Probability Methods in Civil Engineering

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Module No #01
Lecture No #16
Functions of Random Variables- Different Methods (Contd)

Welcome to this lecture, today we are taking this 3rd lecture of this module.In this module, that means the 4th module, we are discussing about the function of functions of random variable.So, basicallyin the previous modules we have seen the different properties of random variable and their standard distributions, we have discussed. Now, from the standard from the standard available random variable, if we generate or if we derivesome other function of those random variables, we have discuss that those functions are alsoa random variable. So, their propertiesof those functions also we will follow the similar properties of those random variable. So, in this module we are discussing this aspect of this property of this functions ofrandom variables.

Nowto start with, what we have done is that,we have discussed the fundamental theory, that is how one particular random variable can be link to the another one through theirfunctional dependence. So, if I know one random variable which isfor which the all the properties, that is the distribution is known and we know its functional relation with anotherrandom variable, then for that newrandom variable that is the function of that,how to get different properties.

Now, we know that if you want know the properties of this function, then we have to first of all know what is their probability distribution andthe probability distribution to get that in the previouslectures, we have shown somefundamental theorem; and that fundamental theorem, based on the fundamental theorem, we havekeeping the basic concept same we have derived different methods.

So, in theSo, in this lecture also we will continue twodifferent methods, one method is discuss in the earlier lecture and in this lecture also we will continue withtwo more differentmethods. So, our basicgoal is to know, what the probability density of this new random variable and once we know this probability density function, then we know that, we can estimate whatever the properties that we need to know, we can estimate that one.

So, the in the in the previoustwo lectures in this module,first we have seen the fundamental theorem and from there we have discuss about the method of distribution function first and in today's lecture,we will discuss about that method of transformation. So, even though we are givingthe different names,I repeat that all this methods are based on the same fundamental theorem only with some special assumptions has been taken for different method, that is why themethods looks different.

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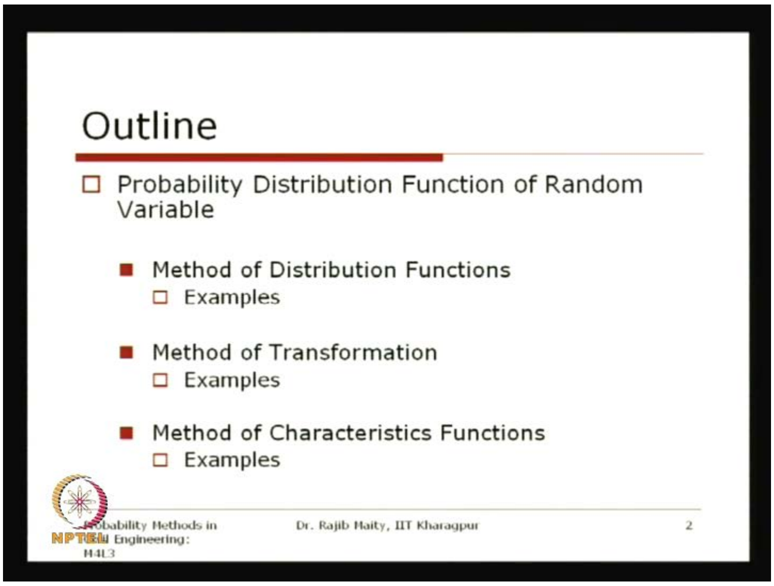

So, we will if I just see the different methods that we have discuss earlier, first in thein the earlier lecture, we have discuss that method of distribution function with some examples. And today basically we will discuss about this method of transformation with examples. Now, when we are talking about this method of transformation, basically what we are talking that, we are talking about that transformation from one random variable to another one.And here the basic assumption that we are following is that,the transformation is known, that is known in the sense that,the unique transformation is generally assume that is one to one transformation. So, for a particular value of theof the original random variable, there is one and only one value for the new random variable, that is derived random variable.So, this transformation is known as one to one.

So, when we are when we discuss that method oftransformation,we will see that how this method of transformation is linked,it is, they can be derived from the fundamental theoremfor this one to one transformation. A little bit of discussion was there in the previous lecture as well, sowill followthat one is details in thisin this lecture.

And finally, there is another methodthat is known as this method of characteristics function. So, we knowthat for the for the standardrandom variables, if we know the PDF then, we can estimate their characteristics function. SoAnd we have seen that, usefulness of this characteristics function is that, once we can the identified the characteristics function, then using their relationship with thedifferentorder of moment, that is first order moment, second order moment, if you can link those things then thoseif those moments are known, if those initialthen the then theproperties, theirlocation, their central tendency, their dispersion, sothese properties also will be known.

So, now if we know the characteristics function, then from that functional relationship for this derived random variable, we can also get the probability density function for thefor the functions of the random variable, so that we will discuss alsounder the method of characteristics function. So, before weproceed we will quickly see that, what we have seen in the fundamental theorem as well as for this method of distribution function,we will justrecall, sothat it will help us to understand that,what is there in the in the method of transformation.

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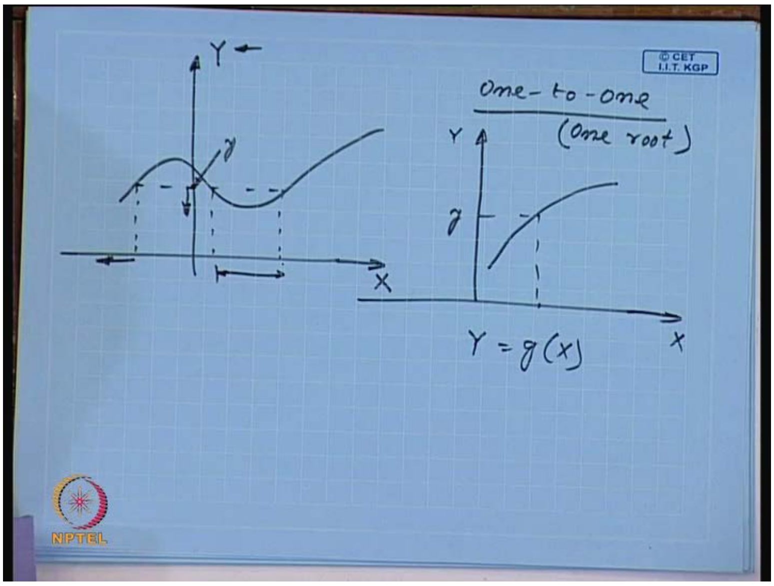

So, what we have seen earlier is that, if this is onerandomvariable, which is the original random variable denoted by x and there is another random variable, which we can derivethrough some functional relationship withthe original random variable x. So, if the if the functions can have any this type of this type ofthis type offunctional relationship, then for a particular value of y,what we have to do, we have to find out the what is the representative set in this in the originalrandom variable that is your x here.

So, if I know this one, then the then the probability of this y less than this specific value y, sotowards this one, then this corresponds to this zone as well as the zone between this one. So, this thing if you just add,if we get then in this way we can get what is the what is the distribution of this y, for each and every possible value of thisof this new random variable y. Now, when we are talking about this method of transformation, first case that we are considering is that one to one. So, when we are talking about thisone to one transformation, that meansthat, the relationship is such that the for a specific value of y, there will be only one possibleonly one possible set for this original random vanoriginal random variable x can can have.

So, for this one, for this kind of relationship, from the fundamental theorem we have seen that, this will be just only one root, sowhat it is reduced to the fact that,it thenin the relation to the fundamental theorem is that, only one possible root for a specific value of y.So, with that root, with the fundamental theoremreduced to only oneonly only one root and using that relationship,we will see how we can get the PDF for this newrandom variable y. Here, we are assuming that functional relationship,that g x is having only only single root for a specific value of y.

