Probability Methods in Civil Engineering

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Lecture No. # 11

Probability Distribution of Continuous RVS

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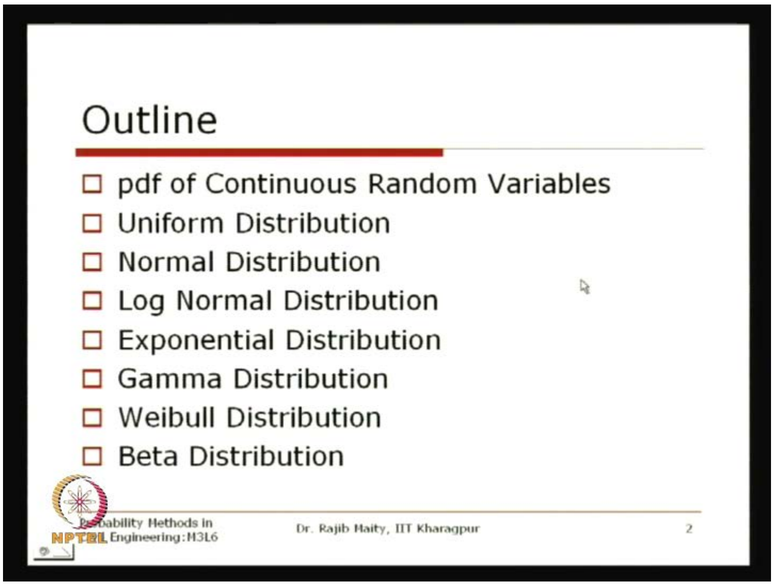

Hello and welcome to this lecture 6 ofmodule 3on random variable.In this lecture,we will cover the probability distribution of continuous random variable. In the last lecture, we covered the discrete random variables.Similarly,for somecontinuous random variable, there aresome standard probability distribution functions are there, which arevery widely used in different problems in civil engineering.We will see those distributionsmaybe in today's class as well as next class we will continue the discussion through differentprobability distributions for continuous random variables.

So, at the startingwe will quickly recapitulate the pdf that is probability density function, you know that for the discrete random variablewhat we refer is the probability density function. So, the probability density function and their requirementfor theto be a valid pdf for the continuous random variable.We will see that very quickly as this was we discussed in the earlier lectures as well; and after that we will go through the differentdistribution function.To list it,we will start with the uniform distribution,then we will see the normal distribution, log normal distribution, exponential distribution, gamma distribution,Weibulldistribution and beta distribution.

Thelist of this distribution may not be exhausted, but these are the distribution, which are generally and widely used for different problems in civil engineering.So, we will cover them 1 after anotherand we will just show there for whichtype of problems,which distributions will be most suitable and that is generally, decided based on their setFor example, when we talk about the uniform distribution this is generally, boundedfrom the lower side as well as upper side and in between that the density is uniform.Similarly, the normal distribution isthe supportof normal distribution is a spanning from the minus infinity to plus infinity.

So, which the entire range of the real axis and log normal distribution, exponential distribution and gamma distribution generally, are lower bounded have been some lower bound. And thesebound is generally, at the origin that is 0 and then we will see the weibull distribution, beta distributions this beta distribution is again,is bounded distribution.And we will see the differentpossible application of this 1 in different civil engineering problem.In this lecture and as well as,in next lecture that is in this module, what we will discuss, is their basic properties of this distribution andwhat are their characteristics.

Applications of thisany specific distribution to some specific problem of civil engineering we will be discuss, in and the subsequent modulesmost probably it is in module 5.So, we will start with this very brief recapitulation ofprobability density function for the continuous random variable.

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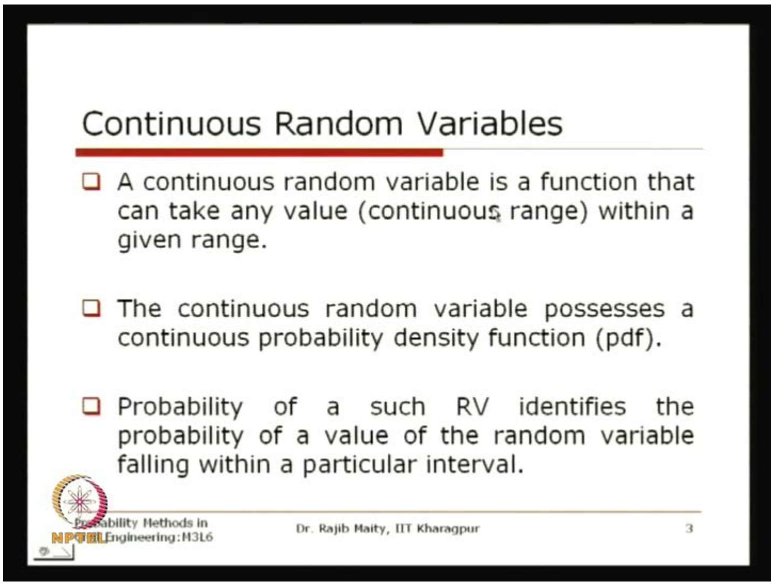

So, as youdiscuss, in the earlier classes earlier lectures that a continuous random variable is a function that can take anyvalue in the sense of the continuous range with in the given range of that random variable.The continuous random variable possess a continuousprobability density function. So, this continuous probability density function we know thatover the range of this possible range of this random variable over thatrange this function is continuous and; obviously, some properties should be satisfied by thisby this function to be a valid pdf.Now, probability of such random variables identifies the probability of a value of the random variable falling within a particular interval.

So, this is basically, the difference between the discrete and the continuous random variable here as we are talking the function as a density functions. So, at a particular point at a particular value of this random variable, if we see thepdfthen what isit is giving is the density.Now, when you are talking about a small interval and small range of this one, then that area below thatpdfis the probability. So, thethat is why the probabilitythough what it identifies is the probabilityof the random variable falling within the particular interval. So, one small interval is needed to define what is the probability for thatrandom variable falling within that particular interval.

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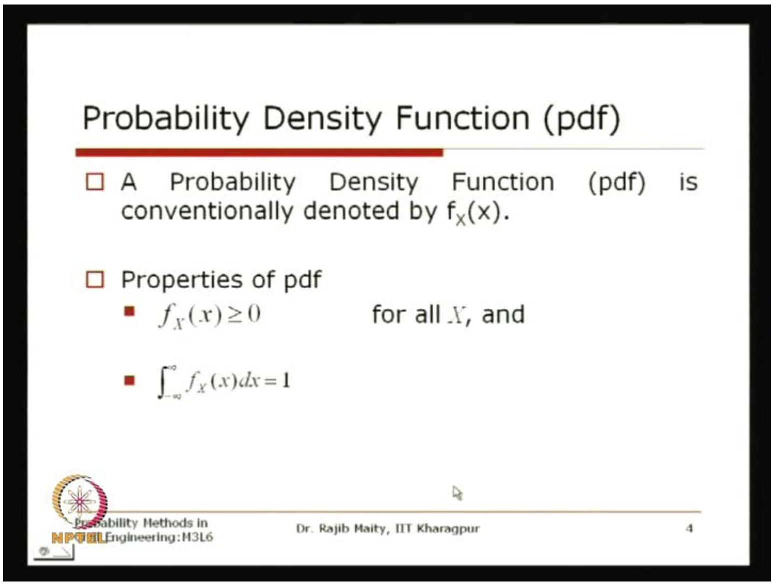

Then the these, are the properties that should be followed to be a validpdfthat is a probability density functionpdfis conventionallyas we have seen earlier that is it is denoted byf X X, where this is the capital x, which is denotingthat random variable this is small x, which is denoting that dummy variable are theparticular value of that random variable. So, there are two properties that it should follow we know that at any for all values of all feasible values ofthis x this value ofthis function should be greater than equal to 0. And it should integrate to unity tosatisfythat second axiomthe total probabilitywithin theall possiblethat feasible over the feasible range of this random variable, which is known as the support of the random variable over this support it should be equal to equal to 1.So, withthis now we will seewe will go throughdifferent probability distribution function.

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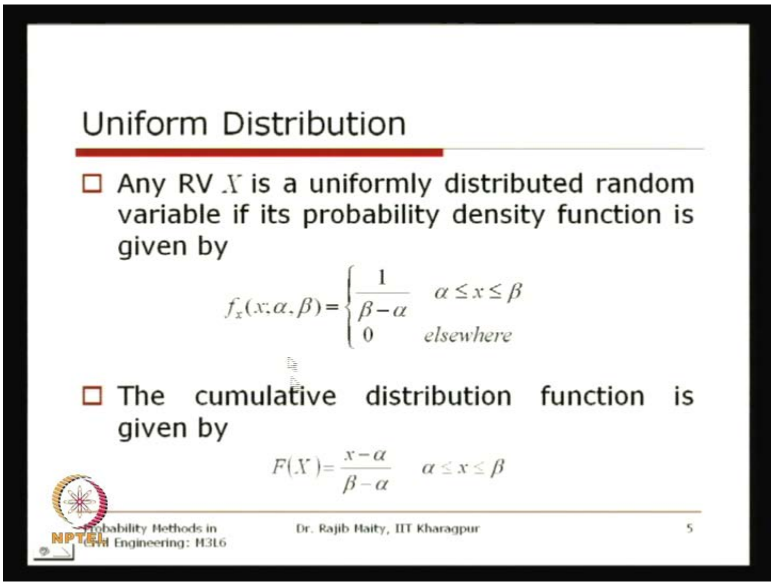

Andfirst we will start with the uniform distribution. So, any random variable Xis uniformly distributed random variable, if it is probability density function is given by this equation that is F Xthis will becapital X F X with this variable x and thisalpha beta or any such thing, which isshownhere are generally, the parameter ofthe distribution. So, here the parameters arealpha and beta. So, with this parameter thisdistribution is expressed by 1 by beta minusalpha for the range,when this x lies betweenalpha and beta,that is the lower limit isalpha and upperlimit is betaoutside this range anywhere, the value of this function is equals to 0.Now, if we see thisfunction then before we call that this is a probability density function we have to see that whether the those two properties are followed or not.

So, we see that as this beta is greater thanalpha. So, this quantity is greater than 0 always and in the outside this region this is equal to 0. So, the first property that is it should be greater than equal to 0 is satisfied and; obviously, this 1 by beta minusalpha, if we now integrate it over thisalpha to beta. So, this range from this beta toalpha, which is again,this beta minus beta minus alpha this will be equal to 1.So, this is the complete form of this uniformly distributeduniform distribution and we know that, if we say that the cumulative distribution function will be given by this x minusalpha divided by beta minusalpha.

