Ground Water Hydrology
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Module No # 02
Lecture No # 09
Ground Water (GW) flow rates and flow directions; general flow equations through porous media

Welcome to this lecture number 9 which is in which we will continue with our previous class previous lecture as well that is on heterogeneity.

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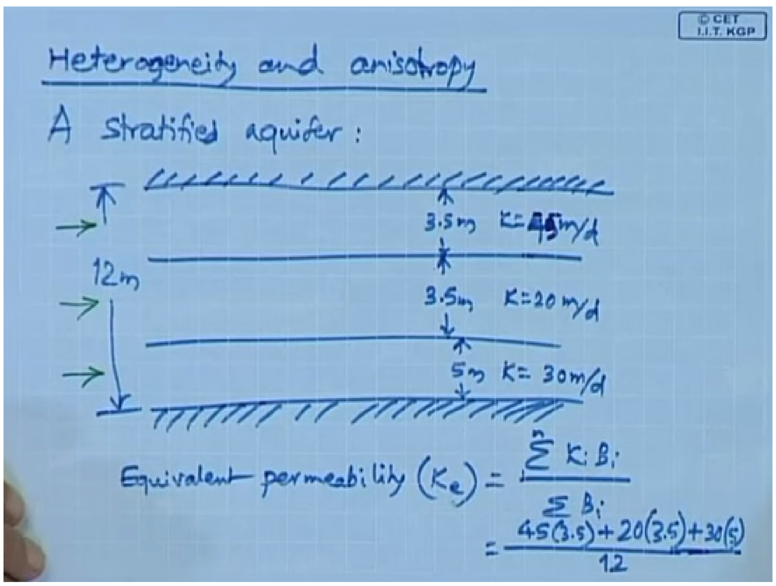

We will take a numerical problem heterogeneity and anisotropy and isotropy in the aquifers and let us consider say for example the stratified aquifer. A numerical problem of stratified aquifer of say thickness 12 meters. So this total thickness is 12 meters and it consist of three layers as shown here.

And this is the flow direction the flow is along the layers or stratifications or strata and the bottom layer has say this is the bottom say it has K value of 30 meter per day and it has a thickness of say 5 meters and the middle and to layer have permeability of say 20 meters per day. This stop layer also have a permeability of 20 meter per day I am sorry top has let us consider take it as 45 meter per day and are of equal thickness.

So obviously here if you deduct the 5 meters thickness of bottom layer from the total thickness of 12 meter. So we are left with 7 meters and that 7 meters is equally distributed between the top layer and middle layer. So that is 3.5 meter is the middle layer and so is the thickness of top layer which is 3.5 later 3.5 meter. So the now we know and the flow along the stratification or layers or strata and for this let us say find out what will be the equivalent permeability as well as what will be the transmissivity.

So let us come to the equivalent permeability so KE in this case is given by summation KI BI from 1 to N / summation BI. So in this case we have to sum there are three layers so there will be 3 terms in the numerator so that is 45 into 3.5 + 20 into 3.5 + 30 into 5. So this is at the numerator and in the denominator the total thickness which is say 12 meters.

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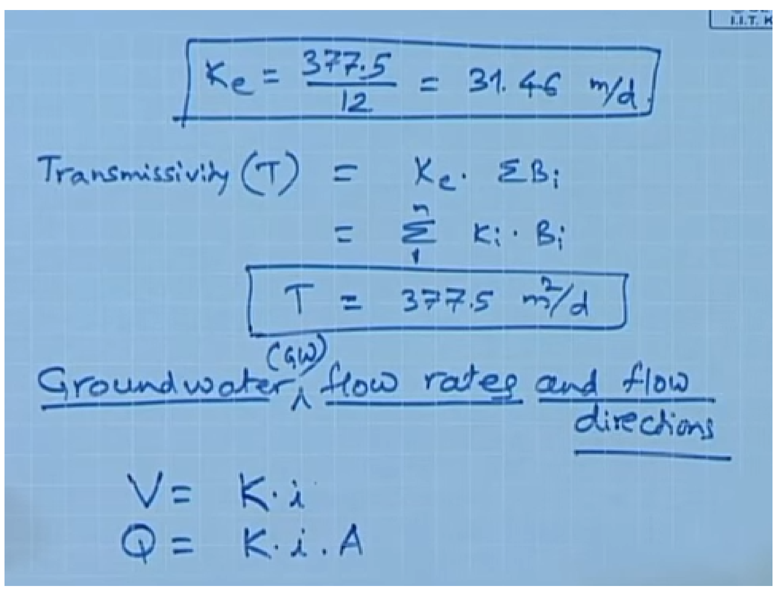

