Probability Methods in Civil Engineering

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Lecture No#20

Marginal Probability Distribution

Hello and welcome to this second lecture of our new module, that is started with the last lecture and this module is on the is on the multiple random variable. And in the last class, we discuss about theirjoint distribution, joint PMF, PDF, and we concluded that last class saying that, we will be starting the marginal probability distribution in this class. So, before that we should take first that one example, that we that we did not do in the last class. So, that example we will start with today's lecture, which is on that joint PDF and after that, we will start the details of this marginal probability distribution.

The example that we are going to discuss, it will also be useful to understand that marginal probability distribution as well, because the same probability distribution is used just to just to demonstrate that how it is related, that the joint probability distribution andthe marginal probability distributions are related to each other.

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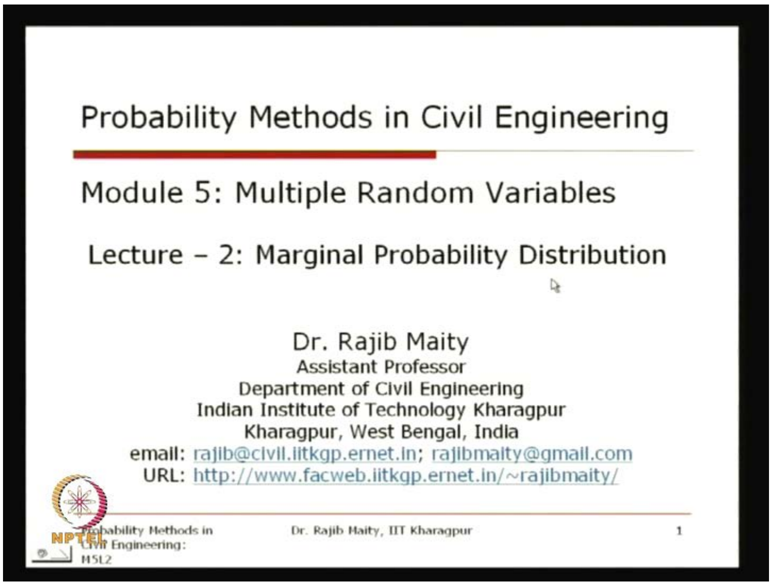

So, our today'slecture is on this marginal probability distribution, which we are going to take that outline of today's lecture as follows that multiple, in this multiple random variable that we discuss in this last class is a joint probability distribution.

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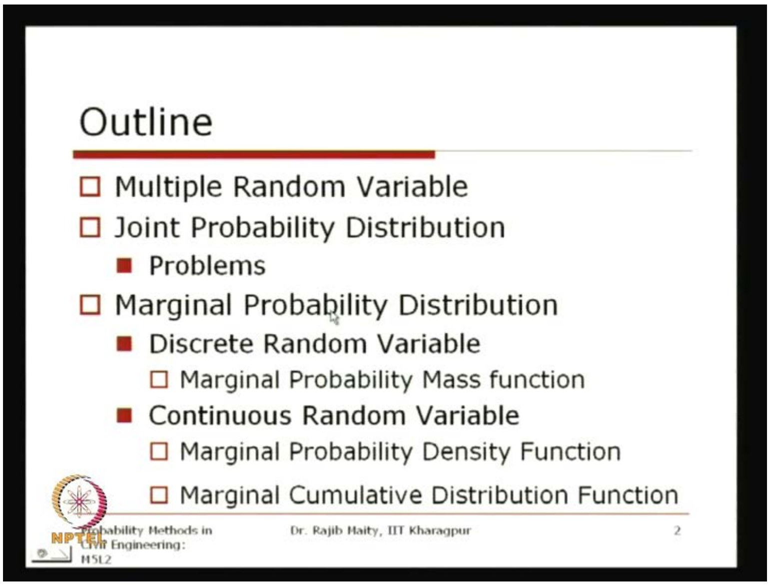

So, we will discuss the problem on that first and after that, we will come to this marginal probability distribution and its details about the thing.So, here one thing before I go through that problem, I just want to give the overall concept and on that concept basis if that is clear, then means the relation between this, that is joint PDF and its marginal will be very easier to understand. So, if you now see here. What we are looking at this joint and this marginal is like this.

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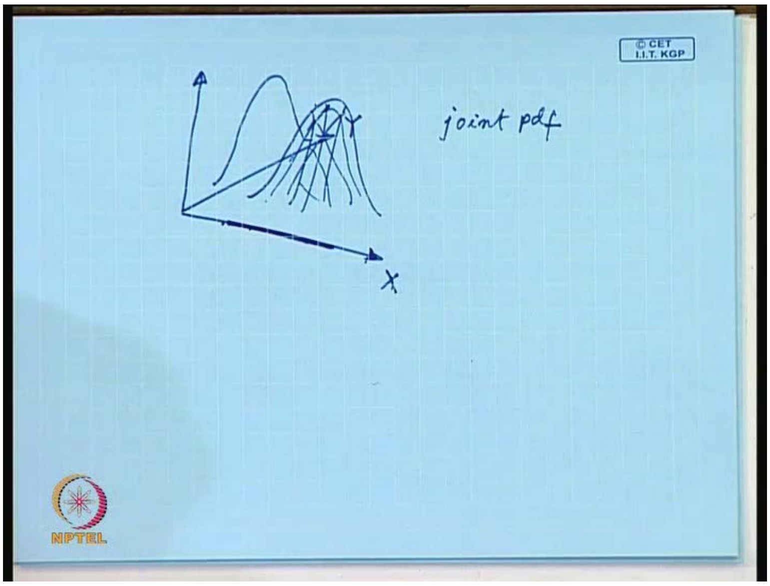

So, as we have told in the in the last class, that if suppose that there are these are the two two we are just now, because whenever I told that, whenever the pictorial representation is there, we can go for this bivariate one. So, at the concept can be extended to this, more than two variables as well.

So, here what we have seen that, these are the two axes for this two random variable and depending on whether this random variables are discrete or continuous, we will get whether the probability mass function that is concentrated at a particular point or this will be a surface kind of, is isis a three dimensional surface kind of thing, that we have seen it earlier just to give some.

So, this kind of surface also we can get for the for the joint distribution. Now,so this is what we can say that, this is when we are saying that this is a joint PDF that we discuss in the last class. Now, where it is related that, through this marginal is that, I just want to know the thethe variation or the distribution of one random variable, that is interest to me, suppose that I have this joint probability distribution and I want to know, what is the distribution over this over this axis, over this this axis means for this random variable X.

Now, it is like this, that sometimes irrespective of the distribution of the y, I just want to know the, what is a distribution of Y.The joint PDF, what it is showing that as we discuss in this last lecture also that, this is the behavior of the of the simultaneous occurrence of both the both the variables, it is true that in whenever if, but it should not be understood that whenever we are talking about the marginal, it is not at all related to this joint distribution or it is the similar to the single random variable, not like that.

So, what is that joint pdfis there, I am just interested to know for a particular value of Xof x irrespective of the distribution of this Y, what is the distribution of this X. Now, when we are talking about the irrespective of the distribution of the other random variable, that means I am just I am just marginalizing out the other random variable, just to get what is the distribution of a single random variable.

Now, very crudely which is definitely or the mathematically not true, I repeat before I state what I am I am going to say, mathematically that is not true. But very crudely, if you just want to visualize it in this way, that suppose that you are taking a source of light parallel to this X, so that for this surface you will be seeing a shadow of that of that 3D object on this plane, that is Y on this this f x, that joint probability density.

So, the shadow that we will see it here,so one shadow you will see see it here. Similarly, if you put the source of light in the parallel to the Y this this way, then you will see a shadow on this X axis. Now, this shadow is obviously, not the not the marginal distribution, but you can see it in that way the, how it is distributing only with respect to the Y alone or with respect to theto the X alone, that is only for the one distribution.

So, that you can say that is the marginal distribution, but obviously, mathematically we will see that how this how this marginal distribution can be obtained from this joint pdf, that we will see.We will first that take up a particular problem, where we are we are discussing about the joint pdf of one one joint pdf problem from the, so that the the problem we did not cover in the last class. So, after that, we will take up the same problem, same joint distribution to find out their marginals so that we will just see how they are related. So, this will be, so taking the reason for, taking of the same problem is that, we will be seeing that whether how this two things are joint pdf and the marginals are related to each other in a mathematical sense.

