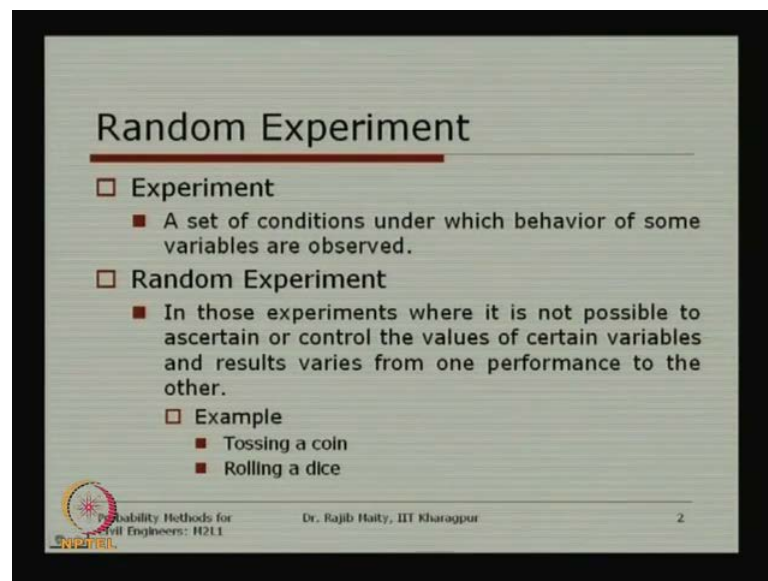


**Probability Methods in Civil Engineering**  
**Prof. Rajib Maity**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture No. # 02**  
**Random Events and Probability Concept**

Welcome to the second lecture for the course probability methods in civil engineering, this is basically the starting of module two, in this module two I will cover the concept of random events which is very useful in the probability methods. in this module, there are couple of lectures for 5 lectures will be there in the very first lecture I cover the random events and probability concept.

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**Random Experiment**

- Experiment
  - A set of conditions under which behavior of some variables are observed.
- Random Experiment
  - In those experiments where it is not possible to ascertain or control the values of certain variables and results varies from one performance to the other.
  - Example
    - Tossing a coin
    - Rolling a dice

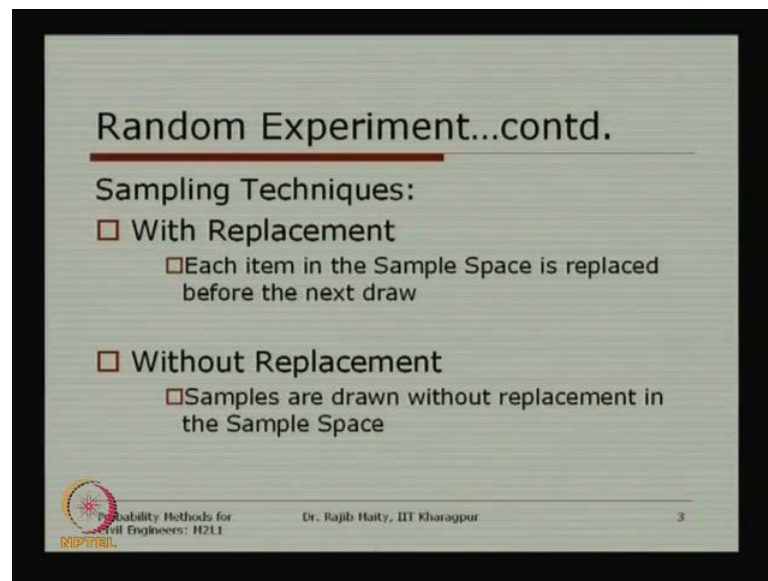
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Basically we have to know the concept or random experiment first, and before that what is an experiment? It says a set of conditions under which behavior of some variables are observed, and when you say that a particular experiment is random that is random experiment in those type of experiment, where it is not possible to ascertain or control the values of certain variables and results varies from one performance to the other performance. For example, if I say that if I threw a coin, and I want to see what is the

outcome of the throwing of a coin, I cannot say with full certainty that first to be outcome whether it will be the head or it will be the it will be a tail.

So, this kind of example that is tossing of a coin, and similarly a rolling of a dice, where I cannot say the particular outcome from a from that particular experiment, then the outcome of that experiment becomes random. And this kind of experiments is known as random experiment.

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**Random Experiment...contd.**

**Sampling Techniques:**

- With Replacement**
  - Each item in the Sample Space is replaced before the next draw
  
- Without Replacement**
  - Samples are drawn without replacement in the Sample Space

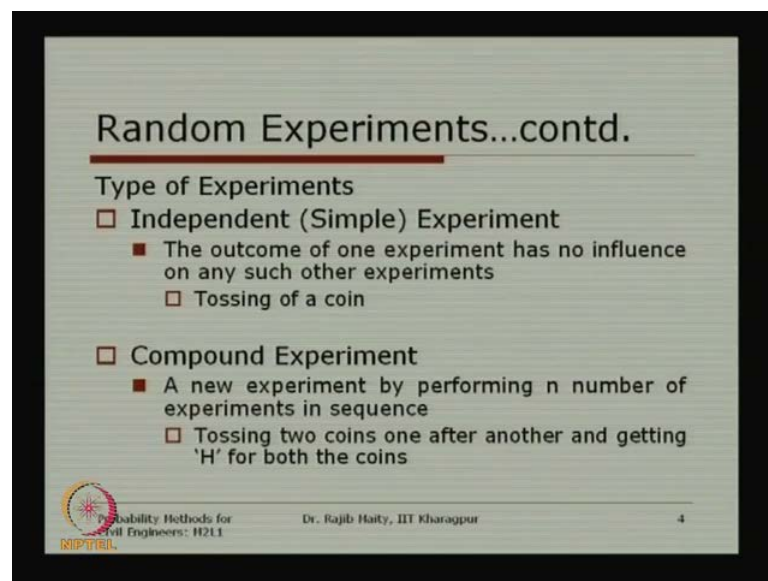
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There are when we take this random experiment, there are different types of sampling techniques. One is with replacement and another one is without replacement, in the case of with replacement, each item in the sample space is replaced before the next draw. Say for example, if I say in a box there are few balls, and the balls are of different colors and I want to take out one ball in random, and I want to observe that what is its color. Now, if I take out one ball and place it back to the box again, and do the next trial after replacing the ball that is known as the with replacement.

If I do not go for this example, and if I give the example of the tossing of a coin, this is also can be treated as with replacement it is sampling technique, because if in the first trial I get one head, and I am going for the next trial that means all the options that is of all the possible outcome head and tail both are with me.

So, this kind of experiment is known as the with replacement experiment; there are there is also possible that we can do the experiment without replacement also in the example of drawing balls from a box, we can just kick out one after another a particular color of ball and keep it outside. So, that the we will know later gradually in this course, that this two kind of sampling techniques is different when we are going for the definition of or the computation of probability for that particular experiment.