So this will be the equivalent permeability and in this case so this equivalent permeability KE will be simply that is that is 45, 65 into 3.5. So that is here I am computing here so this is 65 into 3 into 3.5 + 150 which is 377.5 / 12 which is 31.46 meter per day so this is the equivalent permeability. And let us also determine the estimate transmissitivty or it is also known as transmissibility.

So this is simply = KE into Sigma BI or which is simply = this is summation 1 to N, KI into BI so this is as will be numerator of this 1 and so this is 377.5 meter square per day. So this is the value of transmissibility so here as mentioned here there were three layers the top layer and middle layer having 3.5 meter thickness and bottom layer having 5 meter thickness each with the hydraulic conductivity or coefficient of permeability of 45 meter per day, 20 meter per day and 30 meter per day respectively.

So in this case and the flow is along the stratification so in this simple we just apply the formula and get the equivalent permeability which is say 31.46 meter per day and the transmissivity for this entire aquifer consisting of three strata is 377.5 meter square per day. So this is just a illustration of one simple numerical example consisting of an isotropic aquifer confined aquifer with three layers or three stratification and flow along the this one.

Now let us come to the ground water flow rates and flow directions of course so this ground water we have we have been abbreviating as GW so we know that with by Darcy's expression the ground water flow velocity is given by the hydraulic conductivity K into I and here and the ground water flow rate obviously is given by the velocity into the area flow which is K into I into A.

So accordingly we can if we can estimate all these three parameters the hydraulic conductivity K the hydraulic gradient I and the area of cross section area of flow for the ground water. So if we estimate each of this parameters then the product these three estimated parameters will give us the estimated ground water flow rate.

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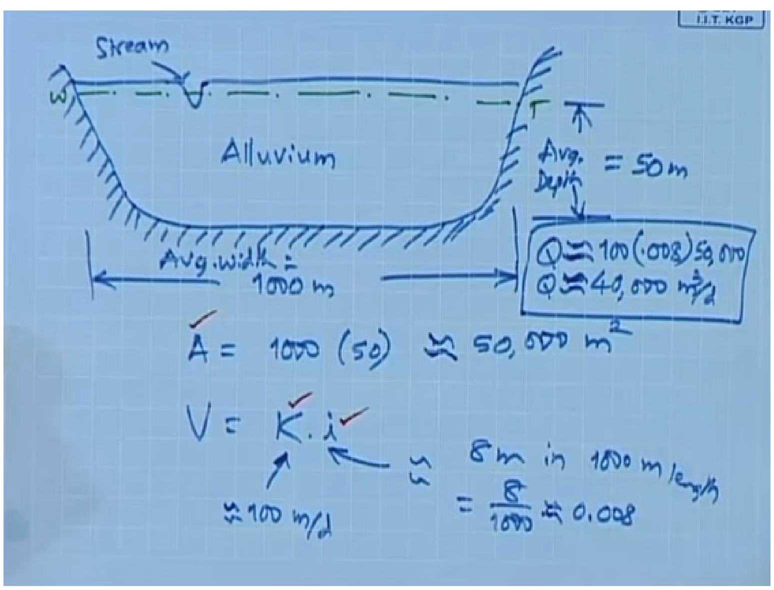

Now let us consider say for example a stream or a river which is flowing above and alluvium something like this and here let us say this is the water table and this is the stream let us say this so this is the alluvium which is essentially an aquifer and say let us say the width of alluvium is something as the order of say 1000 meters in the average depth of this alluvium.

