Probability Methods in Civil Engineering

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Lecture No. # 19

Joint Probability Distribution

Welcome to this lecture, this lecture is basically the first lecture of a new module and this in this module we are dealing with some multiple random variables. So, you know that, so far as whatever the lecture we have covered we have covered the single random variable; we have covered their different properties of the probability distribution and we have discussed some standard distribution that is available and their properties as well.

So, these lecture this module onwards we are starting that multiple random variables this multiple random variable as its name states that, there are more than one random variable is there. So, in many cases where we are dealing with two random variables it is known as bivariate random variable and if it is more than two, then generally we call it as multivariate random variable.

So, this entire module we will discuss about it's about different properties of more than one random variable. Particularly in cases, when we deal with that different module, the different random variables which are associated to each other, this is important in many application field including, civil engineering and we will see that in in many cases that, for the single random experiment it is possible that, there are more than one random variable are associated to each other. So, when we are saying that, these are associated to each other that means that, the thethe they are their behavior are somehow related to each other, somehow associated to each other. So, we have to study study all the random variable associated to each other jointly.

So, now when we are talking about that, this joint behavior of this random variable that time this multiple random variable, the properties and their analysis and all this related issues is comes into picture and that is our focus throughout this entire module. So, in this very first lecture what we will discuss about their first we will start with that joint probability distribution.

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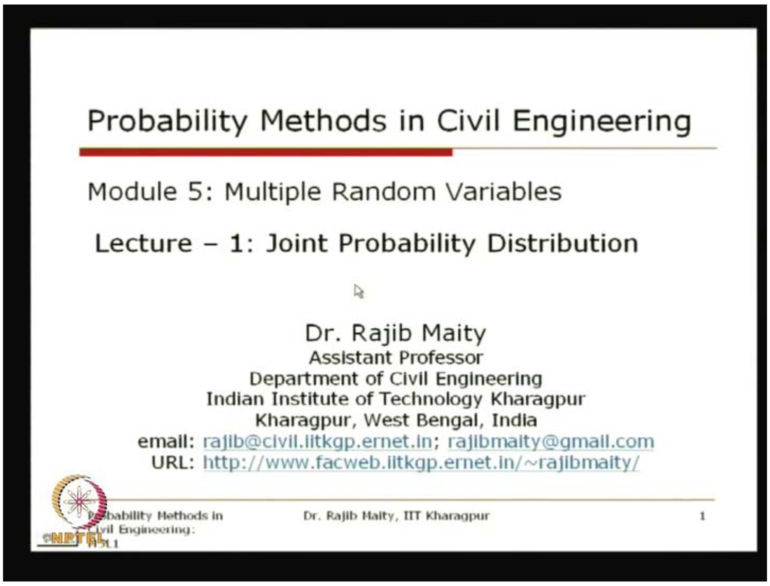

That joint probability distribution here what we mean is that, if one one random variable is associated to one random variable that is one particular outcome is is associated to the other one, then thenhow means how their behavior we can we can we can we can study jointly with the behavior of the adjoining random variable or the or the or the associated random variable here. So, here also that line of discussion we will we will maintain the same or in similar path that we discus for this single random variable that, first of all we will discuss about its definition, how it is, how it is different or how it can be how it can be related to that concept of this random variable. We have discussed the concept of the random variable for a single random variable earlier. Now, we will see that, how the same concept or how the concept differs when we are talking about more than one random variable that concept we will discuss first, then we will discuss about is definition and first significant thing, that we will see is that, how the probability is distributed, because when we discuss about that its single random variable then, we have told that we have we have seen that over entire range of this random variable, how the probability is distributed.

Now, in these case what we will what we will start that, if there are more than one then, how the distribution will look like and how will be their properties. So, this we will take up in today’s lecture.

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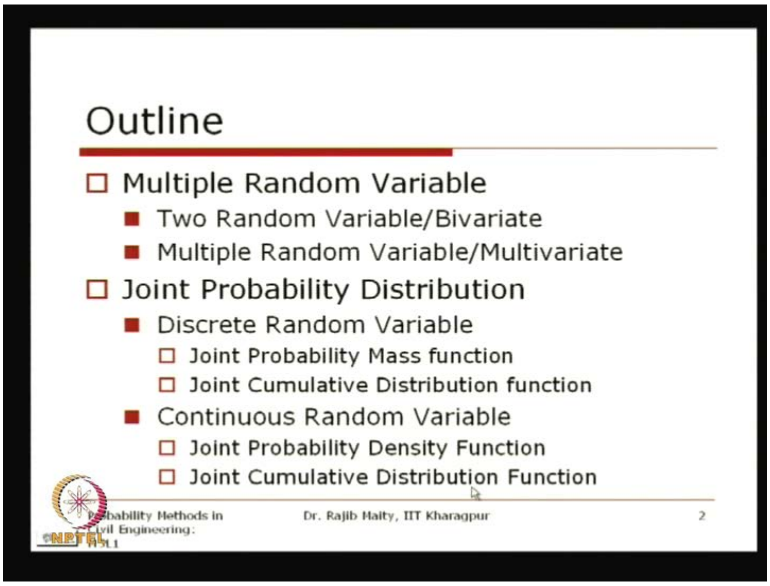

So, the outline of today's lecture is that first, we will discuss about this this multiple random variable and as I have told that, if there are two random variables which is which is very common and many cases we have, we generally deal with this two random variables. So, that is that specially, we know this two random variable as the bivariate random variable this by means you know that, there are two random variables are associated. And sometimes that more than two we call that multivariate random variables, so multiple random variables this is multi multivariate analysis, it will go under that.

So, after this concept, we will discuss about their joint probability distribution and this joint probability distribution again similar to the single random variable; here also, there can be two possibilities; one is that, discrete random variable and you know, for a single random variable this discrete random variable when you are talking about the discrete case that time what we did is that, some specific value over the range only some specific value is possible for that that random variable can take.

So, here also means when we are talking about two or more, so a specific combination of three or two random variable outcome is possible, not the entire range, so that time this will be consider as a discrete random variable. So for as, that the concentration of the mass, this mass means the mass of this probability you know that for the discrete random variable we call it as a probability mass function. So, here also those mass will be concentrated at some particular point, in case of this discrete discrete random variable, discrete multiple random variable as well.

So, this is for the forfor the discrete and then, from this mass function we will also discuss the concept of the cumulative distribution function, how we can integrate that mass function to get that joint cumulative distribution distribution function. And these two things, you know for the continuous random variable we generally call it as joint probability density function that is joint PDF. And similarly, the joint CDF cumulative distribution function.

Basically, here the concept is important in the when we are talking about more than one that time the concept in the sense that, when we are doing some integration and when we are referring to some particular particular value of the probability then, what should be the related graphical representation that is important and you know for the graphical representation, we can only define we can only show only upto the two random variable. So, the if the concept is clear for the two random variable that is bivariate case, then the similar concept can be extended to the more than two random variables as well, so that concept will basically or the pictorially we will try to understand for the for the bivariate case first.

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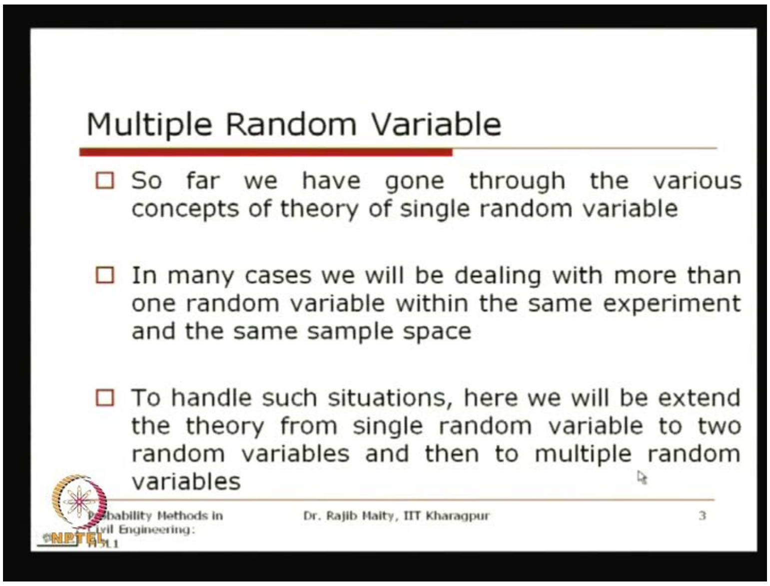

So, as I told that so far, we have gone through the various concept or theory of single random variable and in many cases, we will be dealing with more than one random variable within the same same experiment and the same sample space. So, if I before I go to this civil engineering problem, if I want to give a give an example of their throwing of a dice then, what I can say is that, instead of taking one dice I can take two dice and through them simultaneously, and I can observe that what is the outcome of each dice.

So, at for for a single random experiment, there are two possible outcome is there or that if I take the two coins and toss them together then, the outcome is a is a bivariate random variable. So, I can relate that outcome to a bivariate random variable, because you know that this outcome of a random experiment is some is related to the real line, some outcome on the real line through the through a functional relationship and that function indeed is the random variable. So, the traditional examples are this, so the experiment experiment is such that, that there are two two outcome or the more than one outcome is feasible in that experiment. So, those outcomes are generally considered in case of this multiple random variables.

