothese two variables are assumed to be independent; that means, I have decided from some from the analysis that the this time the supply will be this much amount. Now, this one whether the whether the distribution system needs the maintenance or not sothat a due to the sudden maintenance andthis type of reason we we need tochange the supply, but the demandis always,whatever the requirement of thewhatever the requirement of the community sothat is whyit is declare that if we if we assume that this two event are independentto each other than how this expression is for the simplified.

So, that is why it is explain that, if the available supply X, either x 1 or x 2 is assumed to be independent of thatY1 and Y 2 thus it can be written that; this Y 1 less than equals to x 1,Y 2 less than equals to x 2 that condition that that x is equals to x 1 is now,omitted which ismultiplied by theirmarginalprobability that is probability x equals to x 1.

So, this is the cumulativedistribution for this Y 1,Y 2 which is in case of this x 1; it is multiplied by the marginal probability of x 1 and other one, if it is x 2 then it is multiplied by its marginal probabilitymarginal probability x 2.

Now, if we assume that this Y 1 and Y 2 are also independent;Y 1 and Y 2 independent means whatever what is the demandduring the 7 am to 8 am and these twodemand between the 8am to 9 am, if this two if this two demand are independent then we can write that this this cumulative distribution that is for anything, that is for, if it is the now it is the explained here for the case of x 1 is equals to probability of y 1 less thanless thanx 1and probability Y 2 less than x 2 and we know from the independence just now we discuss that. So, this is the joint distribution which should be equal to theequal to the their marginal; that is probability of Y 1 less than x 1 multiplied by probability of Y 2 less than x 2.

Sothat; that means, these are that cumulative distribution for the Y 1 and Y 2, sothis cumulative distributionthese multiplications should give that that cumulative distribution for the joint if this Y 1 and Y 2 are independent.

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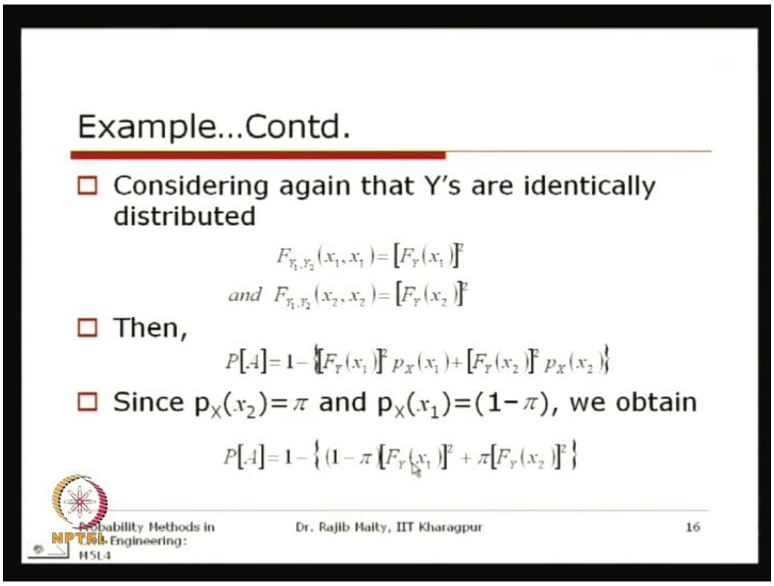

So, this expression is now replaced in this form and what we will get the probability of failure is that, thisnow again; this cumulative distribution when we say that probability of y (x 1)multiplied by Y1 and Y2 and in the problem also for the simplicity sake we assume this two are following the same distributionsame exponential distribution.

So, with the with the identical parameteras well sothat is why here we are just expressingthis is to be square. So, multiplied instead of multiplyingthat f (y 1, x 1)and f (y 2, x 2) it is f (y, x 1) square and this the other one that is f (y, x 2) square.

Now, this two expressions are replacing that expression for the probability of failure which isequals to 1 minusf y(x 1) whole square multiplied by the probability of supplying x 1 amountplus probability of y (x 2) that is a cumulative distribution of y for x 2 square multiplied by theprobability of supplying amount x 2.