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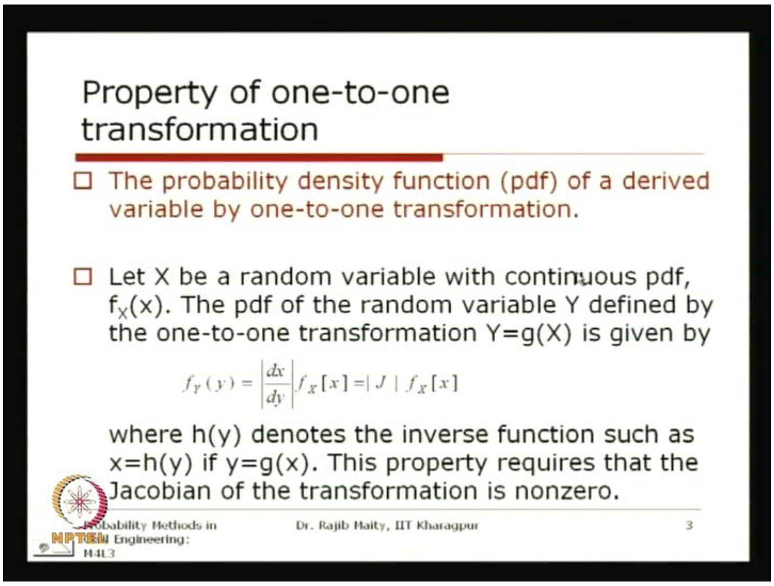

So, herethewhen.So, thehere theassumption that we are using that we mention is that, one to one transformation. So, if the x is a random variable with continuous PDF,having the PDF is f x x. The PDF of the random variable y, which is defined by this one to one transformation, this one to one transformation is important, that what we are mention here is that y is equals to g x.

So, this is the functional relationship that is having between two random variable x and y and for this random variable, this distribution is known that is f x, then from the fundamental theorem, we get that this f y y should be equals to this f x xand multiplying by a factor that d x y, this we discuss in the last lecture as well, that is this one when we are taking this derivative.

Thisderivative is the, we are taking the absolute value and this derivative is known as the Jacobin of this transformation, which is denoted as j. So, where thisdenotes the inverse function, so now, when we are we are we are expressing this one, that f x xwe should express this one in terms of the new random variable y. So, which is the inverseinverse function such as x equals to h y, if that y is equals tog x, so sowe can also write that x equals to as we as discuss earlier lecture that, x equals to g inverse of y, that g inverse of y is here that h y.

So, this propertyrequires that, the Jacobin of the transformation is nonzero,now this is for the continuous random variable that, what we have started with and that is why we are we required that this transformation should be the Jacobin of the transformation should be non zero;and we have also discuss that if it is if Jacobin equals to 0 what does it mean?So, this means that therewill be a spike of the of the probability distribution that is, there will be a discreteprobability for that zone where this, this, this derivative that is a d x d y is equals to 0. So, d x d y equals to 0 means the rate of change of x with respective y is equals to 0 that means.

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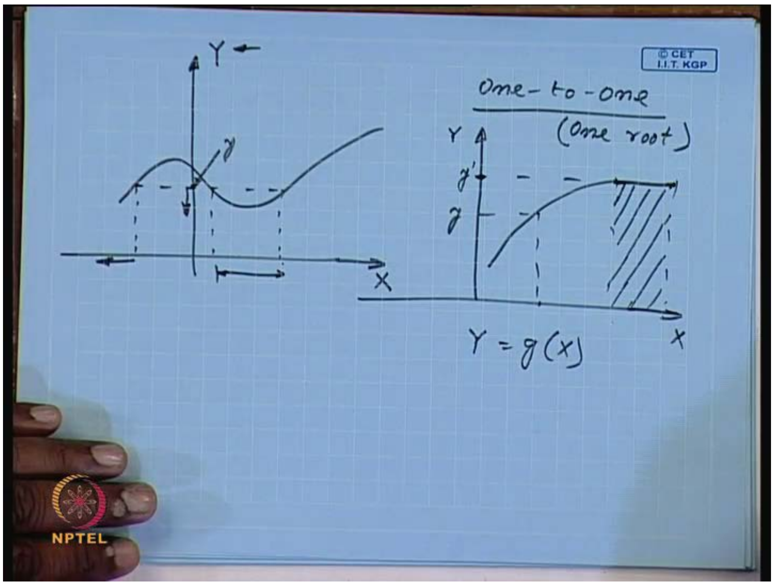

So, whenSo, if the this thing if some functional relationship, it becomes constant for this zone,then we can say that at this point of this value if thisspecific value y prime here, this rate of change ofx with respect to y becomes 0.So, if it is becomes 0 that means, the this probability, that is probability of this y is equals to y prime should be the, it should be equal to the probability of this total area, so that means, here this is be this is become a discrete probability at this point some probability mass should be concentrated for this specific value of y, this is what iswhat isshown here this what explained in thisin this statement.

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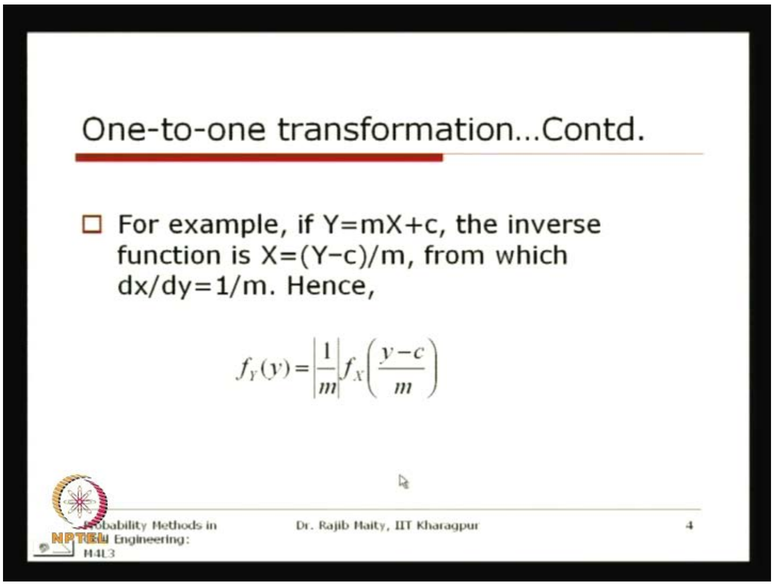

Now, if we take a very a very simple problem if you start with a very simple transformation that is, that y is equals to m x plus c and you know that, this yis equals to m x plus c is a is a transformation of a of a straight line now this.

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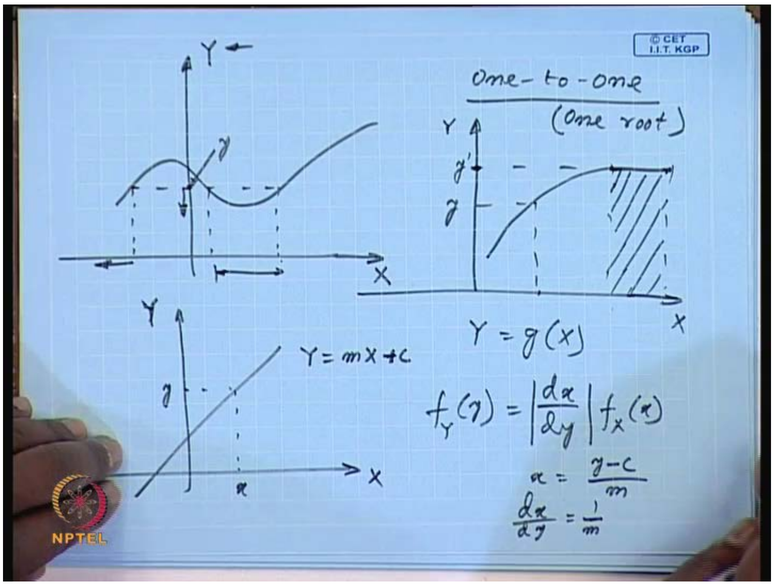

So, the transformation will look will be a will be a straight linelike this. So, where, sofor this one, if this is your x and if this is your y, then the relationship is that y equals to m x plus c, which is the equation for a straight line for any specific value of this y, there will be a one and only one value of this earlier that original random variable x.