So similarly, that to handle this situation this situation means, where they are more than two possible outcomes is there. So, we will extend the theory from the single random variable to two random variable first, this bivariate case as I told that this is the pictorial representation is possible only upto two random variables and then, the same concept in the similar way can be extended to the multiple random variables as well.

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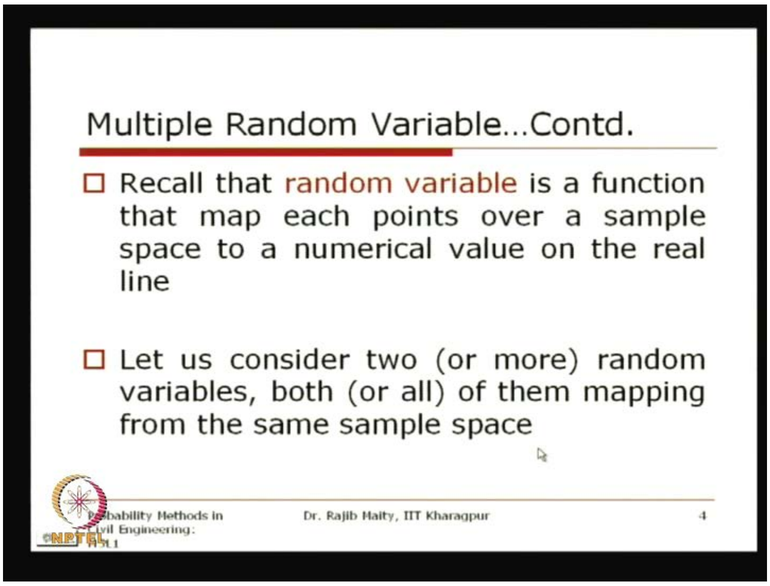

So, now if we recall from our earlier lecture that, the random variable is a function that map each points over a sample space; each points here you mean is that, each outcome of one random experiment to a numerical value on the real line. And this function means this functional correspondence between this outcome to this real line value is youris your random variable what we have seen.

Now, in case of if you consider that two or even more random variables; that means, two or more outcome we are dealing with together then, then both of both or all of them, if it is more than two then both of them mapping from the same sample space, from the sample space to that to that real line to some to some numbers.

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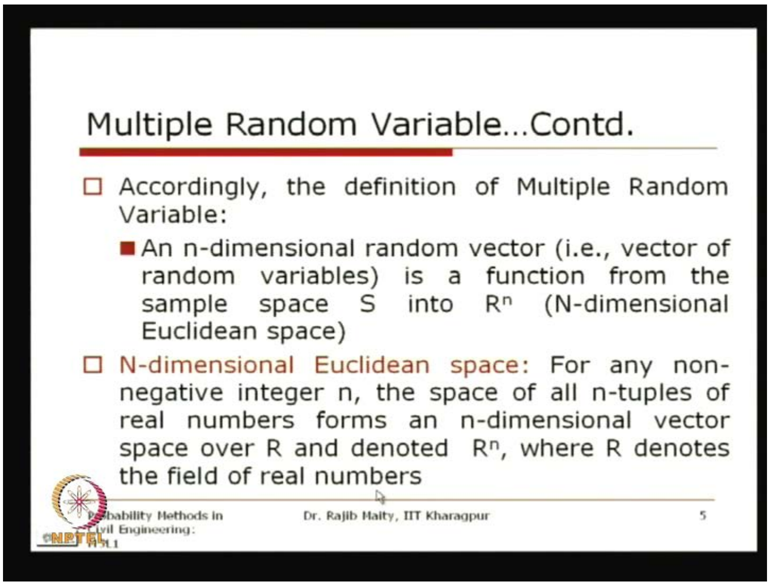

So, accordingly the definition of multiple random variable can be stated as, an n-dimensional random vector that is, the vector of random variables is a function from the sample space S into R n, where this R n is denoting that, n-dimensional Euclidean space. Now, as I told that it will be easier for us to first conceptualize it, in case of the two dimensional here two dimensional case.

So here, if we just want to say that, bivariate case first then, it is a it is a two dimensional random vector having two random variables in it, is this is a function from the sample space S into ainto a two dimensional Euclidean space. So, here we will show that, it is how the pictorially it it looks like in a minute. So, what is meant here is this is this N-dimensional Euclidean space is that the for any non-negative integer n, the space of all n-tuples of the real number forms an n-dimensional vector space over R and denote is as R n, where this R denotes the field of the real numbers.

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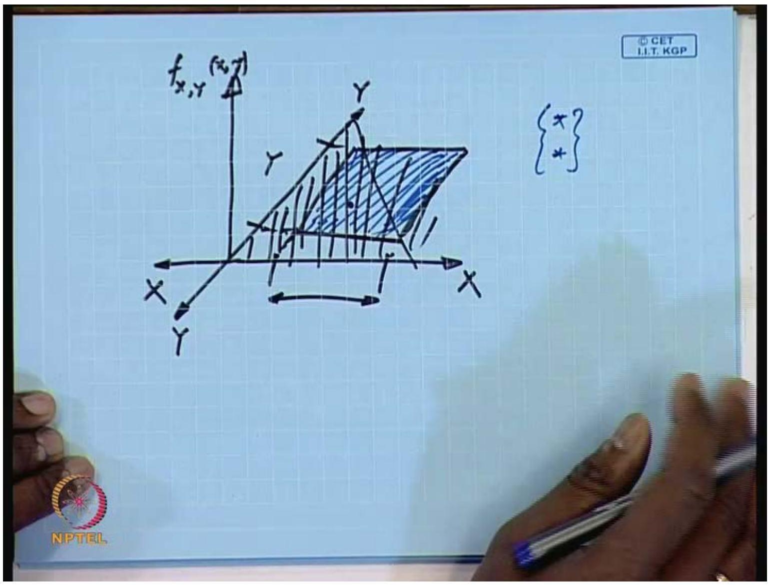

So, what we are just if we look at here on this that means what we are basically looking for is, some two axes we are just looking for. So, earlier this was the axis that we considered here that is for a single random single random variable x and we have absorbed that, how the probability is distributed over this.

Now, what we are doing here or what we are trying to explain here is that, there is more than one random variable and for the pictorial representation we are considering only two random variables. So, there is another axis, where the other random variable which is associated is represented, say that other random variable is y. Now so this is the, so this is your that this is your that surface or the sorry or the plane over which I am so some this, some area is youris your feasible space and over this, I will I will describe how this probabilities is explain.

So, this is your the axis of this, either whether it is a p the p d f or p m f or this can be even the cumulative distribution function. So, now the possible suppose the possible range of x is from here to here. So, we know that, this is called the support of the random variable and say that, this is the this is the range for this y or a support of this y then, basically this joint distribution is defined over joint distribution is defined over this area.

So, this is the area now what we are what we are talking about is that, show this is the area where that that probability is distributed (Refer Slide Time: 14:03). So, in case of the single random variable, we have seen that this the probability distribution what we are considering over aover a line over one axis and here in case of the bivariate case, this is over and over a two dimensional two dimensional space, so over this space how this probability is there. So, that that distribution will now look like a surface over this over this area, so this is what and in and similarly, what you can do is that you can extend this idea to the three dimensional or even that more than three random variables case.

So, those are accordingly that, over which the probability is explained that is defines. So now, so here we just used one word n-tuples, so incase of the bivariate case, there are only two only two possible things are areare associated, because this one that two points will that, x equals to some value and y equals to some value we will refer to one point here. Now, this is for the three random variables together, then there will be three such value and this will refer to a point over aover a space in this area and in that point, how much, what is the probability, how the probability distributed that can be that can be analyzed. So, this is what is meant here in case of this n-dimensional n dimensional random vector and n-dimensional Euclidean space is referred to.

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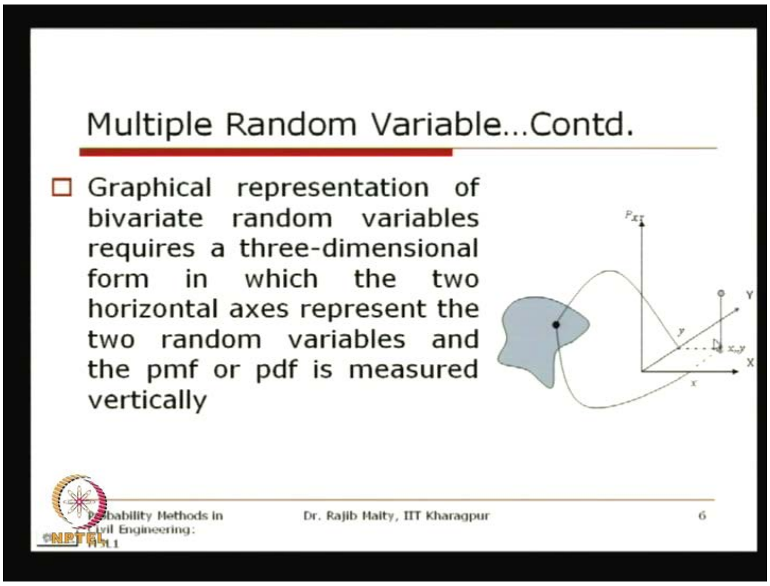

Now here, as I was just showing here in this in this case is that, this graphical representation as I told that, this is for the bivariate random variables case requires a three-dimensional form in which the two horizontal axes represent the two random variables and the p m f or p d f is measured vertically. So, here you can see that, so this is my sample, this is my sample space of this, which consist of this all possible outcome of this one random experiment. Now, each outcome of the random experiment now it can be is there is that random variable that is the y is the random variable, which is a function which is taking that outcome to a point over the over some point over the range of this y. Similarly, the same outcome is also referring to another point over the random variable x.

