Probability Methods in Civil Engineering

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Lecture No #21

Conditional Probability Distribution

Hello.Welcome to this third lecture of this module.You know, this module, we are discussing about multiple random variable. So far, in the last two lectures, we have discussed about the joint probability distribution and we also have seen in the last class that, what is the marginal probability distribution.In this class, we will we will take up another concept, which is also very important for this multivariate statistical analysis, which is known as the conditional probability distribution.

So, this conditional probability distribution is generally can be obtained from the joint probability distribution and its concept is very important to make it clear so that, its application will be easier, for in different fields including the different fields in civil engineering.

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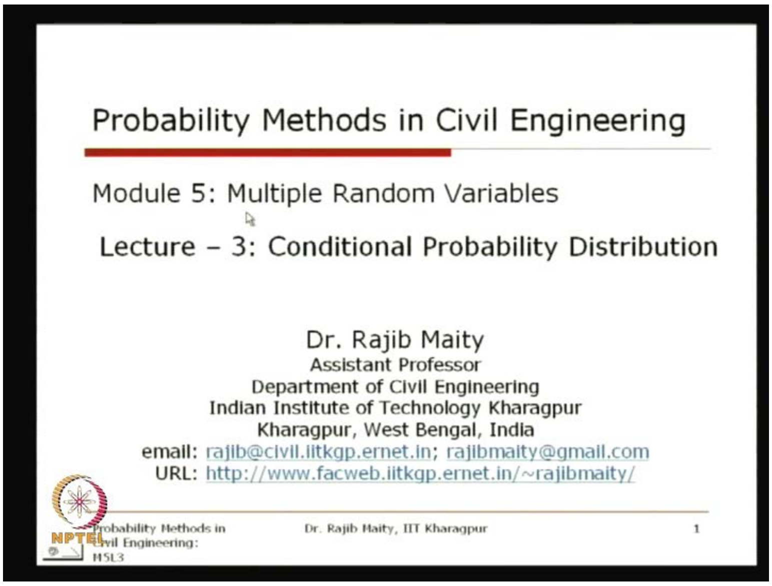

So, our today's lecture will be on this conditional probability distribution.As I told, that this conditional probability distribution and for all the multiple random variable, the concept when we are discussing, will be discussing first for the two random variables, which will be easier to understand the concept, back ground concept.After we learn that concept, and that concept can be extended for the higher number of random variables.

So, and also one more thing that I want to tell at the first.That this, when we are, when will be discussing this conditional probability in today's lecture, we will first take up the, take up that discrete random variable first and from the discrete random variable, we will then,we will go to that continuous random variable. So, why that discrete random variable will take first is that, on the discrete random variable that this concept, will be, I feel it will be easier to understand. So that, with the help of some example, we will discuss that the discrete probability distribution first and then, we will use the same concept for the continuous random variable.

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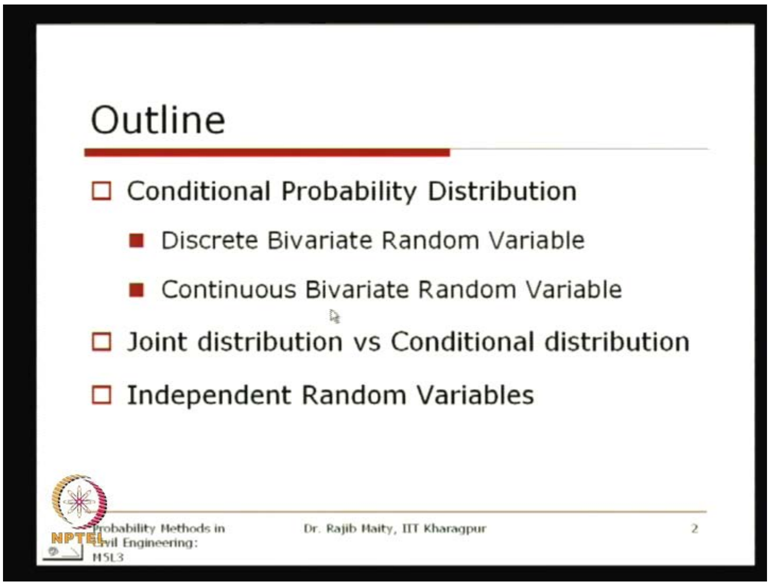

So, our outline for today's lecture is this.In this conditional probability distribution, first of all, we will discussin general what is what is this conditional probability distribution is and how this can, this is obtained in in the general sense.Then, first of all, we will take this discrete bivariate random variable.This discrete, it is and also we will talk about this two random variables, which is bivariate and after that we will discuss about these continuous bivariate random variables.We will see the example problems as well, both for the discrete random variable as well as for the continuous random variable.We will see that their their usefulnessof of this conditional probability distribution concept and then, we will discuss about, means it is not that separate, but, simultaneously we will also learn the that joint distribution verses conditional distribution, in the sense, that how these two are related.means what is. So, we have in the last class, we have discussed about the joint probability distribution and in particular, for the bivariate random variable.We have seen that this can be shown in a pictorial form in a three dimensional space.In the sense, on the on the paper, we can show and from that from that joint distribution, how we can get that conditional distribution or this, what this conditional distribution means. So, that, we will discuss under this joint distribution versus conditional distribution.Another thing which comes from this conditional probability distribution is the independent random variables.This independent random variable, means as the as thethat language refers to, is that two random variables which are independent to each other, that is, outcome of the two random experiment, outcome of one is not influenced by the outcome of the other. So, that is the independent random variables.And, from this conditionalprobability concept and the joint probability concept, we will also discuss mathematically, what this independent random variable means.

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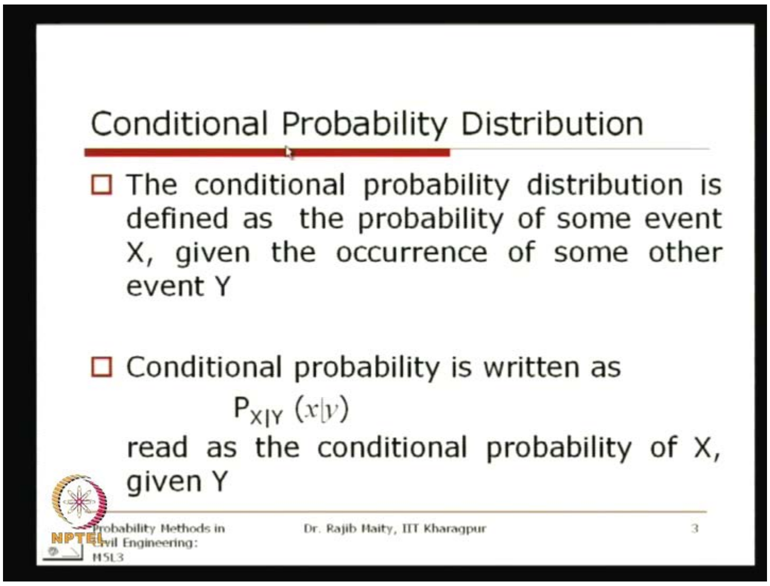

So, this is overall the brief outline that we will fallow in today's lecture. So, first of all, we will start with that conditional probability distribution.The conditional probability distribution is defined as the probability of some event X, given the occurrence of some other event Y. So, here we are discussing with respect to two random variables, one is that X and other one is Y.