So this is the average width and the average depth say that is say of the order of say something like say 50 meters then in that case our cross sectional area A will be 1000 into 50 which is 50,000 meter square and if we multiply this by the ground water flow velocity given by Darcy's Law which is K into I. And suppose the hydraulic gradient is a say the hydraulic gradient is say 10 meter or say it may be 5 meters say.

Let us take this to be 8 meter in 1000 meter length so this will 8 / 1000 which is .008 and obviously that is hydraulic gradient and then the coefficient of permeability if we take this as something like say 100 meter per day then in that case simply these are the estimations permeability or hydraulic conductivity is estimated as 100 meter per day. And the hydraulic gradient is estimated as 0.008 and the area again this also an estimation only that is why I am using this approximately is equal to sign.

Because this is based on the average width and then average depth so in that case so now we can write this Q = 100 into 0.008 into 50,000. So this will be 50,000 into 0.8 so this is of the order of say 40,000 meter cube per say. So this is out estimated value of ground water flow this is just to give an idea so basically here we need to estimate each of the three parameters. The first one is the area of cross section of flow area of flow cross section of ground water the second one is the hydraulic conductivity K.

And the third one is the hydraulic gradient I so this estimate as realistic as possible and based on that once we estimate these of this three parameters then simply take the product of this. So that will give us the estimation so that is why let me use the approximate equal to sign here. So this alluvium with say these dimension average depth of 50 meter and average width of 1000 meters which is essentially the aquifer below the stream.

So we heal a ground water of approximately 40,000 meter per day when the estimated hydraulic conductivity is 100 meter per day through this the material of this aquifer or alluvium constitute this aquifer or alluvium as well as the hydraulic gradient which is estimated as 8 meter and 1000 meter length along the flow. So this is just one to give one idea of this estimating the ground water flow rates.

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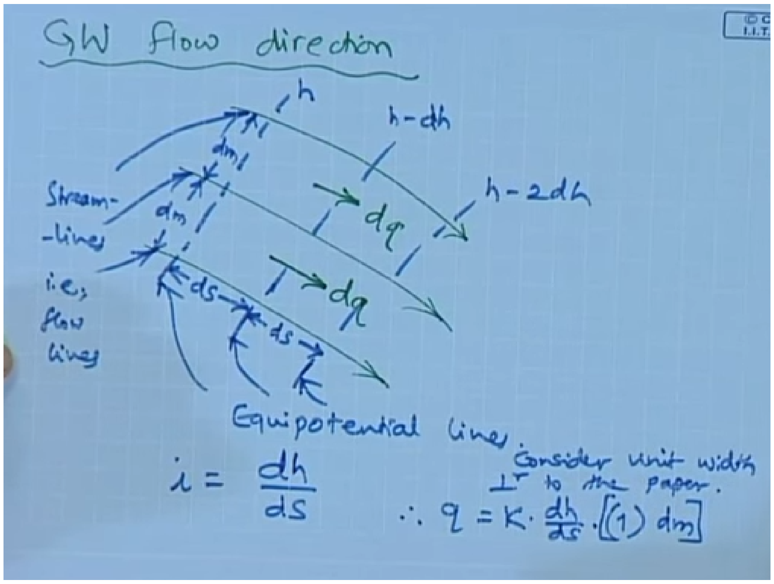

Now let us come to this ground water flow direction so here so we need go by what is known as the flow net which is essentially a network consisting of two seats or orthogonal lines the stream lines shown by the green color this one and the equipotential lines let me show them by the so these are the equipotential lines and these are the stream lines or the flow lines also known as flow lines.

Now let us consider the stream wise let us partition this flow net into square grades and say let us say the total flow between two neighboring stream lines this PDQ and here also this is the flow is DQ between the bottom and middle stream line as well as the middle and the top stream line and let us write down the head values or the total hydraulic head value along each of the equipotential line.