Now, these combinations for this bivariate case, there are only two are two values; the x and y. So, this x and y refers to a point over this x y plane. Now, at this point now if we have the way it is shown is that, it is ait is a p m f that is probability mass function. So, at this point how much probability is concentrated in this in this in this point, so that is what is explain is as the probability mass for the outcome for the specific outcome here as this shown for the for the random experiments, for this outcome the probability that is concentrated is here.

So, there are such so over this entire space, there maybe there may be many such outcome and each outcome will correspond to two points; one is on the x and another is y and that will correspond to point over this a plane and that will have some different different probability. So, instead of distributed over a line for single random variable, this is now distributed over a surface over a over a surface again, when we are calling that surface; that means, I am referring to the bivariate case, so over a surface, how it is distributed that is the p d f or p m f for a bivariate case.

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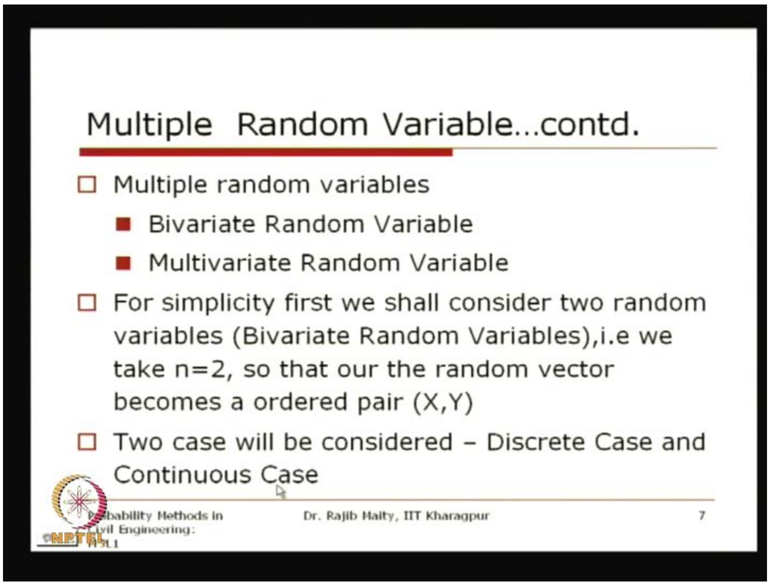

Now, the multiple random variables as I told that, there are that this two random variables is given a special is specially categorize as a bivariate random variable and more than that is all are multivariate random variable. So, for simplicity first we shall consider, the two random variable that is a bivariate random variable that is when we take that n is equals to 2, so that our random vector becomes an ordered pair of this X Y, that what we discuss through that through that pictorial representation.

Now, when you are talking about this again this bivariate random variable, we are when we are taking it, now this also we will be we will discuss incase of, two different cases that is the discrete case and the continuous case; you know the discrete case, it can take only the specific values and the continuous case, it can take any value over the entire support of the random variable.

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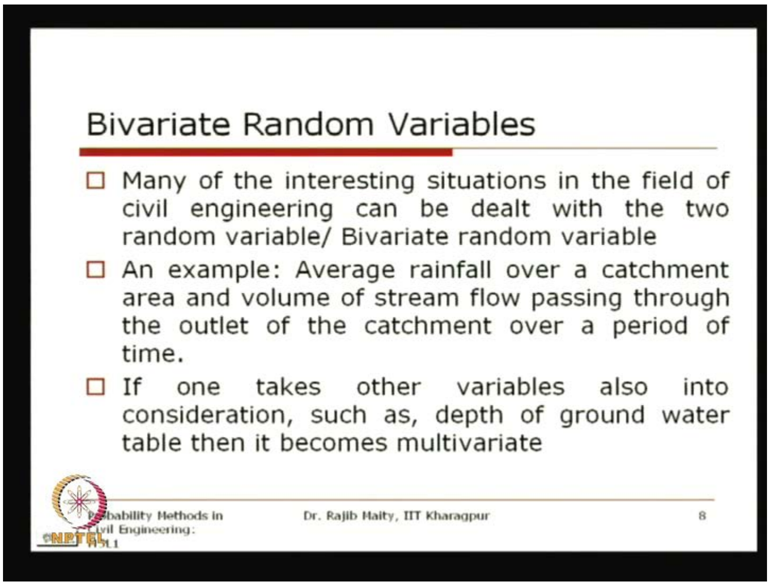

Now, this many of the interesting situation in the field of civil engineering can be dealt with the two random variables. So, these are the bivariate random variable case. So, an example we can we can talk about that, average random sorry average rainfall over a catchment area and the volume of the stream flow passing through the outlet of the catchment over a period of time.

So, now this rainfall what is the rainfall that is occurred over a catchment and if I take that, what is the stream flow for that time that has pass through the outlet. So, this two can this two can have some association to each other and this two can be if we just if we want to analyze this two simultaneously then, we can even we will get a bivariate random variable case, but here one thing is very important that, we should we should mention here, because this should not create the confusion, that whenever we are taking more than one random variable, it is not necessary that, they will always be associated to each other; that means, what I want to mean here is that, that the random variable that we are consider can eventually be independent as well.

So it is not that, always means association does not mean that always there should be some some kind of relationship or some kind of mathematical relationship should exist, these two random variables can even be independent to each other. So, the association when we are meaning, when we are when we are referring to the word association, that independent is also one type of association between the between the random variable.

Coming to the civil engineering problem that, the first example that we have discuss is that, is the rainfall and stream flow from a catchment. And second for and we can even think of the other examples for example, the the traffic volume and average speed of the road. So, average speed at a particular section of the highway if we see that what is the average speed that the vehicle can pass through and what is the traffic volume to that. So, this will also be a bivariate random random variable case.

So in such way, there could be found, there could be many such application can be found out for this different civil engineering problems and here, we will we will discuss about some problems in this lecture as well, and before that some of the properties of this joint probability distribution and there, there is some more that, how it is it can be conceptualize both in case of the discrete as well as in case of the continuous random variable that we will see first.

So, the example that we are talking about is the rainfall over the catchment area and volume of the stream flow that is passing through the outlet of the catchment, this is there are two random variables. So, this is a bivariate random bivariate case and if also in this two, if we are also see the take the ground water tables such as, depth of ground water table also if we consider along with this two random variable then, then it becomes the multivariate multivariate case. So, all three random variables can be analyzed together.

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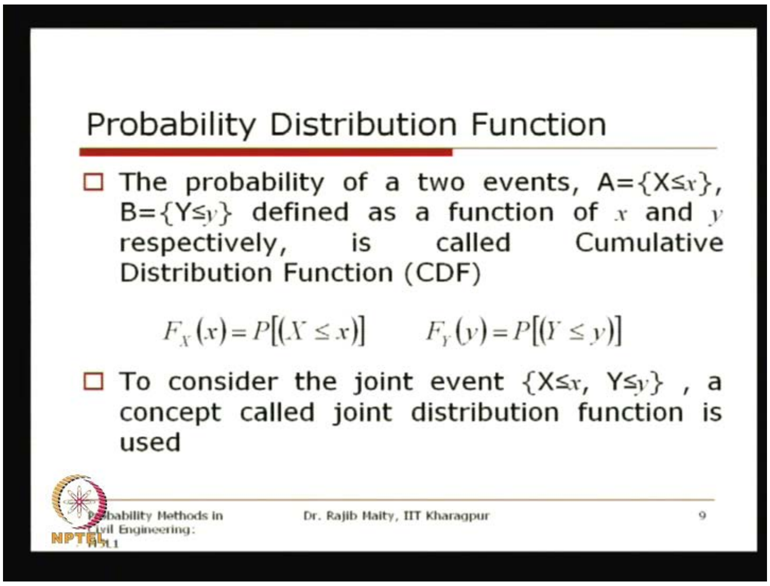

Now, when the probability distribution function when we are talking about you know that, this is also refer to as a as a cumulative distribution function that is, what is the probability when particularly in case of the single random variable you know that, when we are talking about that time, that this probability distribution function or the cumulative distribution function when we are talking about; that means, the how much probability is there is therefor a for the random variable less than equal to a specific value, so that we referred to as this cumulative distribution function or probability distribution function for for that point for that proper point. So, when we are talking about that f x of x; that means, that random variable x is less than equals to the specific value x.

Now, when we are talking about this multivariate case, specially here that bivariate case, so this has to be referred to as that that that both random variables will be less than some some specific values.