So, what is supplied means, that if we want to knowthe, what if we just want to take the what conditional. So, what is the condition to this, is that the condition of the y.That is, the occurrence of the other event Y has already occurred and we know what has occurred for that random variable. So, so that is the condition.

So, once we give that condition, so, there will be some change of the distribution of the other random variable. So, what I want to mean is that, we know the joint distribution. If the X is this and Y is this, means I am referring to some sets. So, X is in this set and Y is in this set and for the discrete random variable, X is having a specific value and Y is having a specific value.Then their probability, we we know from the PMF that probability mass function, means the joint probability mass function.

Now, the here, the condition is that one variable, I already know, suppose that y, I know. So, the occurrence of the Y; the outcome of the Y is already known.Then, if it is known, then what is the distribution of the of the other random variable.That is, what we are referring to as the conditional probability distribution.

This conditional probability distribution is expressed as this, that P X on condition Y.and andThese are the variables, these are the specific value that this random variable can take and this is the way we express that function and this is read as the conditional probability of X given Y. So, you can see that notation that X slash Y; that means, the X given Y.

So, this is the way we express that conditional probability distribution.Now, you can recall that, when we are talking about this joint probability distribution, we are referring to the simultaneous occurrence of both the random variables.We generally use the X and Y, both are occurring simultaneously.

Here, what we are what we are what is the difference with respect to the joint probability distribution is that, this Y has already occurred; what is the distribution of X? So, the distribution of X for a for a specific outcome of the Y. So, this is the, this is our interest, when we are referring to the conditional probability distribution.

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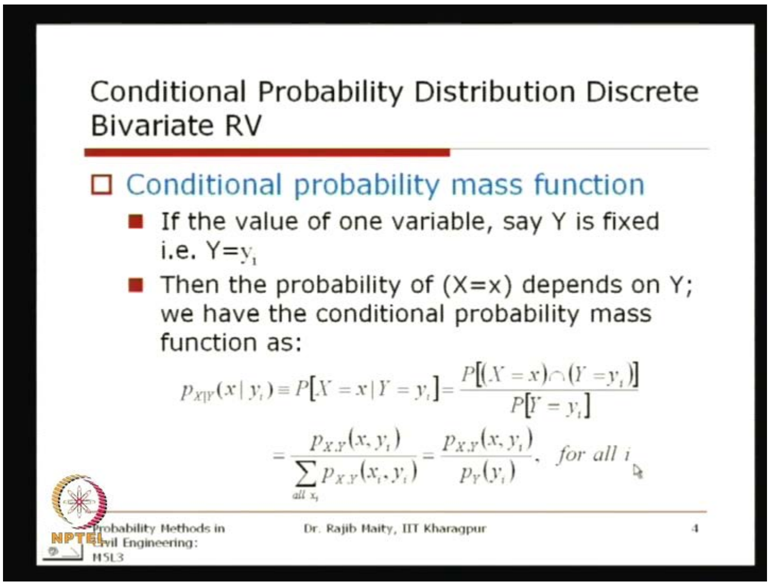

Now, now if we take up that discrete random variable and and continuous random variables separately, then the first of all, if we take that discrete bivariate random variable, that is, two random variable X and Y and both are discrete random variable. We have also shown in the last class, how their PMF looks like in a in a tabular form.That is, X are taking some values and Y are taking some values and for each of their intersection. Suppose that x takes that value, some some value say 1 2 3 4 and Y also can take those specific values, then for each combination of this two, that probability mass is concentrated and that is referring to thus joint P joint PMF.Now, from this one, if you want to know what is the conditional then, it is expressed like this.

If the value of one variable, the value of one variable, say the Y, so, y we are taking that y; that means, that y is the condition is fixed, that is y is equals to y i. So, y i is some specific value that has already occurred for the random variable y. Now, so, this is, we are using as a condition, for the probability of X is equals to x, some specific value of the random variable x that will now depend on the outcome of this y, that is y i here.

So, we have the conditional mass function, conditional probability mass function as the p x on condition y, x on condition y i, that is y specific value is y i has occurred, which is also expressed as that X is equals to x and Y is equals to y i, which is expressed as the probability of X equals to x intersection Y equals to y i. So, here we have to know that when we are taking about this x is equals to x; that means, whatever the possible values of x that can occur in within concurrence of this y is equals to y i.Suppose, that x can take the values of, some set of values 1 to n and y can also take the values from y 1 to n, but, we are talking about, for the y we are talking about one specific values, say y equals to 3.

Now, for y equals to 3, now x equals to, x means, x starts from whatever the possible values that can occur when y is equals to 3.If the x can take all possible all possible values for y equals to 3, then I should take those combination, that is y equals to 1,xsorry x equals to 1, y equals to 3 and x equals to 2, y equals to 3, x equals to 3, y equals to 3, x equals to 4, y equals 3; that means, I am not changing the y, that is y has already occurred, but, what I am looking for is the all possible values that x can take, when the y is fixed for that particular particular outcome.

So, this is, this probabilities we we will see, as well as what we will see, we will just, this quantity is generally divided by probability of y equals to y i.Mathematically, may be little difficult to start with, that why we we need to why we need to divide it, this quantity by this, by this amount, that is probability y equals to that specific value y i.In this case, that we are giving the example y equals to 3 that will be clear in a minute, when we are talking, when we will discuss this thing with respect to the properties of the probability function. So, you know this is also a probability function. So, those properties of the probability function that we discussed earlier should also be satisfied by this probability probability function.

Now, when we are talking about that in a minute, I will just come back to this, why we need to divide by this quantity will be clear. So, now, this thing, how we can express it again, is that, so, this is, now what happens is the is the joint distribution for the specific value of y. So, this we can write that p x y, this is the joint distribution for x equals to general values x and y is the specific value y i.Given it by, now when we are talking about the probability of y equals to y i; that means, I am not giving any restriction for the x; that means, what we discussed in the last class is the, this is nothing, but, the marginal probability marginal probability at y equals to yi. Now, how we get this marginal probability?You know that we have to sum up the quantities of this probability for all x i for the specific value specific value y i. So, this is nothing, but the marginal probability.So, that means, this is the joint probability probability x y for the specific value y divided by that mar their marginal probability for all i, this will give that conditional probability of x given y.

This will be more clear when we are talking about in terms of some some example.Basically here, when we are, may be the discussion when I was continuing, I was just referring to a specific value of y and I specifically mention that this y equals to 3. But, remember that it need not be the specific values only.What we are referring to a subset of the possible value of the y that is on which it is conditioned on. So, it can be a range also. So,what I can say that the y greater than some specific value. So, condition is that y greater than 3. So, that is also a range. So, this also can be conditioned up on.We will just see in a minute through one example.