Now, we know that, if we just want to know what should be therethe distribution of this y. So, from theequation we get that this f y of y, that is PDF of this y is the Jacobin d xd y of this f x of x. Now, this x I have to write in terms of this inverse relationship, sothis x will be equals to your from this relationship can be written that y minus c by m. So, this, sofrom here also we can also write that d xd y is equals to your 1 by 1 by m, because c by m is constant. So, this becomes 1 by m. So,sowe can just replace this part here and this one here to get this f y.

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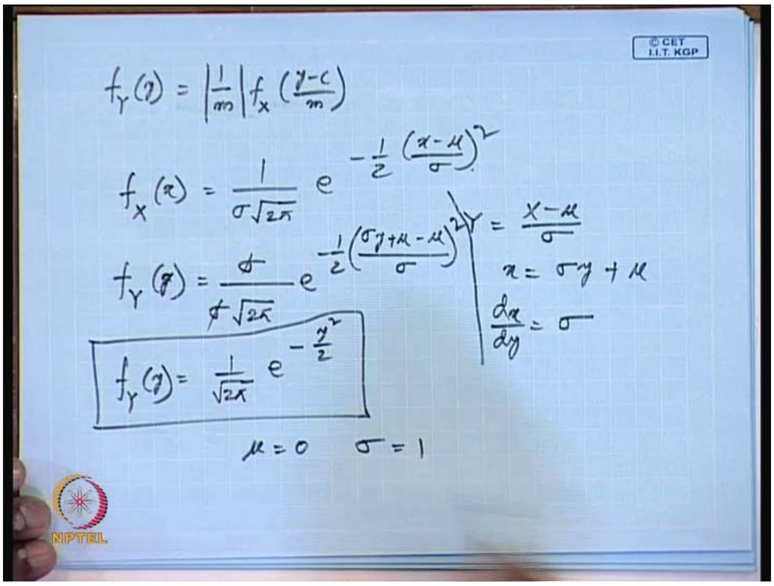

So, in this what we will get is that f y is equals to 1 by m this should be taken the absolute value depending on what is this,the value of this m the slope of this straight line, this one multipliedby fx of this y minus c by m.Now, if we know this form, that if you know the PDF of this x will just replace this value and will get the PDF of y, sothisexpression.

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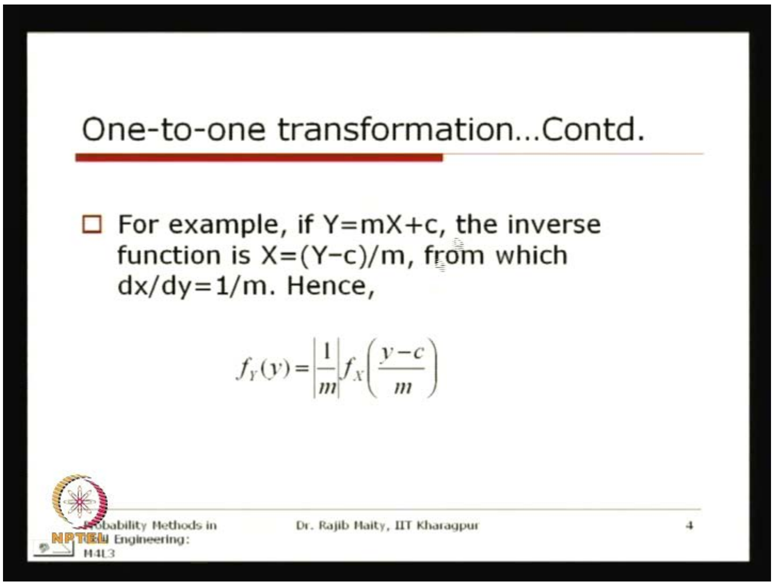

So, this is what is explained here, sofor a relationship, which is a straight line relationship y equals to m x plus c, then the inverse function that we are that we are telling in the last slide it is the y minus c by m from which the d x d y is 1 by m thus this f y y equals to 1 by m absolute value multiplied by f x of y minus c by m.

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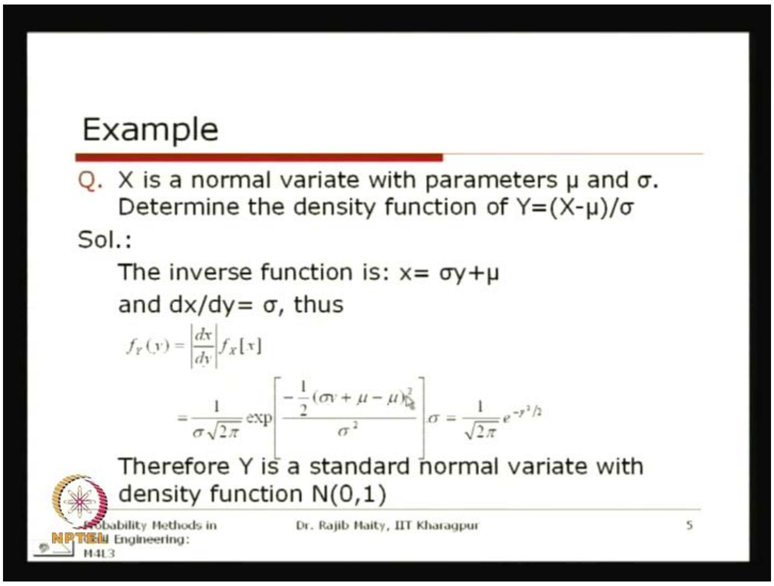

Another example, we also can see is thatstandardnormal distribution and normal distribution. So, here if where the x is a random variable, which iswhich the normal distribution is which follow the normal distribution with its parametermu and sigma. So, if the relationship now determine thedensity of the function y, which is having a functional relationship with x is x minus mu by sigma.

Now, when we are we have also discuss earlier that why this example is taken here just to show that, how we can get these one, because both this distribution we know that is x as well as y, sox we know because this is standarddistribution that we discuss in the lastmodule, this is normal distribution or Gaussian distribution with two parameters mu and sigma. And for a Gaussian distribution if it transferthat random variable through thisthrough thisrelationship that is x minus mu by sigma, we know that this the reducedvariety which is also a normal distribution having its mean equals to 0 and standardization equals to 1. So, this is nothing but,this will follow a normal this will follow a standard normal distribution, standard normal distributionmeans its mean is 0andstandardizationis 1.So, if this one is and we know the distribution for both therandom variable x, which is a normal distribution and the y which is the standard normaldistribution, sohere now,the inverse relationship we will see first, where this x equals to sigma y plusmu, this is the inverse relationship; and from here we should calculate thatJacobin,Jacobin is your d x d y, sohere the d x d y is equals to sigma. So, againusing that samerelationship that is f y y isequals to this absolute value of thisJacobin d x d y multiplied by this f x of x, express in terms of the new variable y,express in terms of new variable y means we have to place this one this instead of this x,we have to use that sigma y plus mu.

Now, this f x of x now,we know that. So,we know that for a for a for a normal distribution that f x of x is equals to 1 by sigma square root2 pi e power minus half x minus mu by sigma whole square. Now so, this has to be this has to bereplaced by this as, sothis y if when we are talking about that is a when this relationship, that we are talking that y is equals to x minus mu by sigma and this x equals to sigma y plus mu just now we have shown. So, here that d x d y is equals to yoursigma.

So, this f y is nothing but,now thatsigma we should multiplied that sigma thatdivided by this original one sigma square root 2 pi exponentialhalf in place of this x, what we arewrite in is that your sigma y plus mu minus this mu divided by yoursigma whole square. So, this is becoming that one bysquare root two pi exponential minus. So, this sigma cancels, soit becomes y square by 2. Now, sothis one is that f y of y we got. So, this is also a normal distribution, which is having thatthatmu equals to that mean equals to 0 andsigma that is standard deviation is equals to here 1, sowhich is nothing but,standard normal normal distribution.