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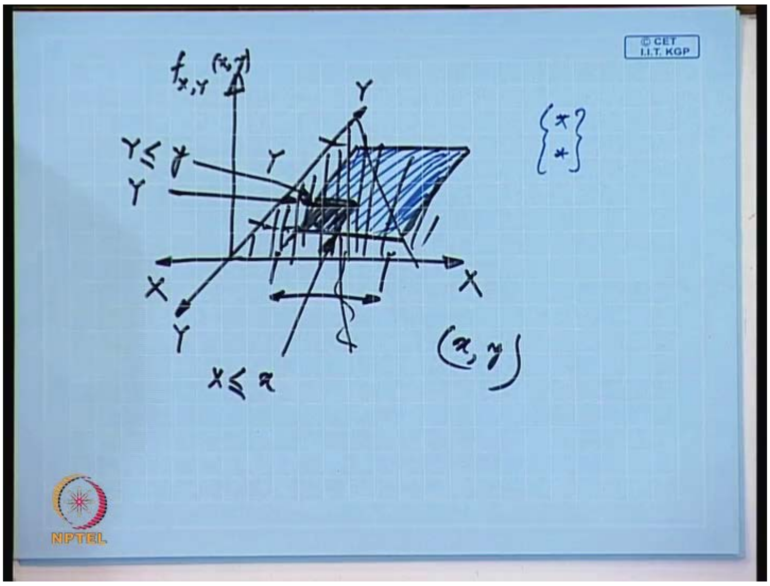

So, the occurrence of both the events should be simultaneous, means what I am talking about is that, if you just see it here that suppose that, this is the this is the specific value of the x that I am talking about is this is the x and if there are some value here that we are talking about is that is that y then, that for the single random variable from the left extreme of the support to this specific point we have we have integrated.

Now, when we we are talking about this bivariate case; that means, both thus cases that this is suppose this specific value y and this is suppose the specific value x that I shown it here. So, now the both the cases so that x should be less than equal to x and this Y should be less than equals to y. So this area if we take so this area if we take if we just integrate what is the total probability in this area, that will give you that the probability distribution upto the point x comma y. So, what I want to trace here is that, occurrence of both the events is important, it is not that either of this two it is that, this and this, this as well as this when both things are satisfied then, is that that is the basis of that defining that that cumulative distribution function or the probability distribution function.

So, now here if you consider the probability of two event, A equals to X less than x and B equals to Y less than y defined as a function of x and y respectively and is called the cumulative distribution function, C D F. cumulative distribution function C D F Now, this is for the single random variable, if we consider them separately. Now, to consider the joint event when we are talking about the joint probability distribution then, this X less than equals to x and Y less than equals to y this concept is called the joint distribution function.

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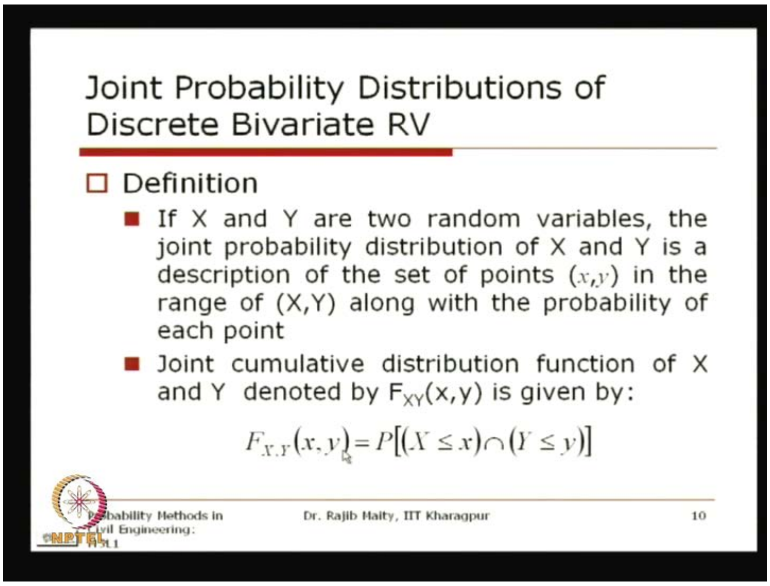

So, how this is done is this one that is, if X and Y are two random variables, the joint probability distribution of X and Y is a description of the set of points x, y in the range of the random variables X and Y along with the probability of each point. The joint cumulative distribution function of X and Y is denoted by that F this capital F X comma Y for some specific value x y is given by that, this this probability distribution is equals to the probability of that X less than equals to the specific value x intersection Y less than equals to the specific value y.

So, here it is referring to that the intersection when we are talking about that is the joint occurrence of both the events is required. So, when this two events are occurring jointly, that probability is equals to the probably that the probability at the point x y.

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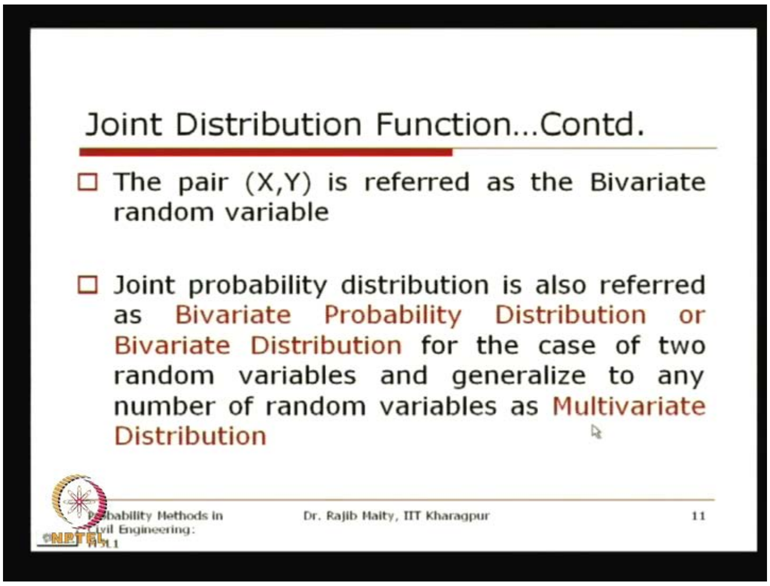

Now, the pair that X, Y is referred to as the bivariate random variable. Joint probability distribution is also referred as bivariate probability distribution or bivariate distribution for the case of this two random variables and generalize to any number of random variables as the multiple as the multivariate distribution. Now, when we are talking from this bivariate distribution that, what we have discussing now, if we just want to if we if we had that more than two random variable then, we can say that, so the simultaneous suppose that there are suppose that there are three random variable X, Y and Z.

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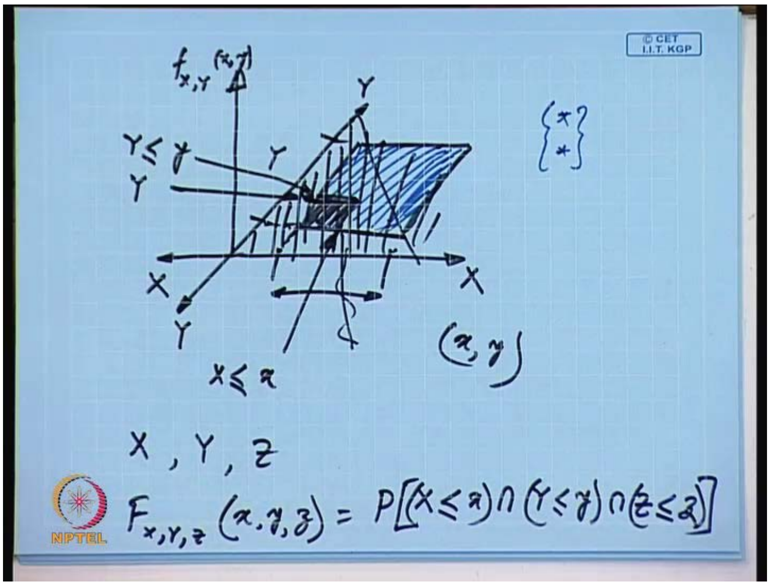

So, that F that is the cumulative distribution of this X, Y, Z of that some specific value of these random variables which are lower case letter x, y, z should be equals to the probability of that X less than equals to x this is one event intersection with that Y less than equals to that specific value intersection with Z less than equals to that specific value of that z. So, this three simultaneous occurrence of this one, that probability is your that probability the distribution of the cumulative distribution function at that particular point x, y, z.

So, here three points are three values are considered, which is referring to a single point in a in a three dimensional space, extending this to that from the two dimensional surface to the three dimensional space that, we are referring to.

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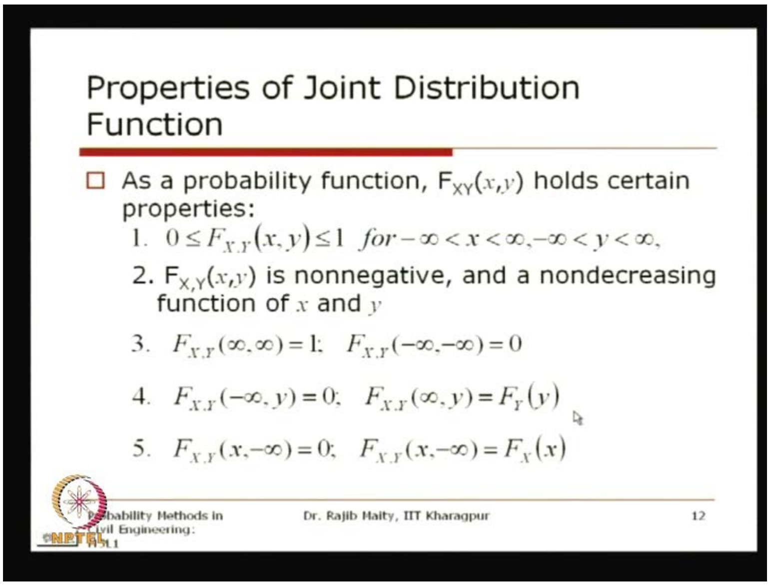

Now this function when we are talking about, then this function should have some properties to satisfy satisfies as we also seen in case of this single random variable. Now, this these distribution are areare limited in the lower value and the upper value, similar to the single random variable, its lower limit is 0 and upper limit is 1 and this x and y cannot can take any possible value of this minus infinity to plus infinity. So, in this. So, these this is are the specific values are over the over the entire range of this real axis, so this can take this values.