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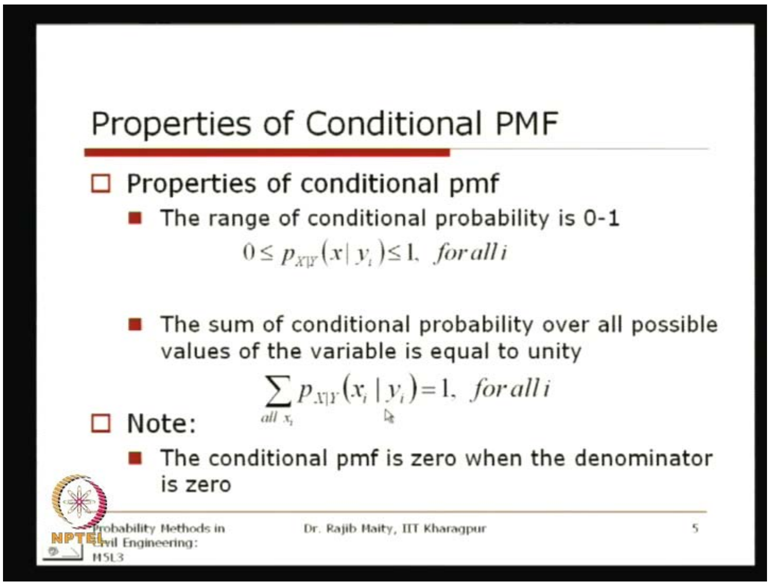

Before that, if you just see that what are the what are the properties, then the properties of conditional p m f and you know that for a p m f, the range of conditional probability is 0 to 1.That is, this is the conditional that probability, which is also a probability function, either this is now, this is we are talking about the probability mass function. But, it can also be a for the continuous case, it will be probability density function.You know that this are this is always that between this is a this is a positive number, this is a nonnegative number.

So, it can it can take from this 0. So, it should be greater than greater than 0 and I thinkthis is a mistake; it can take any any value. So, the probability mass function, the value of the mass function or probability density function whether it is a conditional probability or not, that is, in general, that this should be a this should be greater than 0.

Now, for now as we are discussing this one with with the specific to that probability mass function and we know that this cannot be more than 1, because the second property is that summation of all this probability should be equals to 1. So, this automatically implies that one individual value should be less than equals to 1, because their summing up should be equals to 1. So, which is the second property of the probability function, probability mass mass function also the probability density function.

So, for that conditional probability, that is x given y and y has a specific value y i.Then, for all possible x i, if we sum up the probabilities, that probability should be equals to 1.We will just use this property to explain that why we need to divide it by this, by the marginal distribution.This is the this is the mathematical mathematical clarifications are why we need to need to divide it by their marginal probability. But, conceptually if you see, then then this can be stated like this, that when we are talking about that this is condition on some outcome of the other random variable.Here it is y that means that has already occurred. So, if we say that that as already occurred; that means, I am now my irrespective of fine.

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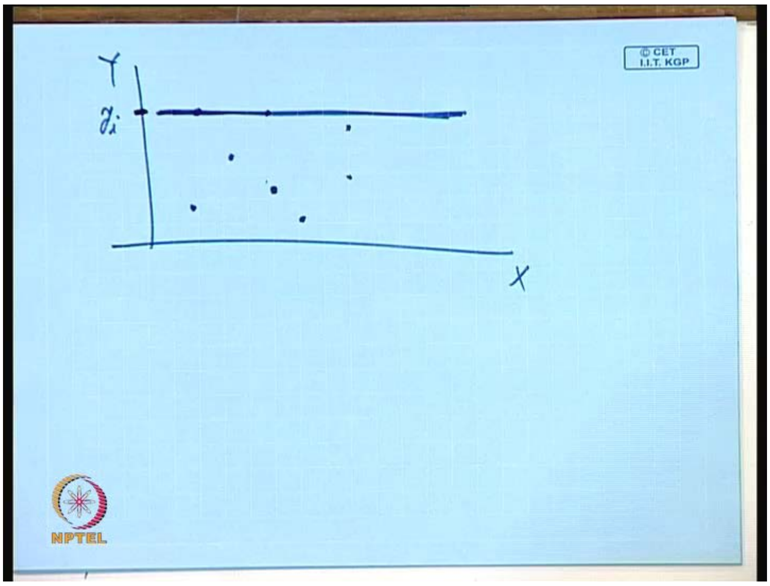

So, if you just draw this diagram, this will be I hope that this will be clear.Suppose that, this is the two dimensional space that we are talking about.Now, there are for example, for the discrete random variable, there are some probability mass are concentrated at some specific points.

Now, when we are talking about that it is conditioned on y and y is having some specific value that we have decided that it is y i; that means, I am already in this. So, the outcome can never be on this zone, outcome should be on this on this zone, on this line, in this case. Because, this is a specific value we are talking about or if it is a range, then on that range it will occur. So, the inter probability that we are talking about, that their sum up, that condition when I am talking about conditional probability, the conditional probability the entire thing that sum up should be equals to 1; that is our requirement.

So, what we are talking that this is that this has to be divided by its marginal density of this conditioned, that random variable to satisfy that that properties, because we are already in this.We are we are already in this subset of the, of this random random variable. So, this will be I hope this will beaga even more clearer to you when we are discussing with respect to a with respect to a problem.

One more thing here, that is, under this note, that if we just see that this is the expression and this is the marginal probability that we are talking about.If the marginal probability for some y i, if it is 0, then what will happen because this is in the denominator. So here, now from this picture also you can see that, if the y i for this, on this on this writing pad if you say, see that on this, if we are talking about some y i, where the marginal probability is 0; that means, that is that is a that is not a feasible event for the for the y. So, this might be this is the total space, if this the total range or the support of this random variable.

So that, I am talking about some y i, which is outside that range or which is taking some set, where there is no outcome; that means, that marginal probability is 0. So, if the marginal probability here is 0 itself, then there is no meaning of this, the condition conditional probability. So, in that case, the conditional probability will obviously be 0.

So, this is what is what is written in this note that conditional p m f is 0, when the denominator is 0; that means, the marginal probability if it is 0.

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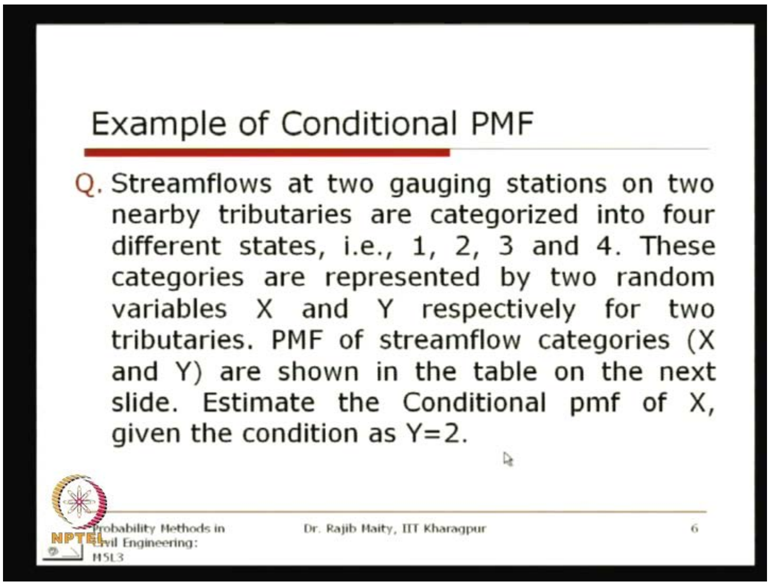

Now, let us take one example on which we will discuss these things of this, we will discuss the concept of this, the conditional PMF. So, we are taking the similar problem that we have taken earlier in last class.Also, just to maintain the similarity on the same same type of problem, we are just taking to discuss their joint. We have discussed the marginal and now we are going to discuss the conditional probability distribution.