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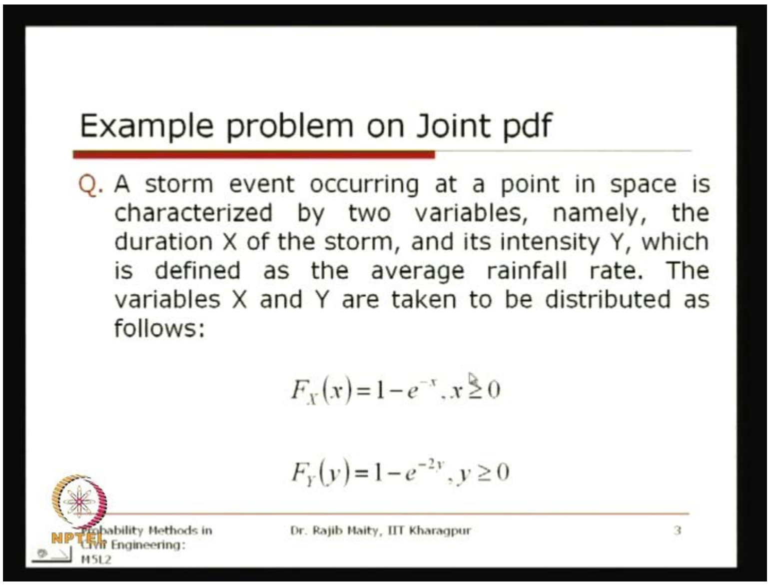

So, we are starting with that problem first as I am telling, that problem is states like this, that a storm event occurring at a point in a space is characterized by two variables namely, the duration of the storm and the intensity of the storm.

So, again I should say here that a particular storm event need not be characterized only by two random variables, there could be some other other random variables as well.For example,the total amount of this rainfall is also onealso one variable that generally we take this threethis three variables together to characterize oneparticular storm event. But here as we are discussing the bivariate case, we have taken that two random variables together.

So, at this point of time, I just want to tell one thing that even though we are saying that this concept can be easily extended to that higher number of random variables, that is more than two, we get the which is possible no doubt, but the it may not be mathematically always very easy. And that also we will just discuss in the light of this marginal distribution, that we are going todiscuss today’s lecture.

There are some very new concept has come up, say thefor example that copula, where we can, still the research is going on how to take care more than two random variables to see, to study their simultaneous behavior. So, the concept can be extended easily, but it may not always be mathematically a very a very easy to easy to capture their behavior together.

But here, obviously, we are taking the bivariate case, that is why the two random variable which which characterize a storm event, we have taken it in this problem. So, those twothose two random variables are one is the duration,the duration of the storm event and another one is the intensity of this, of the storm event has been taken care. One is denoted as X andother one is denoted as Y and there are several such such storm events such that such storm events are taken and their duration is obtained.

So, separately and from the duration, it is found out that it is following a cumulative distribution of this 1 minus e power minus x, which you know that, this is an exponential distribution, this is cumulative distribution from the exponential with the lambda value is equals to 1.Here, and also that rainfall intensity is also analyzed and seen that, which distribution it is following and that we have found this is also exponential distribution with the lambda value is equals to 2 and its, so its cumulative distribution is 1 minus e power minus 2 y, both this x and y are greater than 0.

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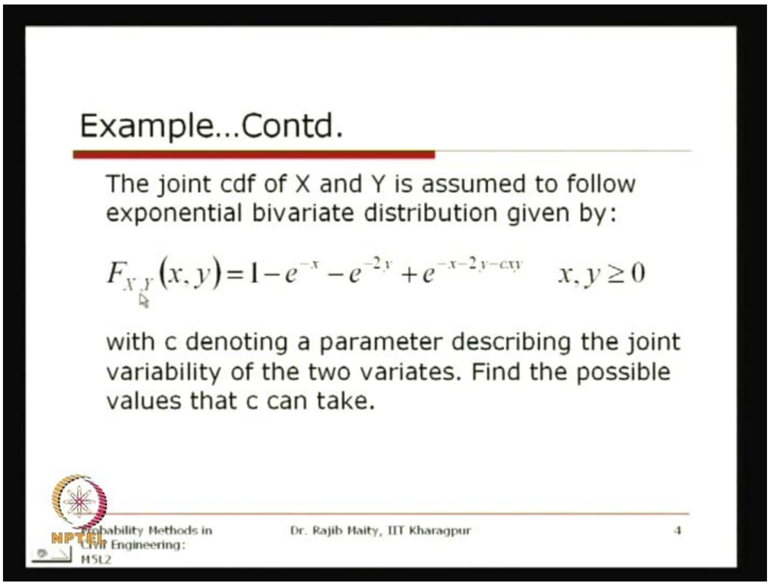

Now, the question is from the the joint cdf of this X and Y is assumed to follow an exponential bivariate distribution given by this this distribution. So, their joint distribution is like this, 1 minus e power minus x and minus e power minus 2 y plus e power minus x minus 2 y minus c x y. Now, thing is that, here lot of question make it can be raise that, how we know that this is the joint distribution and how we can say that those are the marginal distribution.

Those discussion will come up in a minute after this problem while we are discussing the marginal distribution, but here what is our focus of this problem is that, those are the those separately has been seen that, that is following an expositional distribution. And this joint distribution if it follows like this, it is the assumed that is why it is very clearly stated. If it is assume the joint distribution of this x and y is following this one, here the question here is that, is on the is on the parameter c the which is shown here, that c.

Now, what is the possible possiblethis range of this c, that c denoting a parameter describing the joint variability of the two two variates. So, how this two variates, that is that x and y, the duration and the intensity, how they are varying together, what is theirassociation to to each other. Obviously, this is mostly the linear association that we are taking about, this will be more clear when you will be in the module seven, the last module when we discussing about this that correlation or the covariance then it will be clear, but here you can just see that the c is a kind of parameter which is describing the joint variability of the two variates.

Here, so here our question is very simple in the sense, we will just find out the possible values of the c that, it can take for this distribution using the different properties of a jointpdf, that we have discussed in the last class.

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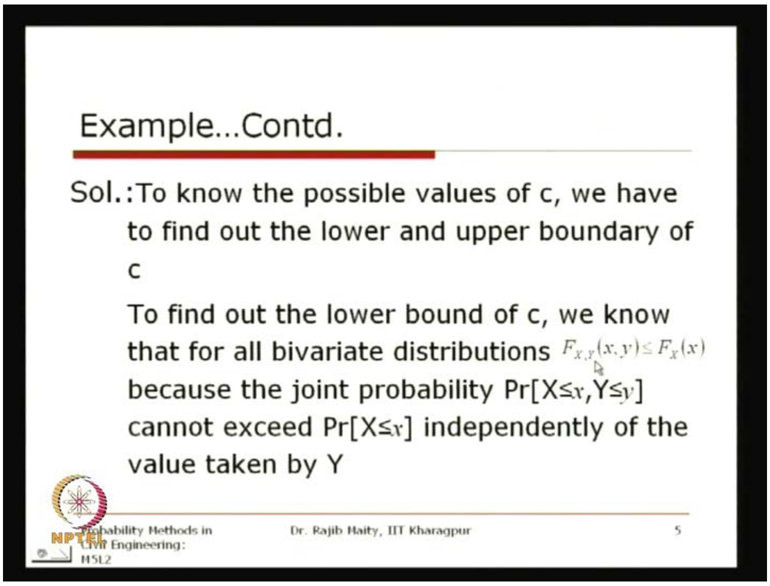

So, to know the possible to know the possible values of c, we have to find out the lower and the upper boundary of the c. To find out the lower bound of the c, we know that all bivariate distribution, that is this F x y and this F x, which is less than equals to this F x, now this when we are talking about the, this is the this is the property that this density will be less thanequals to less than equals to this one, this is the property when we will discuss in terms of this marginal probability distribution.

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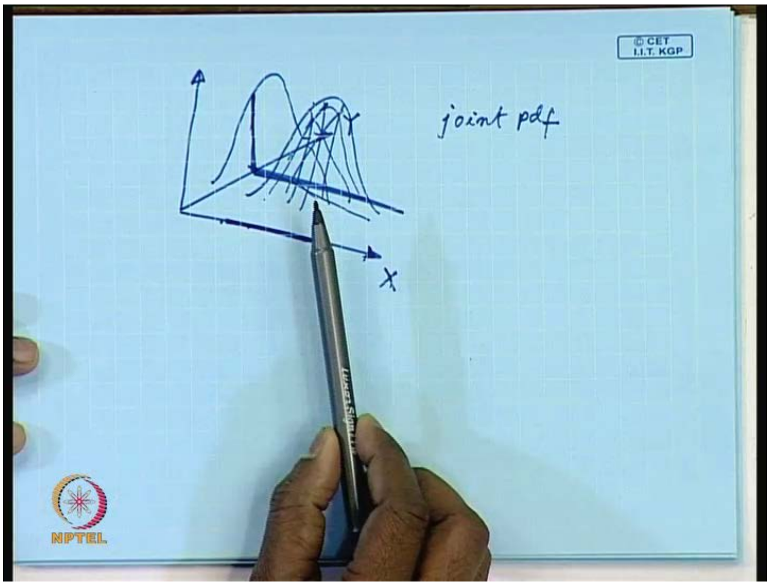

So, in this context, just I can once again take you to this ((pad)) that I just road it, rotate it here that a few moments back, that I was just talking about that if you just put a put a light in this location and the shadow of this one can be a crudely assumed to be the crudely assumed to be the marginal distribution of this joint distribution over this axis.