Now, this F X and Y is a is a nonnegative, and a nondecreasing function, this nondecreasing function of this x and y is important, because so, that if we get some two values that is, the x 1, x 2 and y 1 and y 2 and if this x 2 and x 2 is greater than x 1 and y 2 is greater than y 1 then, this value so this F value at x 2, y 2 should be greater than that x 1, y 1 for any specific, for any for any case of over this entire range. So, that is what is mean by the nondecreasing function.

You can recall from the description of the single random variable that the p d f, that is sorry that c d f, that is cumulative distribution function is also starting from 0 and going to 1 and that is also a nondecreasing function, non decreasingfunction because you know that, this function actually is obtain by the integration of the p d f that is, probability density function and when we are integrating it from this left left extreme and gradually going towards the right extreme; that means at each, for any small step over this over thisthe over the range of this random variable we are adding up gradually to the previous value so obviously, this will be a nondecreasing function.

Now, at the infinity when both are on the right extreme, this x and y should be should be equals to 1, and when it is on the left extreme this will be equals to 0, when either of them to the left extreme, so this function should be equals to 0 whether x is on the minus infinity or the y is on the minus infinity, this values are 0. When either of this on the right extreme then, it gives new distribution which is F Y y and when that y is equals to their infinity, so this should not be minus infinity, this will be plus infinity.

So, when the y is the plus infinity then, this function reduced to the F x of x. Now, this two again we will basically we will take up this two in detail in the next lecture, which is known as the marginal distribution. So, irrespective of the distribution of one random variable, if we consider the what is the distribution of the remaining random variable, irrespective of the distribution of the other one then, what the distribution we get for a for the one random variable that is the marginal distribution that we called and this we will take in detail in the next lecture.

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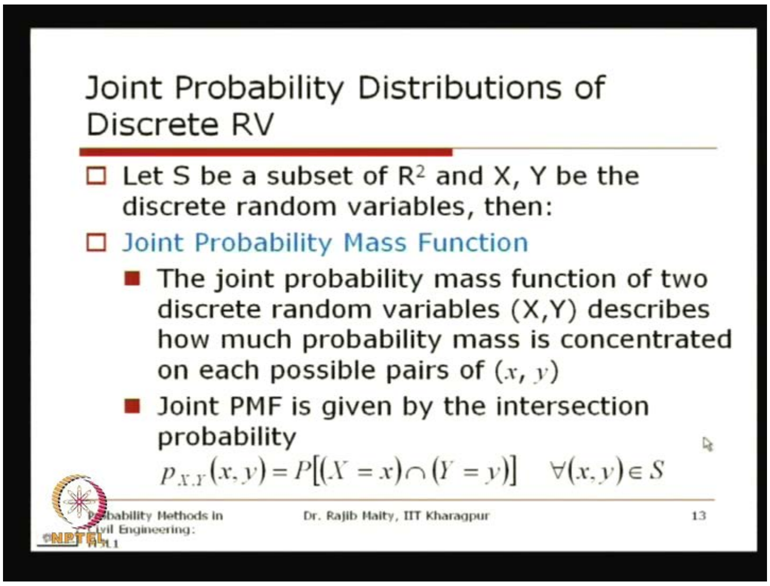

Now, joint probability distribution of the discreterandomvariable discrete random variable, so the discussion that, we have done is the basically for the for the general case of this random variable. Now, if we are talking about the discrete and continuous, there will be there will be a little difference as you know that, discrete can take only the specific some specific values over the range and that continuous random variable can take the any the any possible value over the range can be taken.

So, for the discrete case if that the R square is the is the subset over this that plane X, Y plane then, it will be discrete random variable if it is a discrete random variable then, you know that we generally called it as a mass function. So, here it is joint probability mass function or joint PMF, the joint probability mass function of two discrete random variable X and Y describes how much probability mass is concentrated on each possible pairs of x, y.

So, there are some possible pairs are only possible, so at that point, it is not the density as against to the continuous random variable, this is the probability mass that is concentrated.

As we have discussed earlier, in the in the pictorial view that is at that point some probability mass should concentrated at that point. So, joint PMF is given by the intersection probability that is, p X x you know that, when we are talking about this mass function, our notation is the small p between that bivariate case, so this is the bivariate random variables are X Y and when we are talking that some specific value that is x and y is equals to that, probability X equals to the specific value and Y equals to that specific value y and there when both the things are occurring simultaneously. So, this one and this x and y can form the sample space S.

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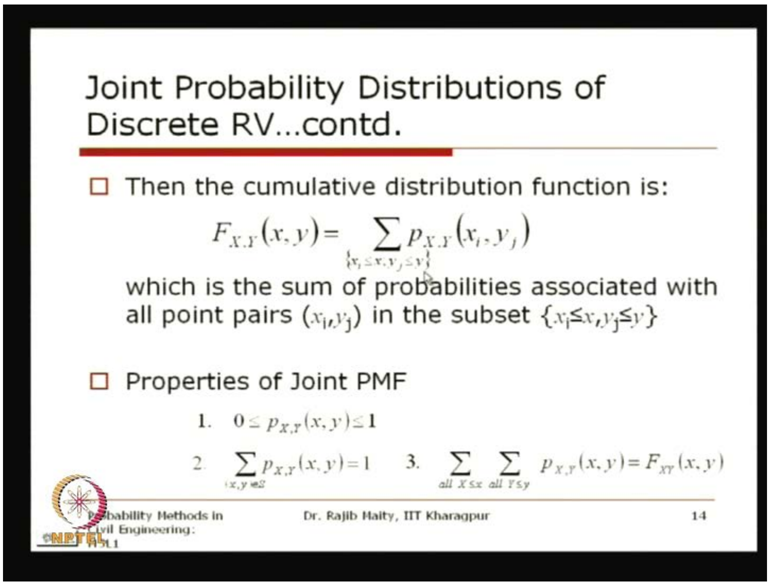

Now, when for this discrete case, when we are when we are considering that cumulative distribution function that this cumulative distribution function, now is the summation of that all such x i y i, where this x i is less than equals to some specific value, x and y i is less than equals to some specific value y. So, what we are, so if you again refer to this figure suppose that, in this zone now if this is now if I just redraw the same figure here, in case of because this is the discrete case.

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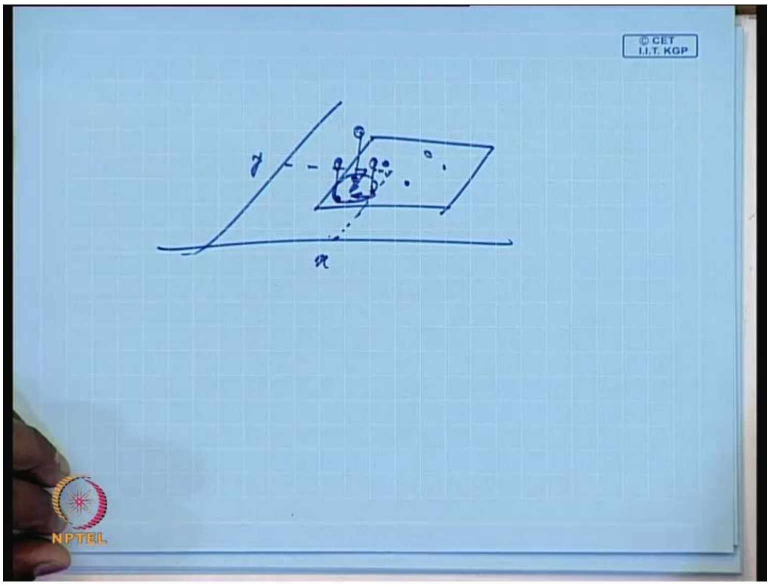

Now, in this in this suppose this the this is the possible entire zone and there are some specific values are possible. So, at each point at each point some probability mass is concentrated here. Now, if we are taking some some particular value of this x and some particular value of this y then, this is the x and this is y and this the cumulative distribution function that we are adding up means less than this this and less than this. So, this three probability mass should be added up to get the cumulative distribution for this uptoup to this range that is, from the x and y this is what is explained and as these are the these are the discrete case we are just adding them up through summation, which is the sum of probabilities associated with all pairs x I, y j in the subset of this case that is, x i less than x and y j less than y.

Now, this properties of joint PMF, again this PMF this this value that is PMF what we are talking about, this can have a lower limit 0 and upper limit 1, if you just add up this probabilities then this will be having the it should be equals to 1 and if we just add uptosome specific value of this x and y of this function then, it will be that that cumulative distribution function, which is denoted as capital F X Y x comma y.