So, stream flows at two gauging station on two nearby tributaries are categorized into 4 different states. We explained earlier that what are the states means, how we just, why we need to categorise in to different states. Say that this states are named as this 1 2 3 4, maybe you can also say that this is the 1 is the low flow, 2 is the one is very low, 2 is low, 3 is high and 4 is very high kind of things. So, this type of categorisation what we are referring to here.

So, this categories are expressed by two random variables x and y. So, you know the, as as I told earlier of course, that this stream flows are generally the continuous random variable that when we are when we are doing the analysis with the categorised stream flow; that means, these are these are discrete numbers. So, these are discrete random variables. So, here the X and Y, that is the states for this two nearby tributaries at two gauging stations, this this X and Y are the discrete random variable.

So, the PMF of the stream flow categories X and Y are shown in the table on the next slide.This we have also shown in the in the context of the other concept in the earlier lectures.Now, here what we are supposed to know is the estimate, the conditional PMF of X given the condition as Y is equals to 2.

Now, what we discuss before I go to the solution, what we discuss so far, the conditional distribution is that, here you see that when we are when we are not telling anything about Y or anything about X, what we are having is their joint PMF, the joint probability mass mass function what is available to us. So, we know for a specific combination of X and Y that is, if X equals to 1 and Y equals to 2 or some other if X equals to 3 and Y equals to 1.

So, for such cases what are the probability that we know from this joint probability mass function.Now, here for the conditional PMF, we are saying that the Y is equals to 2; that means, suppose that this type of situation when it can happen is that, suppose the one tributary is accessible and other was not for some reason.

Now, if we have the first record, if we have, if you know their joint distribution, now if we are, if we know that what is the status of the stream flow for one station, so that means, I know the status of the one station which is in category two.Then, I have to conclude that what could be the possible state for the other station. So, this is the, this is you can just think of this real life situation, where this conditional PMF for this case I am talking of about this case that can be applicable.So, here we have to we have to estimate the conditional PMF of X given that Y is equals to 2.

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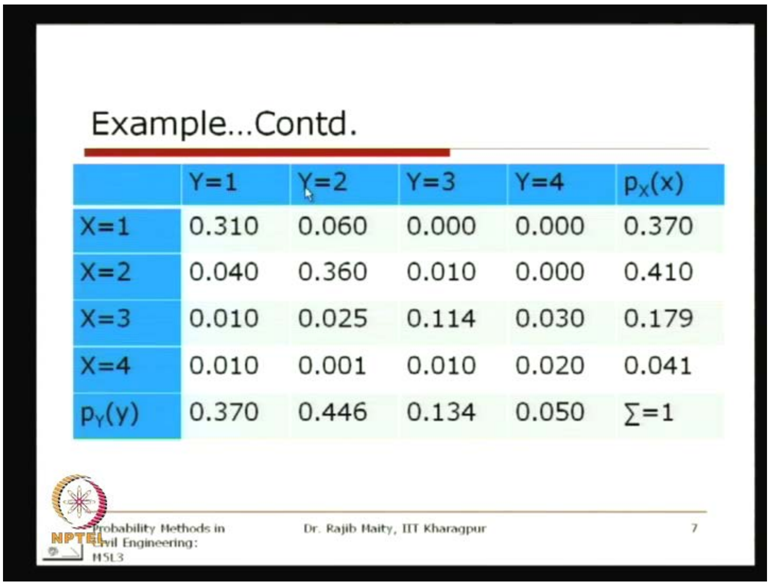

Example...Contd.

	Y=1	Y=2	Y=3	Y=4	$p_X(x)$
X=1	0.310	0.060	0.000	0.000	0.370
X=2	0.040	0.360	0.010	0.000	0.410
X=3	0.010	0.025	0.114	0.030	0.179
X=4	0.010	0.001	0.010	0.020	0.041
$p_Y(y)$	0.370	0.446	0.134	0.050	$\sum=1$

So, let us first see about that PMF. So, this is a given in a in a tabular form.This 4 by 4, if you just see this 4 by 4 matrix, then this is actually giving this entire joint PMF, that is 0.31 this is nothing, but, the probability concentrated, the concentrated probability for the case X equals to 1 and Y equals to 1.

Similarly, the 0.025 is the probability in case X equals to 3 and Y equals 2. So, this is the joint PMF and we have also seen that, if you just add this up that is 0.31, 0.06, 0.00,0.00, then this means what; this is X equals to 1 for all possible values of Y. That means, the irrespective of the values of Y.

So, if we sum it up, then this is nothing, but, the marginal probability of X.Marginal probability of X that we discussed in this last class that is irrespective of the value of Y. So, this is the last column what is shown is that the marginal distribution of X, that is, this is 0.37 is the probability of X equals to 1, 0.41 is the probability of X equals to 2, 0.17 is probability of X equals to 3 and so on.

Similarly, for the Y also the marginal distribution we can get for for a specific value.If you add for all different possibilities of X, then for example, this 0.37 then this is the marginal probability of Y when Y is equals to 1.Similarly, 0.446 is for the Y equals to and so on.So, this is the entire description of this joint PMF and marginalmarginal probability distribution that we discussed in the last two classes.

Now, what here the new thing that we are talking about is, that it is it is conditioned on that Y equals to 2. So, as we are conditioned and Y equals to 2 then that that means, now our concentration is entirely on this column entirely on this column.What we are meaning is that for a specific subset of Y, now we are talking about whatever the distribution for the X for this case only. So, that is, this case means, here it is Y is equals to 2.

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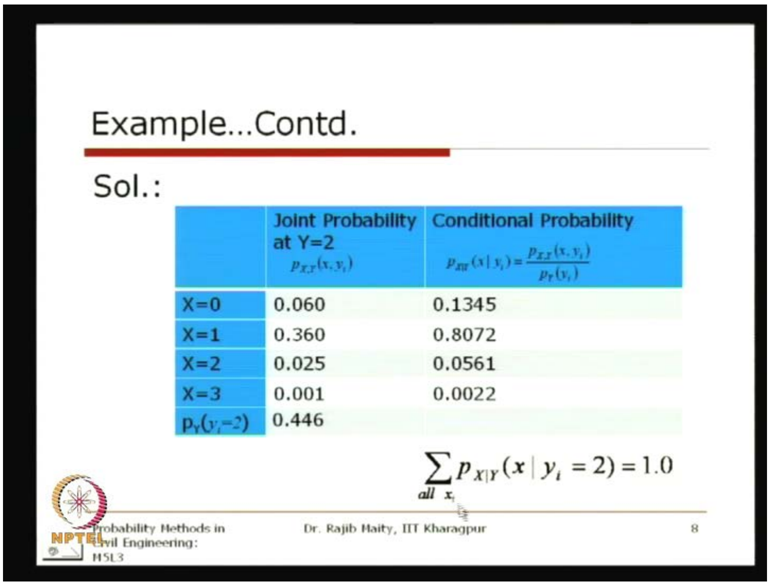

So, in the next slide towards the solution of this thing, so, this is the this is the joint probability values.That is, what we have seen for this different combination of X and Y and Y is equals to 2 here, that we are talking about 0.06, 0.036, 0.025, 0.001.