Now, if you just instead of, if you just little bit, we can we can extend our imagination that, instead of just throwing the light and light is producing a shadow, instead of that if you just assume that, when this light is going here and this this a particular value of this Y, if you just this is whatever the ordinate that we are getting here, is nothing but the summation of this this line,sorry this line is parallel to the X axis. So, this light is coming through this one, when it is penetrating through throughthis through this joint pdf surface. Now, whatever the ordinate that it is getting, it is going on adding them and at the end in terms of its shadow, it just is producing one ordinate which is the summation of all the ordinate that it comes.

So, I am just putting the same concept here, it is the producing one shadow, but instead of this shadow you know the, from the physical thing, it will the maximum height wherever it is getting, it will just produce that one.

Instead of that, I am just asking to extend your imagination that when the light is passing through, it is going on adding the ordinates and producing the sum of of this one. If can assume this one, then this is perfectly the marginal distribution in terms of the mathematical description as well, that we are going to discuss now.

So, what is now here is important, is the now we can see that the marginal is, obviously the obviously the greater than the, what is their theirtheirjointjointpdf. So, that is what the, that property is being used here, that this joint pdf is is for the any of this x and y should be less than equal to its marginal.

So, using this property what we canwhat we can do is that,the because a joint joint probability cannot exceed the probability of this probability X less than x, so this is also can be clear. That is, so as we are taking this marginal; that is the concept, just now I have told, so the using that one, it is easy to understand that the joint probability, that is a probability of joint occurrence X less than equals to x and Y less than equals to y cannot exceed the probability of X less than equals to x, irrespective of what is the distribution of the Y, obviously is independently of the value taken by the Y, that means irrespective of what is the value taken by Y.

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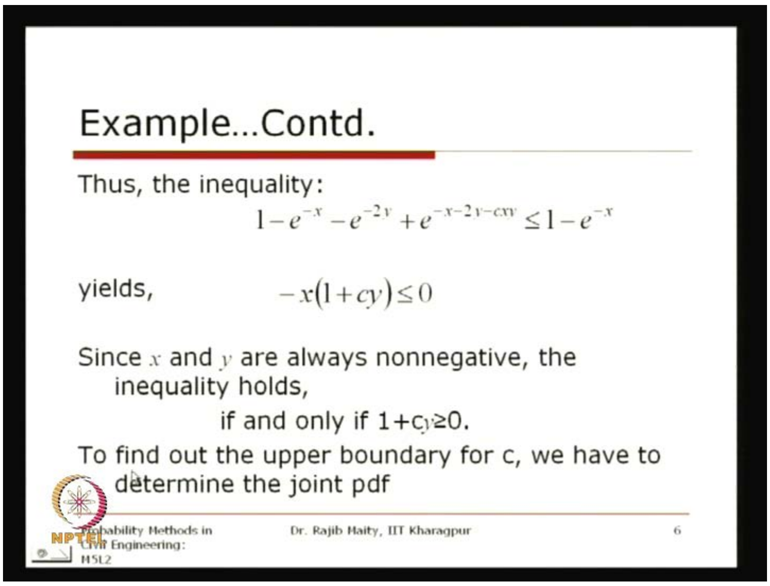

Now, if this is the condition we use, then it comes that we have seen both the things, this is the joint distribution and this is the, this is its marginal that is f x x.Now, following this inequality, we can directly say that from this one, and since x and y are always nonnegative, the inequality holds if and only if that 1 plus c y is greater than equal to 0.

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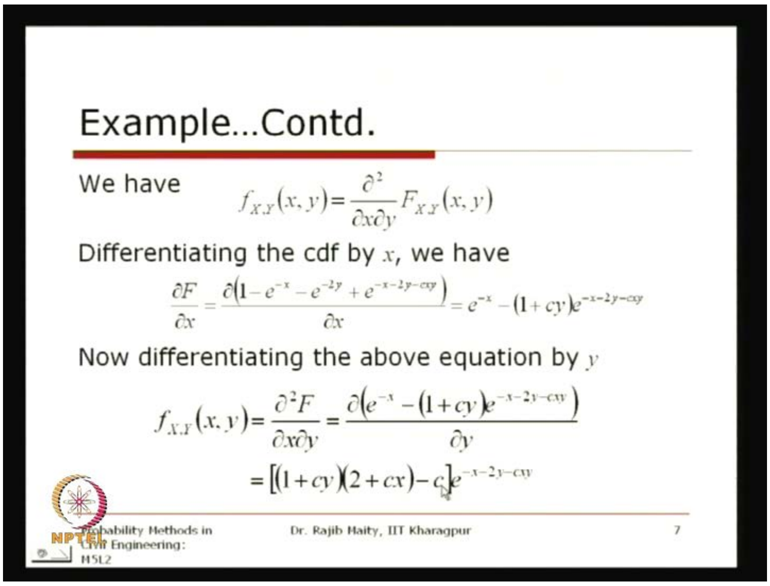

So, now if we want to know what is the upper boundary of this c, then we have to determine it from the joint, if we have to determine its joint pdf and you know that joint pdf in the last lecture, we have defined that this pdf is the double integration with respect to x and y of this cumulative distribution function. So, if you just take this joint distributionof this, if you take the first derivative it yields that e power minus x minus 1 plus c y e power minus x minus twice y minus c x y.

Now, from this one, if we take one more differentiation with respect to y, then it comes that 1 plus c y multiplied by 2 plus c x minus c e power minus x minus 2 y minusc x and obviously, the x and y is greater than equal to 0.

Now, this is that,this is the joint pdf that we got for this random variable x and y. Now, the thing is that, another property that we have seen that for all possible values of x and y, this should be a, this should be nonnegative, so this should be greater than equal to 0.

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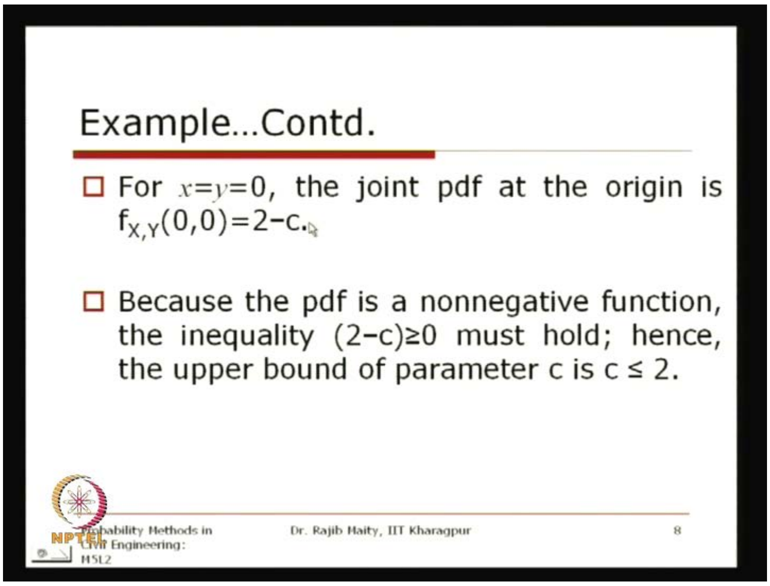

Now, you can see that it will be having the minimum value at, that at, so at x equals to 0 and y equals to 0, at the origin we will check that what this value and from there, we will get that its its upper limit; that is for x equals x equalsto y equals to 0, that is the joint pdf at the origin is given by 2 minus c. So, now this, because this thispdf is the nonnegative function we know, the inequality 2 minus c greater than equal to 0 must hold; hence, the upper bound of this parameter c is less than equal to 2.

So, we this one what the, we got the possible values that c can take, now we will take again this problem later, where we will be we will be discussing that from the joint pdf using that concept of this marginal whether we are getting to that one. Now, we will start that marginal probability distribution which just what we have discuss now, we will just see how we can we can obtain it from the joint pdf or pmf as the case may be whether it is continuous or discrete random variable.

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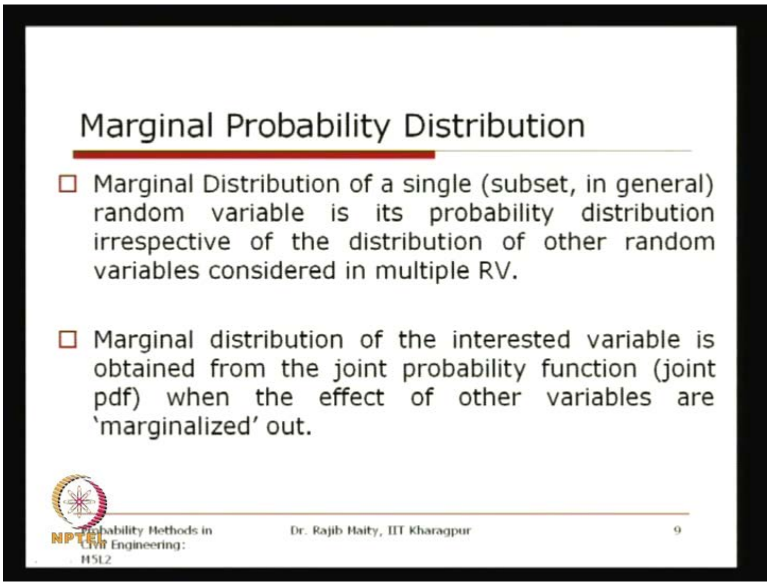

So, marginal distribution of a single single or the subset in general, I will just come to this subset what I just meant. So, for the time being we can just read that marginal distribution of a single random variable is its probability distribution, irrespective of the distribution of the other random variable, considered in the multiple random variable.