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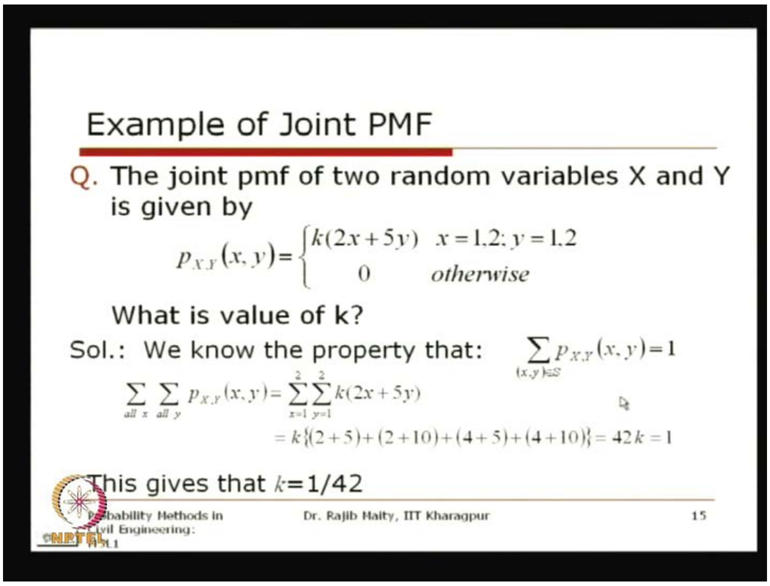

So, using those properties here we can quickly see one problem that, joint PMF of two random variable X and Y is given by this equation that is, P X Y is equals to k multiplied by 2 x plus 5 y, where this x is equals to 1 comma 2 and y is equals to 1 comma 2, so that means that, x y can take the values only 1 and 2. So, here you can say that, this the joint random variable that is the bivariate random variable x y can have a four possible outcome. So, those four possible outcomes are what these are 1 1, 1 2, 2 1 and 2 2. So, these four cases it can it can it can take and otherwise, it is this value of this probability mass function is 0, for all other cases.

Now, the question is what is the value of k? So, we have to find out this value of the k, because it has to satisfy the properties that a PMF can take, so here you know the the properties are that, if we add up this probability mass function over the entire range then, it should be equals to 1. So, using this property if we just add them that is the x can take the value x equals to 1 to 2 and y can take the value 1 to 2. So, if you just take this function and put their values, then we will get this this expression putting that the first one is the x equals to 1 and y equals to 1, second one is x equals to 1 and y equals to 2 and similarly, it is going on for this four different cases as I told that which is are the four possible outcome that can take.

So, all this four possible cases is taken and it has in found that 42 k and you know that, this 42 k should be equals to 1, so from this 1 we can say that this k equals to 1 by 42 to this be a valid PMF. So, this value of k we got it from using this property of this is of this PMF.

So, this kind of problem we have seen, earlier in case of the single random variable as well, where we are just considering only one output or only one possible value of a single random random variable. And here we have seen that, when it is coming that multiple random variable, where it is bivariate case, so two random variables are there. So, both the outcomes we have considered in this in the in the properties of this PMF.

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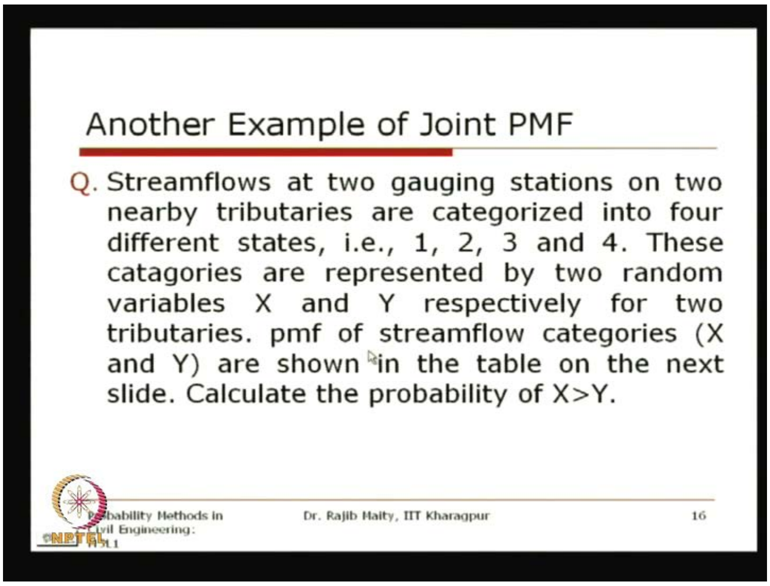

Now, if we take another another problem on this, the stream flow categories some times what happens just to give a little background is that, sometimes we are interested to know the thethethe behavior of the stream flow of a of a new river stretch, where there might be the data was not that reliable, but the adjoining data from the from a adjoining river gauging site is available to us. So, we just want to know that, what is thewhat is the that probabilistic relationship how their bivariate how their how their bivariate relationship between the thestreamflows between two sides.

So, if the sides are nearby then, we can assume that there we wewe can analyze some sort of some sort of probabilistic dependence to each other. So, to do that here we have consider the case, because the streamflow can we know that it is a it is generally consider to be a continuous random variable, but in many cases for example, that the when we are talking about some you will know that mark of case. So, there we generally consider we generally categorize the streamflow into different categories and those categories are now now important.

So, with this categorization means that, if the streamflow is below a specific value then we categorize as one, then if it is between this ranges to this range then we categorize as two. So, this the way we can categorize the streamflow before the analysis. So, once we are categorizing the streamflow in this way, basically this is become a discrete a discrete random variable.

So, here such example is taken here that, the streamflow at two gauging stations on two nearby tributaries are categorized into four different states that is, 1, 2, 3, and 4 for the both cases both the tributaries. So, these categories are represented by two random variables X and Y respectively for two tributaries. The PMF of the streamflow categories X and Y are shown in the table on the next slide. Calculate the probability of X greater than Y. So, the PMF of the streamflow before I go to the next slide that PMF of the streamflow categories X and Y are shown on the table on the next slide.

So, what we are expecting in the PMF that is supplied to us and we have already told that, there are four such categories is that 1, 2, 3, 4. So, PMF means the how much probability mass is concentrated for each possible outcome, so X equals to 1 and Y equals to 1 this is one possible case, so what is the probability mass is concentrated at this point. Now again so 4 by 4 so total 16 such combination is possible. So for, all this 16 such possible combination what is the probability mass that is concentrated at each possible outcome, so that is what is represented through this PMF. Now, once we know this PMF then, we are we are interested to know that, what is the probability that X is greater than Y, X is greater than Y means the the first tributary is having the higher category of streamflow compared to the second tributary. So, if we want to know this probability, then we have to pick up those combinations, where the X is greater than Y. So, such combinations means what we have to we have to take the first possible case is 2, 1. X equals to 2, Y equals to 1.

Secondly, that X equals to 3, Y equals to 2, X equals to 3, Y equals to 1 both the cases, when X is taking three. Similarly, when X is taking 4 then Y can take 1, 2, 3. So, all such cases whatever the probability is there if you add up then, we will get the required probability which is asked for.

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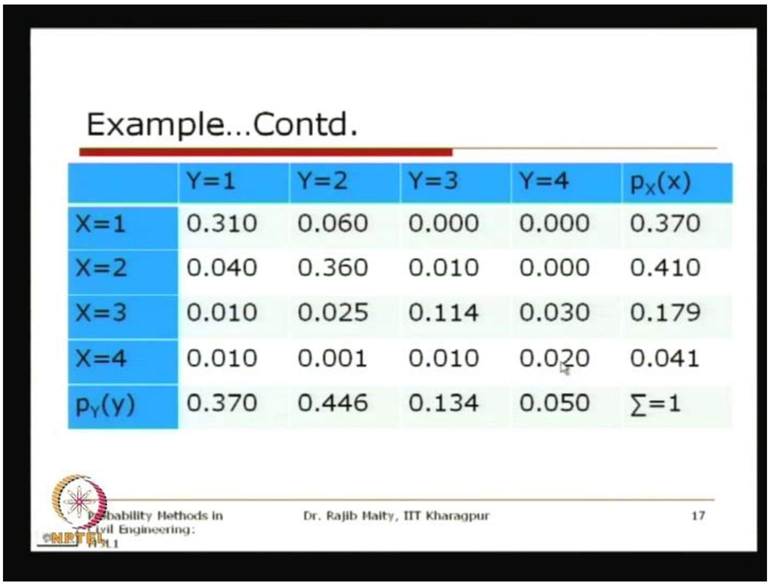

So, first of all we will see the PMF here, you can see that PMF when X equals to 1, Y equals to 1 this is 0.31. This X equals to 1, Y equals to 2 then it is 0.06 and here one thing you can observe that as this streams are adjoining means nearby to each other, so it is highly possible that, both the steps should be equal and that is here reflected reflected through this heavy diagonal, you see this diagonal terms are generally more than their adjoining adjoining terms.

So, that is why, what you can say that, that both the streams are having the tendency to be in the same categories and also both the streams when it is 4 and 4, there is very less less probability in the sense that, both the streams does not go to the very extreme extreme rainfall, so that probability is less.

Now, to solve the problem only this 4 by 4 matrix is sufficient for us. Now, there is one more additional information is given here which is here this P X X and which is here the P Y Y, these are nothing but the summation of these individual rows and these are the summation of these individual columns. So, this is basically, so basically what we can say that, irrespective of whatever the distribution of one random variables, so if we are considering this one that is Y is equals to 1, when we are talking about the Y is equals to 1 we are considering the irrespective of this value of this X, this is what we are just few minutes back was discussing about the marginal probability distribution and this will be taken in this next lecture to discuss that, what is the marginal probability distribution, but here what this last column as I have just included intentionally just to tell you the concept that this can be added to get that the what is the probability concentrated is X equals to 1, in this case irrespective of whatever may be the value of Y, this will take up later.