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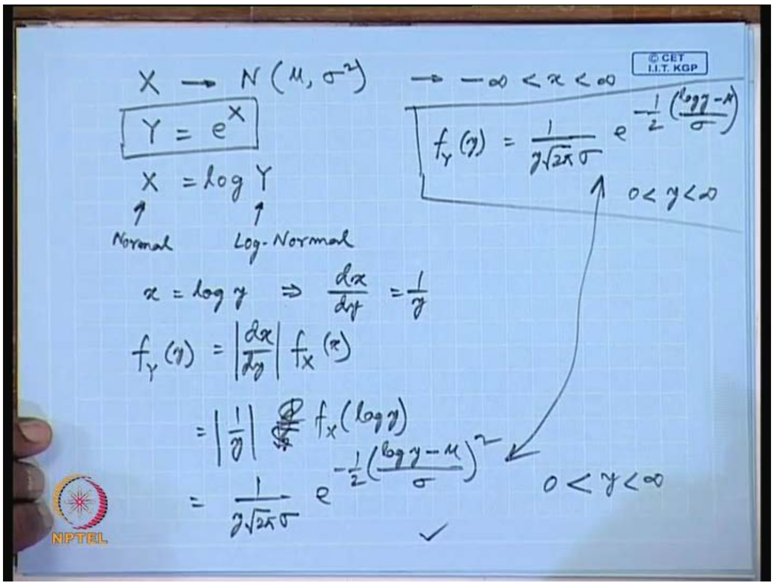

We can alsobefore you go to a specificcivil engineering problem, we can also cheek one moreone more relationship this we have discuss earlier is that, is thatif say that x is a is a normaldistribution with the two parameters sigma andsay mu and sigma with the sigma square is the variance, for which that you know that support is from the entire real axis minus infinity to plus infinity.Now,if we have a new relationship say that y is equals to epower eepower x, if this is the relationship holds then,we can also that express that x equals to your log of y.

Now, if we recall from our from our previous module lecture that, when this x isx is normal distribution x is your normal distribution, that is for a random variable if I take the long it becomes normal distribution that means, this y is nothing but,your log normal distribution.

Now, if this one, sowe know the distribution of x and we also know the distribution of y, sodistribution of y means the log normal log normal distribution from ourearliermodules lecture, we know that for thelog normal distribution, the distribution looks like this that is 1 by y square root 2 pisigma,sigma is out this rootexponential minushalf of this log y minus mu by sigma and this one is that 0 to y infinity, sothis is a nonnegativenumber, sothis distribution we have seenearlier.

Now, if we know this one, thisdistribution the normal distribution if you know that,this is the functional relationship that holds between this x and y can wecan we obtain this log normal distribution from this normal distribution or not that we will see from this one, because this transformation is also one to one transformation.

So, now to getthis one, we have to find out that this the inverse relationship, that is x equals to log y that we have got. So, thatd x d y equals to your 1 by y now thisx is a.Soif I just want to know from here that, what is this. F y y is yourJacobin d x d y and f x of x.Now, from this one, sothis will beyour 1 by y 1 by y and multiplied by 1 by this one we know that this is normal distribution square root 2pi.

So, this x now will be replaced by your this log y, so I can just write that once more one more step, that is f x is of log of y; so, which is nothing but,1 by this y comes square root 2 pi sigma from this normal distribution e power minus half log of y minus mu, sothis log of y we are writing in place of this xminus sigma whole square this zone as we know that this we are taking thisrelationship this one is valid forthis 0to infinity as we have taken thisrelationship. So, this can never come below 0,sothesothis is now this one is nothing but is equals to this one.

So, if this kind of relationship holds, we can see that if x is the normal distribution y is your y is your log normal log normal distribution, which we have got from this one to one transformation method andusing this Jacobin of this transformation.So, now.So, this is the standard, soall this distribution just now what we discuss is that normal thenstandard normal distribution and after that we havediscuss that from the normal to this log normal distribution. So, all this distribution that we now that, these are somesomestandard distribution. Now, will take up oneexample from this civil engineering, where the distribution of the originalrandom variable is known, sowe have to find out that what is the distribution of a of a derivedrandom variable.

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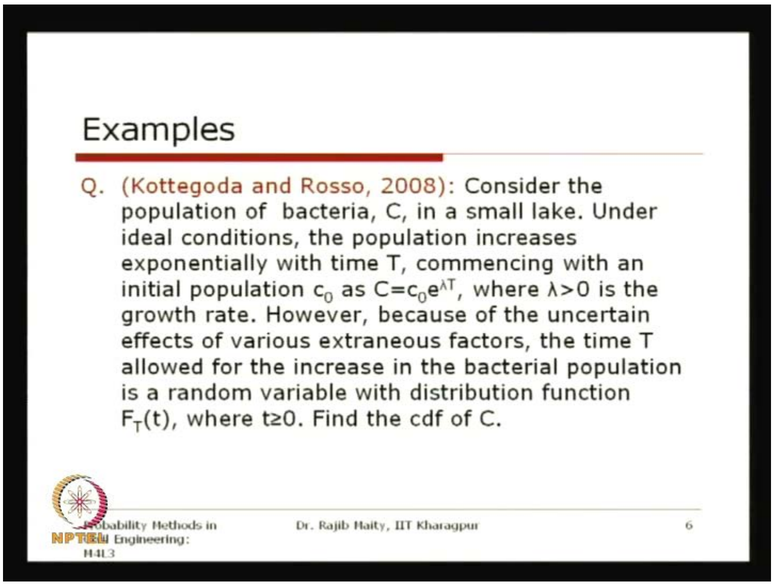

So, this problem is taken from the book Kottegoda and Rosso. So, there are this is on the growth ofthepopulation of bacteria in a lake. So, let us consider that the population of the bacteria that is C that is in terms of concentration in a small lake. So, this Cthe population denotedby C under ideal condition the population increases exponentially with time T, soas time increases this population of bacteria will also increase, commencing with an initial population c naught. So, c naught is the, at T equals to 0the concentration is c naught.

Now, at any time T the Cthe new concentration is equals to it is increasing exponentially. So, the relationship is c naught e power lambda T, where lambda is greater than 0. So, lambda greater than 0here means that this one will increase as the time pass that is as Tincreases this C will also increase, sothisgrowth rate.

However,because of uncertain effect of various extraneous factor, the time T allowed for the increase in bacterial population is a random variable with the distribution function Ft. Now, this T that is the time that is taken for this increase in the population, this is we arethis is this is we are for from the real condition we have seen this is also a also a random variable, that random variable this F T that is the cumulative distribution function of that one is known, some standard distribution is known. So, if this distributionis known then, that we have to find out the CDF of C. So, basically there is a functional relationship is there, this T distribution of this T is known we have to find out, what is the distribution for thepopulation ofbacteria C at a time at time T?

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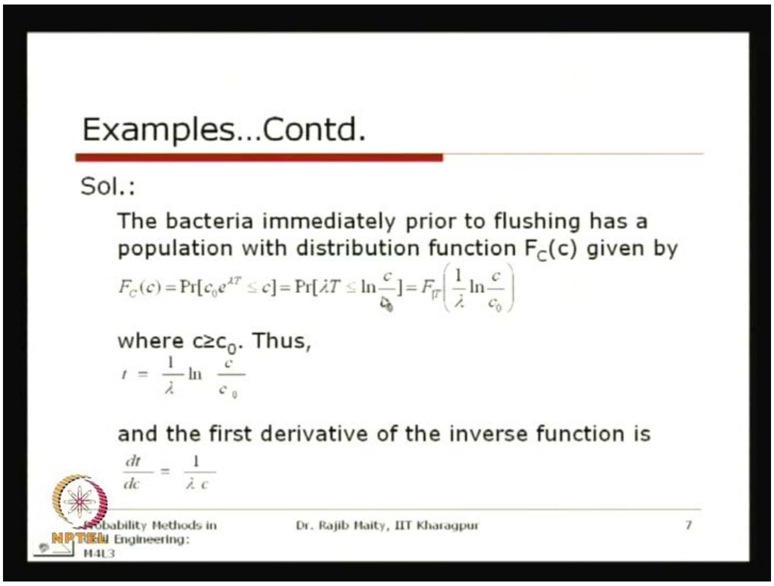

So, to solve thisproblem,first of all we have tofind out that what is thisdistribution for the concentration? Thebacteria immediately prior to the flushing have a population with the distribution function F c c. So, this population which is we are talking about this should be meansthis should be express in terms of the this is a this is from the from the standardproperties of the CDF that we have discuss in earlier lectures, that is the F c cthe c is thevariablehererandom variable is c is nothing but,the probability of that thisconcentration at any time, that is concentration at any time now here is that c naught e power lambda T. So, this one should be less than as specific value which is nothing but,the expression of this cumulative distribution function.