Now, if you see this one, this will be very easily you can imagine it for the bivariate case which we have shown in this in this pictorial diagram as as well, but this is not only that, this is not only for a single random variable.For example, suppose that there are there are more than two, if the case is more more than two random variables are there, that three random variables are there. So, say that x y z and we are interested to know that what is the distribution of the x y irrespective of the distribution of z, then then also you can say that this is the marginal distribution,marginal joint that marginal distribution of x and y irrespective of the value of z.

So, that is why to make the statement general, the subset is is used here. So, out of the n random variable, we can just make the subset with respect to a single random variable or more than one, but less than the total number of random variable, that is there in that multiple random vector.

Now, this marginal distribution of the interested variable, that for which variable we are looking for its marginal distribution is obtained from the joint probability function or the joint pdf.When the effect of the other variables are marginalized out, so here again the marginalized out, the word that we are using here, that means that it is that irrespective of the of the distribution of that other random variable in in in concept.

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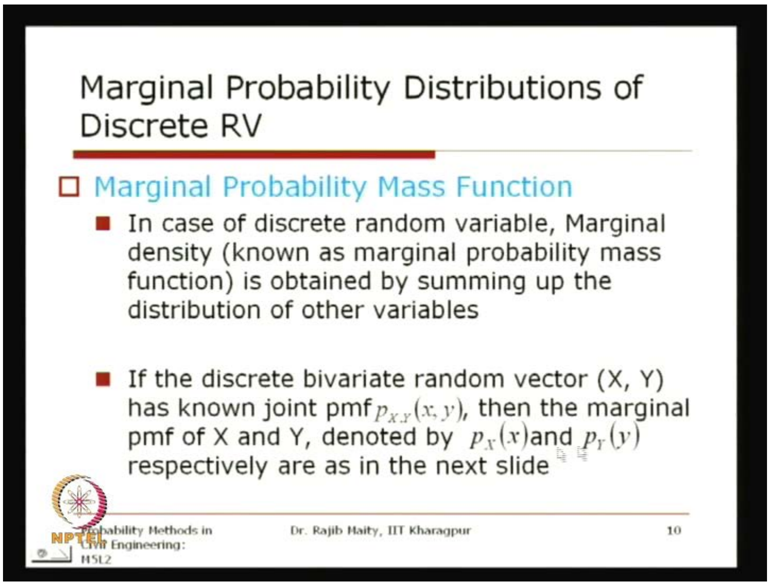

So, now the marginal probability mass function, this is for the discrete random variable. Now, if we just separate out the separate out the two random variables, one is that discrete and another is the continuous. So, if you just take this discrete case first, then what happens again, if we just take that the last lecture example to start with that, we are throwing that two dice and the the outcome of the two dice that is that is consist of two out come and we can say that. So, you know that a last class also we told that, always the association need not be in such a way that it should be related to each other in some sense, the independence is also one association, so there is independently associated to each other.

So, you know that for throwing a two fair dice together, so the outcome of one dice is the independent of what is the outcome of the of the other other one. But here, that for the marginal distribution if we want to know, that means the outcome of one dice irrespective of the other one, whatever the distribution that we are getting here, it should be it should be the same if we just throw a single dice, because they are independent in the sense that outcome of one dice does not have any effect on the outcome of the other other dice.

So, here the marginal distribution, that is for this type of cases where the outcome is having some specific distribution sorry the outcome is some specific values only. So, that time we are getting the discrete random variable and that marginal distribution is known as that probability, marginal probability mass function which we can, which we will get that direct summation of that of that values, over the other random variable, this is what is explained here. So, in case of the discrete random variable, the marginal density known as the marginal probability mass function is obtainedby summing up the distribution of the other variable.

So, this we will see again, because you can recall that in the last lecture, we gave one example and wejust show that that summation and what does that summation mean, that we did not explain properly in this last lecture. So, I just told that, it will be explained in this, in the next lecture in the context of the marginal distribution. So, to describe this fact, that is what the discrete random variable, we had some only few specific values is the outcomes, so there how we are getting the summation to marginalized out the effect of the other random variable, that we will that we will discuss and we will see that, how the how the other one is, how we are getting this through the through the summation.

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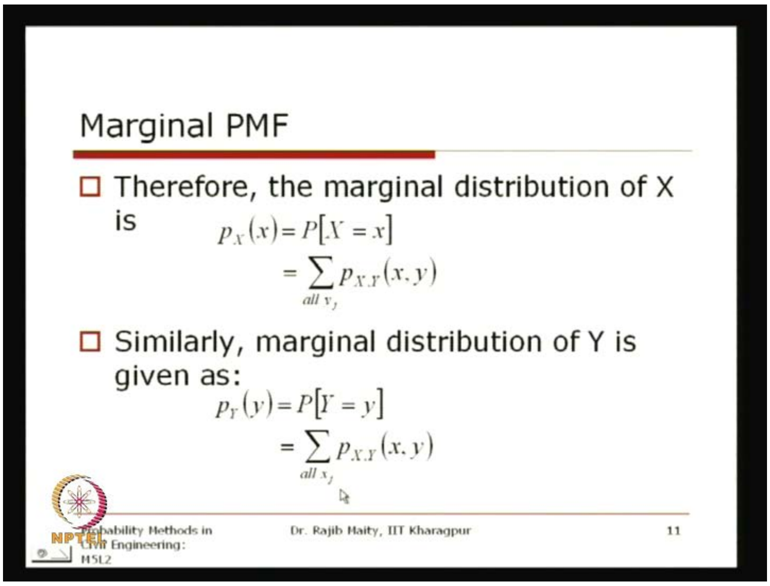

So, if the discretebivariate random vector, one is X and other one is Y, is having the joint pmf p X Y, then the marginal pmf of x and y, these are denoted by p X x and p Yy is a, is shown the how to get this to the marginal from this one, is just the summation of the other one. So, this one the p Xx as I told that, this is the marginal distribution of the X,from this p X Y of this joint pmf here, when we are getting we are summing up for all Y, this is important that when we are getting that marginal distribution of one random variable.

we are summing up forthe other one. So, here it is for the x the marginal distribution for x we are summing up the joint PMF for all y. Now, similarly for the marginal distribution of Y when you are getting, we should sum up for all x, this will be more clear with respect to the to this example that we are going to discuss after this.

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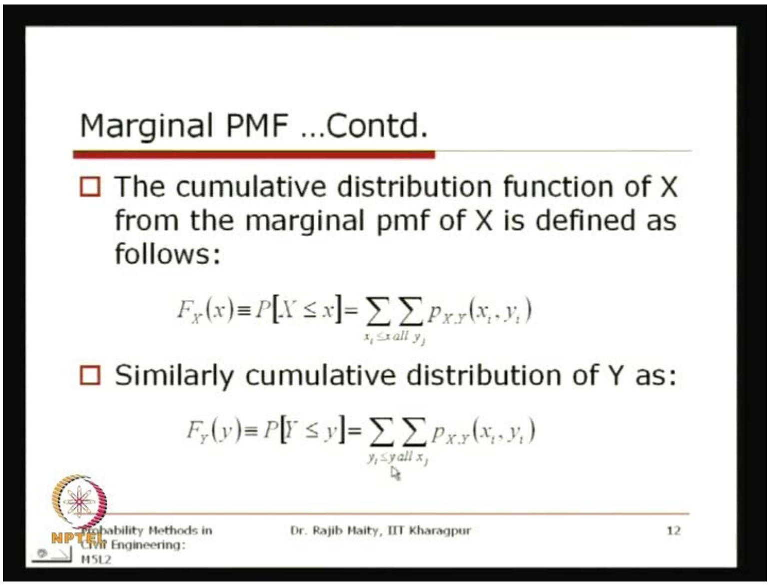

So, before that if we just take the cumulative one, so those are the, again that we can say that those are the, that pdf of that x and y. Now, for that x and y that is the individual random variable for those marginal, if we want to know that, what is the cumulative density distribution function. So, you can following that same thing that, we discuss for this single random variable that is once you once you know that what is their what is theirthatpmf.And then, you can just summing upwith you know that how to get from the, from this distribution to that to that cumulative probability distribution, you have to just go on summing up.

So, same thing is done here also, that is when you are taking that cumulative distribution function of x is nothing, but the X less than equals to some specific value.Now, when we are writing that X less than equal to this small x, and we are not writing anything about y, that means that that is irrespective of the value of the y. So, if we want to get this quantity then what we have to do is that for the summation when we are doing, that is for all x i which is less than equal to x and for y this should be for all y.Now, we have now, also you can see that for all y, when you are summing up, you are basically getting the marginal distribution of x. Now, with that marginal distribution of x, if you just go on summing up up to some specific value x, then you are getting its cumulative distribution.