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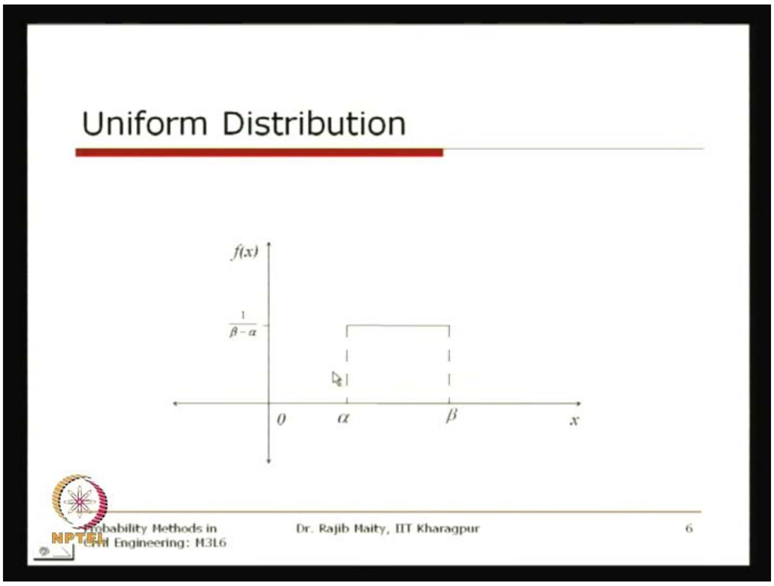

Here the diagram or the how this distribution looks like is shownhere this is youralpha limit and this yourbetalimit.

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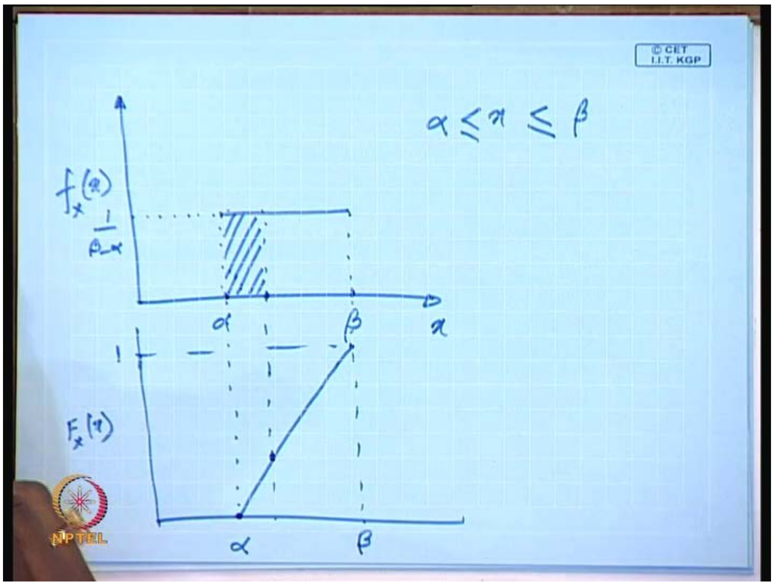

Now, if you just see it once here that now this is what weare calling as thisalpha and this what we are calling as beta and now, this side is your that value of that of that functionf x and this is your the x. So, here the possible range is betweenalpha and beta and this is aclose boundary,close boundary means, thatless than equal tosign is shown. So, this is the close boundary; that meansit is inclusive of these two values. So, add this value the probability is 1 by from this point to this point this probability, if I havejust shown as aindicative line is 1 by beta minusalpha. So, from this range to this rangethe density is uniform all over this all over this region that is why the name isuniform distributionand just for this reference we are just noting thesetwo lines from thisalpha to beta.

Now, when we are talking about what is it is cumulative distribution; that means, any value with in this range up to the x we have to calculate, what is the total area up to that point as we have seen in theearlier description as well from for the cdf?So, at this point Ihave topoint that what is the value for that region in this. So, this point when you are talking about this cumulative distribution functionthis point implies that the total area covered up to that point. So, obviously, the total area covered atalpha is nothing 0. So, thisshould corresponds to this point and when it is in this way, if we justmove this x up to this beta; obviously, from this secondpropertyit should go it should touch the values of total area will be equals to 1.

So, this value should come as 1 and as this is uniformthis should be a straight lines starting from 0 atalpha and 1 at beta. So, this will be your c d fthat iscumulative distributionfunction for uniformdistribution.So, which is here shown here we can see from thisthat cumulative distribution function that is x minus alpha by beta minus alpha.Now, if you put that x is equals to alpha here then you are getting it here to be 0 and if you put that x equals to beta then you are getting this value as to be here1. So, which is shown in this diagram as well?

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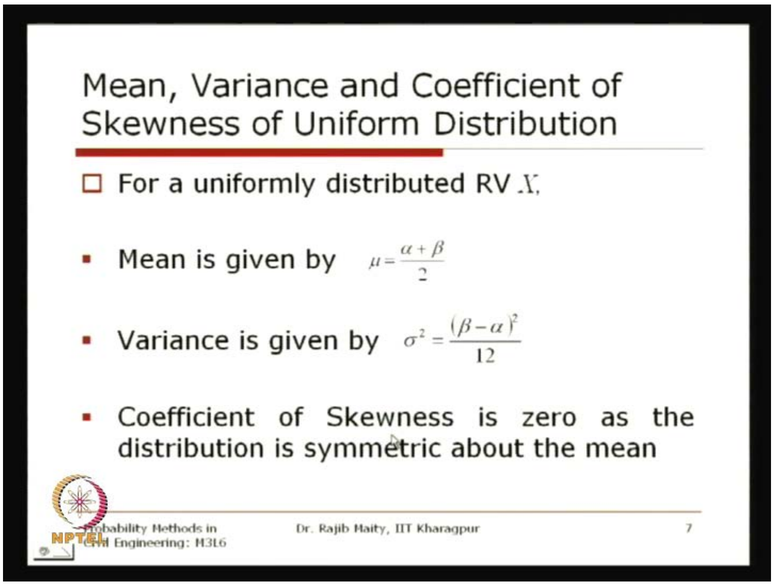

So, this is thatdensity that is the density function now,the from differentinitialparameters that we have discuss,in the earlier lectures that is mean variance and coefficient. If we calculate these things for the uniform distribution is comes as the that ismean is given bymuequals to alpha plus beta by 2, which is obvious from this diagram as well. So, that mean value of this one as this distribution is as these distribution is uniform somid mean valueshould be equal to should be at the midpoint of this two ranges. So, that midpoint is nothing but youralpha plus beta by 2.

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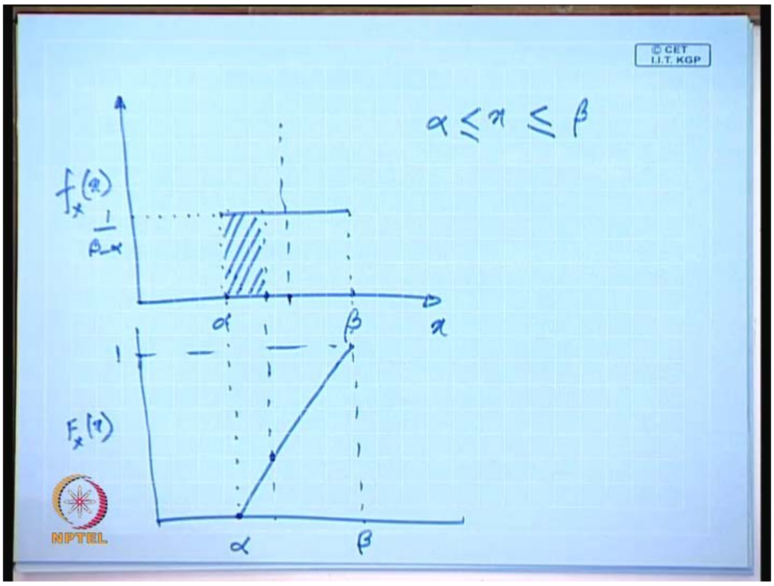

Now,before coming to this variance, if Isee thecoefficient of Skewness then coefficient of Skewness,if we see then we know that, which is the measure of the symmetry with respect to the with respect to it is mean. So, we see that this is also symmetric with respect to the mean that is why Skewnessalso will be 0.So, this issomean, is given by this average value of this lower and upper bound of that distribution and the coefficient of Skewness is 0. As the distribution is symmetric about the mean and variance also theequationthat we use that it should befrom the mean and if we take the square and then wemultiply it with that probability density function and integrate over the entire support. Then we get that the variance should equals to beta minus alpha whole square divided by 12. So, this is your variance and; obviously, Skewness is 0 as discussed.

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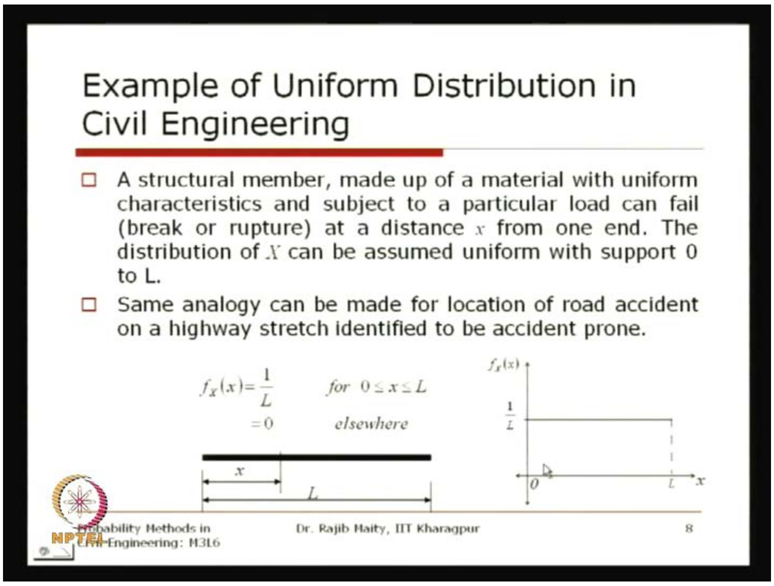

Now,any specific civil engineering applications sometimes, if we see thenif we take the example of a structuralmember, which is made up of the made up of a material, which is a uniform characteristics all over and it is subject to a particular loading condition, then that structural member can failfailing. Means, it can break or rupture at a distance x from the 1 end.Now, the distribution of that Xcan be assumed to be uniform with the support 0 to L now, if this 1.So, this is the structural member now the location of the failure from the 1 end of thatmember that, if it take that particulardistance from 1 end is the random variable then it can we canit can happen anywhere. So, the probability can the that distribution of the probability for this random variable capital x can beassumed to be uniform and this is equals to 1 by L.

So, and this is valid from the 0 to the entire length of thisstructural member. So, that is why this is 1 by L and it is 0 a 0 elsewhere now, if we draw it ispdfit looks like this that is from 0 to L it is uniformly distributed, which is equals to 1 by L.Considering the fact that the total areabelow this curve below thispdfshould be equals to 1.Now, this analogy this example can be analogously can be extended to the other examples like thatroad accident on a highway stretch identified to be the accident prone now, if the total length of this road is equals to 0. So, that location of this accident at can happen any point over this stretch. So, that is also can be followed as a uniform distribution.Now, we will showto start with we started that a uniform distribution, which iseasy in the sense from this mathematical concept.