So let us this be H here and obviously due to friction and other losses here will be continuous head loss. So the along the next equipotential line let the head be H – DH and in this case it can be H – 2 DH okay. And now and also let us consider this stream wise length as DS and let us consider the length that is the DS is the distance between two adjacent equipotential which is along the stream line that is why it is denoted by DS.

And this perpendicular or the meridional so which is which represent the different between two neighboring stream line let this be DM okay. Now we know that by the definition of hydraulic gradient this is DH / DS basically the change in the head per unit displacement along the stream lines therefore here if we consider unit width perpendicular to the paper. So therefore this total discharge Q is simply given by the hydraulic conductivity K into the hydraulic gradient DH / DS.

So this is the velocity multiplied by the area so this is simply given by 1 into DM so this is the area okay.

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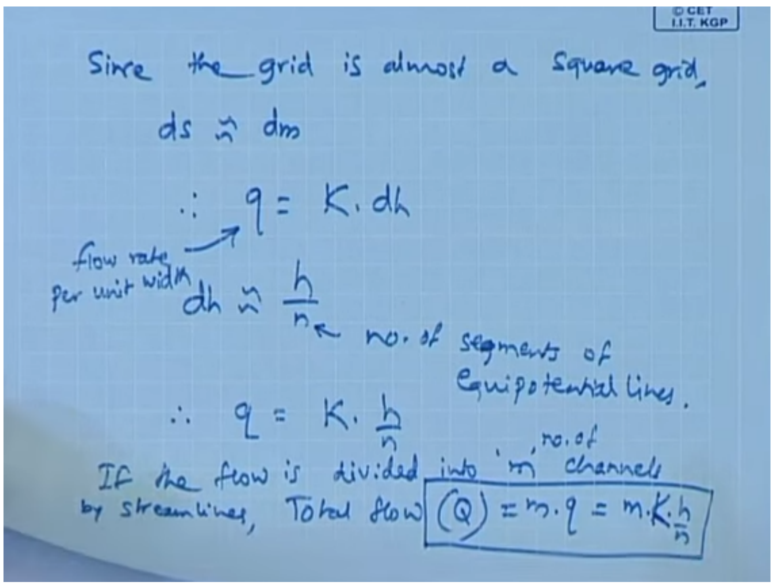

So therefore this the ground water flow rate and here because since the grid is almost the square grid. So we can say this DX is approximately = DM that is the stream wise distance that is approximately is equal to the perpendicular distance along the flow net.

So therefore we can write down the expression for Q is K into DH and suppose this DH the total head H which is there in the up along the upstream most equipotential line if this is and along the down steam most point if the head be 0 if the entire head is lost through friction and other losses in say N along N such equipotential lines.

So in that case so this DH can be approximated as H / N where N is the number of segments is equipotential lines therefore Q which is the flow per unit rate this is a flow rate per unit width is given by K into B / N and the total flow rate for this we have to if the flow is divided into M number of channels and the stream wise direction by stream lines then total flow.

So that is Q = M into small Q which is simply = M into KH / N so this is the expression for the total flow rate. So here M is the number of channels along the stream wise direction and K is the hydraulic conductivity H is the total head in the upstream most along the upstream most equipotential line N is the number of segment in the along the of equipotential lines. So in that case the total flow is simply given by this MK into H / N.

So this is how we can estimate the total flow and also we can estimate using the direction so we can estimate the groundwater flow direction also. So like this we can estimate the flow rate as well as the flow direction.