Similarly, in case of y also this is just the just opposite, when we are taking the cumulative distribution of y, then you are just first of all summing up the pmf for all x. And then, so then you have the resulting sum is your the marginal distribution of y and then you are summing up that marginal distribution to aup to a specific value y to get its cumulative distribution.

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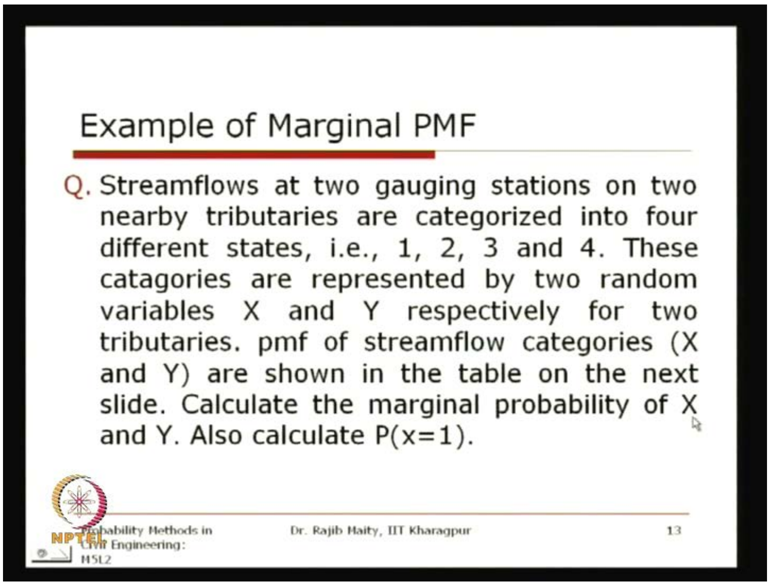

Let us take one example, that the example that you that, we took in the last in the last lecture on this streamflow, there are two stations nearby stations and those the streamflow at those stations are categorized into different streamflow category, and here the four such categories are are considered to describe this problem.

So, the problem once again it states that the stream flow at two gauging stations on two near by tributaries are categorized into four different states, these states arenamed as 1, 2, 3 and 4, so for the both the sides, this states are there. So these categories are represented by two random variables X and Y,X and Y means for the twotwotributaries, first one is

X and second one is Y, in the PMF of streamflow categories X and Y are shown in the table on the next slide. So, what we have to calculate here, now is the, calculate the marginal probability of of X and Y, and also also calculate what is the probability of X equals to 1.

So, you recall that with this problem in the last lecture, we we calculated some some probability that is where the X is greater than, that X is greater than Y, that probability we we got from that from thatPMF what was was supplied, at here we are looking for from that joint PMF, how to get their marginals, that we will discuss in this one. And then from that from that marginal probability, we have to calculate that what is the probability x equals to 1. So, the probability x equals to 1, we will get the marginal distribution of the probability of marginal distribution of X.

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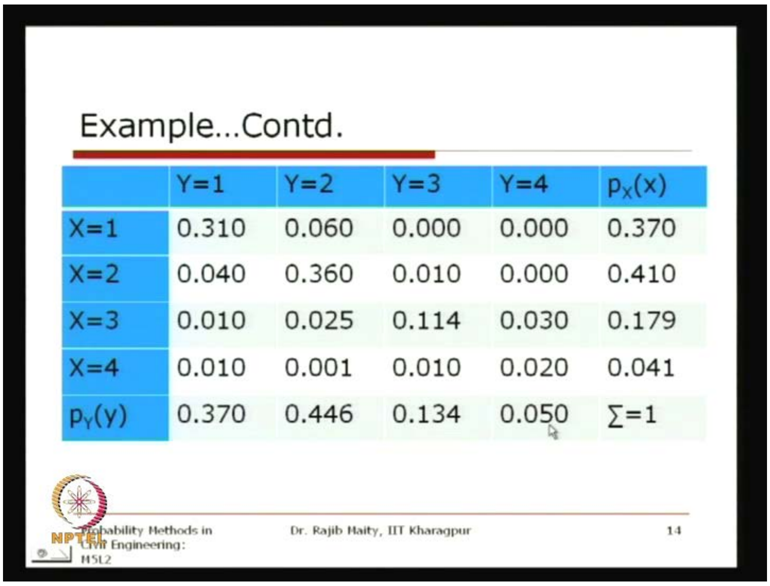

	Y=1	Y=2	Y=3	Y=4	$p_X(x)$
X=1	0.310	0.060	0.000	0.000	0.370
X=2	0.040	0.360	0.010	0.000	0.410
X=3	0.010	0.025	0.114	0.030	0.179
X=4	0.010	0.001	0.010	0.020	0.041
$p_Y(y)$	0.370	0.446	0.134	0.050	$\sum=1$

So, this is that PMF that is there, these are the different states that is X equals to 1,X equals to 2, 3, 4, and similarly that Y equals to 1, 2, 3 and 4. So, this is a 4 by 4 matrix and each matrix, so the possible outcomes are some combinations. So, this is, so this 0.31 means thatprobability of X equals to 1 and Y equals to 1 is equals to 0.31.

So, we discuss while describing that joint PMF that, this occurrence should be joint, this occurrence should be simultaneous occurrence. So, X equals to 1 and Y equals to 1 for this particular case, the probability mass which is concentrated for this outcome is

0.31.Similarly, the similar expression can be given for all this cells here, say for X equals to 4 and Y equals to 2, the probability is very less, which is 0.001.

Now, this 4 by 4 matrix is showing that joint PMF. Now, if I want to know, what is the marginal probability distribution of either of X or Y, then what should we do? So, this cell this last column, what you see is that is the marginal distribution of X. Now, when you are talking the marginal distribution of X that, what we are doing here is that, what is the probability, the asking here, the question that what is the probability that X is equals to 1.

Now, when we are saying this this question which is also asked in the in the in the problem is that X equals to 1, that means irrespective of the value of the Y. So, it can take either, so X can be 1 in the case when Y can be 1 or Y can be 2 or Y can be 3 or Y can be 4. So, X equals to 1 it is when we are taking the irrespective of the value of the Y, that means,any one can can happen.

Now, so that probability of X equal to 1 is nothing but the summation of four events, which are X equals to 1 and Y equals to 1 plus X equals to 1 and Y equals to 2 plus X equals to 1 Y equals to 3 plus X equals to 1 Y equals to 4. That means, these four probabilities has to be added up to get the answer, what is the probability that X equals to X equals to 1, so this is exactly what is done here.

Now also you can think about that pictorial representation, that we give that if we are just putting a source of light on the one side and when the shadow is creating, the shadow is nothing but the summation of all such ordinates, this is exactly is following here. So, here this is your, in this table, in this in this in this matrix form, this is your x axis, this side is your x axis, this side is your y axis. When you are throwing the light here, it is basically going on adding up and creating the shadow here. So, this this this quantity, so this 0.37 is the probability for X equals to 1, this 0.41 is the probability for X equals to 2, this 0.179 is the probability for X equals to 3 and 0.041 is the probability for X equals to 4.So, this 4 is showing you the marginal distribution of X.

Similarly, if you want to know the marginal distribution of Y, then for marginal distribution of Y, that is the probability of Y equals to 1, then we have to add up again that four such events, that is Y equals to 1 and either of this four such events X equals to 1 or 2. So, this should be added up like this, to get that the probability of Y equals to 1 is equals to 0.37, probability Y equals to 2 equals to 0.446, and this Y equals to 3 is 0.134 and probability Y equals to 4 is equals to 0.050. So, what we are what we are getting here is that, now if you go back, if you just recall our once again the problem that we state, that is the, this is the joint PMF that we started with for the two nearby tributaries.

Now, that joint PMF was there, then what is the what is the implication of this marginal distribution, that implication of the marginal distribution is that now for this two tributaries, I do not know what is happening on the other tributaries, I am only interested to know that what is happing in this tributary, and how we can how we can describe their probability.So, if that is the case then, so irrespective of the knowledge of the other other tributary, I am only interested in this one. So, that is reflected in this in this marginal distribution.

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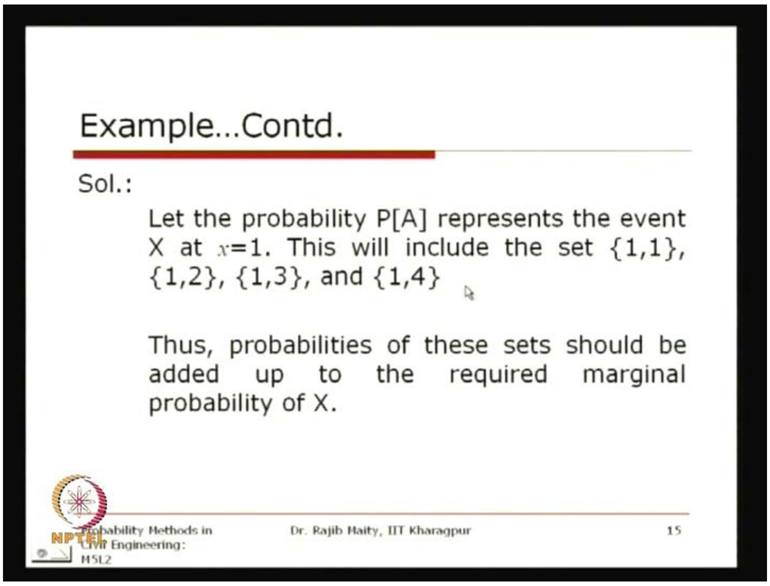

So, to complete this problem, again this is explain here, let the probability represents the event that x equals to 1 for this four such event that is 1 1 1 2 1 3 and 1 4 and this probability if you add up, you will be get that get that probability, that is your point.