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**Random Experiments...contd.**

Type of Experiments

- Independent (Simple) Experiment
  - The outcome of one experiment has no influence on any such other experiments
  - Tossing of a coin
  
- Compound Experiment
  - A new experiment by performing n number of experiments in sequence
  - Tossing two coins one after another and getting 'H' for both the coins

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There are different types of this kind of experiment, one is that independent or simple experiment, and second one is compound experiment. In this independent and the simple experiment it says that the outcome of one experiment has no influence on any such other experiments or also it is also not independent, it is also not dependent on whatever the outcome it just replace trail. Say for example, if I say again the tossing of a coin, then if the outcome for a particular trail, if the outcome is head it does not influence it, it does not have any influence on the outcome for the immediate next trail.

So, this kind of experiment is known as independent experiment. Second one is the compound experiment, it says a new experiment by performing n numbers of experiments in sequence; for example, tossing two coins one after another, and getting head for both the coins.

So, I just want to show experiment here is that I throw two coins, and I want to see the outcome for both the 4 coins it comes head, if I give the example of rolling a dice where

the possible outcome is 6, that means the number 1 to 6, I can say that what is the um what is the possibility that the summation of the outcome of both the dies will be greater than or less than a particular.

So, this kind of experiment it says that it depends on the outcome of one particular - one particular experiment is depends on the total performance or the success rate of the experiment depends on the mutual performance of both the all the trails that is that we are performing, so this kind of experiment is known as compound experiment.

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**Simple Experiment: Trials**

Number of Possible Outcomes

- Bernoulli's Trials
  - If a simple experiment has two outcomes, independent replication of the experiment
  
- Multinomial Trials
  - If a simple experiment has k number of outcomes, independent replication of the experiment

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There are different possibilities of the outcome for a particular experiment and based on the number of possible outcome it can be divided into two different parts. The first one is the Bernoulli's trails it is a if a simple experiment has only two outcomes for example, as I said that in case of tossing a coin there are already two possible outcome head and tail and independent replication of the some experiment, then this kind of trials as known as Bernoulli's trials.

On the other hand, if a simple experiment has k numbers of outcome, and independent replication of this experiment this kind of trials is known as multinomial trials.

So, these things are important to know the basics of these random events which will be useful when we are going to give the concept of probability, so if we know that if we know the basics of this experiment and based on this experiment. If we know what are

the all possible outcome then we will this will be helpful to understand and to and interrupted the results of the probability and an analysis, and if we assign some particular probability. It will help us to inform from the results in hand, so gradually I will show in this how this kind of concept, this kind of interpretation of the um probability is based on this kind of random experiment.

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**Sample Space**

- A set  $S$  that consists of all possible outcomes of a random experiment is called a Sample Space, and each outcome is called a Sample Point
- Example
  - If a dice is tossed, one sample space, or set of all possible outcomes, is given by  $(1,2,3,4,5,6)$

The diagram shows a 2D coordinate system with a grid. A red oval labeled 'S' encloses a collection of black dots. An arrow points from the text 'Sample Points' to one of these dots.

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So, first one important terminology that we should understand clearly is sample space is set of A set it is generally a designated as a capital S, A set is that consists of all possible outcomes of a random experiment is called a sample space and each outcome is called a sample point. Example; if a dice is tossed one sample space or set of all possible outcome is given by the numbers from 1 to 6, so this scan be treated as a pictorial representation of the of the sample space, and this black dots are the sample points. So, collection of all these sample points collection of this entire possible outcome for a particular experiment is known as the sample space.

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**Events**

- Event
  - It is a subset of the Sample Space or in other words, a subset of all possible outcomes
- Elementary Event
  - An Event that consists only one single point of Sample Space is called Simple or elementary Random Event

The diagram illustrates a sample space  $S$  containing several dice faces. A red oval labeled "Event" encompasses a subset of these dice faces. One specific die face is labeled "Elementary Event".

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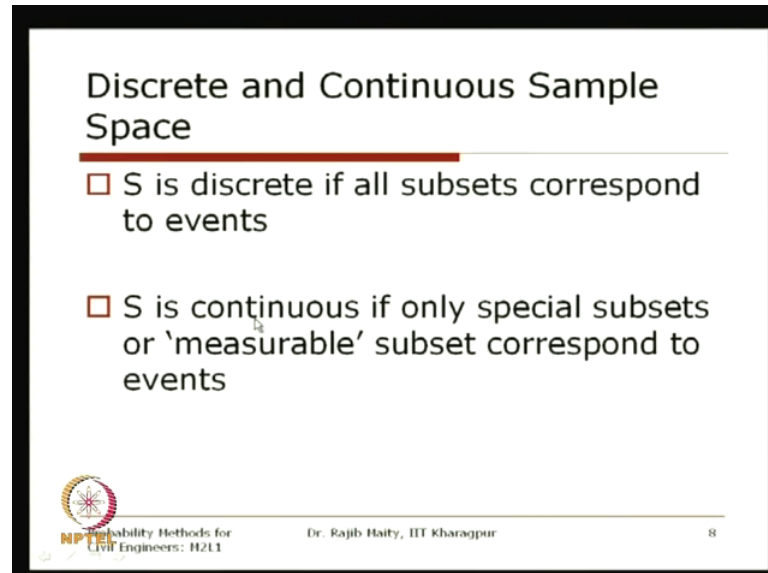
Second thing just after, we know what is sample space it comes as event and an event is a subset of the sample space or in other words a subset of all possible outcomes. So, I know what is my sample space, if I want to show it here the black circle that the black or the or there d area. Here is your sample space and in this sample space, if I select any combination of the sample points then each combination or each at each feasible set is feasible subset of this of this sample points is one in one event there are other conversion like elementary event.

It is very simple, and the sense that if a particular sample consists of only one possible outcome then that is known as simple event or the elementary event. If I give the example again of this throwing a dice here, so a so here the sample space as I told just in the previous slide that the sample space is any integer number starting from 1 to 6 its event can be anything. If I say that 1 2 3, so this is one event, I can say that the number is less than 4, so this is one event. I can say the number is exactly equal to 4 then that is also one event.

So number is exactly equal to 4 that means there is only one possible outcome, and if I say the outcome is one even not number then there are 3; 3 possibilities 1, 4, and 6. Similarly, if I say the outcome is less than 5, so these are all event and one particular event may have one or more than one possible sample points, so concept of this sample


space and event is very important that we will that will help us to understand the concept of probabilities.