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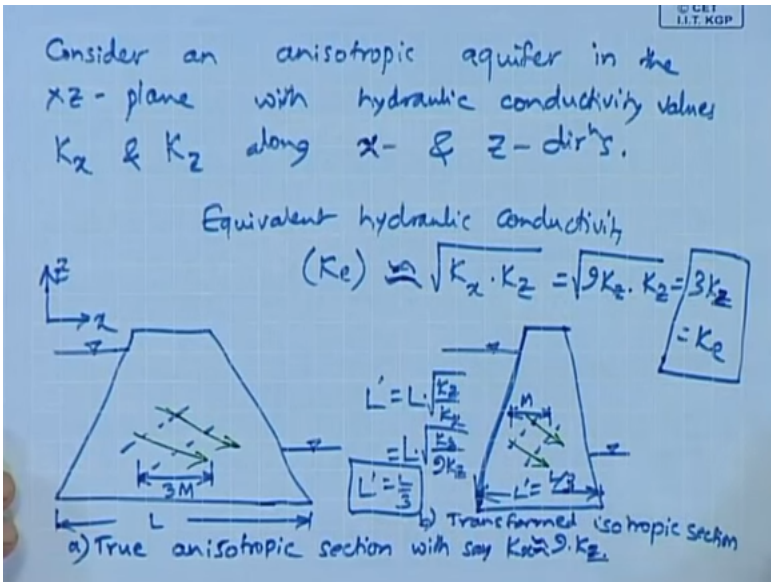

Now let us consider and isotropic case where in say consider in an isotropic aquifer so here which as a in the XZ plane with hydraulic conductivity KX and KZ along X and Z directions and typically in case of the alluvium or aquifers so this KX will be greater than the KZ.

So the equivalent hydraulic conductivity so this is KE is can be approximated the square of the product that is KX, KZ and say in such because of this isotropy which results in different values of the parameters the ground water flow parameters such as the hydraulic conductivity and other parameters and transitivity and so on.

They have aquifer thickness either a geometric parameters or kinematic parameters so what happens is we need to estimate the flow rate through such this one such an isotropic aquifer by assuming by estimating the equivalent hydraulic conductivity and in such case the flow lines that is the stream lines will not be fully perpendicular to the equipotential lines because of an isotropy.

And in such case if we transform this into isotropic condition equivalent isotropic condition in that case only the flow lines will be perpendicular to equipotential lines. So here let us say the flow net analysis of for an earth gram let us u consider say for example this is the and in this case suppose this are the flow lines and the equipotential because of anisotropy they will not be intersecting orthogonally and in this case.

So if this is the total length L so here let me write this is a true anisotropic section with say KX = 9KZ. So this is the downstream and this is the XZ plane this is the X direction and then this is the Z direction. So this is figure which represents the true anisotropic section with say KX approximately = 9 times KZ then so this can be transformed into an equivalent transform the isotropic section for the transforming this into so this is B is the transformed isotropic section.

So in this case so this length L will be so this L dash will be equal to L into under square root KZ / KX and this case this will be L into under square root KZ / 9KZ. So this L dash is simply = L / 3 so this L dash is L / 3 and in such case for this transformed isotropic section. In this case so this distance will be M whereas in this case this distance of a grid so that will 3N so it is only in case of this because of this anisotropy the stream lines as well as equivalent lines they will not be perpendicular to each other they will have this one.

So only in case of transformed isotropic section so it will a transform length which is given by that is L dash will be =L / 3 in this case and similarly in this case since suppose we are since we are taking KX is approximately nine times KZ. So therefore here we can say here this KE is approximately = 9 KZ into KZ which is 9KZ square under square root. So this is 3 KZ so this is the equivalent = KE so therefore so for this anisotropic aquifer with varying hydraulic conductivity values along X and Y directions.

So the equivalent hydraulic conductivity is given by three times the hydraulic conductivity along with Z direction and the base width of the transformed isotropic section through which the flow ground water flow takes place will be given by L / 3 and So this is corresponding to expression this L dash into under square root KZ / KX.