What we are just picking of this column actually 0.06, 0.036, 0.025, 0.001, which is for X equals to 1, X equals to 2 3 4 respectively. Here also, we are taking that, so, this should be 1, this is a typing mistake. So, this X equals to 1, this is X equals to 2, this is X equals to 3 and this is X equals to 4. So, we are simply taking that that column, that is for that set, we had Y is equals to 2.

Now, you know, if you just add it up, then what we will get we will get the marginal probability of Y for Y equals to 2, which is that that 0.446, which we have also seen in this earlier slide that for Y equals to 2, this summation is 0.446. So, this is the summation that we that we get.

Now, if we see that this conditional probability function, that is conditional probability distribution which is equals to their joint distribution divided by their marginal distribution, that is 0.06 divided by this one will will give you this values and andsimilarly, 0.36 divided by 0.446 will give you this and similarly, it will give.

Now, you see that, what we are talking about Y equals to 2 is giving this specific value.Now this one, the thethe first thing is now now clear as you are just making this division, that is 0.06 by this one, we are getting all nonnegative numbers.This is the distribution of X with condition that Y is equals to 2.

Now, as we are dividing it by these values of to this marginal with their sum, basically we are dividing it with respect to its sum. So, then the summation of this this probabilities will be will be 1. So, if you add this 0.1345 plus 0.8072 and these things, we will get that this equals to 1, which is again the second condition for that joint PMF, of that bivariate PMF, that conditional probability function.

So, this we have seen, so here you youyou can now see that mathematically the summation is equals to 1.This is because that we are dividing it by their marginal probability and why we are why we are dividing it with this one, because as we are condition is Y equals to 2; that means, we are definitely for in this column. So, the occurrence of Y equals to 2, we have mix here because that is our condition.

So, that is why in this entire in this set itself, that is, set means here that Y equals to 2.The summation of the probability should be should be equals to 1, because this is a definite case. So, for y equals to 2, it should be either x equals to 1 or X equals to 2 or X equals to 3 or X equals to 4 because there is no other possibilities. So, now, that is reflected in this concept.

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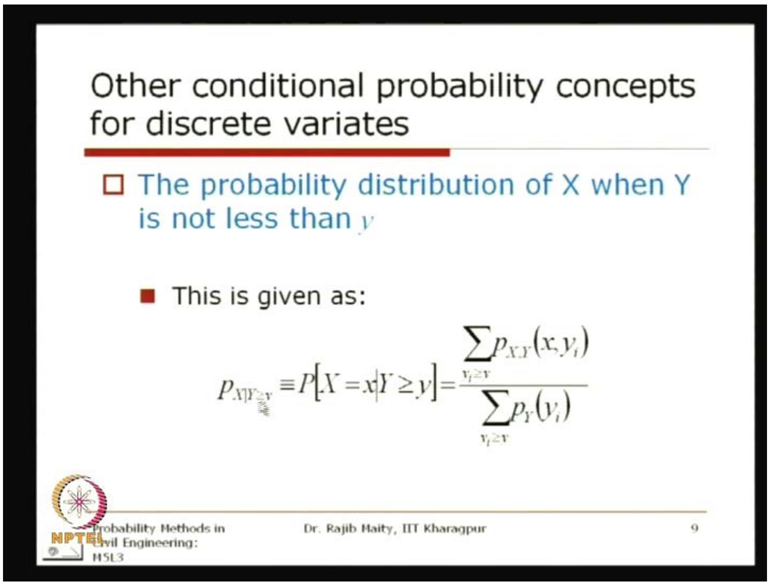

Now, again we will just take that for a for a specific range. So, earlier what we have talked about this is a specific value of this y.Now, if we just want to see that for the range, keeping the same concept background, this can be expressed as the probability distribution of X, when Y is not less than some specific value y; that means, y greater than equals to this specific value. So, this conditional probability mass function is given as, so, probability X on condition Y, the y is a now a range that is y greater than equals to that specific value y. So, probability X equals to x on condition Y greater than equal to y. So, which is equals to this. What now we are taking this joint probability distribution is that, for that range, for Y greater than that specific value, what is the probabilities for different values of x. So, that we are concentrating on the specific range of this specific range of the joint PMF.

Now similarly, for the y, when we are taking this marginal distribution, that marginal distribution is for the specific range.So, again we are taking of that same problem and here the question is that, estimate the conditional probability when Y is greater than equals to 2. So, instead of giving a specific value, we are giving a range.

So, again this is the joint probability mass function and these are the marginals.What we are referring here is that Y greater than 2; that means, we are referring to Y equals to 2, Y equals to 3 and Y equals to 4. So, this range, so, I have to exclude this case that is Y equals to 1. So, Y greater than equals to 2 for this case, the entire PMF part we will take.We will also take what is the marginal distribution for Y, when Y is greater than equal to 2. So, Y is greater than equal to 2 means, I have to take the, take this probabilities only. So, 0.446, 0.134 and 0.050.

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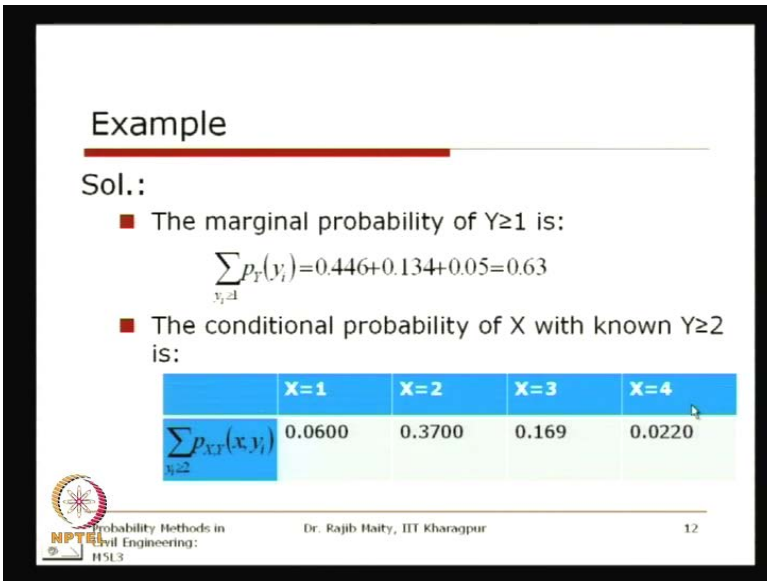

So, if we just see the same thing here that is, for this, so, this one, when we are taking this 3, that marginal probability for this Y greater than equal to 2.sorryThis is Y greater than equal to 2, then 0.446, 0.134 and 0.055, we have added up when we got that 0.63. So, once we get this one, this is our marginal probability for random variable Y for the range Y greater than equal to 2; this will be 2.

Now, the conditional probability of X with known Y greater than 2 is are these values.How we are getting these values, is that, when X equals to 1 for all values which are greater than for all values of Y, which are greater than equal to 2; that means, if you refer to this joint p n f, that is, X equals to 1, all values which are greater than equal to 2; that means, 0.060, 0.00, 0.0.This summation is 0.06 so; that means, we are taking 0.06 for X equals to 1.

Similarly, for X equals to 2 and Y greater than equal to 2, we have to add up 0.36 plus 0.01 plus 0.0 which is 0.37. So, we are just taking that 0.37.Similarly, for X equals to 3 and X equals to 4, we can get.

Now, these are the probabilities and this is the this is the marginal probability for that range of Y. So, next to get their conditional PMF, you have to take this 0.06 divided by 0.36 for X equals to 1.