Now, this two are known that probability ofx 2 equals to pi and probability of x 1 is equals to 1 minus pi. So, putting this expression we obtained the probability of a is equals to 1 minus 1 minus pi multiplied by f (y) for x 1 whole square plus thispi multiplied by f (y) for x 2square.

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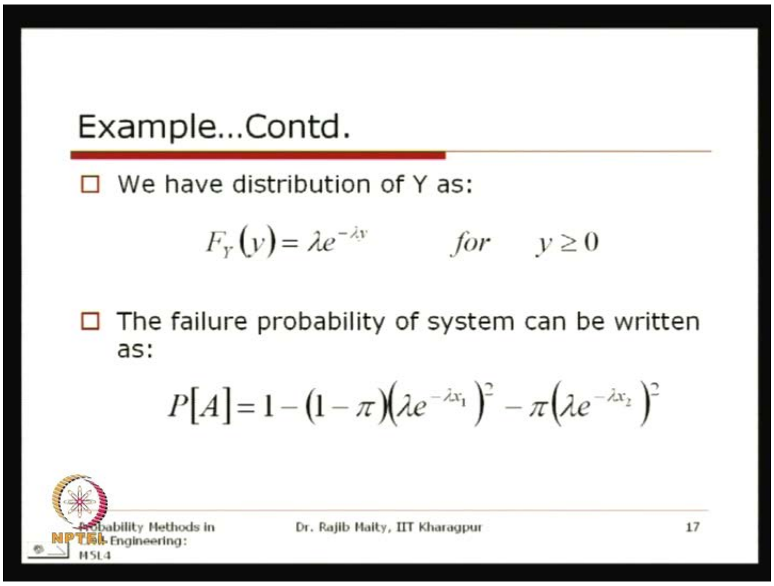

Now, you know this cumulative distributions are for theforthe exponential distribution is alsoknown that is 1 minus,this will be you know that 11 minus e power minus lambda x,if this is the exponential density then their cumulative density you know this will be 1 minus e power. So, this is a mistake it shouldbe instead of lambda e powerminus lambda x it should be 1 minus e power minus lambda x. So, this should be the cumulative distribution and the support is same y greater than 0.

So, this cumulative distribution should beplaced here, that is 1 minus e power minus lambda x 1; this is for the x 1 value square and 1 minus e powers minus lambda x 2 this power square. So, this whole quantity now this parameters we know as we have declare that, we know that distribution; so that means, we know the this parameters also; that means, lambda we know and this pi we know, if we know this parameter we know that this what is the amount ofthat is x 1 and x 2 their magnitude. So, with this magnitude we can calculate that what the probability of this failure issoas asystem designer.

So, this type of probability we shouldwe shouldwe know that this type of probability should be minimized. So, once it is satisfied by the designerthe system designer then thatthe design can be accepted. So, the here the basic idea is that to after getting this probability;this probability should be minimize to the extent possibleso we havediscussed.

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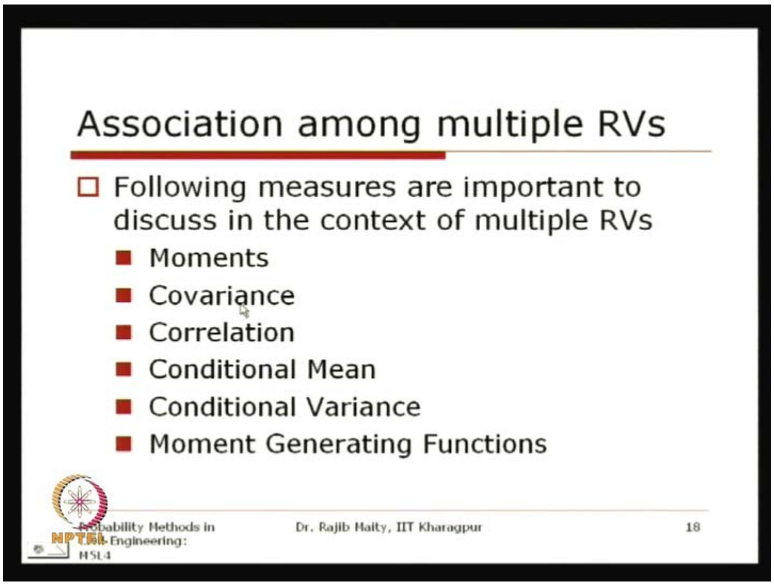

Sofar, the conditional probability and the practical application in some of the civil engineering problem we have seen, and alsonow what the next thing that we just wasreferring to in this at the starting of this lecture is that, this whenever we are talking about this multiple random variable we areinterested to know their joint association.