So, now this one after some algebraictransformation that is lambda T should be is less than equals tolog natural c by c naught, which is also furtherwritten thatis the f t of this. So, this is T less than equals to 1 by lambda log natural c by c naught. So, now, when we are writing that probability of t less than 1 by lambda log natural c by c naught that means, that is the probability distribution of the cumulative distribution of the random variable T and here that 1 by lambda log natural c by c naught has come.

So, now where this c is greater than c naught, soas this cwe have seen that it from the initial population, this c at any time it is always increase as this lambda is greater than

0.Thus this T is equals to 1 by lambda l n c by c naught. So, this is our that that inverse relationship that, we have gotthat is given that is c equals to c naught e power lambda T. So, this is our inverse relationshipwhich states that T is equals to one by lambda log natural c by c naught. So, if you know this one then we can calculate itsderivative that is d t d c which is needfor the Jacobin which is equals to, so1 by lambda c.

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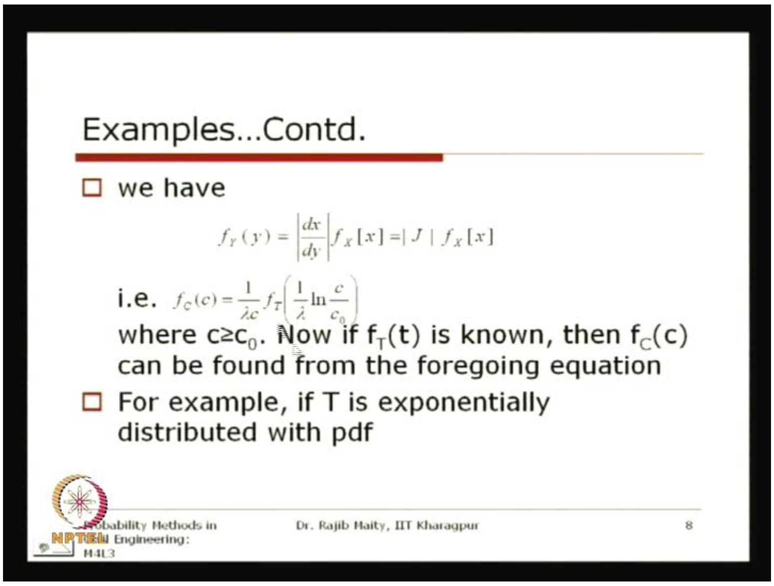

So, once we know this Jacobin then we have to just get that,f t is that is also we know thatthis is the functional relationshipthat is f y y, the new variable is the Jacobin multipliedby f x x express in terms ofy. So, this f c c is nothing but,then one by lambda c lambda is greater than 0 c is also greater than 0, sothe absolute value is equals to one by lambda c multiplied by f T 1 by lambda l n c by c naught. So, this t is express in terms of its inverseinverse relationship, where c is greater than c naught now if this f T is known then f c can also befound from this equation.

Now, what was given in this problem that, if from thiscumulative distribution of this t is known,sosoas this is known then youcan easily get what should be thewhat should be expression for this part now here we can take onespecific example that is if this T is exponentially distributed.Now, if this T is exponentially distributed thenwe have to some assume some we know that how this exponential distribution form is that isa e powerminus T.

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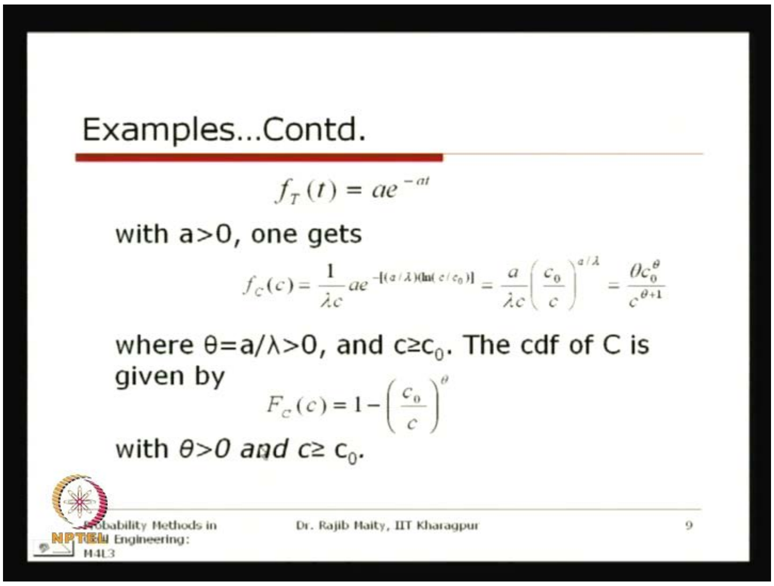

You recall that from our earlier lectures we generally use this lambda as parameter for the exponential distribution, buthere the lambda is used for some othergrowth rate. So, we are using some othervariablethat is a, sothis is the form of this exponential distribution this t is the obviously, greater than 0 that is a e power minus a t with this a greater than 0.

Now, So, once you know this one, thenthis thing can be show f t is if it is given like this then we are just replacing this value with thatexpression, that is a by lambda log natural c by c naught, sowhich can berearrange and express that a by lambda cnaught by c power a by lambda, which is equals to thetac naught power theta by c power theta plus 1 this theta is now a newvariable, which is equals to a by lambda which is greater than 0and c is greater than c naught.

So, the CDF of c is given by f c c is equals to 1 minus c naught by c power theta, sothis is your PDF that is probability density function from here if weintegrate, we will get thatcumulative distribution function for c, which is one by one minus c naught by c power theta and here this theta is greater than 0and c is greater thanc naught. So, similarly if this f t follows some other distribution, sothis expression we willwill change accordingly.

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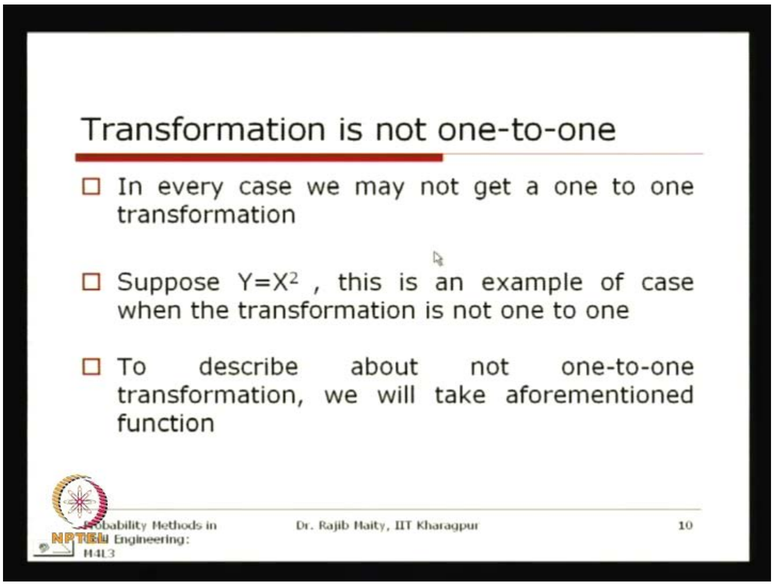

Now, So, far what we discuss is that method of transformation is for thefor the transformation is unique that is one to onenow we will see one more case that where the transformation is not one to one then whatshould we do?So, again we will recall that same the fundamental theorem if the roots are basically roots are more than one thenthen how this transformation changes.