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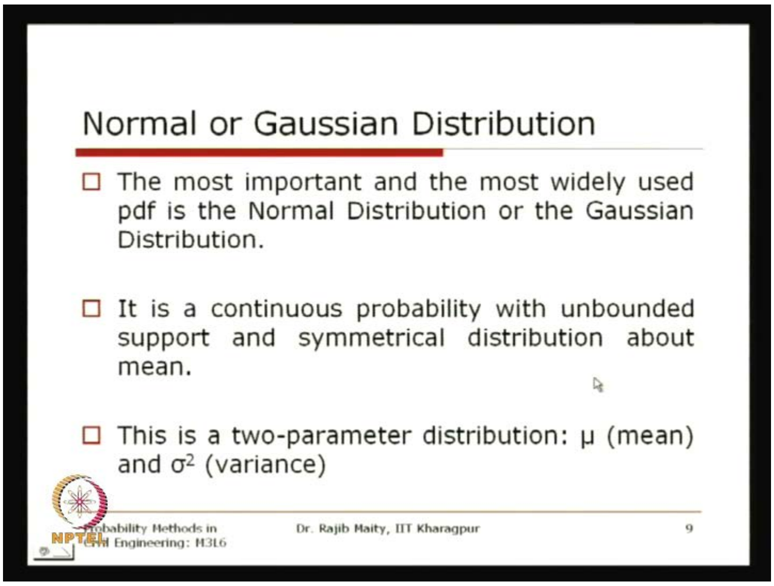

And we will proceed now to the normal or the Gaussian distribution and this isthis normal and Gaussian distribution is one of the most important distribution in every field includingcivil engineering. And mostmany applications has found tobeis very useful this normal distribution is useful for many applications in different research field.

So, that is why this normal or the Gaussian distribution is most popular distribution among theallcontinuousprobability distribution and this is a continuous probability distribution with unbounded support.

Unbounded support means, mathematically it can take the values from minus infinity to plus infinity and it is symmetrical distribution about the mean. So, these two properties are there andthis is a two parameter distributionagain, that isthemean is mu and variance is sigma square for the uniform distribution. Also we have shown that there are two parametersalpha and beta, which are the basically,the bound for the distribution here also there are two distribution, one is thismu and another one is the sigma square. We will see how the distribution looks like and it is different propertiesin the success in this that.

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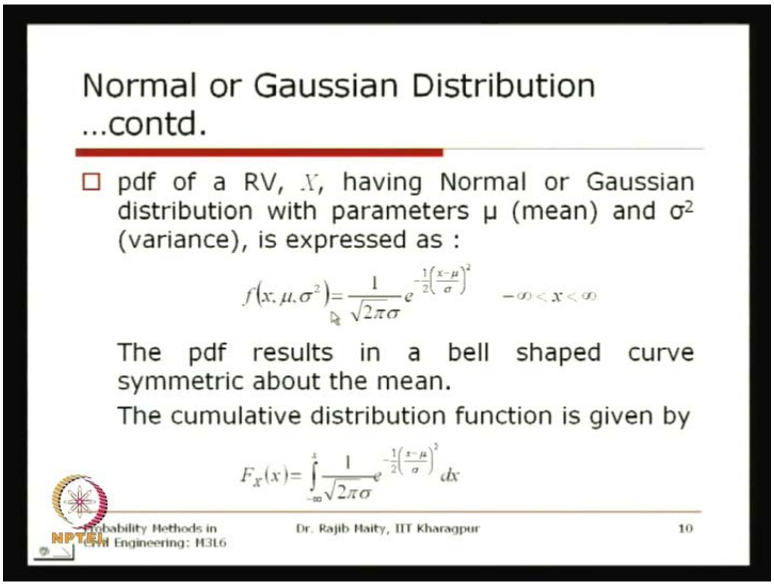

This is thepdffor this normal distribution. So,pdfof the random variable Xhaving normal or Gaussian distribution with the parameter mu andother parameter is sigma square. It is expressed as that F x x with this two parameter equals to 1 by square root of 2 pi sigma remember that this sigma is outside this square root, if it is within; obviously, 1 square will come here multiplied by exponential of minus half x minus mu by sigma whole square. And this support for this x isfrom minus infinity to plus infinity as just now discuss. So, thispdfresults in a bell slap bell shaped curve and which issymmetric about the mean in the earlier cases also, if you see that this distribution generallylooks like this.

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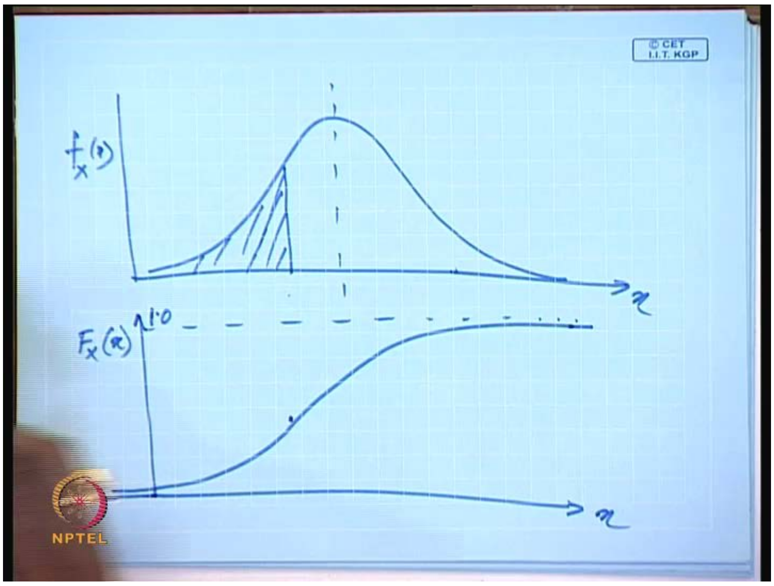

So, this distributiongenerally, looks like this sothis ishavingit ismean and; obviously, the mean mode and median are same here and this is with respect to this is known as the bell shaped curve and this is symmetric with respect to thismean. And if you see again, if you see it is cumulative density function then; obviously, we can say that it starts from this minus infinity andgo up to thisplus infinity. So, if we take that any particular point here and total area,if we calculate and put it some value here then this c d fthat is which is yourpdfand this is your c d fand this generally, goes and become again,it touch this 1 at infinity plus infinity it is asymptotic to this line one and this is asymptotic to the line 0 at minus infinity.

So, this is thehow this cumulativedistribution looks like and this is how thepdflooks like for onenormalor Gaussian distribution.So, mathematically we know that toget that cumulative distribution function we have to integrate from this left hand support that is minus infinity to it can go up to x and this integrationfrom this minus infinity to xof thispdfthis integration of this one from this it will give you thecdf.Now, this integration is difficult and we will see that howthis is over come through this numerical integration and with the available chart that we will discuss in a minute.

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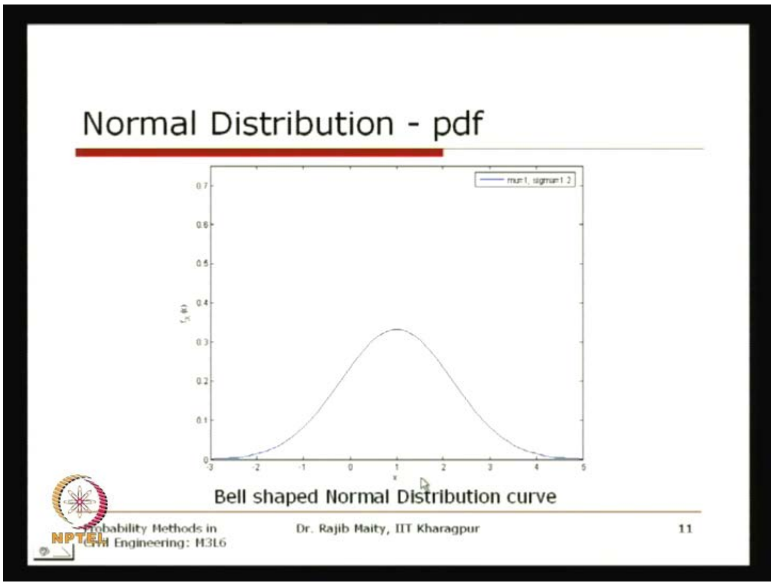

Before that we willshow this the example of this bell shaped curve that it isthat we have shown is the this bell shaped curvethe parameterfor this one this curveis that mu equals to 1 and sigma is equals to 1.2. So, as this parameter mu is equals to 1 you can see that it is the maximum density is at this point 1. So, this mu is basically, the location parameter where the maximum density is located. So, that is signifiedby this mu and sigma also it is generally, showing the spread over this mean and we will show this onein the examples how this things can affect the shape of thisshape of this curve.

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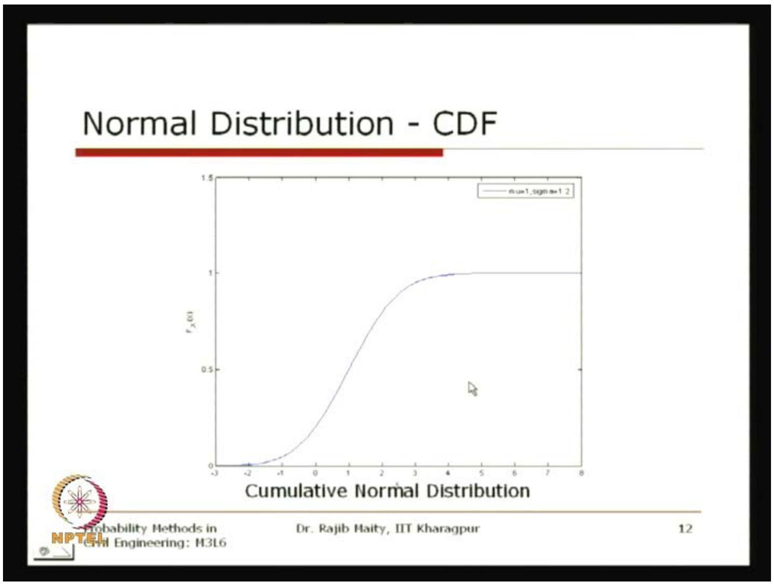

And this is your for the samepdfwith mu equals to 1 and sigma equals 2, if we calculate what should be that cumulative distribution function that isc d fthen it looks like this. And this line you can see that this is asymptotic to 1 at plus infinity and this one towards the left it is the asymptotic to 0 towards minus infinity.

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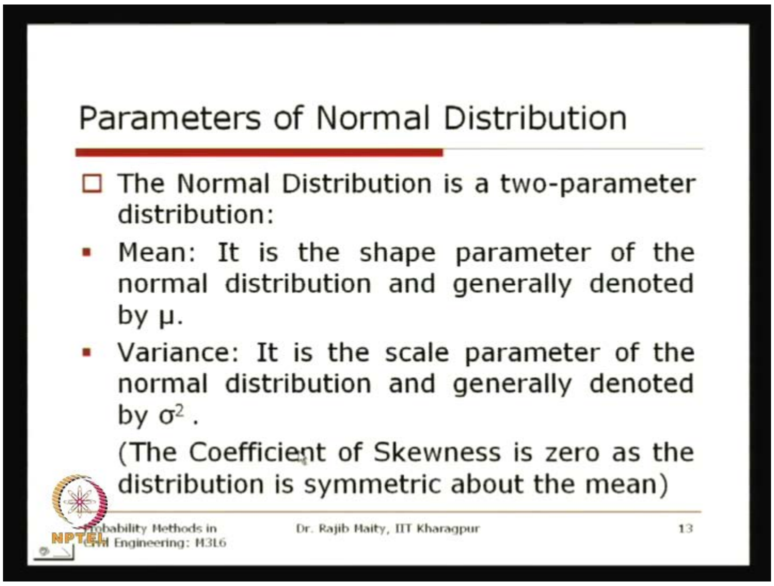

Now, this parameters of thisnormal distribution as wediscuss, that this normal distribution is a two parameter distribution. So, the first parameter is yourmean, it is the shape parameter of this normal distributionand it generally denoted by this mu. And the second parameter is the variance,which is the scale parameter of the normal distribution and this is generally, denoted by sigma square.Now, the coefficient of Skewness is again, 0 similar to the uniform distribution what wediscuss, earlier this is 0, because this isthis distribution is also symmetric about the mean that is why this Skewnessis 0.