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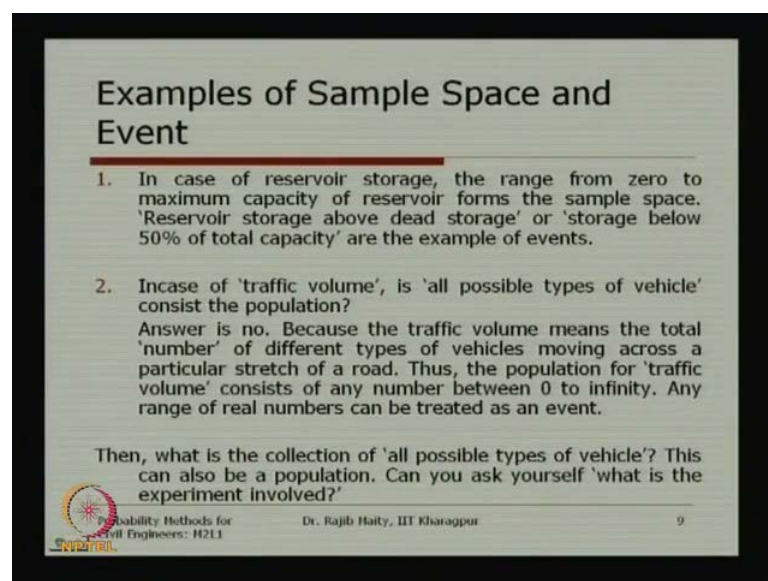
**Discrete and Continuous Sample Space**

- S is discrete if all subsets correspond to events
  
- S is continuous if only special subsets or 'measurable' subset correspond to events

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Another thing and an important thing to know here is the discrete and continuous sample space, so discrete and continuous sample space can be expressed as follows S, which is which denotes the space that S is discrete if all subsets corresponds to events on the other hand S is continuous, if only special subsets or measurable subset corresponds to events.


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**Examples of Sample Space and Event**

1. In case of reservoir storage, the range from zero to maximum capacity of reservoir forms the sample space. 'Reservoir storage above dead storage' or 'storage below 50% of total capacity' are the example of events.
  
2. Incase of 'traffic volume', is 'all possible types of vehicle' consist the population?  
Answer is no. Because the traffic volume means the total 'number' of different types of vehicles moving across a particular stretch of a road. Thus, the population for 'traffic volume' consists of any number between 0 to infinity. Any range of real numbers can be treated as an event.

Then, what is the collection of 'all possible types of vehicle'? This can also be a population. Can you ask yourself 'what is the experiment involved?'

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Coming to the civil engineering here, so the concept of the sample space event and the probability of their particular thing, this seems to be clear first to understand the probability associated with particular event. If I give one example say in case of reservoir storage the range from 0 to maximum capacity of the reservoir forms the sample space, and this is obviously to any value can be taken from this 0 to this maximum capacity say some maximum capacity. If I say it is capital C so 0 to c any value is possible, and this full range is our sample space.

Now, reservoir storage above the dead storage or reservoir storage below the 50 percent of the total capacity are the example of events, so these are for example, the reservoir storage above the dead storage is a particular subset of the total feasible total possible sample space. So, these particular thing is one event, so if I know this is a event generally the probability are given to this particular event. So, I can ask what is the probability that the reservoir storage is above dead storage or I can ask what is the probability the storage is below the 50 percent of the total capacity of the reservoir.

Similarly, suppose if you take the example of another random experiment that is the traffic volume in this transporters and engineering this is mostly used, so in case of traffic volume if I ask that is all possible types of vehicle consist the population that answer is no, because the traffic volume means the total number of different types of vehicle moving across a particular stretch of the road. So, this is the number that I want stretch here, so that number consists of by definition that number is your traffic volume. So, here the population or here the total sample space for the traffic volume consist of any number between 0 to infinity and any range of this real numbers.

So, again here this thing is one example of the discrete sample space, because these numbers cannot be any real number this should be an integer number, so these sample space is a discrete sample space, and any range of real numbers can be treated as an event. So, I can say what is the probability of the traffic volume less than 10 or what is the probability that the traffic volume is greater than 100, this kind of thing, so any subset of this 0 to infinity and it must be a real number and that consist of the sample space for this experiment.

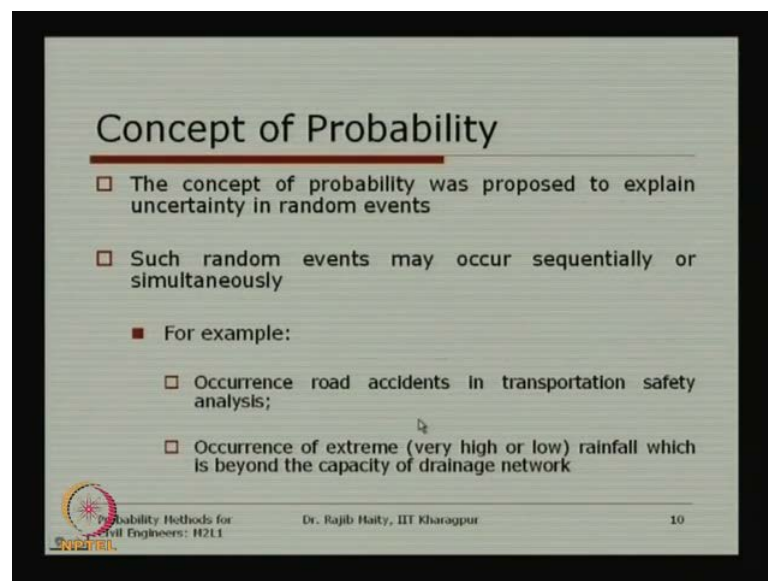
Coming at to this particular thing or before that if, so I hope you understood that in this experiments the reservoir storage this sample. Space is a continuous, sample space. So



the first example is the example of one continuous sample space, and second one is one example of discrete sample space coming to again this particular question that so far in case of this traffic volume. All possible types of vehicle are not, that is not the population of this one. Now, if I ask you that what is then, what is this means? What is the collection of all possible types of vehicle is this not can this not be a population? The answer is **yes**, this can also be a population, so you can ask yourself what is the experiment involved for this if this is the population. We will come back to this particular question again and this is very interesting and this is very important concept at the starting of the understanding of random variable, and which we will cover may be in the successive after some lectures that this random variable the concept of random variable comes, that it does not mean that random variable is a variable which is which is random. This is a completely wrong definition of this random variable and when we discuss about this definition of random variable we will come back to this particular question once again, and we will say that what is that particular what should be the exact definition of this random variable.

Now, we will try whatever we have understood so far about it about the sample space and its event we will try to tell them, we will try to understand the concept of the probability here.

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**Concept of Probability**

- The concept of probability was proposed to explain uncertainty in random events
- Such random events may occur sequentially or simultaneously
  - For example:
    - Occurrence road accidents in transportation safety analysis;
    - Occurrence of extreme (very high or low) rainfall which is beyond the capacity of drainage network

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The concept of probability was proposed to explain the uncertainty in the random events as I told that the outcome of a random experiment cannot be a curtailed. So, I have to give some numbers some belief, and I have to assess that particular belief I have to assess that particular number, and this concept of probability is generally develop to give to give some explanation to that uncertainty in the random events.

Such random events may occur sequentially or simultaneously as we told in just previous few previous lectures few previous slide that this any random event, which is this can be either consist of the sequentially or this can be in a in a simultaneous fashion. Now, in the sequentially we will a later part of this course, we will also understand this sequence can be either of in the temporary direction or in the in the special direction. That is known as the special temporary thing so any particular random variable, we can observe it in the time sequence or in the special sequence and there are a different types of analysis that we can prove and we will take you through all this all this things.

