Ground Water Hydrology
Prof. Anirban Dhar
Department of Civil Engineering
Indian Institute of Technology – Kharaghpur

Module No # 08
Lecture No # 36
Modeling and Management of Ground Water: Ground Water Hydrology

Welcome to this lecture number 36 in this particular lecture, we will cover this modeling and management of groundwater.

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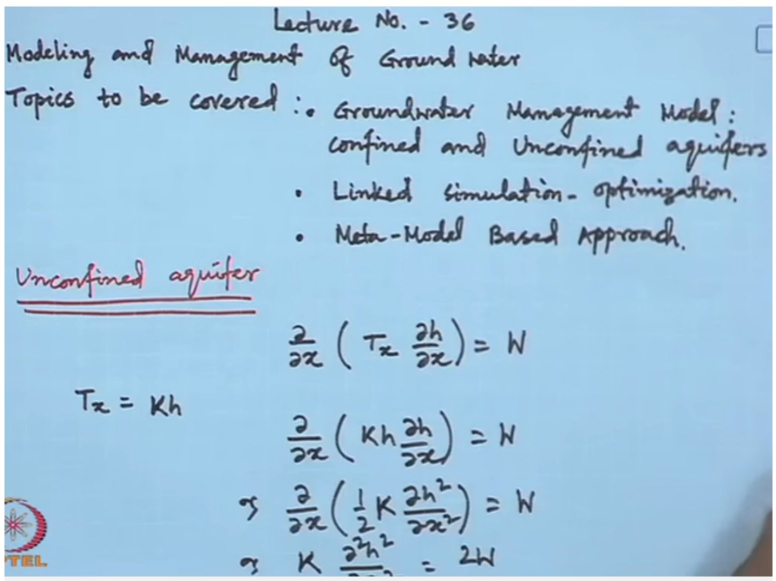

And under this so our main topics is modeling and management of groundwater and under this topic to be covered are groundwater management model, that is for confined and unconfined aquifer and this linked simulation optimization. Also we will cover that Meta model based approach. In the last lecture or lecture number 35, I have talked about groundwater management model for confined aquifers.

That is basically confined aquifers with one dimensional flow situation under steady state condition. In that one we have one linear objective function and we have (()) (02:50) our governing equation using final difference method and we have utilized those equations as constraints for our optimization model and finally the total model or total formulation can be solved using linear programming because all the constraints and objectives are linear in nature.

So today we will start with the unconfined aquifer first and we will try to solve that unconfined aquifer flow management model. This is also applicable to one dimensional flow situation. So steady state groundwater equation for unconfined aquifer can be written. So this is for steady state flow situation where X is the flow direction then TX and this is Del H / Del X = W. So in this one TX can be written as KH, K is the hydraulic conductivity and H is the hydraulic head.

So with this expression if we replace it in our original equation then we can write the whole equation as KH Del H / Del X = W. Or we can write it as Del H / Del X half K and this is basically H square X square. So for homogeneous type of aquifer, we can take this A out of this derivative thing homogeneous uniform condition. We can write it as K and we can transfer the 2 on the other side So Del 2 H2 / Del X 2 and we can write it as 2 W.

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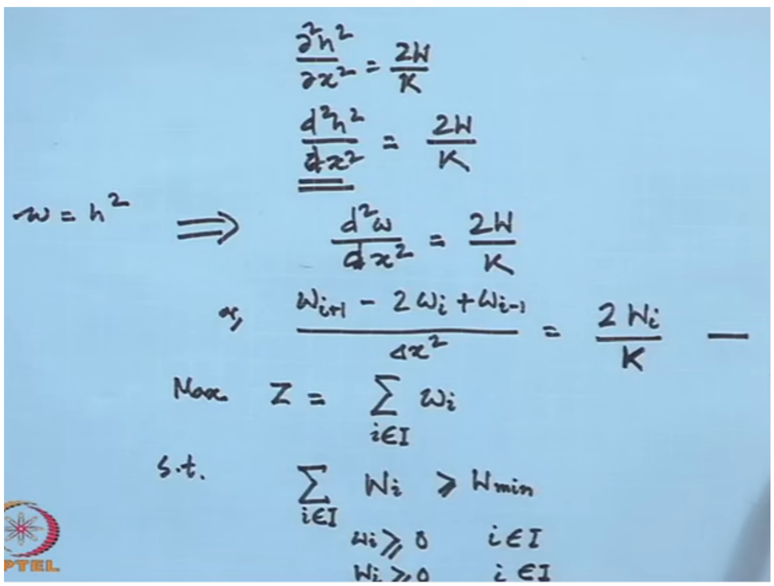

So from this one final equation will be our Del 2 H 2 Del X 2 or 2 WK or simply we can write this as because H is only a function of X. So we can write it as our ordinary differential equation. Now we can apply our final difference discordisation to this left hand portion. But the problem is that it has got this H square term. So what we can do, we can substitute another secondary variable with, let us say this is small W.

We can write it as H square, so with this replacement our equation will become linear equation. So with this substitution our final equations that will look like D to W. This is DX 2 and TW / K.

Finally if we apply final difference method, so we can write it as WI + 1- 2 WI + WI - 1 divided by Del X square and on the right hand side it will be WI divided by K.

We are using single K value for the whole domain because we have considered that aquifer is homogeneous in nature. So now this particular equation can be used as constraint for our optimization problem. So our optimization problem, then it will become maximized our Z which is summation of all small WIs where I belongs to that set capital I which is the state of all wells.