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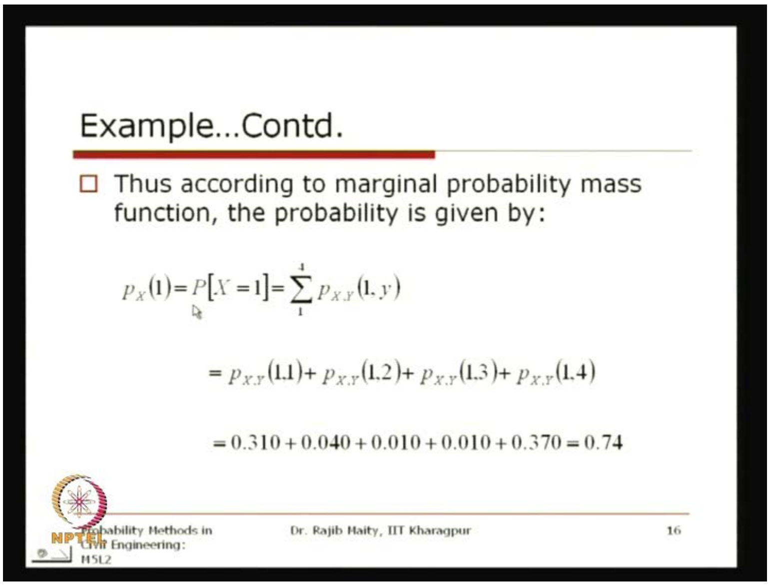

So, these four probabilities are added up and to get the answer is your 0.37 sorry there is a mistake in typing. So, this total probability equals to 0.37, which is that probability of X equals to 1 as we have seen in the table.

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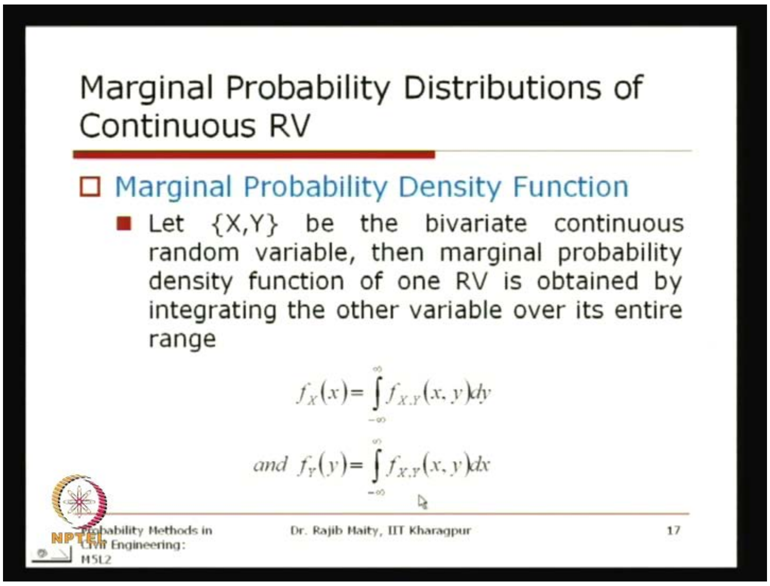

Now, now if we just extend this one to the continuous random variable, then this continuous random variable, incase of the continuous random variable you know such,so this will be the integration. Again, we have to follow the same concept here also, that is let X and Y be the bivariate continuous random variable, then the marginal probability density function of one random variable is obtained by integrating the other variable over its entire range. So, this one, this f Xx is the marginal distribution of x and this f X Y x y is the joint pdf of the x and y.

So, once we want to know what is the marginal distribution of of X, then then we should integrate the, integrate it for the other random variable which is y and this is for the entire range of this of the y. So, if we do this one, then then what we get is the marginal distribution of X. Similarly, we will get the marginal distribution of Y, if we do the integration with respect to the x over its entire range.

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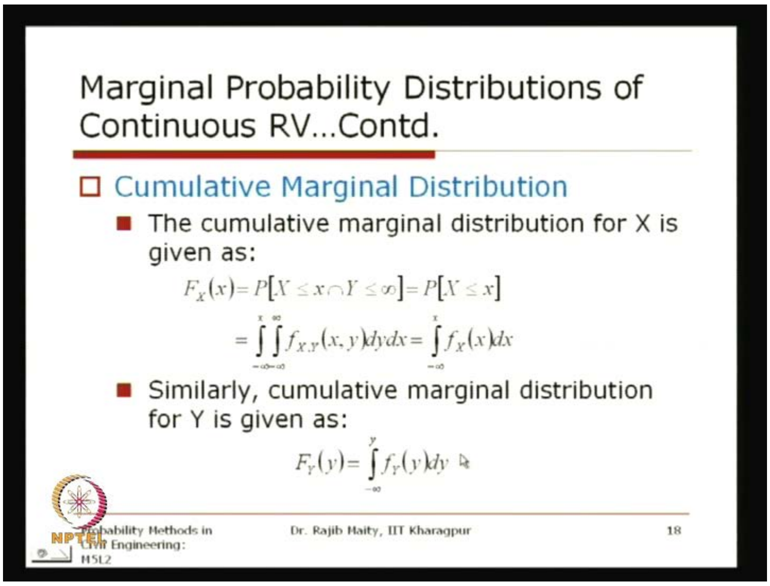

Now, for the cumulative marginal distributions, once we get that F Xx, that is thus that PDF, then we can do its, again we can do the integration from minus infinity to some specific value x to get that to get their cumulative probability distribution function.

Now, here if we see it that cumulative marginal distribution from the joint joint distribution, what does it mean that F X of x is equals to probability of X less than equals to some specific value x, and Y less than equals to infinity; y less than equals to infinitive means it is covering the entire possible range of this Y, and you know that which is nothing but as it is the entire range, which is nothing but is equals to probability of X less than x.

So, this in the integral form, it will be the, for this entire range of this Y, from minus infinity to plus infinity, this one should be integrated out for this for y and you know that this is nothing but the marginal distribution. So, this is replaced by this F Xx and the outer integration is from the minus infinity to some specific value of x which is with respect to the x. So, So, this terms that F x x is the nothing but the, from minus infinity to some specific value to get thatcumulative marginal distribution of X.

Similarly, for the y it is the integration from minus infinity to some specific value of y and it is integrating the marginal pdf of the of the y random variable y. So, this way we will get both that marginal p d f and marginal c d f.

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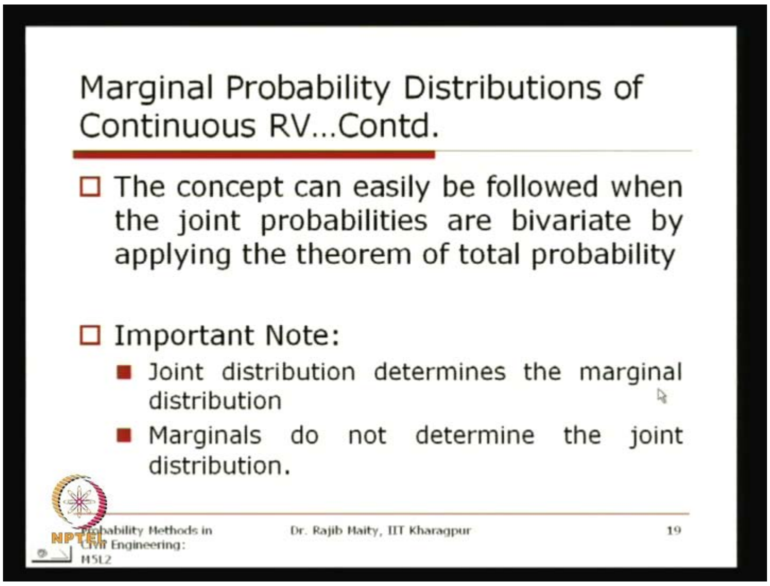

Now, the concept can easily be followed when the joint probabilities are bivariate by applying the theorem to theto the total probability. So, this total probability means, when we will be, you will be discussing that the conditional distribution in mostly in the next lecture, that time this comment will be more clear that, how from the total probability theorem we are getting again this this distributions, from the thatthat joint probabilities, how we get it from there.