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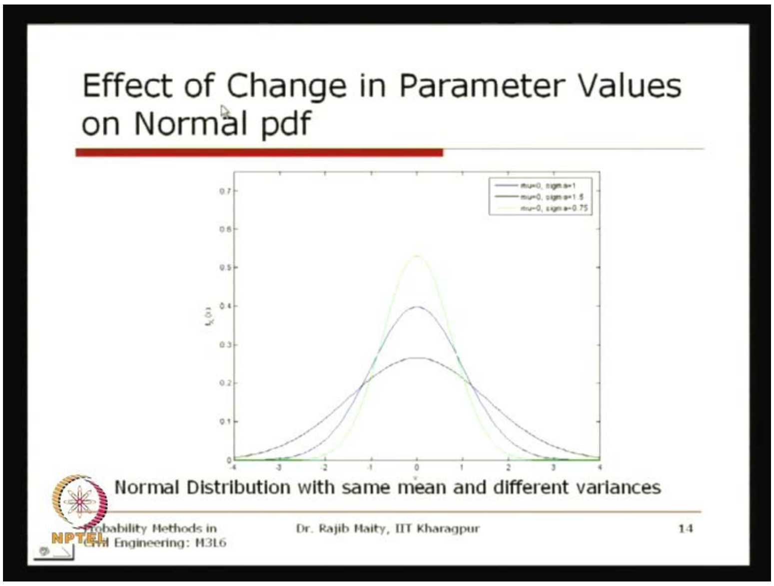

Now, if you see the effect of change of this parameter as we was talking that effect of change in this parameter value on this normal pdf, then it looks like this. We have plotted here three different normal distributions, thepdffor the normal distribution with different parameters. Thethis the blue line that is the middle one, if you see this is for thethis is for the similar value 1.5 for all these three distribution that is shown here having the same mean mu equals to 0. So, that is why for all this distribution you can see that maximumdensity is concentrated at x equals to 0.

Now, this blue curve thisblue one is having the sigma value equals to 1.5whereas, this green one is having the sigma value is 0.7 5 and this black one is having the sigma value is equals to 1.Now, you see for this black one the sigma value is theis1.5, which is the maximum,which is maximum here. So, theso, this is that is why keeping the mean same for all three distribution.This is more spread the spread is more about it is mean andso, it is reflected from it isvalue of this parameter sigma, which is 1.5.Similarly, for this green one thesigma value is the minimum and which is 0.7 5 that is why it isthis spread about the mean is the minimum most among thisthree distribution curve. So, this sigma generally, controls the spread about the mean, which is reflected from this background.

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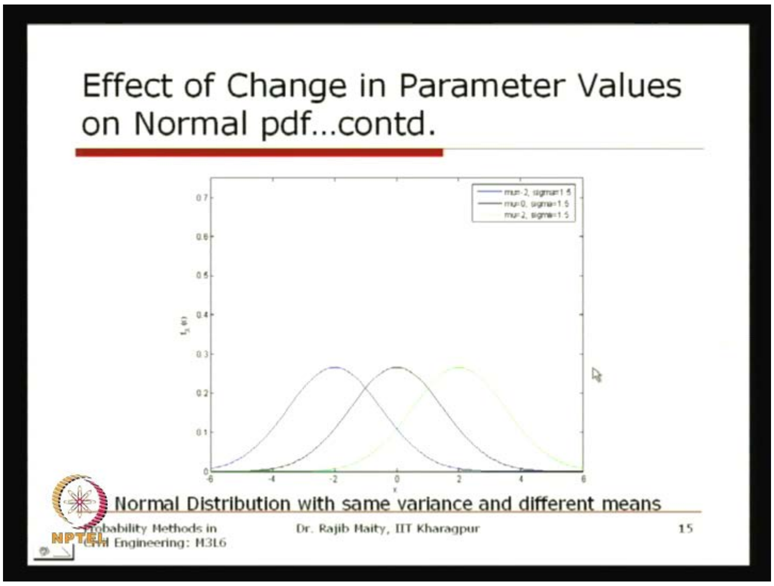

Now, the effect of change in the parameter value the second parameter now here we are taking this is that a mu.Now,in this three plot again, what we have kept same is that sigma now for all this three curves sigma is equals to 1.5.Now,so, as this sigma is same then you can see the this spread about the spread about the mean the respective mean is same for all three curves.Now, as wehave change for this blue one mu is minus 2 for the black one mu is 0 and for green one mu is 2. So, you can see that this is generally, shifted from the one location to anotherlocation for blue one it is it is a centered atminus 2 for black one it is at 0 and for green one it is at 2. So, thatso, where this it is centeredsothat is controlled by this parameter mu.

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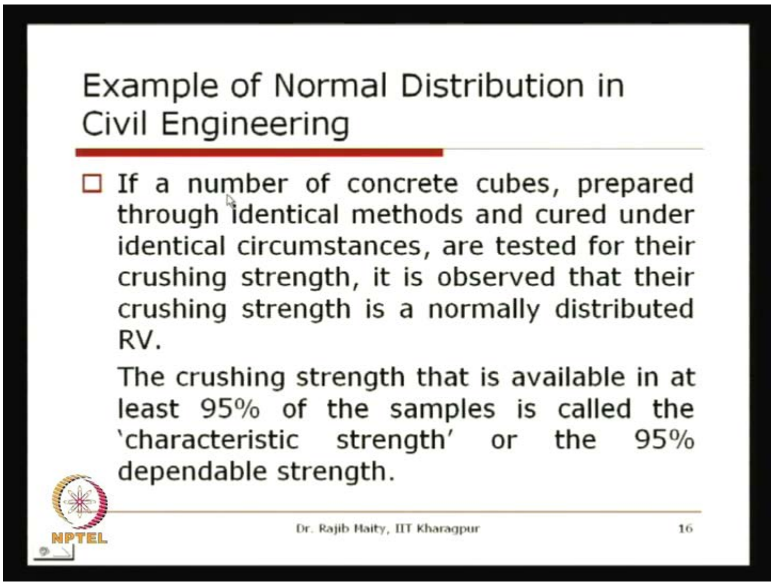

Now,theif we take some standard example of this normaldistribution in civil engineering, if a number of concrete cubes. So,one example, is thatstrength of this concrete, if a number ofconcrete cube prepared through the identical methods and cured under the identical circumstances are tested for their crushing strength it is observed that their crushing strength is ais a normally distributed random variable.Now, the crushing strength that is available in at least 95 percent of the sampleis called the characteristic strength of the 95 percent dependable strength.

You know that example of the characteristic strengthin the for thestrength of the concrete is that it is that in the 95 percent cases weif wesee that the particular strength of that cube is exceeded that isgenerally, denoted by this crushing strength.Now, that how we can say that this is the 95 percent cases it is exceeded. So, it is ifgenerally, found to follow a normal distribution keeping it is mean. And we considered thatstrength, where it should be exceeded at the 95 percent cases to designate that particular strength to be the characteristics strength of the concreteof the particular concrete. So, this is how we define that characteristics strength of the concretecube.

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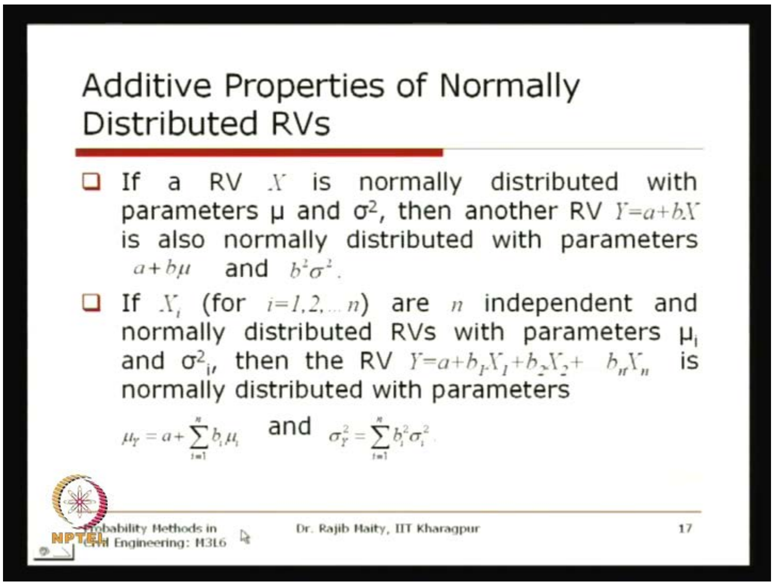

Now, there are somenice properties of this normally, distributed random variable, whichis known as this additive property, if a random variable Xis normally, distributed with it is parameter mu and sigma. We discuss the normal distribution having two parametersmu and sigma then, if we get another random variablethe which is related to this earlier one that is X in the through this equation that is Y is equals to a plus b X that is the random variable is multiplied by b and added witha thenthis Y is also a normally, this y. We will also the normally, distributed; however, it is parameters will change like thisthat it is the first parameter in case, of this mu it will be a plus b mu and it is second parameter,whichis sigma square is equals to b square sigma square.

So, while getting this new parameter what we are doing is that we are justputting the mean value that is thatfirst parameter value here and getting the mean for thismu random variable. And when we are talking about the variance that is the spread around the mean then the constant term thatwas adding that isnot that is not affecting, but what is affecting is by it is multiplying coefficientand it should be squared. So, that is b square multiplied by this sigma square.Similarly, if there are n numbers of suchnormally, distributedrandom variable and if we can say that these are independent to each other than, if you create another new random variable, which is Y is equals to a plus b 1 multiplied by X 1 b 2 multiplied by x 2similarly, up to b n multiplied by X n then this Y will also be a normally, distributed and it is parameterwill be.

So, we will just put the individualmean of this random variables to get the mean for this y which is. So, the mu y is equals toa plus summationof b Imu i. So, a plus b 1 mu 1 plus b 2 mu 2 extra up to b n mu n and for this sigma square it should be the coefficient for those random variable that square times their individualvariance and their summation up should give this 1.This second property generally, leading to the central limit theorem and that we will discuss, in the subsequentlectures we will see that we can relax the requirement of this normal distribution for thisrandom variable, if these are simply, if they are independent and identically distributed itself.

We can say that this Y will have this normal distribution, which is theresult of the central limit theorem will be discussed later. And the first property when we are talking about that is the Y is equals to a plus b X this y can be treated as the function of X, which is thewhich will be discuss, in greater detail in the next module where we will discuss about that functions of random variable. So, while discussing the functions of the random variable we will know, if we know theproperties that parameters for onethis by one random variable how to get the parameters for the or the distribution as well as parameters as well as distribution for it is function. So, this normal distribution is one example that we have shown here in while, discussing the functions of random variable this will be discussingin a general way irrespective of this of any particular distribution of the original random variable.