Suppose that, if we just start our discussion with the bivariate random variable only that is the X and Y there are tworandom variables are there.Now,how they are associated with I cannotsay they arerelated with, because that is I that may not be the right word, but the association when we are referring to that can be the linear association or the non-linear association ortheir their variability with each other their covariability between two random variables all this things are important and this things are also generally obtained from thefrom their probabilistic that their joint probabilisticbehavior.

So, thiswill discuss now, one after anotherand we will go throughthis concept that is the one; is that moment then the covariance correlation conditional mean, conditional variance moment generatingfunctions maybe all this things will not be covered intodays lecture, but we will just start with one after another. And we will know that, what the important information that we can obtained fromthisfrom thisfrom this measures for themultiple random variables.

So, you also know that thismoments and the expectations we have discussedin case of the single random variable and thus; obviously, here also the similar concept will be used, but its has to be extended to the higher dimension for the single random variable, what we have seen that,it is just one one excess if you just go for the pictorial representation on based onthe one axis we havediscuss their area concept, that is the moment when we are discussingwe have discuss in the concept of the moment.

Now, if we just extend it for the bivariate case; that means, it is now a three dimensional space. So, two axes representtwo random variable and other one isthe probability density. So, now, if we suppose that the expectation if we just extend to the two dimensional case. Now, whatever we have consider for thearea in the one dimensional that the single random variable, now the same thing for the two dimensional case it will be the volume.

Now,you can imagine for, if it is more than two then it is something going to be three dimensional for three random variables and one dimension for their probabilitydensity, soit can be extended according it.

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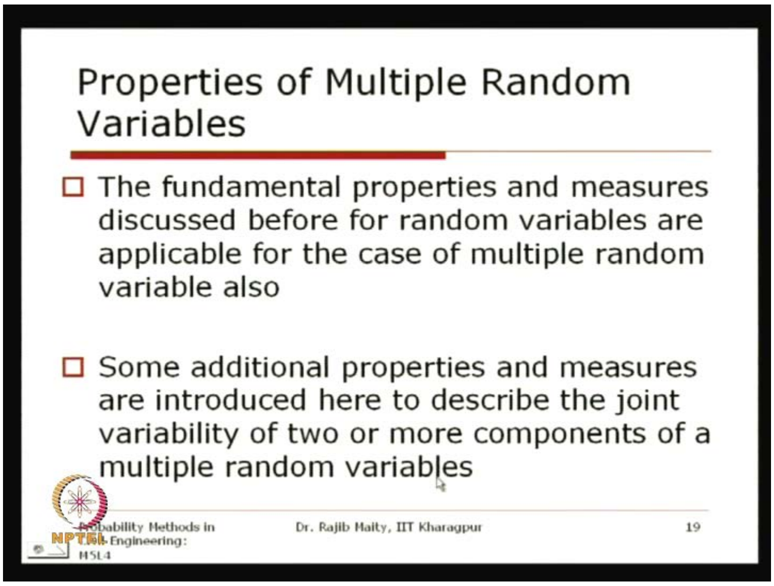

So, any way to start with the bivariate case we can just go for the first is thatthe fundamental properties then the measures, that we have discussfor thefor the random variables these are applicable for the case of the multiple random variable also. And some additional properties and the measures are introduced here to discuss their joint variability of the two or more components of the random variables.

So, as I was just telling thatthe the fundamental concept that we have used earlier for the single random variable will also be will also be applicable, and the same conceptshould be extended.First we understand for the two-dimensional, case and then we will go forwe will take that for the onehigherdimensional; at least that is for the mathematical expression purpose.

So, this detail discussion on this moment, and its expectation then one of the most important things is covariance, and from the covariance we will know that the correlation;correlation is the measure of the linear association.This linear is importantthis, then gradually we will go on the other properties as well for the multiple random variables.And this will be started discussing in the next lecture.Thank you.