So, different conclude this one, there are two important things that I want tomention here, that joint distribution determines the marginal distribution, second marginals do not determine the joint distribution. This is very important in the sense that, if you if you know, how we know that is our, that is another issue. But if we know the joint distribution joint pdfor joint pmf, from there we can determine what is the marginal distribution. But if we know the marginal distribution, then we cannot determine what is its joint distribution from the traditionally, that is that that what we know. If I just give agive a simple example, and that is also very important to remember while applying in the different problems in the civil engineering is that, sometimes we use the concept of the joint normal distribution, you know the normal distribution, we have discussed already.

Now, if we get two random variables and we state that this two random variables are having a joint normal distribution. That means,that that means, we get aaaa three dimensional bell shaped curve as their joint distribution.Now, if we can declare that with x and y follow the follow a joint normal distribution, then we can conclude that both of them are their marginals follow a normal distribution.

But if we get two random variables to different random variables, both of them are following normal distribution, then we cannot say that that they are joint distribution, also will be the joint normal joint normal distribution. So, one side we can do the conclusion, but other side the reverse is not true. This is very important to remember, and that that is basically now, the question is that if we know the marginals, then how we will know that know that joint distribution.

So, that I have just mentioning of the beginning of today's lecture, that there is new, relatively new concept has come on this copula. And that copula theory can be used to find out their joint distribution, with the help of thiswith the,if the with the knowledge of their marginal marginal distribution. In one of the subsequent lecture, we will cover the introduction to this copula theory, for this this purpose in the later stage.

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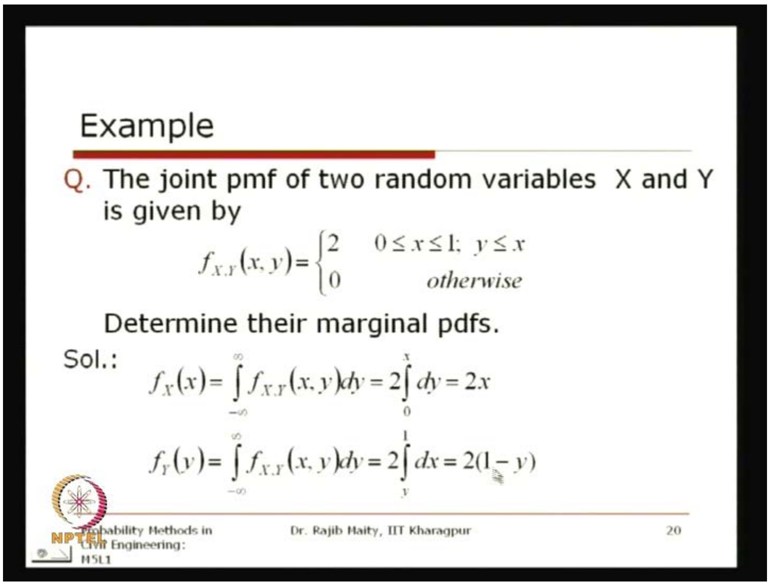

Now, we will take a simple problem to start with, that is if the joint,no this will be pdf, if the joint pdf of two random variables X and Y is given by this expression that is f Xx y equals to 2, for the range 0 to 1 and this y less than x. So, this this range you can see here, this is that x is varying from 0 to 1 and y is varying from 0 to that x, so this y is always less than x. So, this is a kind of the triangular shape over which the the probability is that, the pdf is defined here.

So, determine their marginal pdf and straight forward, if we apply the knowledge, that is from that the integration over this entire range that is f X. So, if we get the, if we want to know the marginal distribution of x, that is the integrated over the entire range of y and which is from 0 to x. So, starting from 0 or it can take a maximum value up to that x., so this is equals to 2 x.And similarly, f Yy is equals to what we have to do the integration with respect to the, this will be x with respect to the x, and this starts from this y to 1 and if we do this integration, we will get that 2 into 1 minus y.

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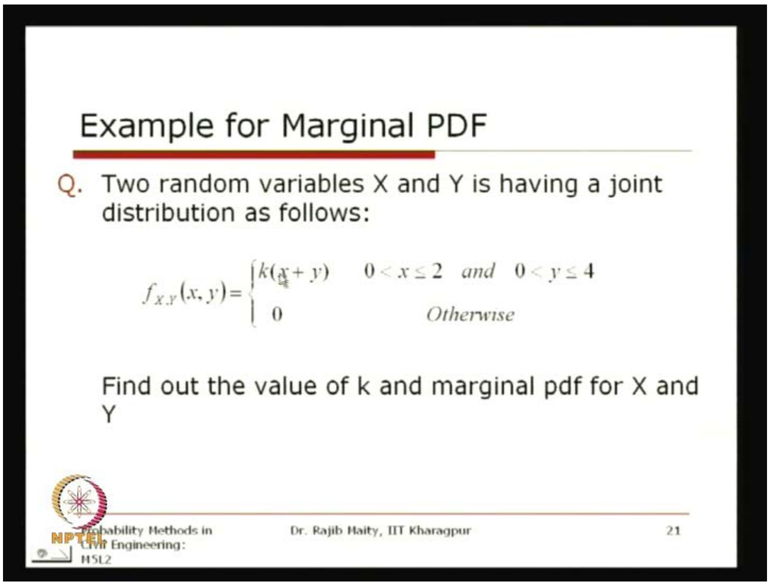

Now, little bitsome more, so that is one, that is the example. So, this is the this is the example of the the uniform distribution that is, that because that is constant PDF form is the is the is the constant which is equals to 2. But if we take it, is the PDF is related to some their their dependent variable X and Y. So, two random variable the X and Y having the joint distribution as follows, that is k x plus y and the range is this 0 to 2 and y ranges is from 0 to 4.

So, if we take this range and, so first of all, if we want to know that find out the value of k. Now, whenever you know that this problem we have taken earlier also means, this type of problem that is to find out k, we have to apply that to be a valued pdf. It has to be equals to the integration of over the entire range should be equals to one and then once we know that k, then what is the marginal pdf of x and y, that is the question here.

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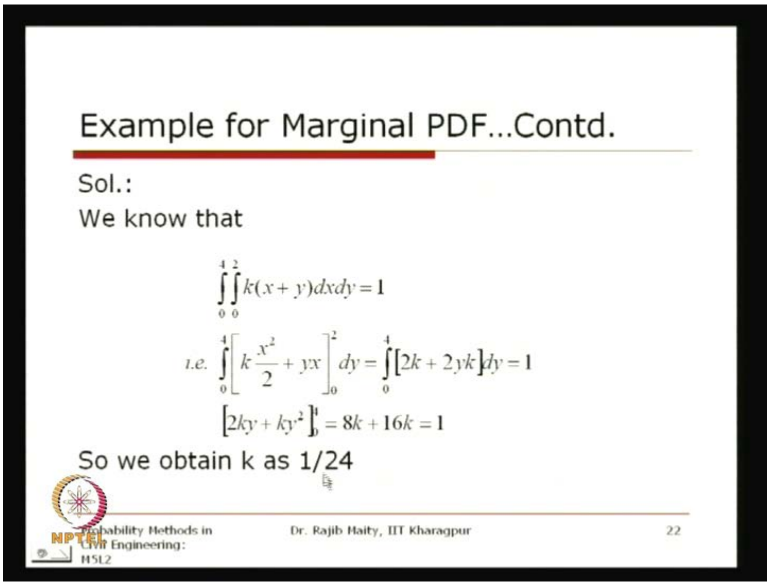

So, here, so over the entire range, if we do the integration for this x and y, so this is should be equals to 1, and this integration from this 0 to 4 if we take, then this is k x square by 2 plus y x which is from this 0 to 2, if we take this one. And this one is, so if we go with this, then we will get this should be equals to 1.

So, ultimately it is coming that 8 k plus 16 k is equals to 1, so you obtain that k is equals to 1 by 24. So, which so, the. So, the pdf is F X Y is equals to is equals to that is your x plus y by 24, sothis is the pdf form.

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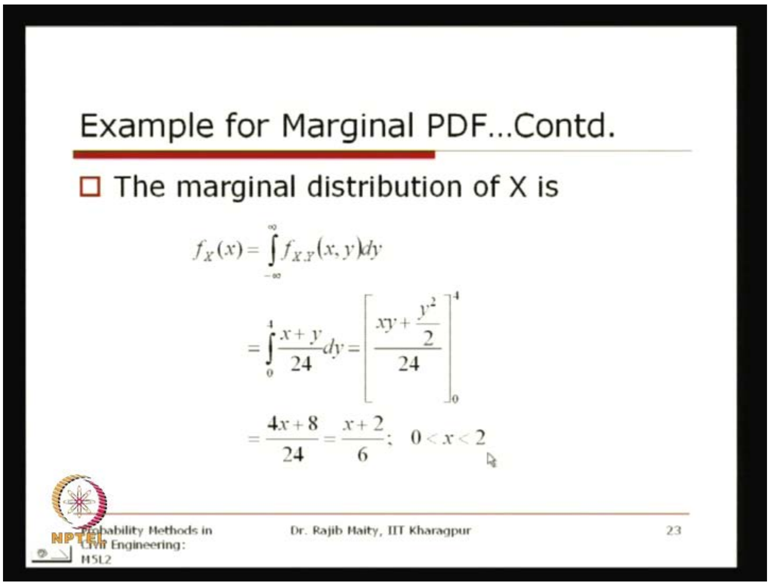

Now, if we want to know the marginal distribution of X, then we know that we have to integrate it with respect to y, and from the minus infinity to plus infinity,throughwe are taking throughout the entire range which is the 0 to 4, if we take this range and we will get that 1 from this 0 to 4 for the y, and we will get the range at x plus 2 by 6, and for the x ranges from 0 to 2.