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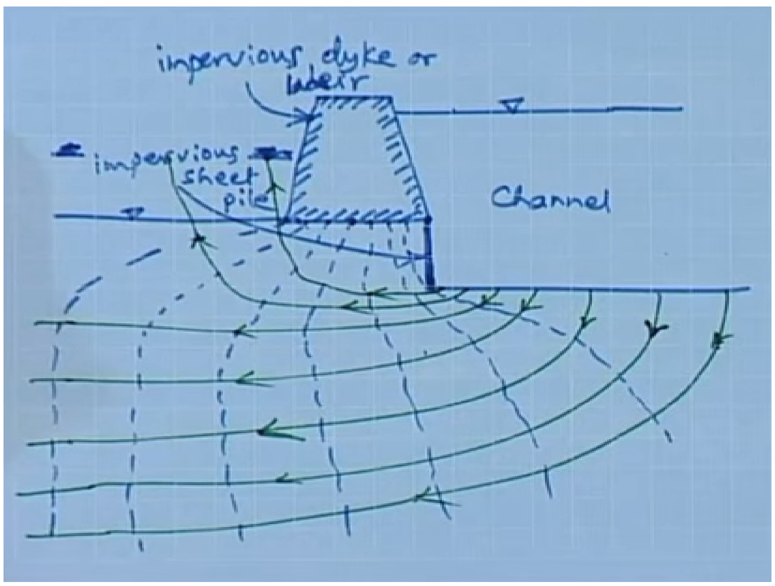

So like this and say let us consider another case where in there is a rear or a barrage with say some kind of upstream and of course here are the let us consider this to be almost impervious and in this case this suppose let us consider this as the channel and in this case what happens is the channel is pervious whereas this is a impervious dike or barrage or rear so in this case the stream lines will be and let us consider this also to be and this is the impervious sheet pile.

So in such a case the stream line will take the shape in if the downstream water level so in this case the stream lines will be having stream lines and the flow lines. So these are the flow lines and we can construct the so if this the head difference between the upstream and downstream. So if we divide this head difference in two so in this case there is 1, 2, 3, 4, 5, 6, 7. Seven stream lines are there and if we divide this into say seven it is a now let us this is not a downstream =.

So this is one of the downstream water level and if we so this upstream water level and downstream water level if we divide this into seven divisions. So here we get our so this will be the downstream most equipotential line then followed by the next equipotential line will be this the third equipotential line like this and the fourth equipotential line in the fifth equipotential line the next equipotential line.

So to get to know the flow direction at any particular direction we need to use the concept of flow net and we need to consider the flow net so therefore the stream line or flow at any particular location will give the ground water flow direction at that particular location okay.

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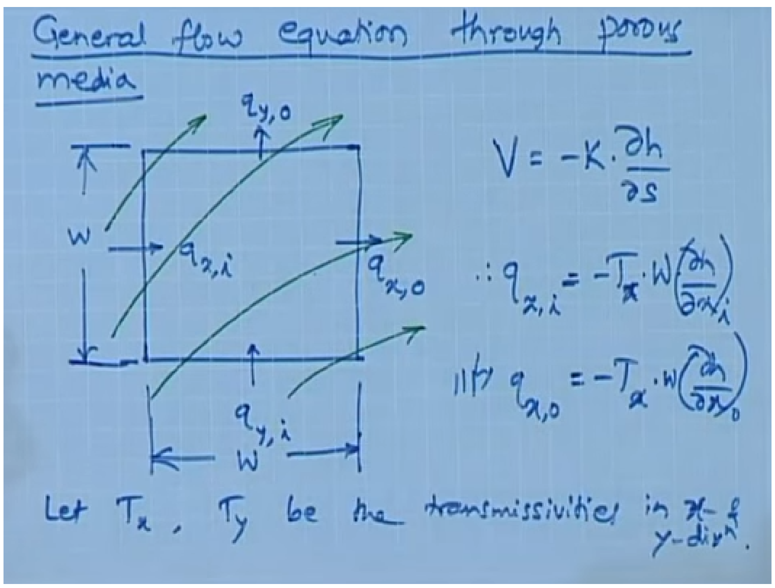

And so this regarding the now let us consider let us go to the general flow equation through porous media so here let us consider through two dimensional flow initially and again later on let us extend it into a three dimensional level flow domain. So in this case this is the flow domain two dimensional flow domain and then say these are the stream lines shown by this green color and let considering the unit width let the flow across the this boundary of this square flow domain the QXI.