So, for the time being we should understand that this random events can happen either sequentially or simultaneously for example, here the simple example that I can say now is occurrence of road accident in transport, and safety analysis or occurrence of extreme. That is very high or very low rainfall which is in case of very high of course, it is that which is beyond the capacity of the drainage network or occurrence of the very low rainfall which may affect the agricultural production or so.

So, this kind of occurrence this kind of random events is very important for our civil engineering purpose, and we will know that what are the way that we can we can assess the probability, and what are the way different methods of the probability can be applied to this particular to this concept of the probability can be used in that kind of analysis.

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**Concept of Probability...contd.**

- Generally for particular system, occurrence of such events approaches to a constant number. With the increase in number of observations, this number remain constant.
- An obvious example of tossing a coin ('Experiment') and counting the outcome as head ('Event'). An estimation of the percentage of heads approaches to 0.5.

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In this concept of probability again generally for as particular system occurrence of such event approaches to a constant number with the increase in the number of observation this number remain constant; so for example, if I take a particular experiment in hand and I want I just see for a very long record very long historical record, and if I see that what is the chance or what is the frequency of occurring that particular event. We can say that it is a generally, we will classify into that success and failure, and we see that what is the chance of what is the frequency of the success or particular thing in the historical record and this historical record is very important. Here this should be sufficiently long to assess some probability of this, before coming to this real life civil engineering problem, I can say you once again a simple example to this example of tossing a coin again.

So here the tossing a coin and observing its outcome is your experiment, and counting the outcome as head say for a particular outcome head or it can be tail, so this is a particular event. So, if I say that the or tossing a coin can observing the head in this experiment, if the head comes then it is a success and if head does not come that is failure, so that is one event and if I do if I follow this approach that is for what is the occurrence of a such particular event approach to a constant number, and by our get to the experiment. I can say we can say that this number for this particular experiment will approaches will approach to 0.5, so these 0.5 generally we assign this particular number generally. We say this is the probability of getting head of one experiment having a

point. We will come to this real life problem of this cyclone event or this thing a little bit later.

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**Interpretation of Probability**

Based on Relative Frequency

□ If an experiment is performed  $n$  times and an event  $A$  occurs  $n_A$  times, then with a high degree of certainty, the relative frequency  $n_A/n$  of the probability of occurrence of  $A$ ,  $P(A)$ , is close to (Papoulis and Pillai, 2002):

$$P(A) \cong \frac{n_A}{n}$$

Provided that  $n$  is sufficiently large, i.e.,

$$P(A) = \lim_{n \rightarrow \infty} \frac{n_A}{n}$$

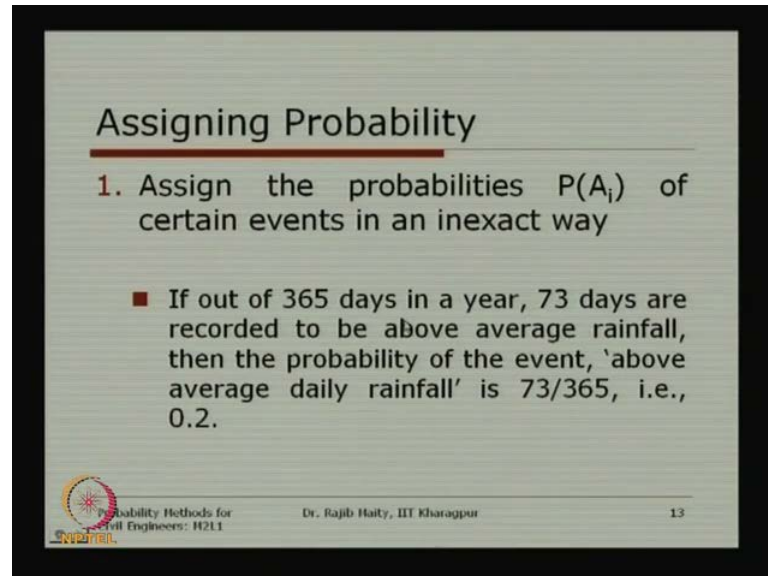
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So, that it will be clear in the in the area of civil engineering problem also before that we should we should know the classification, we should know the definition the classical definition of probability as I told that we have we should have a very long record have known with us or if we are conducting the particular experiment. We should conduct the experiment for a very long time, so that the number will approach to a particular constant value here based on the relative frequency. If an experiment is performed  $n$  times and an event  $A$  occurs  $n_A$  times then with a high degree of certainty this all this chances are very important, so with a high degree of certainty the relative frequency  $n_A$  by  $n$  of the probability of occurrence of a which is designated as  $P$  in bracket  $A$  is close to  $n_A$  by  $n$ .

So, that probability of occurrence  $a$  is the ratio of  $n_A$  by  $n$  here you can see that this  $n_A$  is nothing but the success the number of success, and this is the total number of time, so as I told in that this should be sufficiently long this should be a this should be basically this should be the infinite number of series or sequence should be tested to approach to that particular number. So, this definition, so if I want to know exactly what is this probability with the based on this relative frequency concept, then this  $n$  should be sufficiently large and that is why we write the probability of  $A$  is equals to limit  $n$  tends to infinity  $n_A$  by  $n$ , so this is this definition is generally based on this relative frequency

concept there are other definitions of the probability are also there we will probably will see it shortly.

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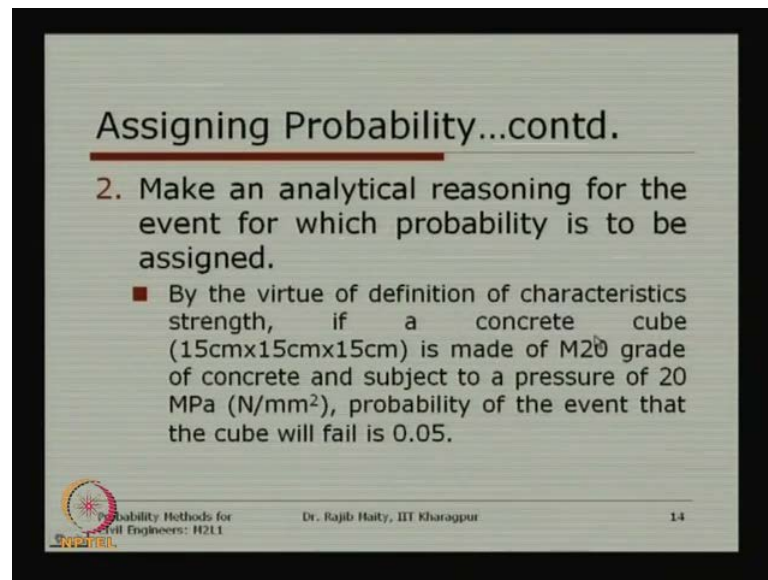
**Assigning Probability**

1. Assign the probabilities  $P(A_i)$  of certain events in an inexact way
  - If out of 365 days in a year, 73 days are recorded to be above average rainfall, then the probability of the event, 'above average daily rainfall' is  $73/365$ , i.e., 0.2.