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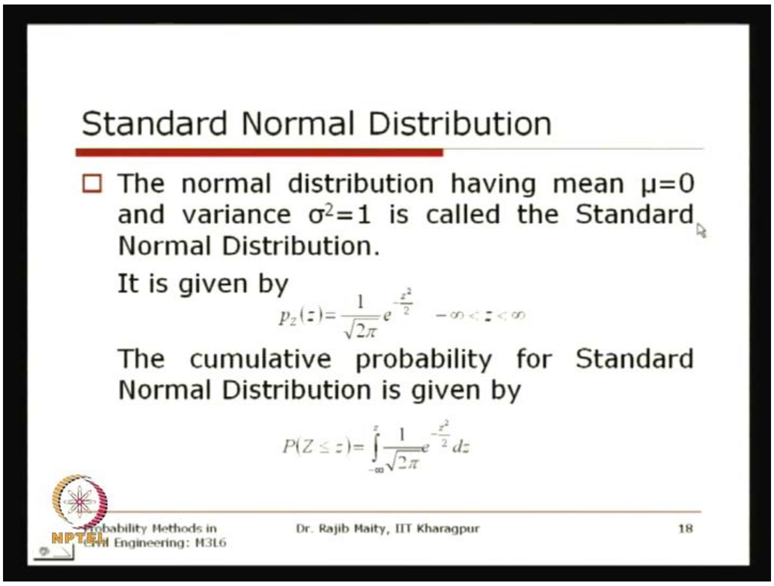

Now, anotherdistribution, which is derived from this normal distribution, is known as the standard normal distribution. This standard normal distributionwhen a normal distribution is having it is mean mu equals to 0 and variance sigma square equals to 1 then this particularnormal distribution with thisspecific values of this parameters is known as the standard normal distribution.So, in this original distributional form that is what we have shown it earlier that this distribution, if you just put mu equals to 0 and sigma equals to 1 what the distribution form we will get that will be the standard normal distribution instead of using x there we are using z as the dummy variable.

So, this isyour standard normal distribution tocontinue with our samenotation this is also as continuous instead of p this will be f andthis is the capital Z and this is the small z. So, this is specific value and this is the random variable. So, whose distribution is 1 by square root 2 pi exponential minus z square by 2and; obviously, the z is having the support from minus infinity to plus infinity. So, this is also normal distribution, which is having the mean mu and variance 1.The cumulative distribution for thisstandard normal distribution again, can be found out from integrating it from this leftsupport to the specific value z, which is 1 by square root 2 pi exponential minus z square by 2.

So, integrating from this left support to the particular value x and this is also will be the capital F and this z less than equals space particular value z, which is giving you the cumulative probability distribution. Now, we should know, what is the use of this distribution and why we need this specific distribution with this specific parameter is that this integration whenever,we are talking aboutthis integration, which is important to calculate the probability it is notthisintegration cannot be done in the close form. So, we have to go for some numerical integration and the numerically integrated values are available for this standard normal distribution.Now, for any general normal distribution we can convert it first to the standard normal distribution and get it is desired probability and that we will discuss in a minute.

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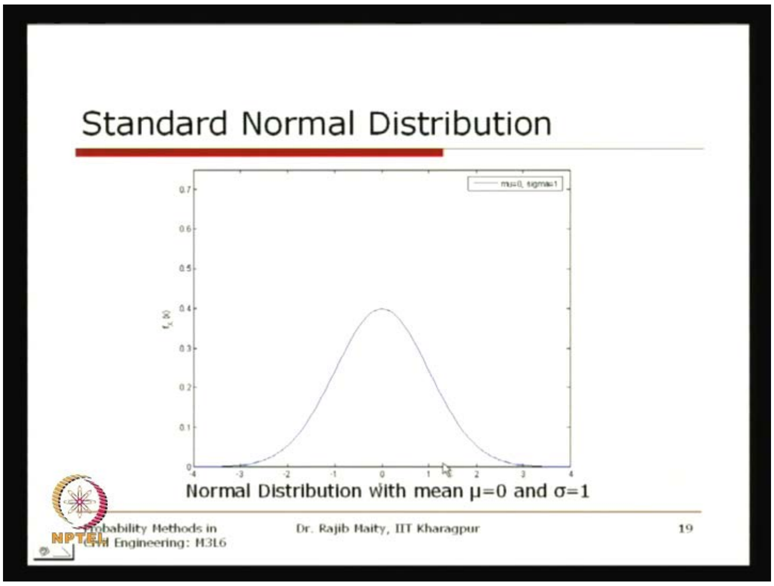

So, before that this is how this distribution that ispdfprobability density function looks like and as you know that it is mean is equals to 0 and sigma is equals to 1. So, that is why it is centered at 0? Basically, itsupport is again that from minus infinity to plus infinity the, but one thing you can see that most of that probabilities is concentrated between minus 3 to plus 3.

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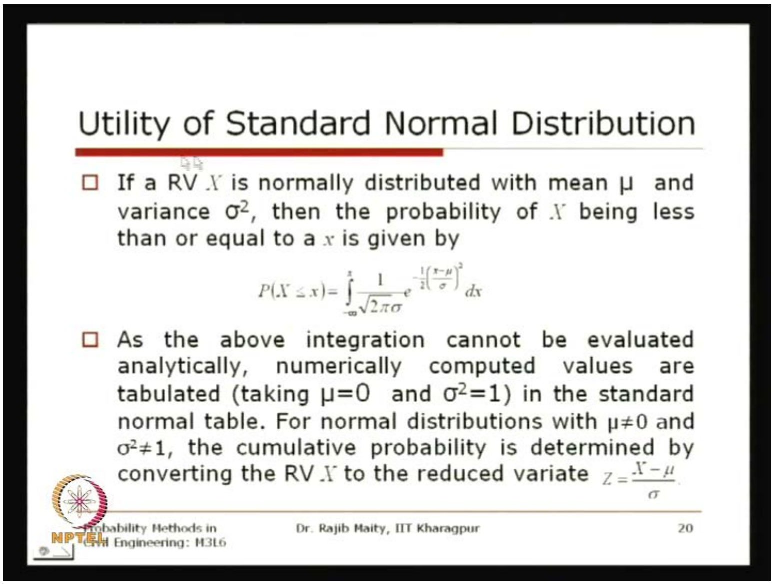

Now, while the thingthat we are discussingwhy we need this standard normal distribution is as follows, if a random variable Xis normally, distributed with mean mu and the variance sigma square then the probability of X being less than or equal to x is given by this particular distribution.Now, as the above integration cannot be evaluated analytically as we discuss just now the numerically computed values are tabulated taking this mu equals to 0 and sigma equals to 1 that is the standard normal distribution. So, for the standard normal distribution these valuesare numerically computed and listed.

Now, for all other normal distribution with any values of this parameter that this if this mu is not equal to 0 and this sigma square is not equal to 1,then the cumulative probabilities can be determined by converting this x to it is reduce variate. So, how it is converted that is this X that particular random variable is deductedfrom it is mean that is the whatever, the value mean is there and it is divided by sigma. So, we get in new random variable which is Z.

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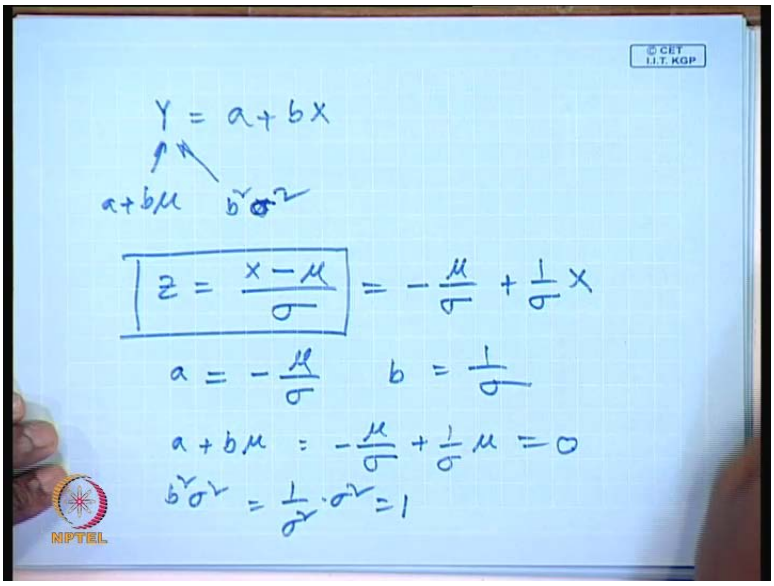

Now, we have seen just now in a minute we have seen that if that y is equals to a plus b X just few earlier, if this is normal distribution we have seen that it is mean the mean of this y is a plus b mu and it is variance is b squaresorry b square sigma square.Now, what we are doing here the conversion is thus z is equals to X minus muby sigma, if Ijust write in that form that is mu by sigma plus 1 by sigma X. So, here we have a is equals to minus mu by sigma and here b is equals to 1 bysigma. So, thatthe mean for this zwill be a plus b mu, if Itake it from here a plus b mu and if Iput this value is equals to minus mu by sigma plus this 1 by this b is 1 by sigma multiplied by mu, which is 0 and for this variance for this z is b square sigma square, which is again this b square is 1 by sigma square multiplied by sigma square equals to 1.

So, we have converted it in suchway that isthat is that is the mean for this new random variable is 0 andvariance for this new random variable becomes 1. So, through this conversion of any normally, distributedrandom variable we can generate a another new random variable, which again,normal distribution with mean 0 and standard deviation 1 and this 1 this conversion is irrespective ofany specific value of mu and sigma thatthat original random variable is having.

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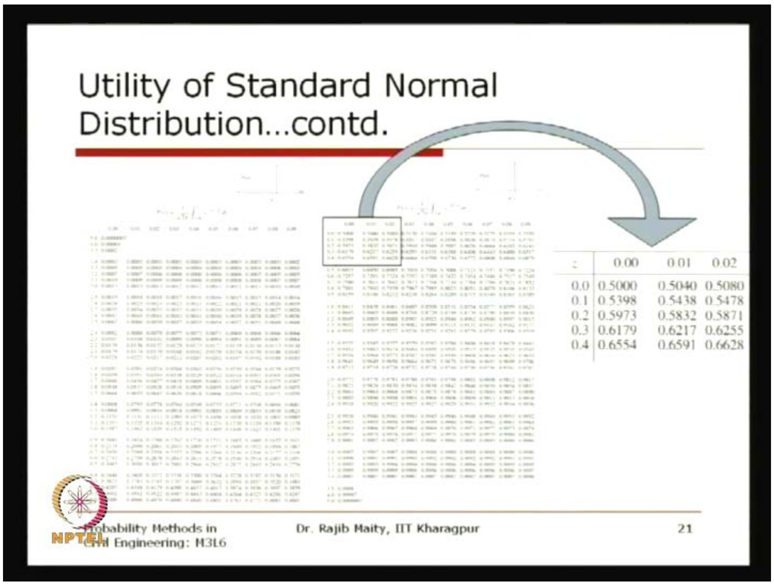

So, once we can convert this one then we can use thisstandardnormal distribution chart, which is generally,looks like this. So, this is your thatpdfof this normal distribution and this cumulative values are listed as followsthat is for any specific value of this z this is basically,started from minus 5 and goes like this and coming and going up to 5. And we know that from this from thiseffectively from minus 3 to plus 3 itself most of the probabilities areexhausted. So, if Ijust zoom it here then Ican see for the value when this z value is at 0 that is adjust at thus at the mean we know that the total probability covered due to the property of a symmetry should be equals to point 5, which is shown here.