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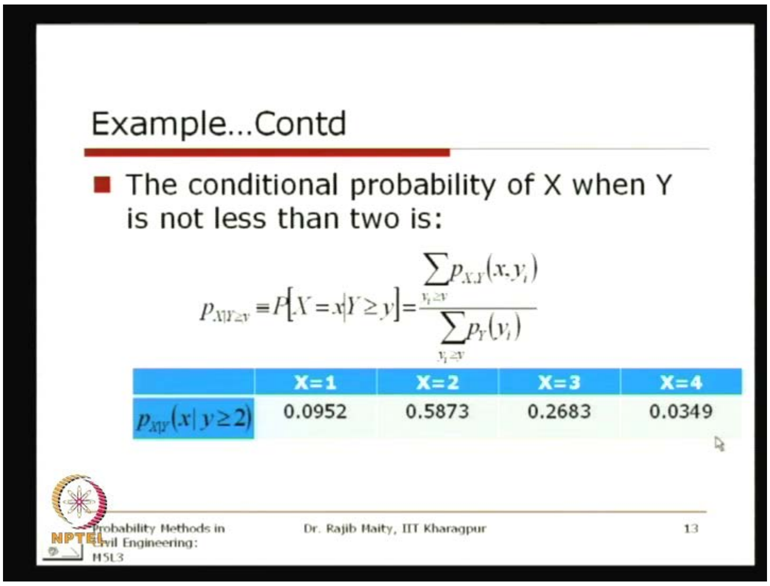

Similarly, for X equals to 2.37 divided by 0.63 and similarly, for X equals to 3 and X equals to 4.This is what is done here that the conditional probability of X, when X is not less than 2 is this. So, this this values that we have discussed and this one that we have we got. So, if we just divide these two numbers, then for that X equals to 1, the probability is 0.0952, for X equals to 2.5873, for X equals to 3.2683 and for X equals to 4.0349.If you add up these probabilities, it will be equals to 1,of course.

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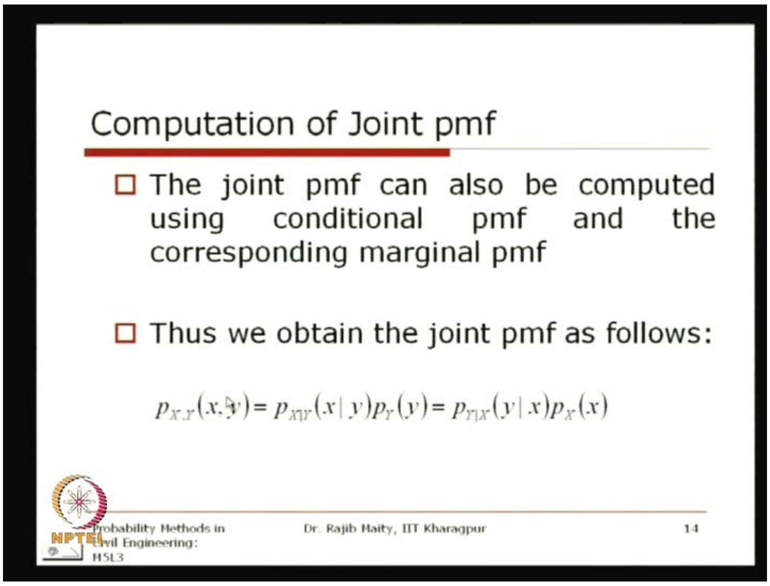

So now, if this one is taken, that now at starting we was just mentioning that how we can relate it for the conditional to the joint.Then, mathematically, the relationship from thatfrom that form it looks like this; that is joint p m f can also be computed using the conditional p m f and the corresponding marginal p m f. So, that means, from this equation actually.

So, from this equation, this is the joint and this is the marginal. So, the marginal multiplied by this conditional distribution will give you the joint. So that, once a joint distribution is equals to this,this is the marginal distribution and this is the,sorry this is the conditional distribution and this is the marginal distribution. So, if you just take these two, if you multiply these two; obviously, this will be equal to its joint p m f.Similarly,so, this says that this x on condition y multiplied by the marginal of y, this can also be expressed that y conditional x multiplied by marginal of x. So, both these things are same and is equal to its joint distribution.

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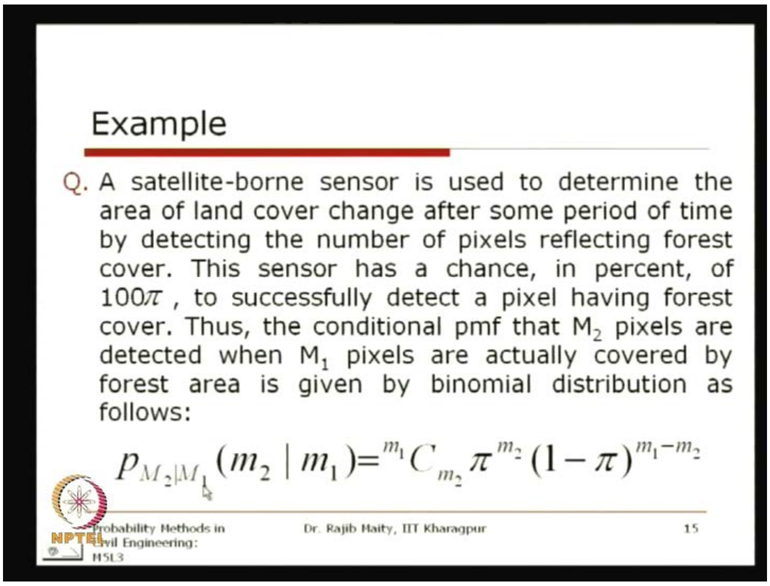

So, if you take one problem related to this,this is related to, sometimes, what happens for this orbiting satellite around the orb, we take the satellite imagery and from that imagery we want to know any specific attribute of this landward land cover on the earth.This generally help us to for differentstudies, different changes start studies, if we want to make some, what we do, we generally take the satellite imagery in near and after some year, we take another one.

So, two satellite imageries are kept for some year apart and we see the change.For example, that deforestation. So, for 10 years how much forest area has decreased or this another application is also, that we want to know that flood affected area. So, if we want to see it from the satellite, then have to see that how much area is flooded. So, when the satellite observe the earth from the space, it looks in terms of the pixels.This pixels means is the small representative area, which is taken as one unit to the sensor on board in the satellite.

So...So, these pixels depending on the quality of the sensor, this pixel size varies.Now, one pixel size is known is representing how much area. So, now we,we if we know that what is the area of one pixel, now depending on the number of pixels for that attribute whether it is forest cover area or the flood affected area, we we know that how much area is under that attribute.

Now, some times what happens this sensors, whether it will successfully detect a particular pixel for that attribute or not, that is, that we can express in terms of the probability. So, here if I if I want to look into it in the concept of the conditional probability, here the condition says that, suppose that the example that we will discuss now is on the forest cover area.Suppose, that a particular pixel is really having the forest cover area. Now this is the condition, that the that the pixel is really having the forest cover area. Now, from the satellite, when we are looking, now the question is whether that satellite can say that whether yes, it is forest cover area or no.So, obviously, the goodsensors will have the probability of success should be very high and the defective sensors may and may not be that high. This is also a chance.