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Before that to assign the probability of a particular event there are there are 3 different ways that we can assign the a particular probability for a particular event say for example, one of the event is  $A_i$  to assign the probability  $P(A_i)$  of a certain event in a in exact way suppose this way of this one this way of assigning the probability. I have to see the historical record of a particular experiment, and based on the historical properties based on the historical success rate of that particular experiment. I have to assess some number so as this historical record is never infinite series. That is why we say it as the inexact way here one example is that if out of 365 days in a year, there are 73 days are recoded to have to be above average rainfall, then we say the probability of the event above average daily rainfall is 73 by 365. So, this 73 for this experiment is the success, and the total number of trial total number of days total number of experiment is your 365, so this is an inexact way of assigning the probability to a particular event a.

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**Assigning Probability...contd.**

2. Make an analytical reasoning for the event for which probability is to be assigned.
  - By the virtue of definition of characteristics strength, if a concrete cube (15cmx15cmx15cm) is made of M20 grade of concrete and subject to a pressure of 20 MPa (N/mm<sup>2</sup>), probability of the event that the cube will fail is 0.05.

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The second way that is to make an analytical reasoning for a for the event, which is for which probability is to be assigned? So, this analytical reasoning again is one of the most important thing to understand a with respect to the different already established you already established concept. That is I generally follow some norms and to develop some particular say particular material in civil engineering or say I have some particular belief in me. So, in this way if I know that the way the definition is given for a particular for a particular attribute. I should say and if I know what was the background to give that give that particular event then I can assess that that probability given related to the that particular event here.

One example is given here that if you already know the basics of the civil engineering, then I hope you know that m 20 grades of concrete. If you do not know this is a particular grade, there are different grades of the concretes are available say m 15 M 20 m 25 and 25 m 0 35 40. It can have so different grades of the concrete is there the by definition of these particular grade of concrete, it says that this is a characteristics strength and it is defined as if a concrete cube and that concrete cube dimension is 15 centimeter by 15 centimeter by 15 centimeter, and if we say that this is made of M 20 grade of concrete and this cube is subject to a pressure of 20 mega Pascal Newton per millimeter square. Then the probability of the event that a cube will fail is 0.05, now where from we get this 0.05 is that is line in the definition of the characteristics strength.

So, this characteristics strength of the definition characteristics strength says that what is the specimen? What is the strength at which I can assure that at least the failure that; that failure of that particular concrete cube will not exceed 5 percent, so that is why so if I take a m 20 grade concrete and the and the pressure is 20 m P A. So that the probability that the event the cube will fail is point naught 5. So this is one analytical reasoning and example of this analytical reasoning of assigning probability to this particular event.

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**Assigning Probability...contd.**

3. Assume that the probabilities follow certain axioms and then by deductive approach determine the probabilities of an event using the probability of other events.

- The probability that a testing device will be rated as very poor, poor, average, satisfactory and excellent are 0.05, 0.15, 0.68, 0.1 and 0.02 respectively, then the probability of the same device will be rated as above average will be 0.12.

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Then the third one is that a kind of deductive approach assume that the probability follows certain axioms will come to this one the axioms of probability, and then by deductive approach determine the probability of an event using the probability of other events.

So, there are different kinds of events is available and somehow say you assume for the time being that somehow I know the probability of a particular event, now using this information that I know the probability of this particular event is this then read the questions come at what is the probability or some other particular event, then I should be able to assess. I should be able to compute the probability of this particular event using the information of the probability of the available information of the probability or some other events. So, this is a deductive approach suppose that here, because this will be more clear when we cover this axioms of probability, but before I cover this one this is a very easy example that must be it should be able to understand is the probability. That a



testing device there are very there are different types of testing device available in civil engineering suppose that testing device is a company has given it to the market, and based on the customer satisfaction that customer will categorize that particular testing device as very poor average satisfactory and excellent, so based on somehow from the market study. Some other way the company knows that a particular testing device to be categorized as very poor the probability of this to be categorized as very poor is 0.05 to be categorized as 4 is 0.15 average 0.6 eight satisfactory 0.1, and excellent point naught two then that if that question is that, what is the probability? That the same device will be rated as above average, so if I say it is above average that means it should be either satisfactory or excellent.

Now, there are some of assumption background to this that a particular customer cannot categorize is at both satisfactory, and excellent if we categorize the satisfactory. So, he is having only one choice, so these two particularly these two events of categorizing as satisfactory or excellent is mutually exclusive again the definition of mutually exclusive is says that the occurrence of a particular event ensures that non occurrence of the another event. So for example, that tossing of a coin, if I say that outcome is head then it is automatically implies that the tail has not occur, so in other words the head and tail both the outcome cannot happen simultaneously this kind of events are known as mutually exclusive, and if again if say that this two things are independent.

So, if one customer as categorizes is as respecter, and other one categorized as excellent and these categorization by different customers is independent to each other then I can if I want to know what is the probability? That the particular device is rated as above average this will be simply summation of the probability, however for the satisfactory this above average satisfactory, and excellent that is point one last point two so here in this way of assigning the probability of a particular event. Here the event is above average this kept this rated as above average, this is particular event is based on not based on any particular experiment, this is rather based on the information that is available on the probability of some other event.

So, this is the thing this is a deductive approach that is why this is deductive approach, and it is determined by the probabilities of the other events for base authorities known.

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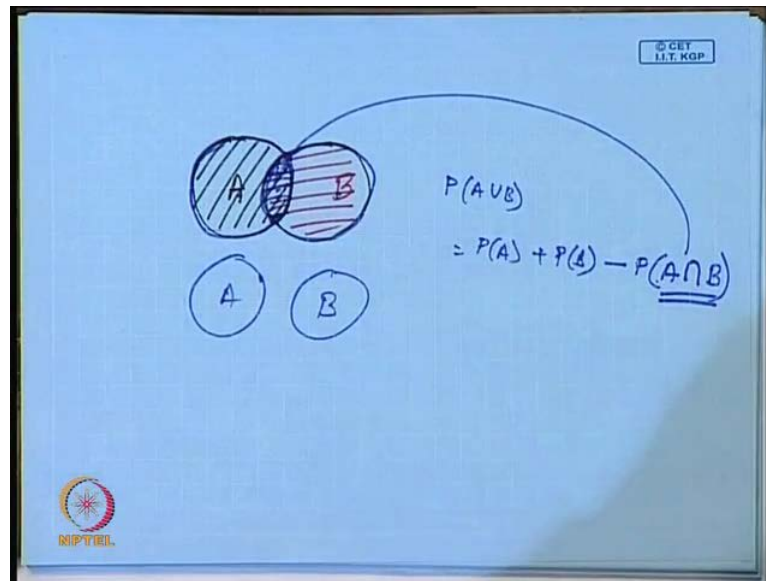
**Axiomatic Definitions**

- If there is a set, S of mutually exclusive (one event exclude occurrence of the other) events,
  - Probability of A, P(A) is a non-negative number:  
 $P(A) \geq 0$
  - Probability of all events in the set, S is unity:  
 $P(S) = 1$
  - Probability of event  $A \cup B$ , is addition of probability of A and probability of B:  
 $P(A \cup B) = P(A) + P(B)$
- Union of two events A and B,  $A \cup B$  defined as an outcome when A or B or both occur simultaneously.