Now, as we are forany value suppose that if Itake that point 0.21. So, this is your point 2 andthis is a second decimal 0.21. So, up to point z equals to up to 0. 21 the probability covered is0.5832.Now, we willsee oneexample, how to calculate the probability for the for any normal any normal distribution that we will see. So, this is how we have to read this standardnormal distribution and these tables are generally, available in any standard text book.

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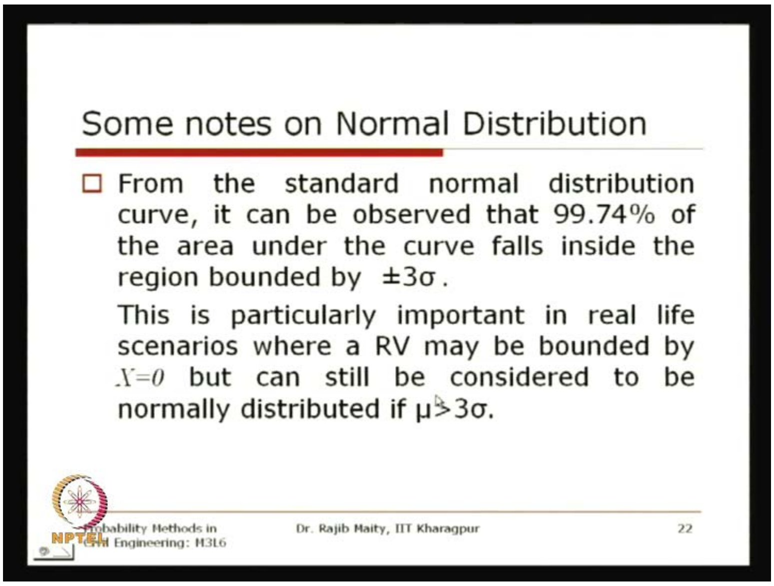

So, what we have found from thisdistribution is that from the standard normal distribution curve it can be observed that this 99.74 percent of the area under the curve falls inside the region bounded by plus minus 3 sigma. So, for the standard normal distribution what we saw that sigma equals to 1sofrom this minus 3 to plus 3. So, this much this much probability, which is almost closed one is already covered in that. This is particularly important in the real life scenario, where a random variable may be boundedby X equals to 0, but can stillconsidered to be normally, distributed if mu is greater than three sigma.

So, in the real life sometimes we can come across to the situation that those random variables are effectively alower bounded by 0, but if we see that it is mean is away from the origin with a magnitude of three sigma. Then we can once see that whether, that can also be considered to be a normal distribution as we know thatbelow this 0 meansleft side of this 0. So, towards a negative value effectively the probability is 0.

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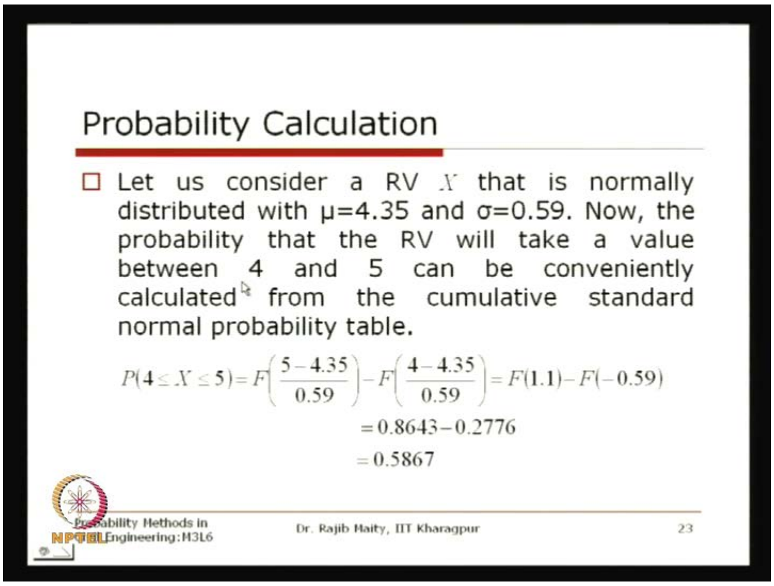

Now, if we just want to do one small exercise how to calculate the probability using this standard normaldistribution let us consider a random variable Xthat is normally, distributed andonce we say that this is normal distributed we have this specified this parameter. So, withmean equals to 4 point 3 5 and sigma is equals to 0.9.So, now, if we look for theprobability that this random variable will take a value between 4 and 5 then this can be calculated from this cumulative get standard normal probability table how. So, our intention to the probability of X line between 5 to 4 should be equals to first what we are doing is that we are we are converting this two limit that is the 5 and 4 this we are converting to it is reducedvariate.

So, how to convert it to the reducedvariate that particular value minus it is mean divided by sigma. So, if we are we reduce itthe upper that upper limit corresponding reducedvariate lower limitcorresponding reducedvariate. So, this is again, 4 minus mean divided by sigma now this value ends up to the 1.1 and this is minus 0. 59.Now, this one as these are reducedvariate. So, this is havingthe mean is equals to 0 andvariance is equals to 1. So, this we can read from this standard normal distribution for 1.1. So, if you refer to this chart then we can see that what this 1.1.

So, this cell, if we can read it that is 0.4838. So, what is meant is that starting from here up to 1.1 total area covered is 0. 8438. So, this is 86 4 3maybe this will be 8 4. So, this might be a mistake that is this might be that 8 4. So, this is how we read thisprobability for a particular value ofthe standard normal distribution. And similarly, we can read this value for the otherlimit and we can deduct thisprobability to get this value. So, what is actually, here is donegraphically is like this.

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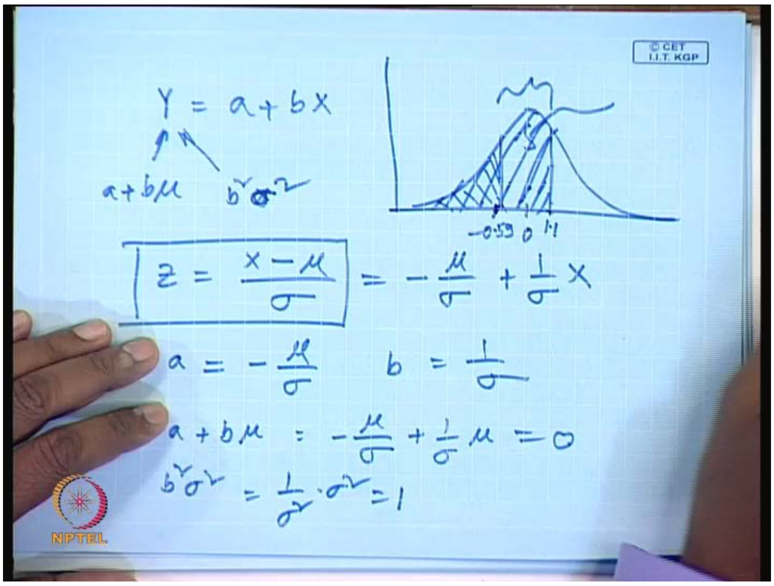

So,first of all this is reduced a variate. So, if this is your 0 then the reducedvariate is looks like this now from the 1.1 what is the value we get from this table is the total area from thisfrom the left support to that 1.1. So, this total area that is the total probabilitywe will get from thatstandard normal distribution table and for the left oneis the minus 0.59 may be somewhere hereminus 0.59. So, what you are doing that up to this much what is this areathat we are deducting. So, that we will get, what is there in this area only. So, what is the total areabetween this two limit we will get.

So, the total area up to this one minus total area upto this point to calculate the probability between this to limits.So, this is exactly is donehere first of all we haveconverted to the reducedvariate, which is 1.1 another 1 is this point minus 0.59 this 5 corresponds to 1.1 and this 4 corresponds to 0.59 and this two valuesare taken from this standard normaldistribution table.

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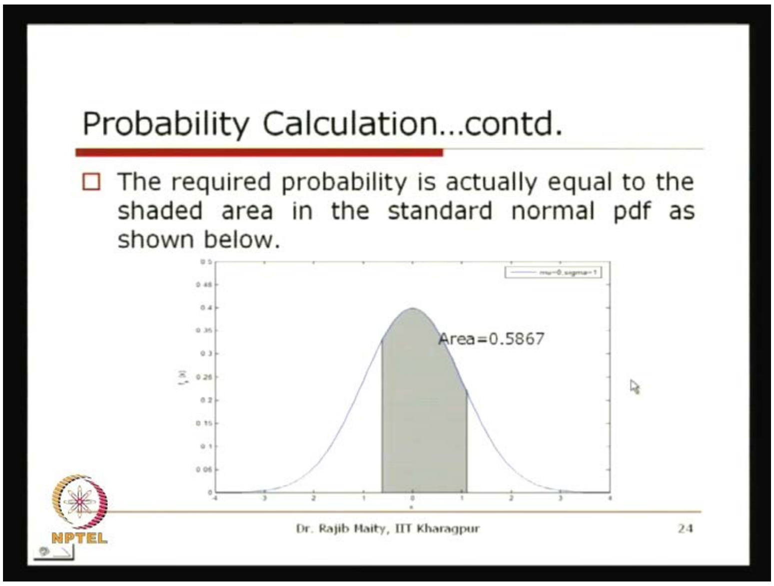

So, here this graphical repartition is shown here again, that is 1.1 this area. So, up to this that value, which is indicating is the total area from this minus infinity to this point. So, that is why we have do to get only thismuch area we have to deduct the area, which iswhich isup to that point of minus 0.59. So, this area should be deducted to get the area in between these two limits.

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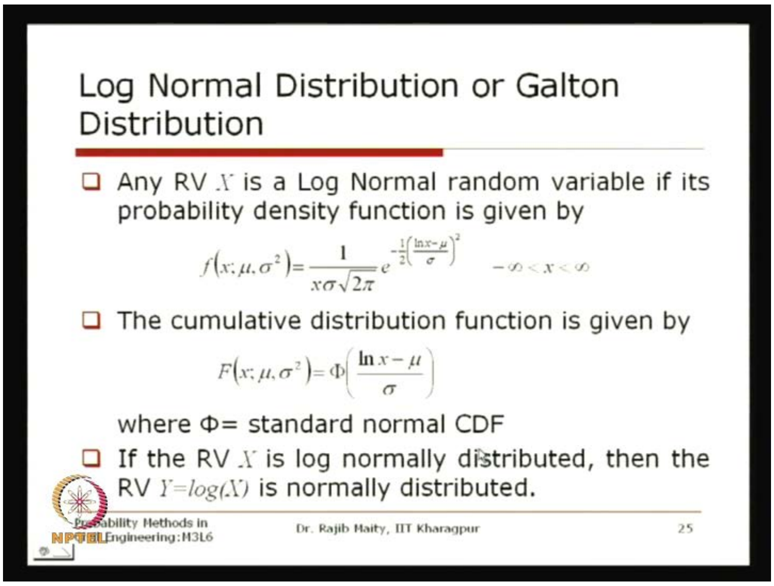

Next, distribution that we are going to discuss, is the log normal distributionthis also known as Galton distribution any random variable Xis a log normal random variable, ifit is probability density function is given by as follows. Again, it is having two parameters one is mu and this sigma square is 1 by x sigma square root 2phiexponential minus 1 by 2 in bracketlog natural X minus mu divided by sigma whole square. And it is limit for the x is fromso, this limit isthis is a mistake this limit from the 0 toinfinity.So, why this is important and what is the difference between this normal distribution is thatfor the normal distribution need variesfrom minus infinity to plus infinity, but this log normal distribution support is from 0 to infinity.