So, here we are taking thatthe transformation is not one to one, soin every case we may not get a one to onetransformation suppose that y is equals to x square if this is the relationship, this is an example of theof the case, when the transformation is notone to one.

So, to describe about that not one to one transformation, we will take this asaforementioned aforementioned function. So, we will take this function that y equals to x square x square,we will define some PDF for this x and we will try to find out what is the PDF of this of this y.

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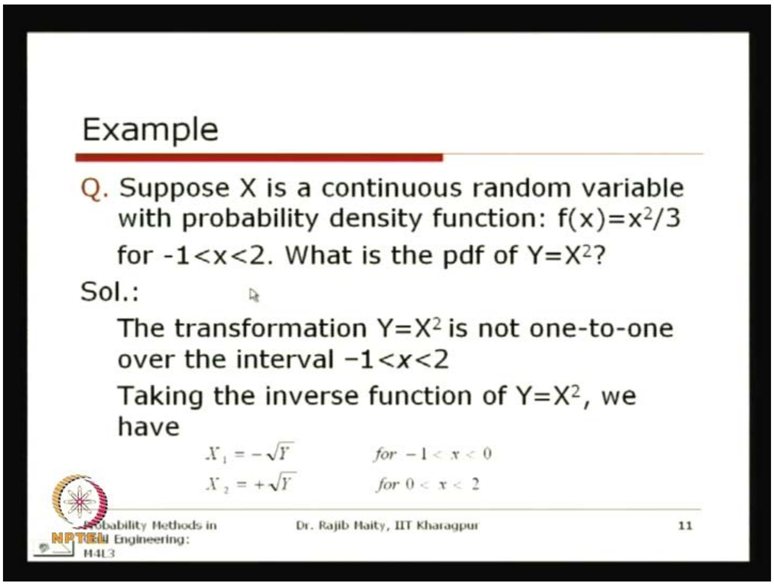

So, here we are taking this example, that is suppose that x is a x is acontinuous random variable with the probability density function fx is equals to x square by 3, for thiszone that is minus 1 x 2, sominus 1 to 2 in this zone this density is defined that is x square by 3.Now, we can check that whether this integration of this total of this PDF is equals to 1,that is why this by 3 is comingas a normalizing factor.

So, this is a PDF and the functional relationship that, we have taken is that y is equals tox square, sowe know what the PDF of x is and we also know that, this relationship is not one to one then, what should be the distribution of this y. So, to solve this one that this transformation isis not one to one to one over the interval of thisminus 1 to this 2. So, taking the inverse function, that isy equals to x square that is x equals to plusminus square root y, sothis x 1 3also two roots we aregetting now the x 1 equals to minus square root y, sothis minus square root y we will get for this x when it is minus 1 to 0;and for the x 2 is the plussquare root yfor the region 0to 2.Now,this can be, soif we just plot this function this will be clearer to express this 1.

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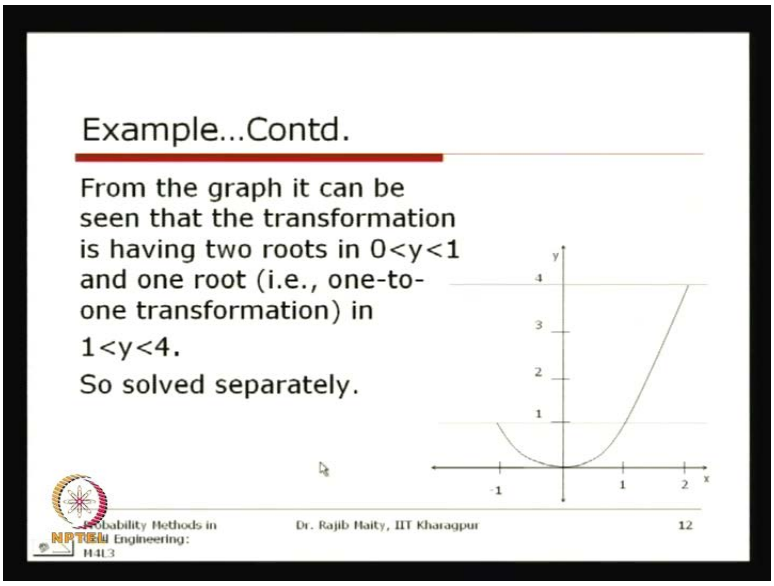

So, here the PDF is shown here from this minus 1 to this two and we have just given two lines here, one is that y equals to 1 and oneis that y equals to 4. So, for this zone that is from this x equals to minus 1 to 1 the root, if I take the square root ofthis one this square root of y that minus will give the this root that is from this minus one to 0; andand for this one from 0 to 2 that this zone thatthere we can take that root of this plussquare root of y that is from this 0to up to 2.

Now, what we can see here thatfrom for this y this variable for this y, there is if we start from 0 to 1, soin this zone there are basically two roots for thisrelationship. So, one is on theon the negative side another one is on the positive side. So, we have to calculate this zone separately that is 0 to 1; and for this 1 to 4 in this zone of for the random variable y,for this zone that is 1 to 4 then this transformation is one to one.

So, from.So, we have.So, we have divided this random variable for this two zone two zones one is from the 0to 1, where there aretwo roots exist and another zone is from 1 to 4 where there one root exists. So, from the graph it can be seen that the transformation is having two roots in the zone 0 to 1 for y and one root that is one to one transformationin the zone 1 to 4, sothese two zoneis solved separately.

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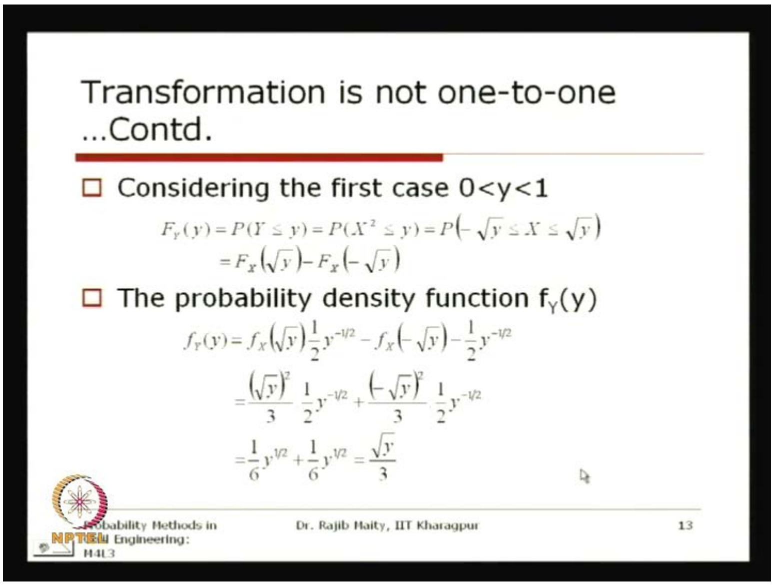

So, the first zone that is when we are taking thatthat y is between 0 and 1 then f y y is equals to probability of y less thany, this specific value should consist of the two zones, that will show that is probability of x square is less than equals to y, which is equals to probability of minus square root y to the plus square root of y. So, this f x of square root of y minus f x of square root ofsquare root of y.