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So, these are the three different ways that we can assign probability to a particular event, so as I have just in the previous slide we discuss about the axioms of probability. So, different type of **different type of** concept of this probability can also have the different way. Just now, we have seen that what is the relative frequency the concept based on the relative frequency, there are other concepts are for example, here the axiomatic definition. So, there are few axioms that is there for a particular probability events, there are basically 3 axioms are there they are probability of A P A is an non negative number, so this P A is always greater than equivalent to 0, this is the first axiom. Second is the probability of all events in the set S unity, so the probability of all possible outcome of a particular event is equals to one, and the third one is the probability of event A union B is the addition of the probability of A and probability of B that is probability A union B equals to probability A plus probability B here, this symbol that is A union B is the union of two events - A and B is defined as an outcome when A or B or both as occurred simultaneously.

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In the concept of the Venn diagram you know that if this is your probability A, and if this is your probability B, so the first circle what I am showing as a black dotted. If this is your and if the red dot that have given, if that is the red settled area if this your B then A union B is the summation of the both. So, this total area is your A union B, so this is the two different events, and so if they are unions of the probability of this total event is nothing but probability of A plus B, but there is a condition here, that this two events are mutually exclusive why, because here if you again see this one. That if I say that this area is your probability A and this area is your probability B, then if I just simply add this one then this area obviously which is common in A and B will be added twice that is why when it is mutually exclusive then I can say suppose, that this is your a, and this is your B, then probability A union B is nothing but probability of A plus probability of B.

Now, based on this 3 axioms that we have discuss with different numbers. We can reduce different numbers of equations where we can use it for the detection of the probability of the others for example, in case of this when two events are overlapping each other then what should be the probability of probability of A union B that you can easily say that probability of A plus probability of B. In this process we have taken this area twice, so this should be minus this area should given as A minus, and that is probability of A. Another new way and tell it is the intersection of B, so probability this symbol this probability A intersection B is nothing but this particular area, where it says that this is A common in both the events. So as we are added in this process, we are

added this area twice, so this should be detected here. So, this is this second one is yours is a these are the 3 axiomatic definition, where we can follow the detective approach you can get the probability of the other events if the probability of the particular events of some other events is known to us.

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**Classical Definition**

- The probability of an event A,  $P(A)$ , is determined, without actual random experiment, as a ratio as follows:

$$P(A) = \frac{N_A}{N}$$

where  $N_A$  is the favourable outcome related to the event A and N is the total possible outcomes.

- It is implicitly assumed that all possible outcomes of an experiment are equally likely.

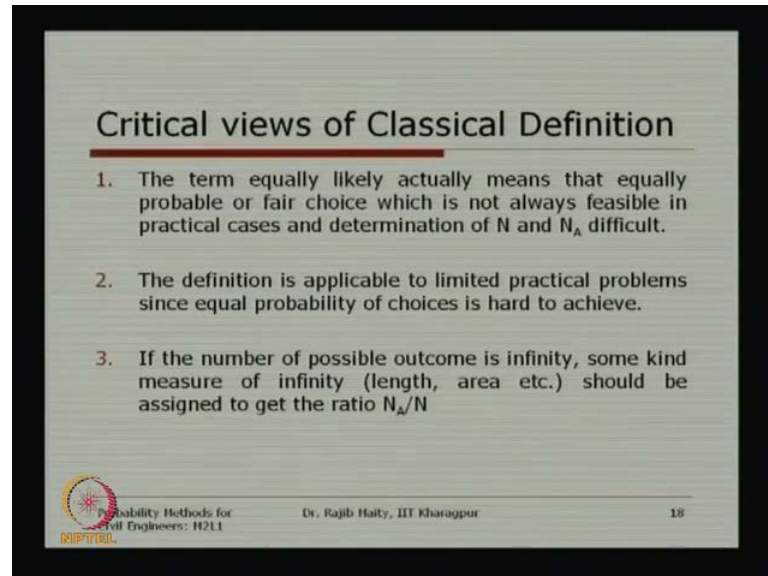
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The third one is the classical definition, the classical definition the probability of an event a probability A is determined without the actual random experiment as a ratio as follows probability A equals to probability  $N_A$  by N, where  $N_A$  is the favorable outcome related to the events, **sorry**; this there is one t to the event A and N is the total possible outcome. That means, we are not exactly following any particular random experiment whatever the observation is available to us based on this whatever the receiver of this success out of the total number of trail, we are assembling this particular number which is the classical definition of this probability. It is implicitly assumed in this event that all possible outcomes of particular experiments are equally likely.

Coming to the throwing of a roll dice example that all possible outcome, that is any integer number from 1 to 6 the outcome should be all this outcomes should be equally likely, so that probability of getting one should be equals to probability of getting two equals to all this numbers should be equals to 1 by 6. So, however day to experience, we generally assign this probability, and these when we say that **yes** all these probabilities

are equal and equal to 1 by 6, then all these events are equally likely basically this is a point where this basic concept of probability is quotient.

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**Critical views of Classical Definition**

1. The term equally likely actually means that equally probable or fair choice which is not always feasible in practical cases and determination of  $N$  and  $N_A$  difficult.
2. The definition is applicable to limited practical problems since equal probability of choices is hard to achieve.
3. If the number of possible outcome is infinity, some kind measure of infinity (length, area etc.) should be assigned to get the ratio  $N_A/N$

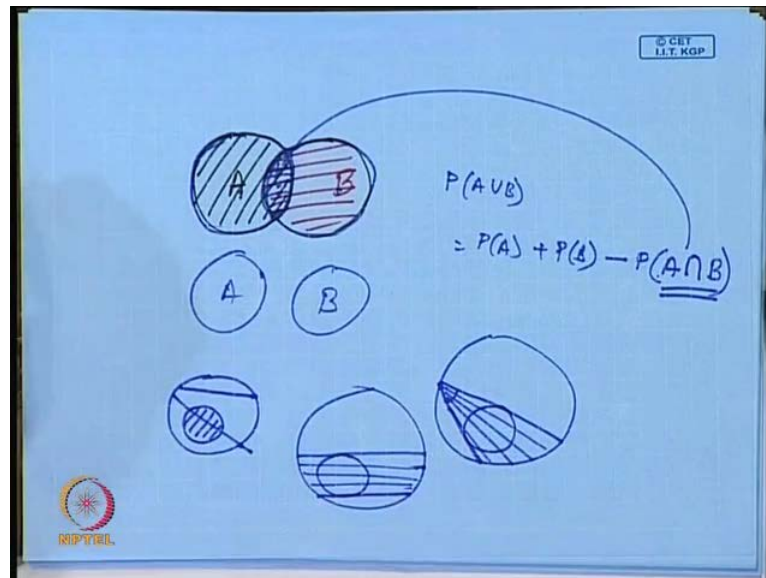
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And sometimes this kind of critical view is thrown to this basic concept of this probability. The first the term equally likely actually means that equally probable or fair choice, which is not always feasible in the practical cases. It is not possible for that for the practical cases we basically having a fair dice of fair, one is impossible even though you go for a very large number of trials, and will how large is large to get that particular probability of a head say 0.5 or for a dice for a particular number is one by 6 is that that whether they are really equally likely that cannot be in sued always. So, this is one this is the first thing that generally face the problem of this classical definition.