Basically, what we are doing again, isthat we are justtaking that one random variable we are taking it is log and it is relatedthrough this equation that is Y is equals to log x. And now what is shown here is that if this random variable Xis log normally, distributed then that Y, if Itake that log X is thenormally distributed. So, if wecome across withany distribution, which is log normally, distributed we can take it is log and convert it to the normal distribution and again, we can use the sameprocedure,which is followed for the normal distribution.

Now,so, as it is related to this as the log normal distribution is related to the normaldistribution through this functional transformation this will again, be clear in the next module where we arediscussing about the functional random variable. So, if thisdistribution is known or if this normal distribution is known, what is the distribution for this xand through that oneit can be usually, shown that this distribution of this x is following is having the form like this, this will be 0 this is not minus infinity this is 0.And again, that cumulative distribution function is given by this one where this is your thatthat cumulative distribution of this normaldistribution and l n x minus mu bysigma.

Basically, if you see this mu and sigma that is the parameter of this log normal distributionthis mu is the mean of the variable X aftertaking it is log. So, after taking the log of X, if you calculate the mean then that is equals to it is parameter muand similarly, if you takethevariance this is this sigma square.

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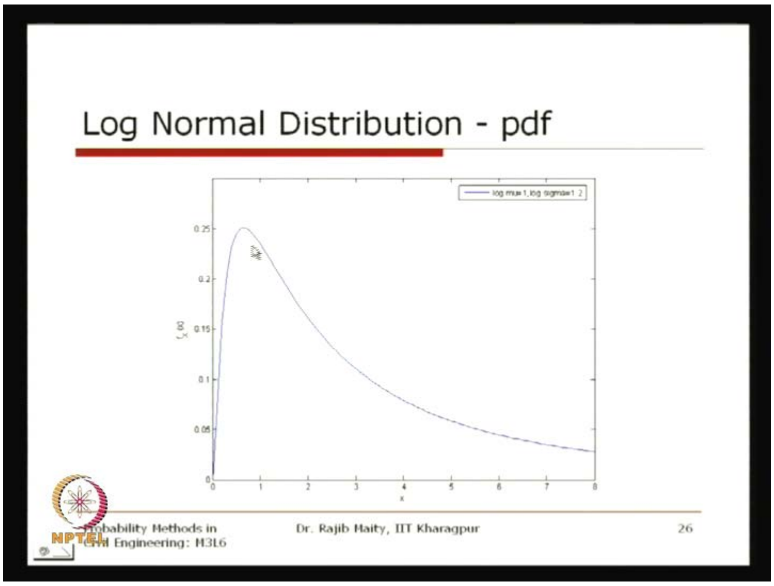

Ah this is how a log normal distribution with this log mu is equals to 1 and log sigma equals to 0.12 looks like. So, this is lower bounded by 0, and go up to plus infinity that, which is the support for this log normal distribution.

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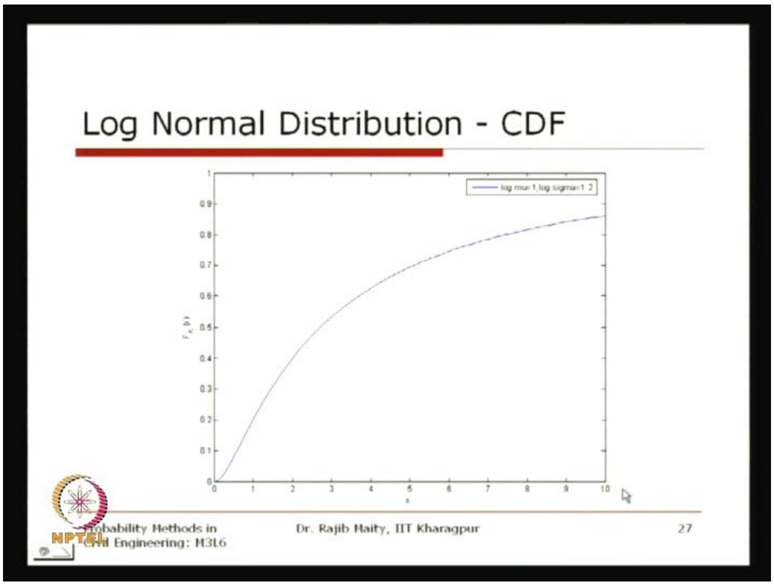

Andsimilarly, this ishere cumulative distribution function for the log normal distribution.

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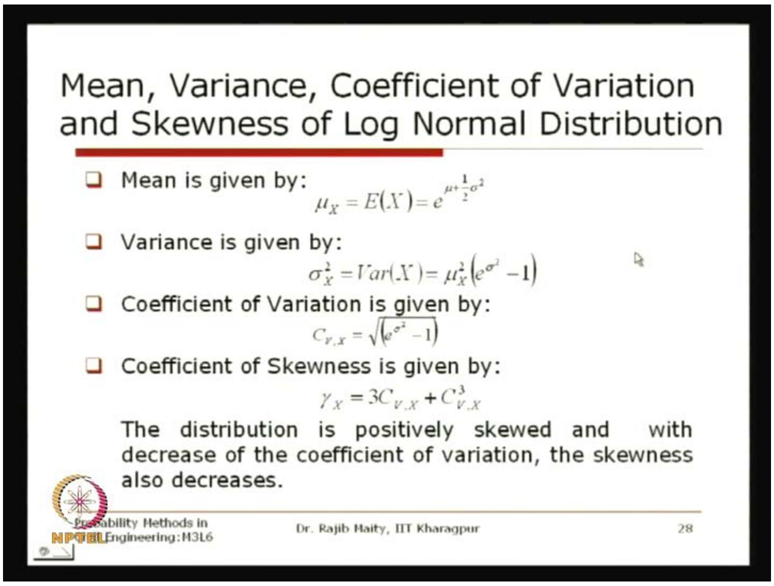

Now, this mean of this log normal distribution is expectation of this X, which can be shown that e power mu plus 1 by sigma square that is this is the mean for that variable X. And this mu this is the mean after taking the log of thisof the distribution of x, if you calculate the mean this is that mean, which was discussed.Again,the variance of this distribution that is variance ofxis equals to mu x now thissubscriptx is shown as this is for this mu for this x this one that square multiplied by exponential sigma square minus 1. Coefficient of variance isagain,given by the C v of this x that is the random variable Xis equals to square root of e power sigma square minus 1. And coefficient of Skewness is given by for this gamma x is equals to 3 multiplied by C v xplusC v x squarethat C v xcube. The distribution is positively skewed and with decrease of the coefficient of variation the Skewness also decreases which can be reflected from this equation.

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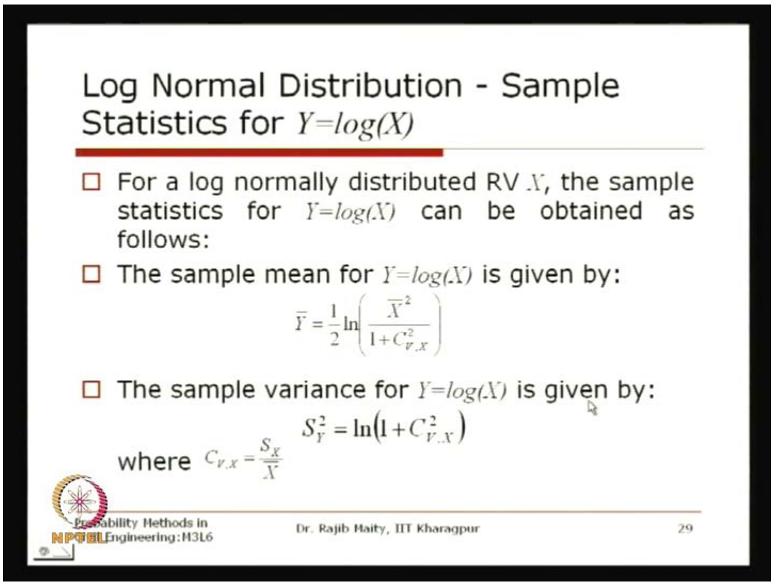

Now, for a log normally,distributed random variable Xthe sample statistics for Y equals to log X that is we have some observation some sample data we have taken it is log, if we take that log then how we can get the mean of that convertedrandom variable. This isobtainedthrough this equation that is Y bar after taking the log it is mean is equals to half l n x bar square divided by 1 plus C v xs square. So, this C v is the coefficientof variancefor thatfor that observation x and this x bar is the mean for that particular observationof the observed value, which is square.The sample variance forthe Y equals to log Xagainthis 1 is equals to S y square is equals to ln 1 plus C v x. So, this is the coefficient of variation with square plus 1 take the log get thatsample variance of y this coefficient of variation as we know is the ratio of thisstandarddeviationofthe X divided by mean of that X.

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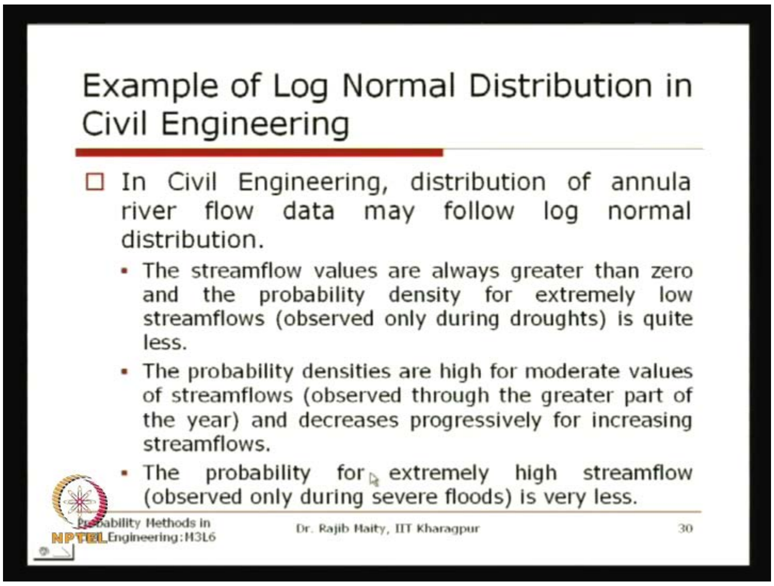

In civil engineeringthe distribution of annual river flow data may follow the log normal distributionthe streamflowvalues generally, are greater than 0 that is it is lower bound is 0 and the probability density for extremely low stream flows are quite less.This is; obviously, for some kind of big rivers andif you see thatit is generallyhaving some contribution from thisground flow or the snow-fed rivers like that. So, generally,insome flow ismaintained throughout the year. So, theit isso, that probability is low for the extremely lowflows then the probabilitydensitiesare increaseswith increase increasing amount of this annual flowfor the moderate values of the stream flow and again, it decreases progressivelyfor the increasing stream flow.