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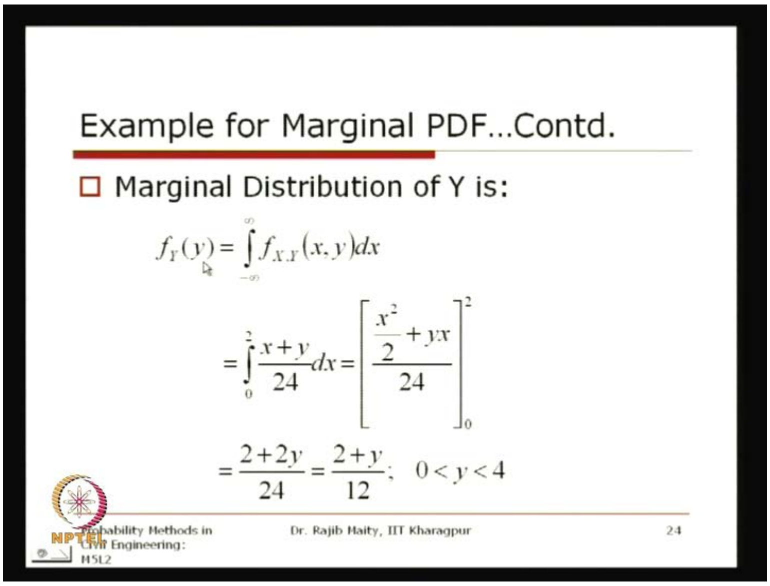

Similarly, if you take the marginal distribution for the Y, we can follow these steps from this minus infinity to plus infinity and we can check that from 0 to 2, it comes as this 2 plus y by 12 for the range 0 to 4, for the range of y. So, one thing we can see that marginal distribution when we are calculating this, the variable x is not present there, because that is integrated out. So, this is only depending on the on the variable y,similarly for the X also, it is only dependent on the variable x.

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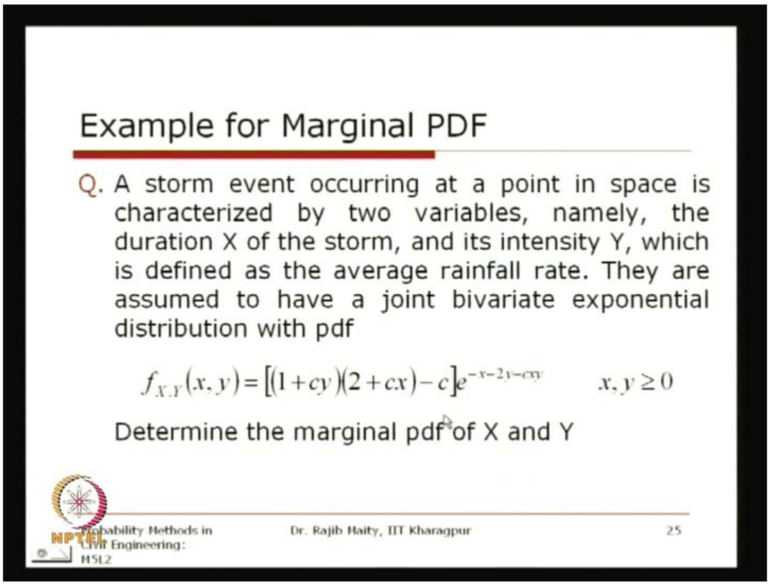

Now, we will take again that, the problem that we have we started with this lecture is the storm event, where this joint PDF is known to us, the storm event occurring at a point in the space characterized by two variables, namely, its duration X and intensity Y. And what we have seen there is that, if this is this is the joint distribution form, then what should be its marginal PDF for this X and Y.

So, in the earlier problem, we have shown that this is the marginal, this is the joint and we are asking for this, the possible range of this value of y. So, now we know only that joint distribution and I and we are interested to know what their marginal distribution is. Again, I repeat that this joint pdf is given here, and we are interested for the, we are interested for the marginal one. But we cannot ask the reverse question that is if this is the marginal, what should be the joint until and unless we are using that theory of the copula that we will discuss later.

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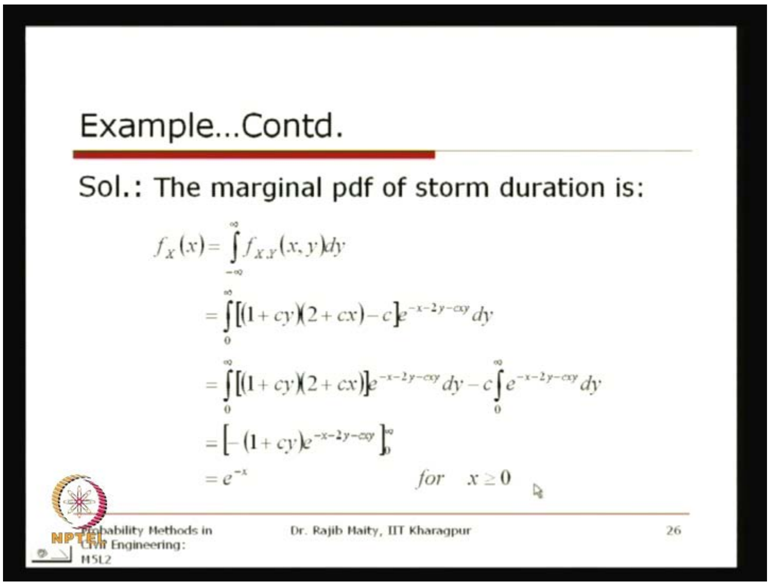

So, here what we have known, this is the joint pdf is given to us, and and I am interested to know their marginal distribution. So, to get the marginal distribution again, that we have to integrate it from this, over the entire range of y and if we follow this integration steps, then we will see that this f X, that is the marginal distribution of x is coming to with that e power minus x for the range x is greater than 0.

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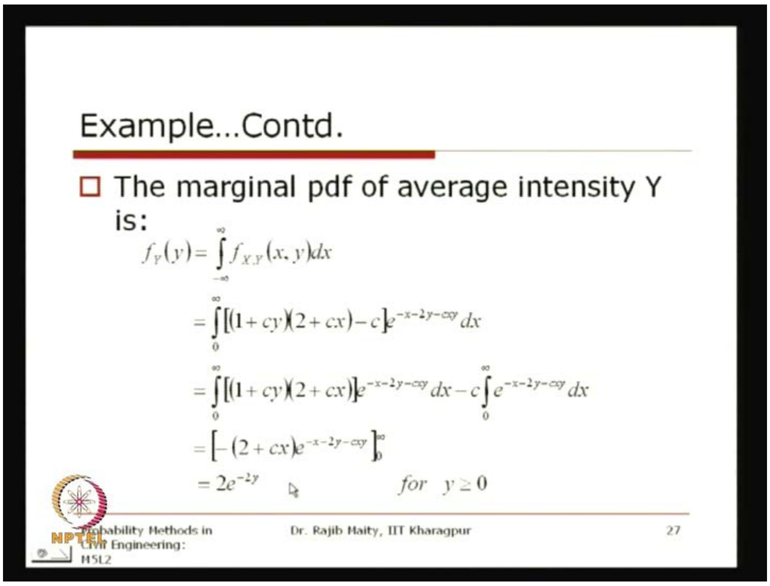

And similarly, for the Y, the marginal pdf of the average intensity Y, it should be equal to that fY from minus infinity to plus infinity. If do this integration step, we will get that this marginal distribution of y is equals to 2 e power minus 2 y, so these are the intensities.

Now, from here if we are, so if we want to know what is theircdf, then we have to integrate it again from the from the lower extreme to some specific value of this y.For example, if you do it here, that is the 2 e power minus 2 y, then you know that this is an exponential distribution and this, it is cdf will be 1 minus e power 2 2 2 y, which is exactly we just stated at the beginning of this class for that problem. And this is also another exponential exponential distribution, which cdf will be 1 minus e power minus x for this range. So, in this class, we have seen that the concept of this marginal probability distribution is discussed in this class and joint probability distribution, we have discussed in earlier.

So, now if we just think about that we know the joint distribution and the marginal is discussed, after that one thing is very, one more concept is important, that is known as the conditional distribution.The conditional distribution means that, I am little, so instead of, for incase of the marginal, instead of saying that irrespective of the value of the other, I just want to know if the other variable is given some condition, that is the the y is this, what is the distribution of the x.

So, instead of saying the irrespective of the other variable, I am saying that the other variable is this, so that is the condition. So, once that the condition is defined, what is the distribution of the other. So, this is the generally known through this conditional distribution and which is also obtained from this joint pdf. So, this conditional distribution will be discussed in the next class. Thank you.