Second thing is that the definition is applicable to the limited practical problems, since the equally probably probability of the choice is hard to achieve, so even though there are some experiments that by choice or by the by the experience to say that this all this possible outcome are equally likely. So far those kind of experiments this definition can be applied, but what about there are many other examples where this kind of equally likely events cannot be assumed, so in such cases the applicability of this classical definition generally throws the question here.

And the third thing is that the number of possible outcome is infinity, now if a particular for a particular experiment, if I say the number of possible outcome is infinity, then some kind of measure to the infinity should be assigned to get the ratio of the N.

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Say for example, if I if that example is given like this that in a circle is there and there is another circle inside this and one random event is that there is a chord here, what is the probability? That this chord some part of this chord will be inside, this in a circle as well. So, there are two chords, I can just draw now one is that this is passing completely outside, this in a circular another is passing inside this one. Now, if I want to assess the probability of this kind of experiment, then some kind of what is the total possible outcome should be there, so thin miss experiment I can draw any line here and say that there are these many these many chords are possible.

So, I had this some kind of some kind of assessment of this area - this area if it goes this is outside, and if it goes in this area this is this is outside. So, this can be achieved in different ways, the first thing is that if I know the origin of this to circle; one way is that we can just start for any point, and say that this is one and there is another chord like this. And I should say that this is the angle, which is my favorable case so all chords in these area will pass through this circle and all others are this one, so that this area divided by the total area can give the probability.

Secondary way of looking the same problem is that there is one outer circle, and there is another inner circle as well here. And I can join two particular chord here like this, and say that all possible chords that is in this area are my favorable case, and all other except all other which are falling outside that is the non favorable case. So, this area divided by total area will give this will give the probability there is a classical example is known that these two probability. Some time may not may not match to each other, so this is also one particular drawback of this classical theory, and when we are going to give that favorable and non favorable to non favorable ratio when we are when we are computing the ratio of this favorable case and non favorable case.

So, that definition of this how this experiment is performed whether with respect to the experiment this particular thing is more feasible or this; one is more feasible since to be justified first, and then the probability should be should be calculated. And this is particularly in case of when the possible outcome is infinite that kind of problem may arise.

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**Validity of Classification Definition**

- In application, the assumption of having equally likely outcomes can be established through long experiment size.
- If the occurrence of cyclonic storm is random in the time interval  $(0, T)$ , the probability that it may occur in the interval  $t_1, t_2$  equals to  $(t_2 - t_1)/T$

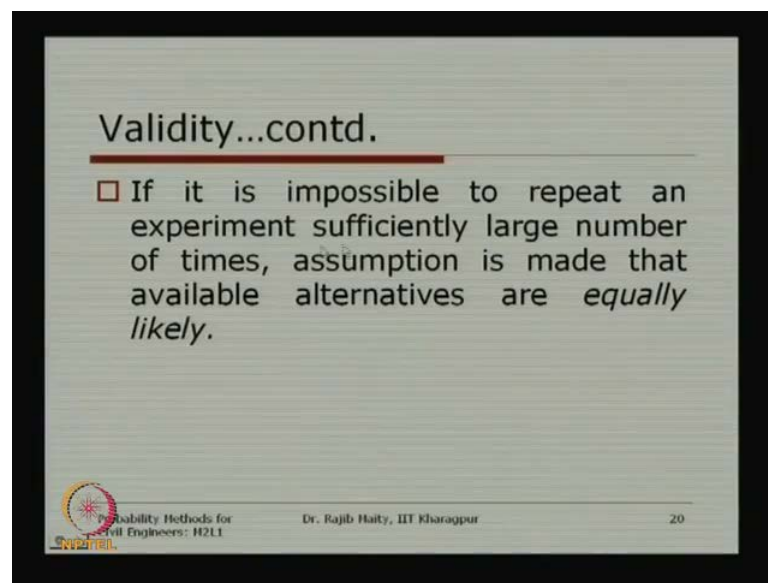
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So, validity of the classification definition in application the assumption of having equally likely outcomes can be established through an long experiment, so an long experiment size; say for example, that if I take an example or occurrence of cyclonic storm is random in the time interval of 0 to t, then the probability that it may occur in the interval  $t_1$  to  $t_2$  is equals to  $t_2$  by  $t_2$  minus  $t_1$  divided by  $t$ , so 0 to  $t$  some time



interval. I am taking and I am saying that the occurrence of this cyclonic storm is random, and I am implicitly assuming that this occurrence is equally likely over any time in this time interval. Then what is the probability that it will obviously this  $t_1$  is greater than 0, and  $t_2$  is less than  $t$  then in this area. Of course is that means that  $t_1$  and  $t_2$  are lying in the range of 0 to  $T$ , then the probability of this cyclonic storm occurring in this time interval is equals to basically this is  $t_2$  minus  $t_1$  divided by total  $t$  capital  $T$  minus 0. So, this is the so this is that, so in this kind of analysis when we are assigning this probability we are implicitly assuming that the outcomes are equally likely.

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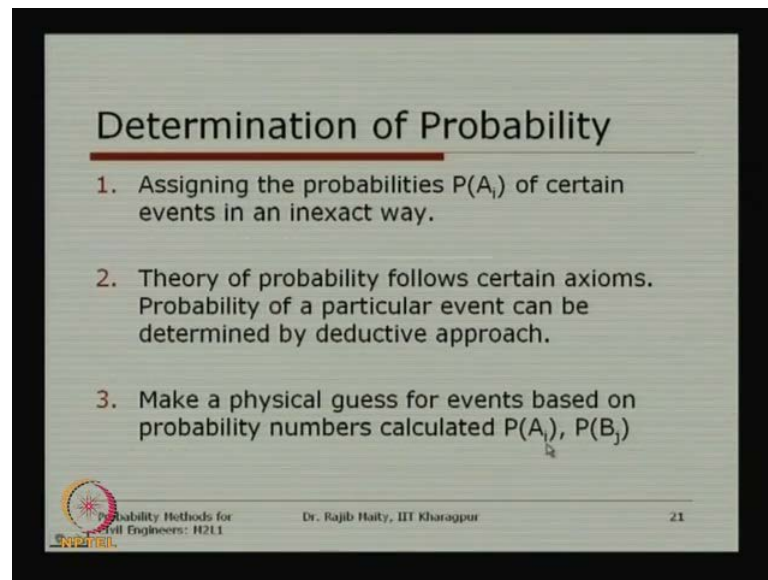
**Validity...contd.**

- If it is impossible to repeat an experiment sufficiently large number of times, assumption is made that available alternatives are *equally likely*.