So, the probability for the extremely high stream flowagain, is very less. So, what we can see is that it is starts from 0 take the peak and again, it is coming down. So, far as the densityof the probability is concern over therange of this annual river flow.So, this can follow we cannotconfirm it and we cannot sayit as ageneral case that all the annualrivers are always follow the log normal distributionthat even though we cannot say that, but there is a possibilitythat this may follow a log normal distribution.

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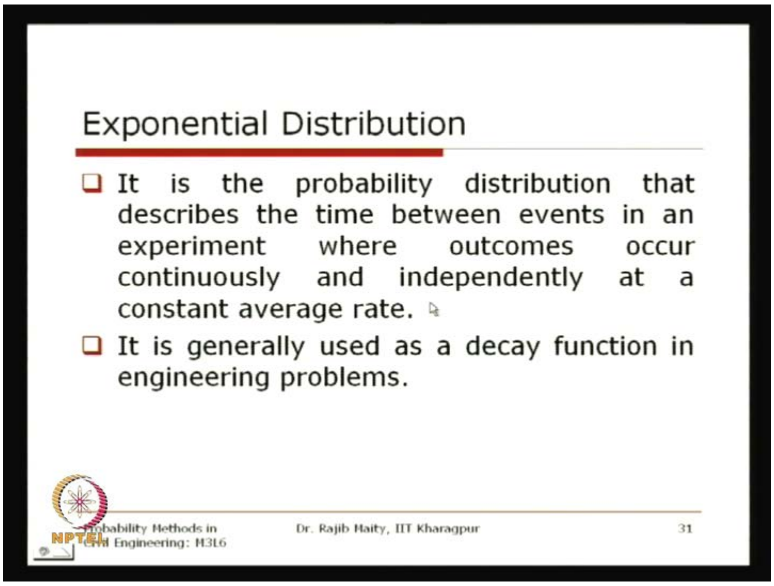

Next, we will discusses, that exponential distributionit is the probability distribution that describe the time between events in an experiment where the outcomes occur continuously and independently at a constant average rate. It is generally, used as a decay function in the engineeringproblems.

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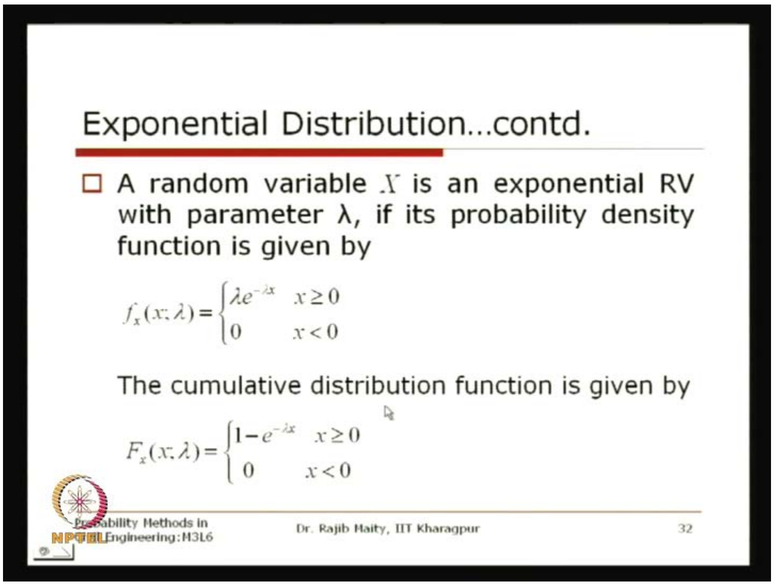

So, this distribution is a mathematicallyvery effective to show and in the previous lectures also we have taken this example, to show it is different properties of thecontinuous random variable. And we know that it is probability density function looks like the lamda e power minus lamda x for this x greater than equal to 0 andelsewhere it is 0.And here it is a single parameterdistribution and the parameter is lamda and if we take we in the earlier lectures also we have seen that this cumulative distribution function is can be shown asthe 1 minus e power minus lamda x.

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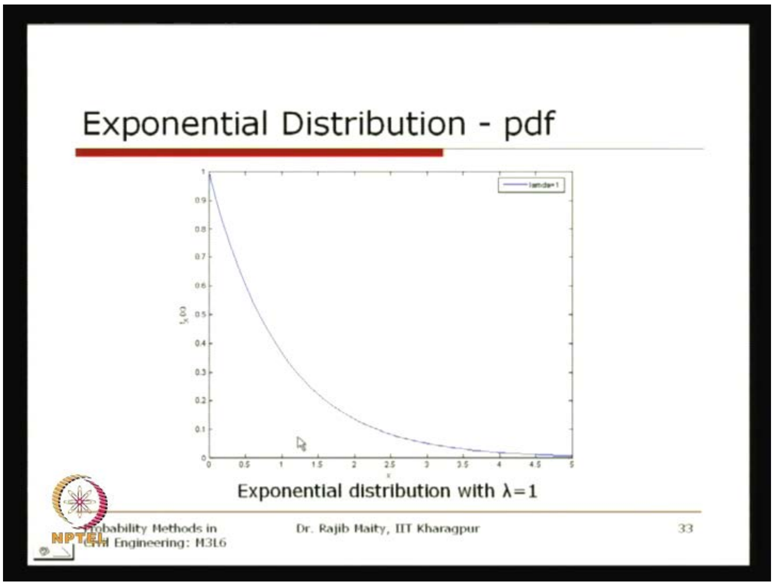

And it is distribution that ispdfthat is probability density function with the particular value of lamda equals to 1 taken here is looks like this.

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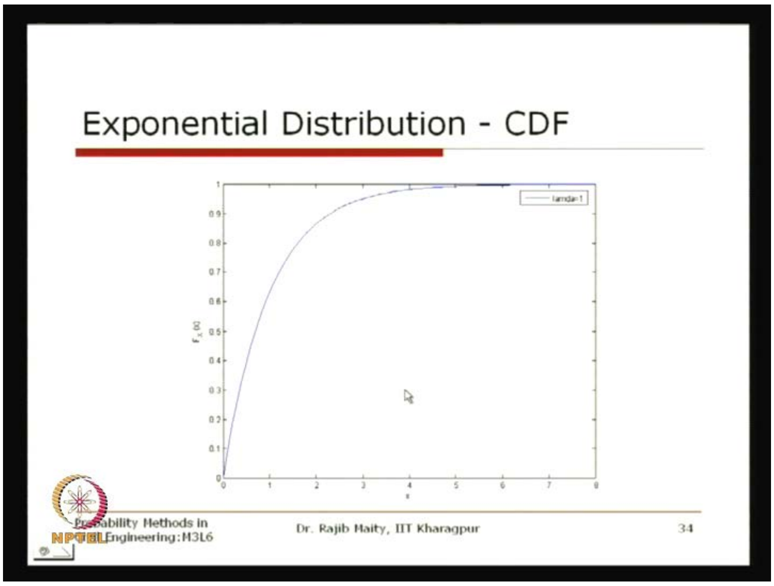

And it is cumulative distribution again, looks like this, which is starting from 0 and going asymptotic to 1atx equals to infinity.

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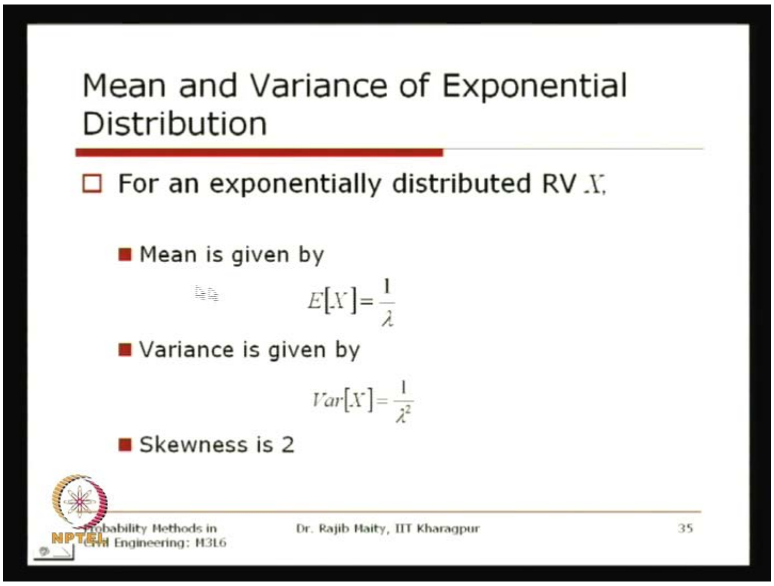

And this things also we calculate earlier it is mean is given by expectation of X is given by 1 by lamda and variance is a 1 by sigma square. It is Skewness can be shownis equals to 2 and coefficient of variance also is shown it earlier and which is equals to 1 in earlier lecture we covered thisdistribution as example.

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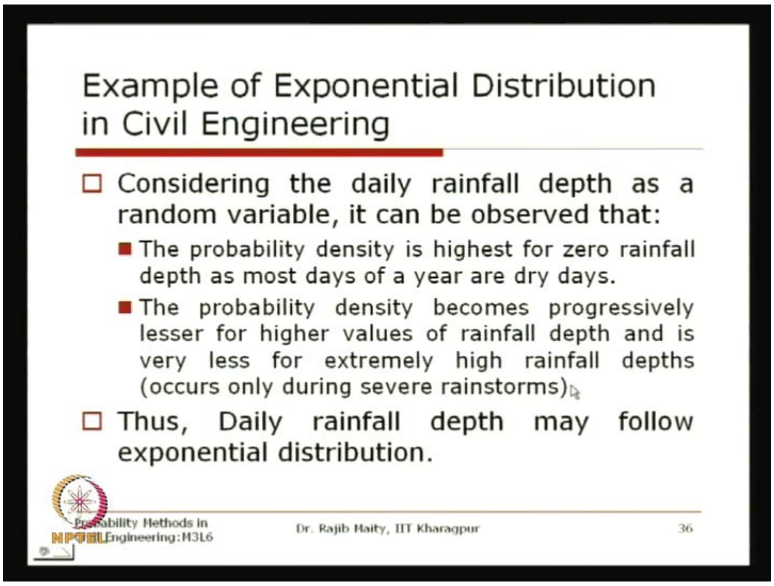

So, if we take that daily rainfall depth and the probability density is highest for this 0 rainfall we know that most of the days, if in case, the most of the days are dry days then it isthe maximum probability is concentrated aszero. The probability density becomes progressively lesser for the higher values of the rainfall depth and it isvery less for the extremely high rainfall depths. Thusthis may follow again, it shouldnow may (( )) should be follow for daily rainfall depth may follow and exponential distribution andwith this we stop with this exponentialdistribution here.

So, in this lecture we cover thatuniform distribution, normal distribution, log normal distribution and exponential distribution there are some more distributions arealso important. And that we will cover in thenext class,and then we will go through the next module and in successivemodules application of this kind ofdistribution indifferent civil engineering problem for different modules will be explained later.Thank you.