So, the condition on the real ground too. So, weather what is really existing on the ground, that is the ground truth that we refer to. So, the condition on the ground truth, what is the outcome from the sensor that we will see through this problem that we are discussing now, in the light of the conditional probability distribution.

A satellite-bornesensor is used to determine the area of land cover change after some period of time by detecting the number of pixels reflecting forest cover.This sensor has a chance in percent of 100 pi to successfully detect a pixel having forest cover or not.Thus, the conditional p m f that m 2 pixels are detected when m 1 pixels are actually covered by forest area is given by the binomial distribution as follows.

So now, that the chance that we are talking about, this 100 by this, is the probability that weather a pixel will be successfully detected or not. So, if the ground, on ground really it is the forest cover area, what is the probability that that will be truly detected by the sensor.That is given by this probability.Now, suppose that there are total m 1 pixels are there; m 1 pixels means m 1 equivalent to that m 1 pixel area are actually forest cover and the sensor has detected m 2 pixels; that means, m 2 is less than m 1.

So, depending on their probability of success that we have discussed in the in the previous module also, you know this will this will fit as a binomial distribution.

So, this binomial distribution, now there are two random variables here.One is that m 1 and other one is the m 2. So, probability of m 2 on condition m 1, these are the variables on which we are representing m 2. m 1 is equals to following that binomial distribution, that is m 1 C m 2, that is m 1 combination m 2, that probability of success power, how many pixels are successfully detected.And, this is 1 minus pi probability of failure, that power how many pixels are not actually detected, that is m 1 minus m 2. So, this is the this is the conditional distribution of m 2 given m 1.

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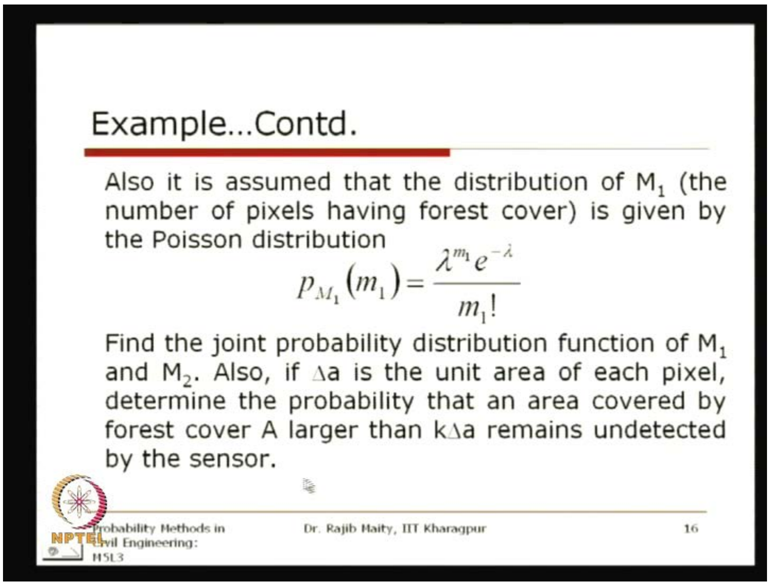

Now, for that m 1, so, how many pixels are there actually covered by the forest.That is, that is now, so, that is basically, when we are when we are considering that how many pixels is forest cover area, for a for a given area.That is now, irrespective of how many will be detected. So, that means, this is equivalent to the marginal distribution and this marginal distribution is having is assumed to have a poisson distribution.

So, here the parameters of this poisson distribution is lambda.It says, that it is assumed that a distribution of m 1, that is the number of pixels having the forest area, actually having the forest area is given by the poisson distribution, which is, expressed as this; that lambda power m 1 e power minus lambda divided by factorial m 1.

So, we have to find out, find the joint probability distribution function of M 1 and M 2; this is straight straight forward.That we have seen in this last discussion, last slide that how to get the joint distribution once the conditional and the marginal is available to us.Also, if delta a is the unit area of each pixel, that just now I explained, that each pixel depending on the sensor capacity,should it should refer some actual area on on ground, that is represented by 1 pixel. So, here it is the delta a.delta a, is the unit area of each pixel, determine the probability that an area covered by forest cover,A larger than K delta a, remains undetected by the sensor.

So, basically what we are looking for is the difference between that m 1 and m 2.m 1 is actually forest cover, m 2 has been detected.Now, when we are talking about that that one unit pixel is having the area of delta a and we are talking the total area undetected is capital a and which is equals to the K delta a. That means, we are talking about that difference is equals to K, that is m 1 minus m 2 is equals to K; that means, the total area will be K delta a.

I repeat the question once again here, the second one is that if delta a is the unit area of each pixel, determine the probability that an area covered by the forest cover A larger than K delta a, remains undetected by the sensor.

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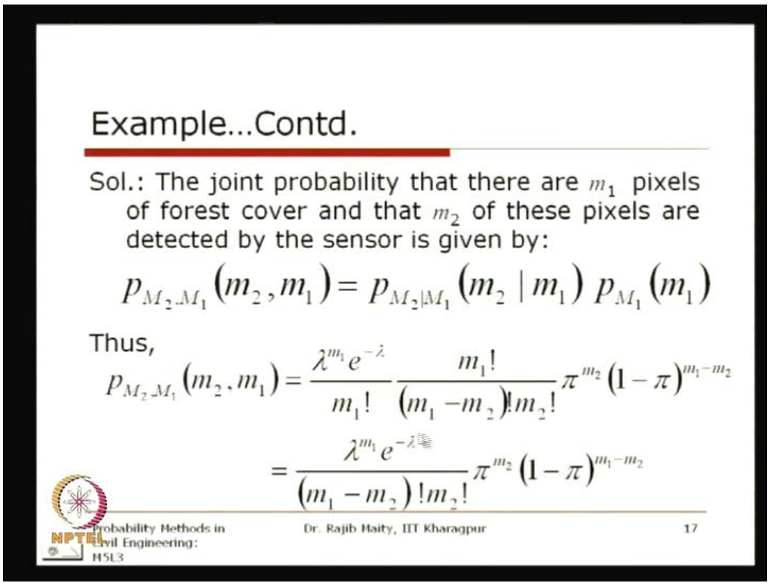

So, to solve this problem first that to get that joint distribution,we will see that joint probability that there are m pixels of forest cover and that m 2 of these pixels are detected by the by the sensor is given by that probability m 2 comma m 1.This is, we are talking about this joint distribution, which is equals to, as we discussed that this is the conditional distribution, that is conditional distribution of m 2 given m 1 multiplied with the marginal of m 1.

So, both this in formations are available to us. So, we will just multiply them to get their joint distribution, which is lambda power m 1 multiplied by e power minus lambda divided by m 1 factorial multiplied by this is, this expression is basically m 1 combination m 2. So, m 1 factorial divided by m 1 minus m 2 factorial multiplied by m 2 factorial multiplied by pi, that is the successfully detected the probability of success power m 2 multiplied by 1 minus pi power m 1 minus m 2.On which, we can after some simplification it comes to this form. So, which is the joint distribution of m 1 and m 2.