So I standing for inlet input inlet section and similarly the flow here per unit width let this be QX comma O then similarly along the bottom boundary. So let the flow be QY comma I and then this is the inlet and then so here let this be QY comma O and let the dimensions let since we are considering this to be a square grid. So let us assume the dimensions of each side and this flow domain as W okay.

And in this case we know that the Darcy expression hold good that is the ground water flow velocity is given by – K into D the partial derivative of H with respect to S. Where H is the head and S represent the replacement along the flow line or stream line now let us consider let this for this flow domain let TX comma TY be the transmissivity in X and Y direction okay. So therefore this QXI we can write this has QXI can be written as – TX which is the transmissivity in the X direction into the width of the flow domain that is W and we are considering the unit thickness.

So the area will be W into 1 into DH / DX in the direction similarly QX the flow the outflow through the right boundary which is denoted by QXO is given by – TX is transmissivty along into W into DH / DX at this outlet section okay.

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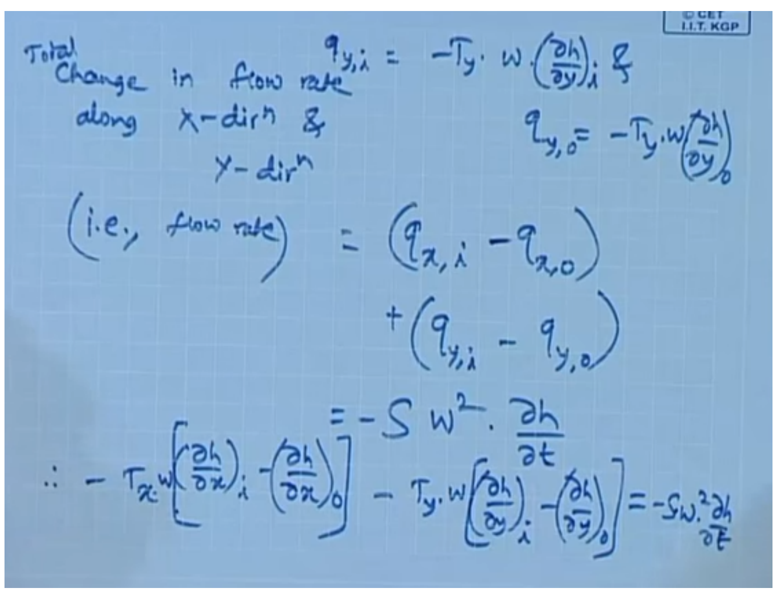

Now let us take down the change in the flow rate change in flow rate along X direction and Y direction so similarly here we can also write down the expressions for QYI and QYO. Let me write here this is QYI can be written as –DY into W into H / DYI and so QYO = -TY W

DH / DYO. So therefore the total change in flow rate along X and Y direction so this is basically this is essentially that is the flow rate.

So this is simply given by QXI – QXO so this is the change in flow along with X direction + QYI - QYO and this must be = this can be expressed in terms of the storage coefficient S multiplied by the area and in his case it will W square this will be cross sectional this one and multiplied by the rate of change of H with time the partial derivative of H with time. So therefore we can write down the – TX into DH / DXI – DH / DXO.

Let us divide the whole thing this is TX into W – TY into W DH / DYI – DH / DYO this must be = - S into W square into DH / DT okay.