Now, as a quick check what we can do is that, this 4 by 4 matrix that you can see for this different possible values of this X and Y, if you just add up then we should get this summation should be equals to 1, if we get this summation equals to 1 then only it will be a valid p d f, because we have already told that, these are the four possible cases that the four possible states that it can take. So, this is beyond this there is no such possible outcome is isis expected is possible, so that is why this summation of this itself the probability should be equals to 1 that is what is shown here.

Now, as we are discussing that, we are supposed to get what is the probability of X greater than Y, so we have to pick up those combinations, so for them X equals to 2 and Y equals to 1. So, this you have to pick up similarly, X equals to 3, Y equals to 1, X equals to 3, Y equals to 2, X equals to 4, Y equals to 1, X equals to 4, Y equals to 2, X equals to 4, Y equal to 3. We are not supposed to take even that X equal to 4 and Y equal to 4, because in the problem it is asked that, X is greater than Y, if it is shown that X greater than equal to Y then, we we have to take even that that equal equal parts as well that X equals to 1, Y equals to 1, X equals to 2, Y equals to 2 we will not take, because we are asked for only that X greater than Y.

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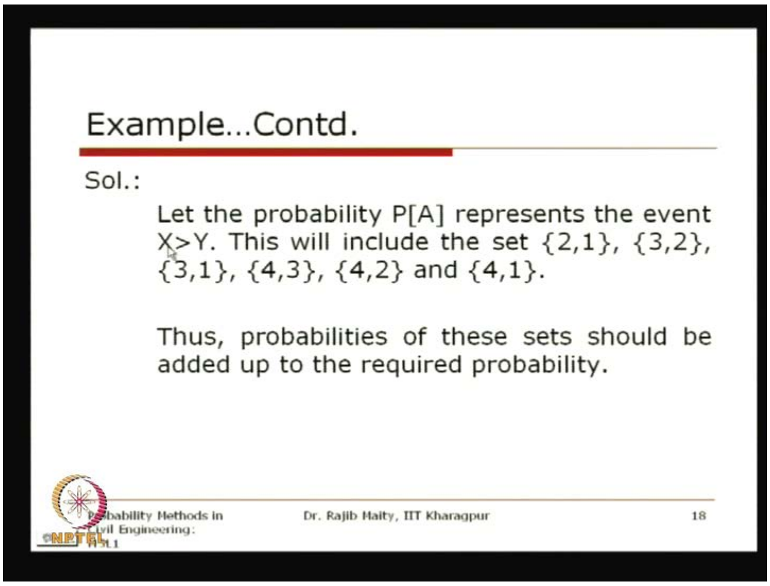

If this is understood then, these are straight forward that this is the event which sets which is the represented by this set 2 1, 3 2, 3 1, 4 3, 4 2 and 4 1, so all this sets are there, where this event, this condition is satisfied X greater than Y. So, we have to just add up these probabilities to get that one.

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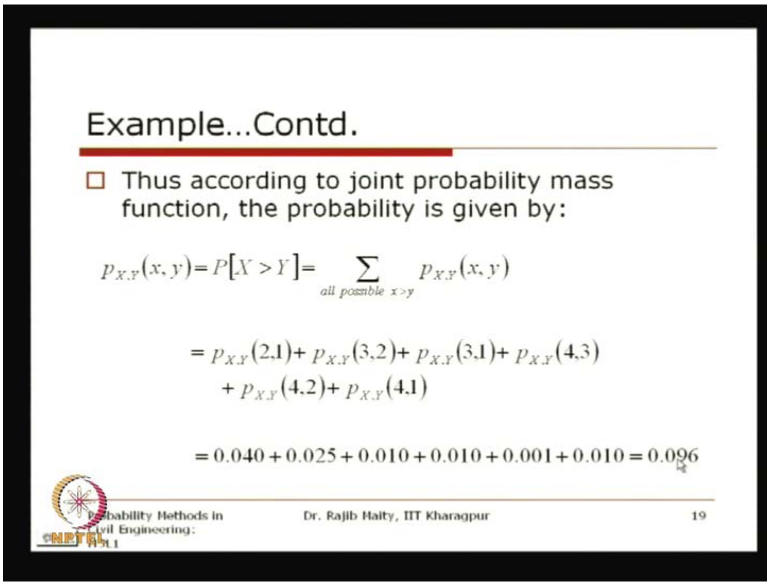

So, we are just taking this one, this summation to do this one and all all this probabilities are added up and this probabilities are picked up from that table and we got that 0.096 is the probability for the event X greater than Y.

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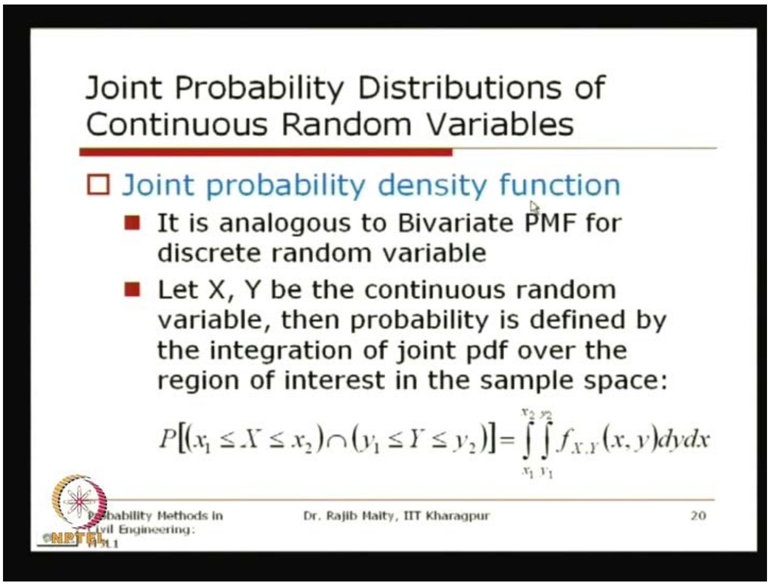

Now, if we take that concept to the continuous random variable, you know that for the continuous random variable as it can take any possible value over the entire range, so that instead of the summation, we have to go for the integration we have to integrate it. Now, for the bivariate case we have to integrate it over the entire area and if it is more than two, if it is three dimensional, then we have to we have to integrate it for the entire volume or the three dimensional space and similarly for the higher dimensional space.

So here, we are discussing this bivariate one, so it is analogous to this bivariate PMF, where we have just use the simple summation. So, if this X Y be the continuous random variable, then probability is defined by the integration of the joint p d f over the region of interest.

So, that this probability of this x 1 to x 2 and y 1 to y 2, when these two cases are occurring simultaneously that means, we are referring to that to that specific area, where this X is between this two range, Y is between this two range, now to get this one we have to integrate the function from this y 1 to y 2 and x 1 to x 1 x 2, now this f X x and f Y y is gives you that what is the probability density function.

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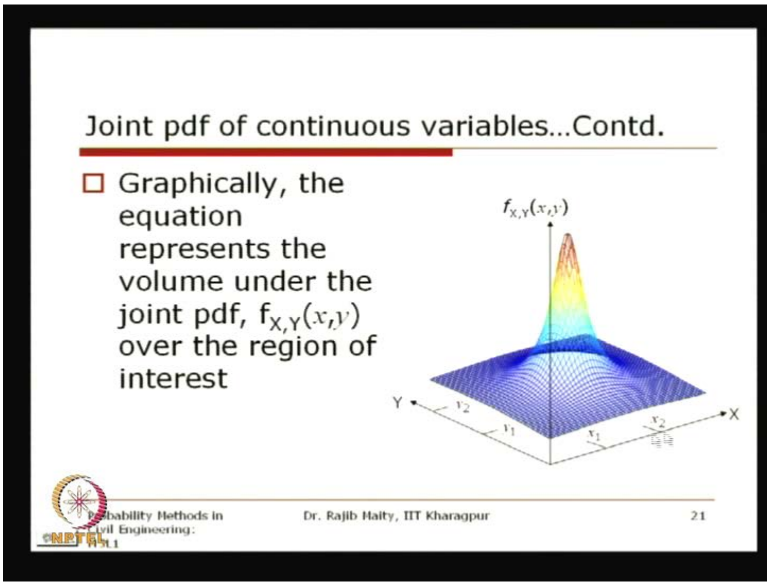

Now, if I just want to show this one as a as a graphically, if I want to see now this you can you can refer as as we are just, so far discussing that this is a this is a bivariate case and sorry this is the continuous case and in this continuous case, it is the not the probability is not concentrated to a specific point, so it can be a entire thing, so entire entire range. So, entire range when it is distributed; that means, this is like a surface now, so this f X Y x y is now a surface which is shown here and what we are referring of this of this earlier case, that is from the x 1 to x 1 some specific range some specific range we are just talking about y 1 to y 2.

So, if you just extend this line, this will consider one one rectangle over this and that over that if we just take that entire volume of this one, so that area or the volume below this curve, earlier for the single random variable we are talking that area below the curve, now we are talking about that volume below the surface. So, if this is the case then, we will get what is that joint that that cumulative distribution that probability, which is there for the event that x in between x 1, x 2 and y in between y 1 and y 2.