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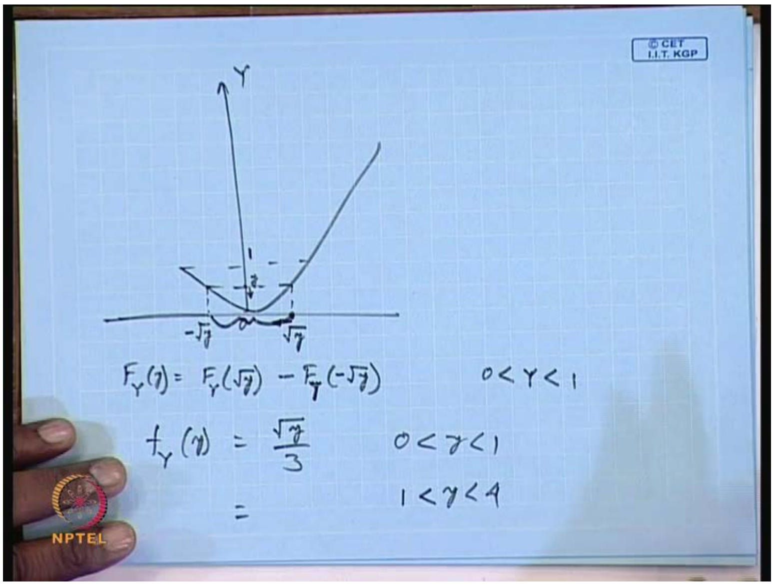

So, what is there actually how we are getting this one is this one this. So, for this zone that is from 0 to 1 for any specific valueif I take y, then what is this?So, for this less than y.So, this is your random variable y, so for this any specific value thaty is less than thisy then what we are getting is that this 2 zone 1 iswhich is explain that this is for this side, sothis zone that we arewe are getting, so now, to get that this zone that is what we need that the square roots of, sothis is nothing but,your square root of yand this is nothing but,that minus square root of y.So, from this 1 this total minus this 1, so we will get this part, sof y of square root y minus f y of minus square root of y to get this zone which is nothing but,equals to your f y yfor the zone 0to 1.

So, this one we are getting, which is shown here in thisin this relationship, now sothis probability density function f y is so, f y we aregetting that is in terms of its now we willnow do that this one this is express in terms of the root and this is you are the Jacobin of the transformation, sothis is half square root ofy power y power minus y power minus half that is 1 by square root of y that is that Jacobin. So, this Jacobin is for this zone this is the positive one and this is the negative one,sorry there will be that absolute value symbol here. So, this minus halfsquare root y once you take thisabsolute value of this Jacobin it becomes plus that half y power minus half.

So, now this f x square root y is express in terms ofthis, that is this cumulative distribution this f x isgiven that is PDF is given to us. So, x square by 3, this x square by 3 this x is now replaced by this square root of this y. So, square root of y square by 3 similarly here it is the minus square root of y square by 3.

So, if we just take this one add this 2, then we get that square root of y by 3, sofor this zone that is f y of y is becoming square root of y by 3 for the zone y this 1, similarly we have to find one more expression for the zone 1 to 4 that that isstraight forward because that relationship is one to one as we have discuss.

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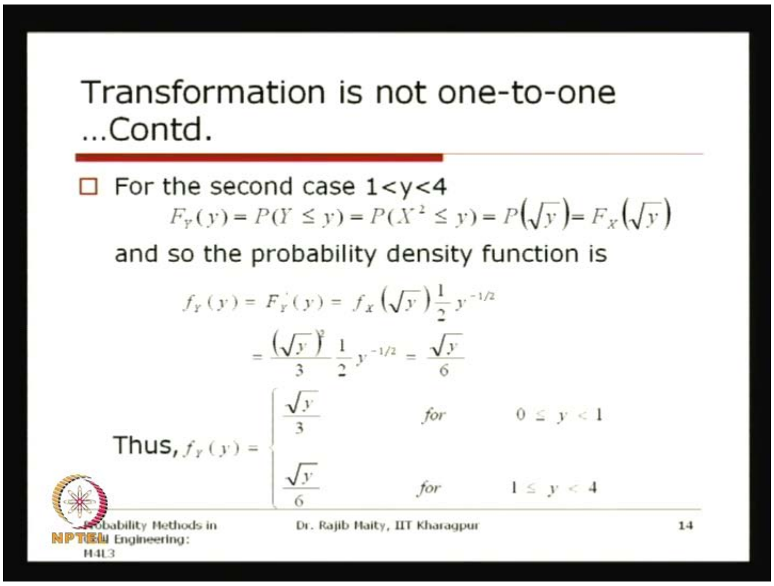

So, for this zone that is for 1 to 4 that f y y is equals to this less than y that is x square is less than y, which is P of this square root y that is f x square rootof y. So, the probability function that f y y is equals tothat,this f x of square root y single root only the positive root, we are consideringmultiplied by this absolute value of thisJacobin, which is half y powerminus half. So, this Jacobin multiplied now we are expressing this one in terms ofthe PDF of x, that is x square by three in place of x we are writing square root of y. So, square root of y square by 3 multiplied by half y power minus half whichis equals to square root y by 6.

So, this expression is for thefor therange 1 to 4, sofor thus 1 to 4 what we got that this is square root of y by 6. So, this is the complete statementfor thefor the distribution of the of y that is f y of y square root of y by 3 for this zone 0to 1 and square root of y by 6 for thefor the zone 1 to 4. So, this is this is shown here. So, here also we can once check that if this if this transformation or if this density is correct then we can if we just integrate it for the entire zone that is 0 to 4; and we take this one then also this will this should equals to the unity that you can checktothethis expression a valid PDF.

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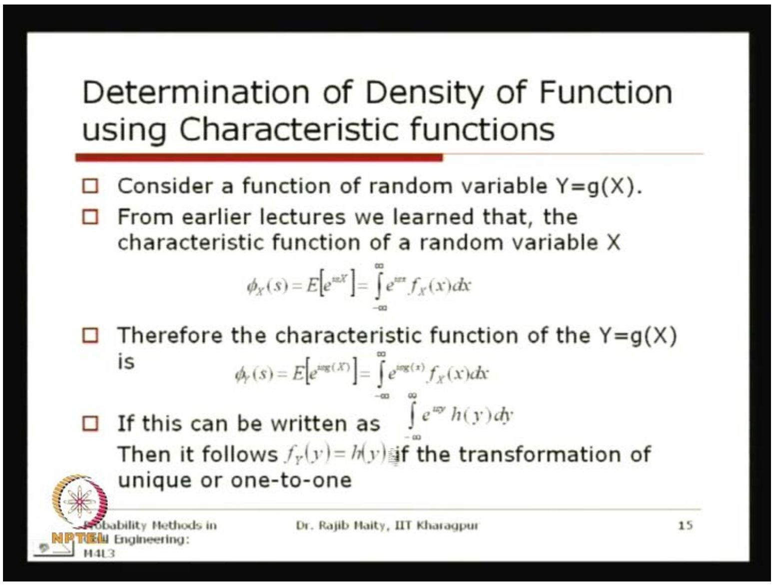

Now, we willdiscuss about theanothermethod, which is the method call that, thismethod of characteristic function. So, we will determine the relationship we determine wedetermine the relationship of the PDF of the new variable, that is function of the random variable through the characteristic function.

Now, this characteristic function we have discuss for afor avariable earlier in the earlierlecture now in this lecture easingthat propertywe will see that howthis that propertycan be utilize, can be used to find out the density of thefunction ofrandom variable.

Now, we have seen that,if there is afunction of random variable like that y is equals to g x, now for these random variable x, thatthe characteristic function of that random variable xcan be shown that, this isf xof s is equals to that expectation ofe poweris xthis I is your square root of minus one that is a complex number s is the variable, that we are using here; and this capital x is your random variable. So, expectation of this quantity is nothing but,the characteristic function that we have explain earlier.

Now, this expectation of aof a function also we have we have discuss earlier that is if. So, this can be express, that is for any function that should be multiplied by the PDF of thatrandom variable and integrate over the entire supportto get theform of thatform of that characteristic function. So, this is integrated from this minus infinity to plus infinity multiplied by e power is xmultiplied byf x xd x, sothis is the expression for thecharacteristic function; and this is obviously, for the continuesone and you can see that if it is a if it is a discreterandom variable then this integration will be replaced by the summationand this f x that is the PDF we will replace by its p m f that, we generally in standard we call that for p m f is for the discrete random randomvariable. So, this isthe thisis the characteristic function of a random variable x.

Now, now the functional relationship as you have decided that y is equals to g x.Now, for this g xnow my goal is to get the that characteristic function for this function g x, which is nothing but,equals to the another random variable y. So, if I want to get thatcharacteristic function then, that f y s should equal to that e poweris y. So, this I s y is your random variable here now this y is equals to g x, we are replacing that y in terms of g x, sothat exponential of is g x now this exponential that exponential ofi s g x this function if I take the expectation then we will get the characteristic function of that function of the random variable x, which is y, sofrom againfollowing the samethat definition of this characteristic function.