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If it is impossible to repeat an experiment sufficiently large number of times assumption of assumption is made that the available alternatives are equally likely, this is the basically the start point of the probability theory when this kind of experiments cannot be run in a large number this is implicitly, assume that all possible outcomes are equally likely.

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**Determination of Probability**

1. Assigning the probabilities  $P(A_i)$  of certain events in an inexact way.
2. Theory of probability follows certain axioms. Probability of a particular event can be determined by deductive approach.
3. Make a physical guess for events based on probability numbers calculated  $P(A_i)$ ,  $P(B_j)$

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Now, the determination of probability the assigning the probabilities  $P_i$  of A certain event in an inexact way, second this is we have discuss just whatever the available record that is available with us and in that what is the number of success, and what is the total number that is available that ratio gives you that that probability for the particular event.

Second theory of probability follows certain axioms. So, just now we have seen what are the axioms that that probability follows, so probability of particular event can be determined by the deductive approach, and third one what we have seen that make a physical guess of the event based on the probability numbers and probability numbers calculated  $P_i$  n  $j$  B.

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**Probability and Occurrence**

- If  $P(A)$  is known, the answer to the question of its occurrence in the next trial can be based on size of  $P(A)$ 
  - Answer 1: If  $P(A)$  is low, e.g., 0.3
    - It concludes *only a certain degree of confidence that the event A will occur*
    - It is a measure of belief, cannot be tested experimentally thus subjective
  - Answer 2: If  $P(A)$  is closer to 1 (eg. 0.999)
    - It concludes that with practical certainty, A will occur in next trial

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This probability and occurrence, I think this is needed at the starting of this course to know that what is this probability, and what is this what is the actual occurrence in the real field, suppose that I say a particular experiment say that the rainfall event say that the rainfall event occurs more than what is my what is the capacity of my drainage in network. So, this is a particular event that rainfall amount greater than the which should be our drainage system can take care, and I say that this probability is say 0.0405 now the thing is that if I say that this probability is 0.05. Now the question is that in the next event of the rainfall what is the chance what is the occurrence that that particular rainfall will really exceed.

So, what does this 0.05 what does this number actually give you to inform this is the topic of this particular size, so the if the  $P(A)$  is known the  $P(A)$  means that probability of particular event is known the occurrence to the question is if it is occurrence to the next trial can be based on the size  $P(A)$ . So, the now this size are  $P(A)$  matters nothing here the  $P(A)$  is low say for example, 0.3 it is given, then it concludes that always certain degree of confidence the event a will occur, it does not mean that exactly you perform ten trials. And exactly 3 trials will be your success that is not the case, so in a long terms that is a with certain degree of confidence, so this is very important the certain degree of confidence that is the thing that we can infer from here.

So, it a measure of belief that I am this in generally cannot be experimentally a tested, so this is subjected thing of the measure of this belief that lead this degree of certain confidence that this a particular event a will occur.

Second thing if that if the probability of a probability is living close to one; that means say for example, 0.999 it conclude that with the practical certainty a will occur in the next trial. So if very high probability, then we generally say that with the practical certainty this will definitely occur when the probability is very high, so the probability when it is very low, it is very unlikely to occur, and when it is very high. Generally we say that it is likely to occur, but it can it never say that exactly if you consider, if you consider some n numbers of trial, and it will happen exactly in the probability number multiplied by this total number of trial, it may not occur in that exact number way. So, the probability and occurrence and occurrence of particular events should be understood in it should be assessed in this particular way.

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The slide is titled "Randomness vs. Causation" and is divided into two columns. The left column discusses randomness, and the right column discusses causation. At the bottom, there is a concluding statement and footer information.

Randomness	Causation
<input type="checkbox"/> Randomness links with probabilistic system	<input type="checkbox"/> Causation links with deterministic system
<input type="checkbox"/> Randomness is stated with certain errors and certain range of relevant parameters	<input type="checkbox"/> Causation is stated with a high degree of certainty if number of outcomes is large enough

There is no conflict between Randomness and Causation since theories are not laws of nature, both statements are true

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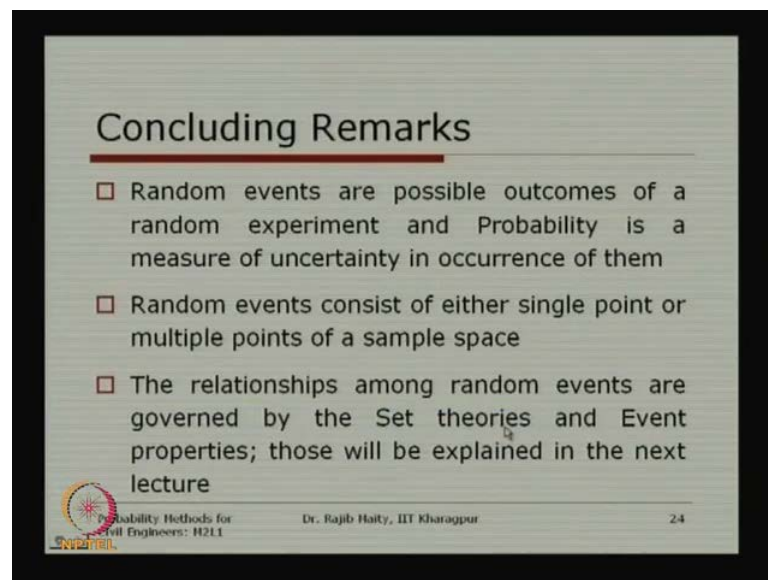
And the last thing, but this is also important the randomness and causation, so this randomness links with the probabilistic say system, and the causation links with the deterministic system. The randomness is stated with certain errors and certain range of this relevant parameters, and causation is stated with a high degree of certainty if the number of the outcomes is large enough, but again basically there is no conflict between the randomness and causation, because the when you say that there are some laws of this

nature, these theories since that it says this since the theories are not the laws of nature both statements are true.

So, even though I say that a particular system is deterministic, that deterministic system may we just in we just for our simplicity sake we have just say that this is a deterministic system, but it will really see whether that particular system is exactly deterministic on a there is a little bit of randomness may be there as well.

So, basically that is why we say the basically the conflict between the randomness, and causation are not are not there not there; these are just basically some concept, and if it is clear then it is generally very useful to that.

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**Concluding Remarks**

- Random events are possible outcomes of a random experiment and Probability is a measure of uncertainty in occurrence of them
- Random events consist of either single point or multiple points of a sample space
- The relationships among random events are governed by the Set theories and Event properties; those will be explained in the next lecture

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The final concluding remarks is this that in this lecture we have seen that random events are possible outcomes of the random experiment, and probability is a measure of the uncertainty in the occurrence of them. Random events consists of either single point or multiple points on a same sample space, the relationships among the random events are governed by the set theory, and the event properties; and this set theory and this event properties will be **will be** explained in the next lecture of this course. Thank you very much.