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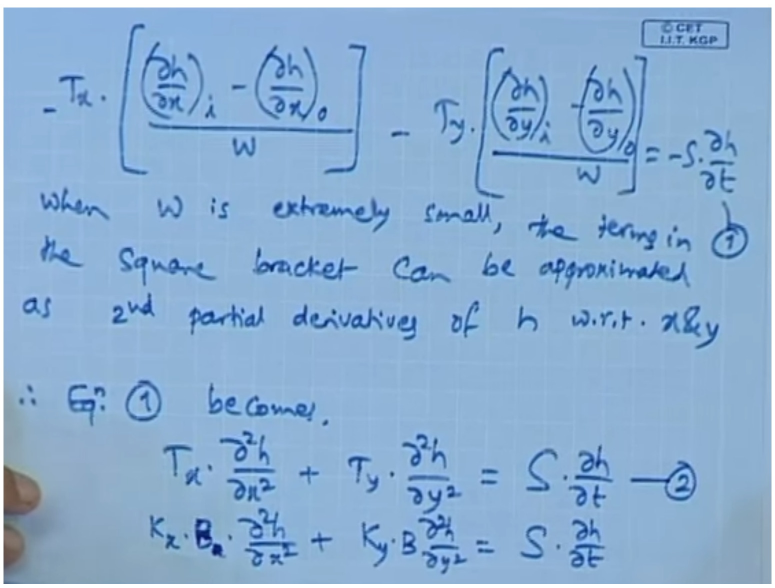

So we can write down we can simply this as TX into DH / DXI - DH / DXO divided by W – TY into DH / I – DY / DYO / W. So this is = - S into DH / DT so here in this when the when this W is extremely or infinite extremely small in this case. So this the turn and the square bracket under the square brackets can be approximated as the second partial derivative of H with respect to X and second partial derivative of H with respect to Y.

So therefore so the terms in the square bracket can be approximated as second partial derivative of H with respect to X and Y. So therefore suppose if we call this as equation one so this equation one becomes DX into D square H / DX square + TY into D square H / DY square. This is equal to the storativity S into DH / DT okay.

And this TX and TY we can be simplified as KX into W KX into say B which is the BX we can say B into D square H into DX square + this DY can be substituted as KY into B where B is a saturated aquifer thickness into D square H / DY square this is = S into DH / DT. Or the same this so therefore if this equation 2 and then this is equation 2 and then this is equation 3.

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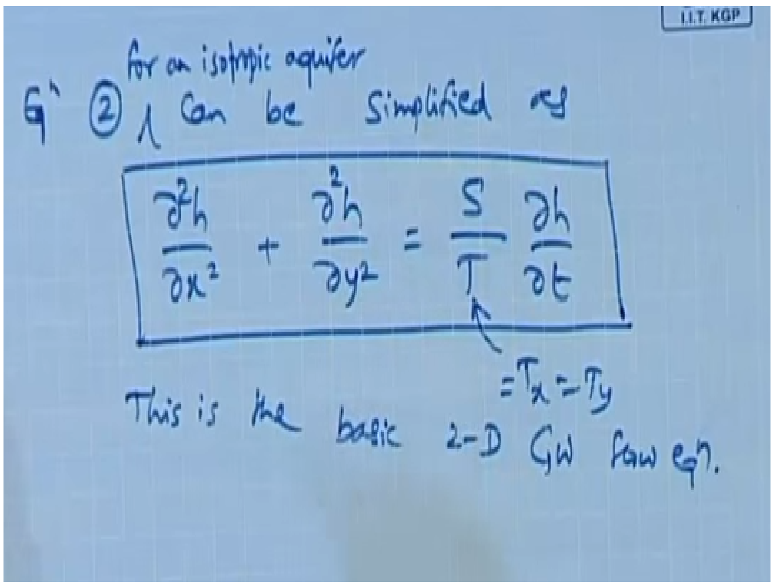

So equation 2 can be simplified as D square H / DX square + D square H / DY square which is = S / T into DH / DT so this equation 2 here this is for an isotropic aquifer. So in this case this is T = say TX = TY okay so this is the basic ground water flow equation in two dimensions and of course if we have the third dimension also so in that case so this is the basic 2D ground water flow equation.

So in the next lecture we will discuss we will extend this this two dimension equation to three dimensional zone and also we will see the it's a variation in the when the flow is steady in this one thank you