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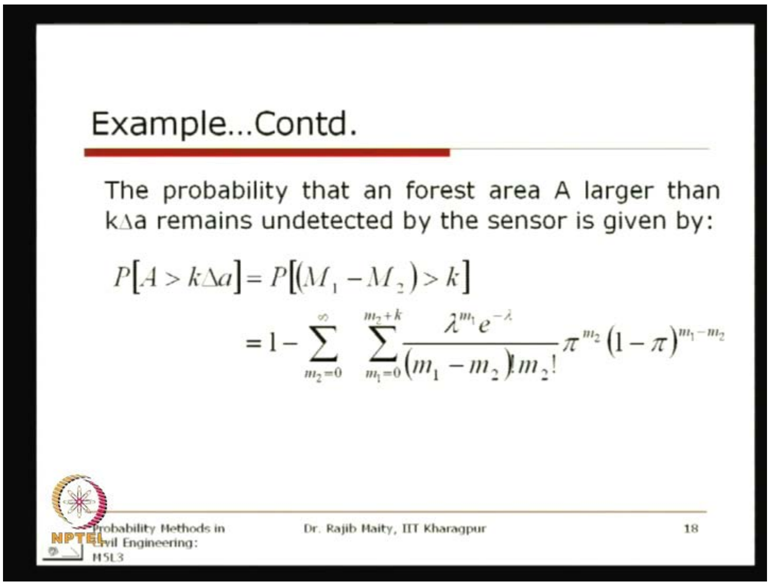

Now, the second question, which is interesting, is this.That the probability that a forest area A larger than K delta a, remains undetected by the sensor is given by; that means, we are looking that probability that A greater than k delta a. So, A greater than k delta a, as we have explained, that is, this delta a is the is the area for one pixel. So, that means, this this is there are k pixels are there which are undetected.Now, the undetected pixels, in terms of the random variable that we are discussing is m 1 and m 2. So, m 1 is the total number of actual pixel, which is having the forest cover area and m 2 is detected. So, m 1 minus m 2 greater than k.This is, so, this two probabilities that is, that we are referring to, that is the m 1 minus m 2 is greater than k. So, have to find out this probability from the joint distribution.

Now, from this stepto this one, how to get to get this this expression, that we have to, it is simple that in terms of the, if it is continuous, then it will have a joint integration or as it is a discrete, then we have go to a summation. But, summation we have do it here, that which area that we are referring to.

Note one thing here that this is greater than k that we are talking about.That equality sign is not there because, we are we are we are we are interested to know that greater than that particular value. So, how if we just want to represent it,in terms of a pictorial view, just know that what are the limits that we get for this summation is this.

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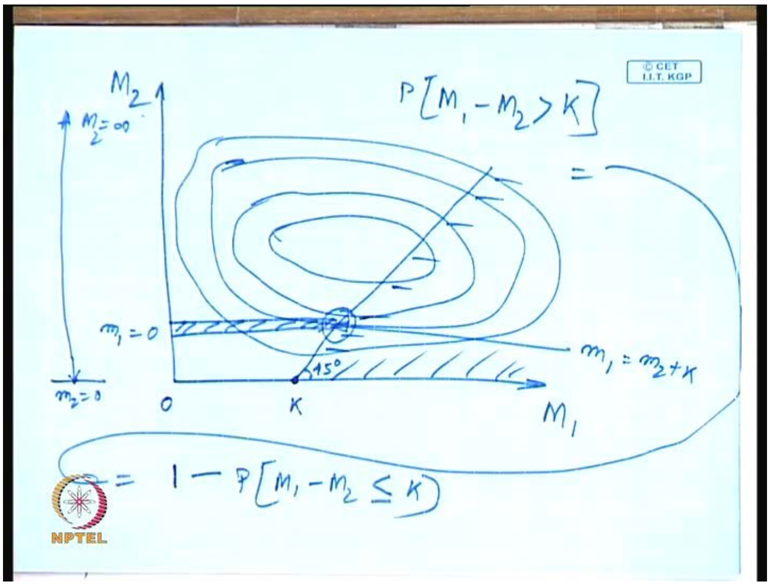

Suppose that, this is your, this your m 1 and this is your m 2.this your m 1 and this is your m 2 this is your m1 and this is your m 2Suppose that this the m 1 and this is your m 2, then there are some so; obviously, this is a discrete probability function.Then, we can have different values of m 2 and one satellite imagery generally have many numbers of such pixels. So, effectively you can say that these are many possible values on this m 2 and many for this m 1.And of course, but one thing is true, from the distribution also, from the present distribution as well as the binomial distribution, you know their support by now and all is starts from 0 to infinity, so it takes in this zone. So, may be the 0 1 2 3 4, it can take in this zone. So, the area that we are talking about, this is starting from 0 and goes infinity towards this and from 0 goes to infinity towards this, taking some specific values.

Now, what we are talking about is that,is that that m 1 minus m 2 is greater than K, that is, what we are referring to. So, which area is referring to this inequality is that, suppose that this your K, that is m 1 equals to k. So, if I draw a line passing through this K and having an angle 45 degree with the m 1 axis; that means, this m 1 minus m 2 greater than K refers to this zone, but, excluding this line.If it is equality sign comes here, then including this line, but, as we are talking about that this zone is greater than K; that means, this is the area that we are talking about, is this this zone,is that is that area of this inequality.

Now, what we can do is that, if we want to know what is this area including this line, there are two things.One is that, I can I can get the summation for this full area and and I can get the, I can get it or as it is there at this line is included, we can do the other side also.We can do the summation for this area and that we can take from this 1 minus of this one, because we know the summation of this full thing is this 1.

Now, suppose there are some probability mass is concentrated for this specific combination of m 1 and m 2 and suppose that these are the these are thethat contours of the of the probability of the joint probability. Now, basically what we are doing is that, if we this is this is; obviously, this a this a discrete random variable that we are talking about, but, you can also imagine that this probability, this is just like a surface and the surface is represented by the contours.This contours, we just want to know the part of the contour to get this probability;probability of m 1 minus m 2 greater than K.

So, which we have to just replace as we have, as you told, which we havetold in this way that this 1 is equals to 1 minus probability of that m 1 minus m 2 less than equal to K that we are talking about; that means, we are interested to know this area in this and then, minus it from 1.

Now, now to know this area, what you do do if you just take say a small strip, if you just take, then the range of m 1 for this one, so, this is your m 1 is equals to 0.This this pointthis point is your m 1 is equals to m 2 plus k from simple geometry.Now, so, this strip that we have to sum it up and we have to move this strip starting from 0 starting from 0 that is, your m 2 is equals to 0 here and it can go up to that m 2 is equals to infinity.Then, we will get this total area. So, m 1 range is from 0, m 1 equals to 0 to m 2 plus k, then, the summation is m 2 equals to 0 to m 2 equals to infinity. So, to know this area, then we will deduct it from the 1 to get what is this probability.

This is exactly is done here, you can see now, this 1 minus the, this 2 summation for this, the joint distribution and this joint distribution for this m 1.First, we are adding up the m 1 equals to 0 to m 2 plus K and this one again, we are summing up from m 2 equals to 0 to the infinity.This probability is given the probability m 1 minus m 2 less than equals to K, which you are subtracting from the total probability 1 to get this probability.

Now, so, this is the solution in the sense, if you know this parameter of those distributions and those things then; obviously, we will get what is the actual actual probability of the area to be remain undetected.

So, in the, so, we will, I will stop for this class here and what we will discuss in the next class is that, how we will we will take this concept to the continuous random variable, how we we will deal that the independence also that in the light of this conditional distribution that we will take in the next class.Thank you.