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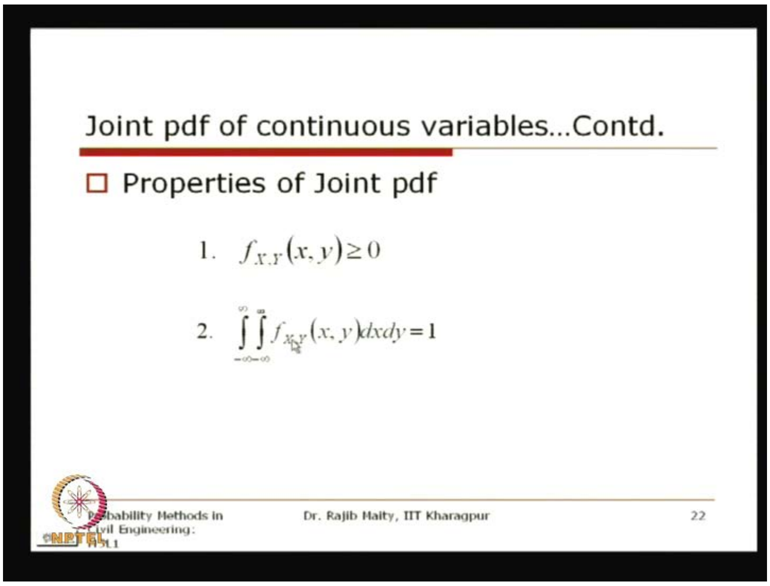

Now, this properties of this joint PMF is that, this f X Y x y should be greater than equals to 0. So, this is non-negative and if we integrate for this entire range of X and Y this should be equals to 1 this is straight forward taken from what was the properties that is total area earlier we call that below the curve, for the single random variable, here the below the surface a total volume should be equals to 1, in case of the bivariate.

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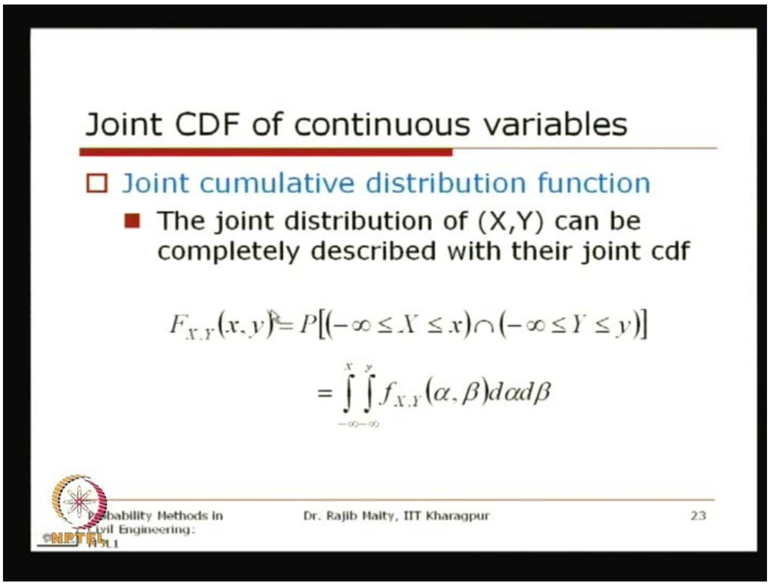

This joint cumulative distribution function now, you know that this is equals to that what is the probability we have discussing. So, from this from the left extreme to some specific value, left extreme to some specific value, if we integrate this value then, what we will get is this joint cumulative distribution function.

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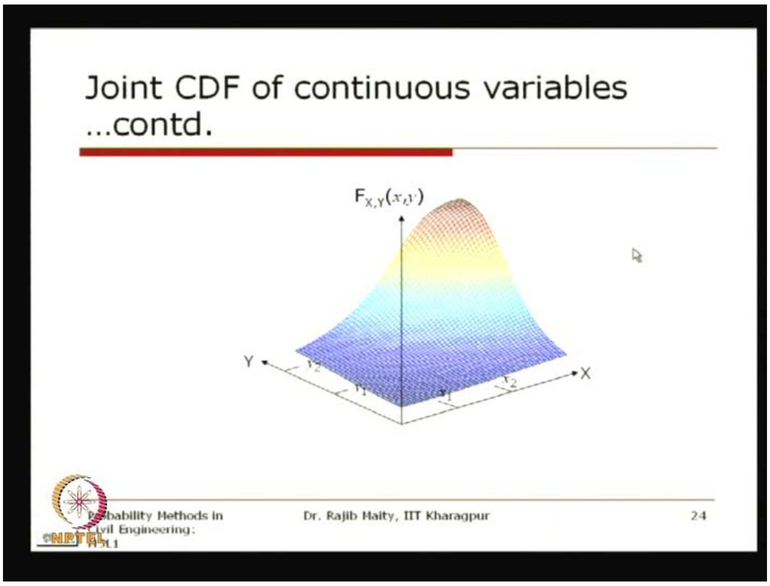

And this joint cumulative distribution function in case of the bivariate one, it will look like this, so we are just adding up the values. So, if we if we just add up this values this will go on adding to it and for this each and every value, when we are adding, so it will be that this surface will be non-decreasing in case of the combination x 1, y 1 and x 2, y 2 as I just told few minutes back, that if the x 2 is greater than x 1 and y 2 is greater than y 1 then, the cumulative distribution function will be more at x 2 y 2 then x 1 y 1, which is reflected from this surface. And you know that, entire integration is equals to 1; that means, this will be go and this will end up at when this F X Y is equals to is equals to 1. So, this will start from 0 and it will end up at 1.

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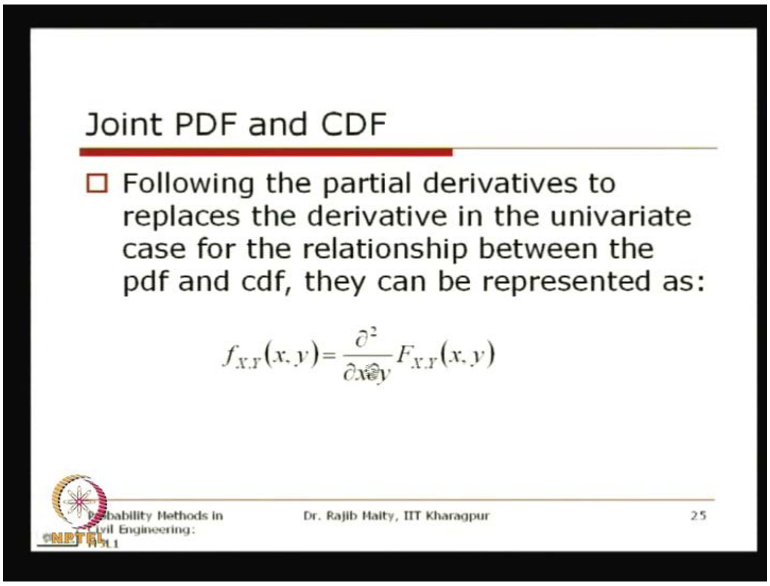

This joint P D F and C D F now it is clear that, how it is related to each other is that f X x that is the joint P D F is the double double differentiation of this C D F with respect to both x and y. So, if we do this differentiation of this C D F, we will get that joint P D F these are the relation between this P D F and C D F similar to the single random variable, where we have taken that only a only a single differentiation with respect to that particular random variable.

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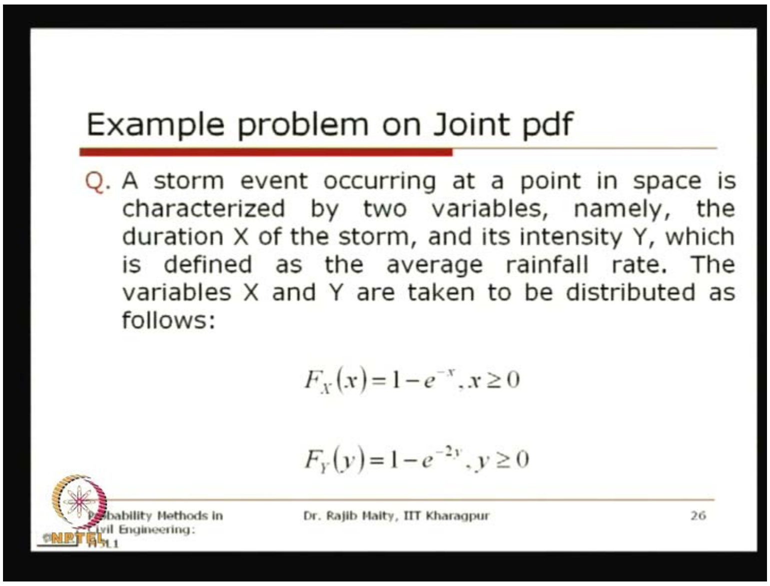

Now, there is a problem on this joint p d f again for this continuous case, but we will take this problem in our in our next lecture. So, what we have what we have seen today's lecture is the concept of the multiple random variable from the single random variable, how we can extend our concept to this multiple random variable we have shown some of the graphically representation, how how this things looks like in case of the bivariate case, because in the bivariate case that two orthogonal axis is referring to one two random variables and the z axis showing it's it is probability density or the or the cumulative distribution.

So, this things we have discussed and we have seen some problem on the discrete joint probability mass function as well and we will see some example on this joint probability density function in case of the continuous random variable in the next lecture and also we will start that marginal distribution function in case of the multivariate random variable, thank you.