We can take this function and this is multiplied by this theprobability density function of this random variable x and if we just integrate it from minus infinity to plus infinity,then will get the characteristic function.Similarly, again if it is a discrete random variable then this can be replaced by its summation sign.

Now, again if I just one to know if we do not want to express is in terms of x, if we entirely express is in terms of y then then this expression can also be expressed as that minus infinity to plus infinity e power is y h y d y now from this two, if you just compare then it followsthat f yy isyour h y if the transformation is unique one to one. So, if we can get this relationship then, this h y that is the inversethat is that that is express in terms of here in this expression cans we can get that PDF of this y.

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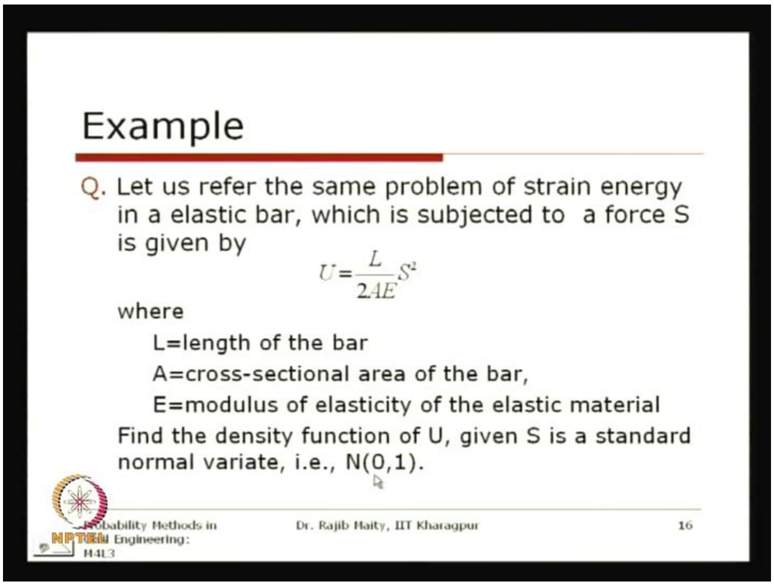

Now, to explain this one we will take onea example and this example, we have solvedearlier also but using the method ofdistribution I guess and now we have taken this same same example and we will solve this example with the method ofcharacteristic function.

So, this one that let us refer the same problem of this strain and energy the problem wason the strain energy of the elastic bar; and that strain energy isgenerallyrelated to it is force that isunder, which the elastic bar is subject to. So, the relationship between this u and this U and thiss is like this that U equals to l by 2 a epower S square now, where this l is length of the bara is the cross sectional area of the bar and e is the modulus of elasticity of the of the material.

Now, find the density of the u, given the s is a standard normal standard normal distribution standard normal variety that is the n equals toa normal de normal distributions, which is having mean 0 andstandards nequals to one now.

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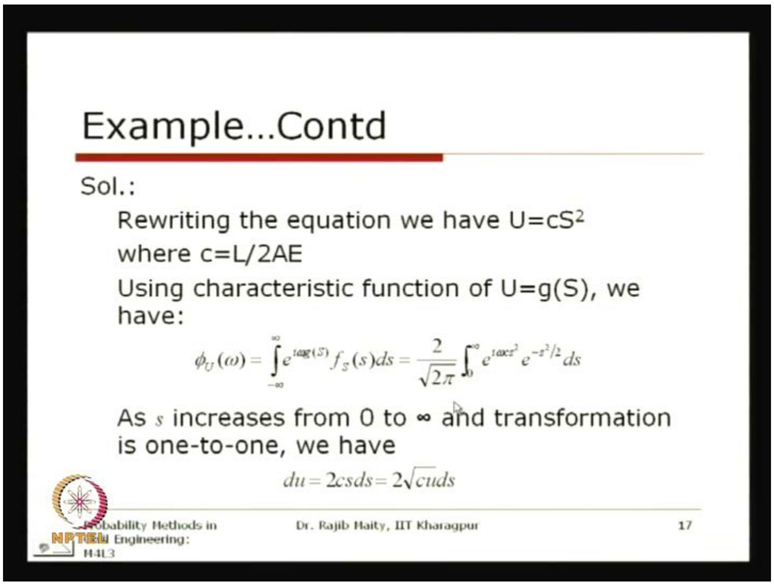

So, this one again we are just clubbing all those constant herethrough this another constant see. So, the relationship is Uis equalsto C S squareusing the characteristic functionthat is U equals tog S, we have that phi of u omega is equals to minus infinity to plus infinity I omega g s and f s isequals to f s of s d s. Now, this is the characteristic function if you justreplace this one this density weknow, which is a standard normal distribution,if you replace this onethen we will get the 2by square root of 2 pi integrate from 0to infinity. This is an even function that is why multiplied by 2 and taken from this 0to infinity e poweriomegac s square multiplied by e power minus s square by 2 d s as s increases from 0to infinity the transformation is one to one. So, we have that d u equals to 2 c s d s that is 2 c square root of c u d s.

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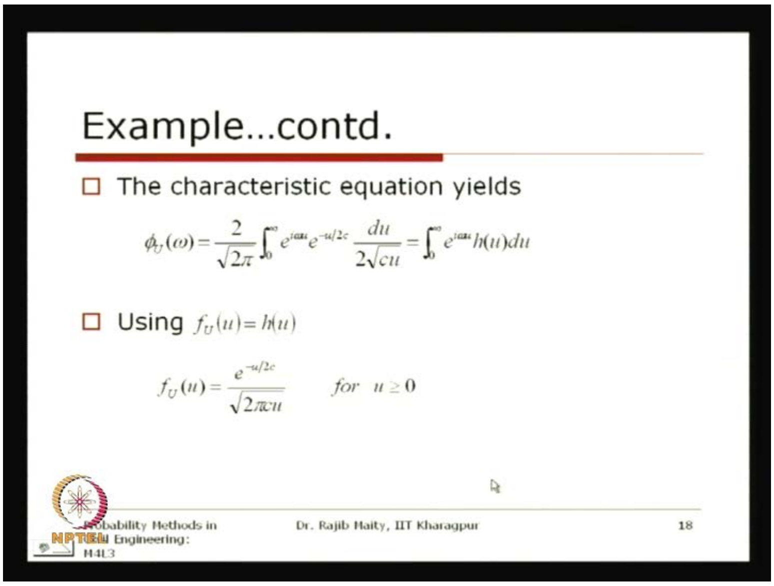

So, the characteristicequation yields that phi u omega is equals to 2 by square root 2 pi integration 0to infinity e power iomega u e power minus u by 2 cd u by 2 square root c u. So, we arewe are expressing in terms of u here, sowhich is equals to 0 to infinity e power iomega u h u d y now now if we d u.

So, now if we call if we compare then this f u c that is the density of the random variable u is equals to h u here. So, if you just compare this one it comes that f u u is e power minus u by 2 c divided by square root 2 pi c u, for this u is greater than equal to 0.So, we got the samedistribution that same PDF that you got in the earlier using the method ofdistribution. So, here also we have seen that usingthe method of characteristic function we got the samesame expression as well.

So, starting from thefundamental theorem we have discuss that method of distribution method of transformation one to one as well as not one to oneas well as not one to one then we discuss that method of characteristicsfunction with examples.

Now, one thing we have to keep in mind that even though this transformation, we have explain particularly in some of the non-linear transformation, which is very common in the civil engineering problems here; that kind of transformation may or may not always in this kind of close form salutation and they are what is more what will be useful in such cases that if we come to know some of the properties of those derived variables that is its expectation that means, we get the mean variance and it is obviously, if we get the moment generating function. Then we canwe know the required information of thosethe function that is the derived random variable. So, these things we will discuss in thenext class how we can without knowing the PDF how we can know thoseproperties of the derived random variable thank you.