Now this is subject to our total pumping for all pumping wells that should be greater than equal to minimum pumping value and other constraints that we need are small W should be greater than equals to 0 it cannot be negative and also our pumping cannot be negative. So these are the constraints which need to be satisfied for our objective optimization problem.

We have our particular equation that will act as binding constraint for the optimization problem or equality constraints. This approach is basically our embedding technique approach where we directly utilize the governing equation within the optimization problem as constraints. So with this HI or H.

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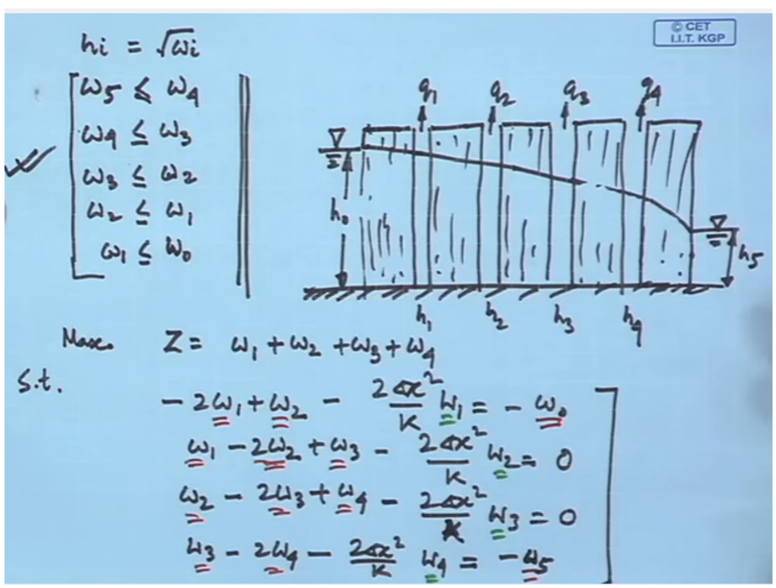

At a particular location that can be determined from this root over w small WI thing and we can also satisfy additional constraints because for this particular location we can have constraints like W5 that should be less than equal to W4. Small w similarly, our W4 should be less than W3 and

W3 should be less than W2. Similarly we have W2 less than equals to W1 and W1 is less than equals to our W not.

So like our previous confined aquifer problem, we can set another problem where this is your imperious datum and we have four wells. So this is the four well situation and both the left hand and right hand boundaries are defined in terms of hydraulic head. That is basically H 5 and H not, so these are basically specified boundary conditions or specified head boundary conditions and we have Q 1, Q 2, Q 3 and Q 4 for this situation.

Then corresponding to this we have H 1, this is H2, H 3, H 4. So we can see that this particular equation is set of constraints can be directly utilized within the optimization model to get the solution because the head here at this downstream order down gradient point should have less value compared to one up gradient point. So in this particular problem this is the aquifer portion, we have the problem as maximized Z where small W1, W2, W3, W4.

This is our objective function subject to final difference equations, that is 2 W 1 + W 2- Del X square / KW 1 - W not and W1 - 2W2 + W3 - 2 Del X square / KW 2, this is 0 again. We have W2 - 2 W 3 + W4 – 2 Del X square / K and W3. This is 0 and final equation we can write it as W3 2 W4 - 2 Del X square / KW4 = - W5.

All these are small W, these two are small W in this side. These are basically small W and these are basically capital W wells or pumping wells. These corresponds to our hydraulic head, now this set is the binding constraints and this particular set which, we have already written that is valid for the optimization problem that will give a proper result.

Because any down gradient hydraulic head cannot be greater than the up gradient or cannot be greater than compared to up gradient well. So what are the other constraints that will be required for this 1.

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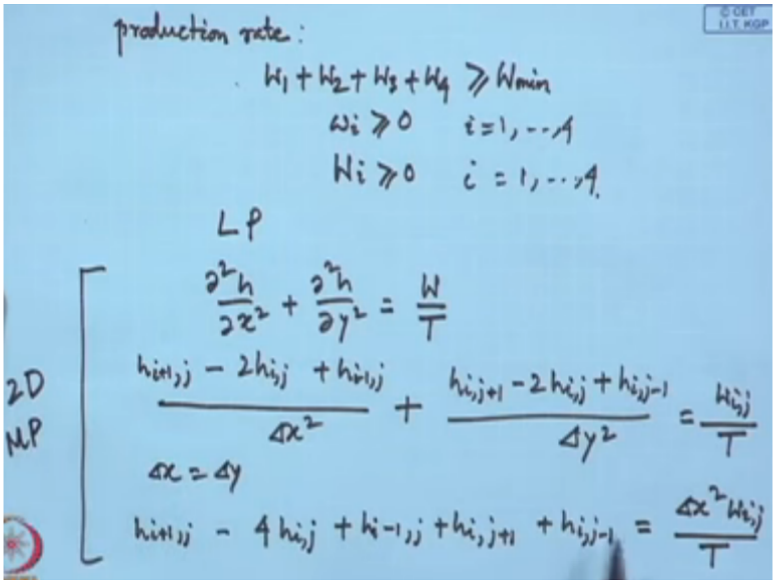

Those are related to production rate, in case we have W1, W2, W3, W4. This should be greater than W minimum and small W i should be greater than 0 for I = 1 to 4 capital WI greater than equals to 0 for I1 to 4 again all the objectives and constraints are linear in nature. So we can directly solve it using linear programming algorithm. Now we have already covered this confined and unconfined one dimensional flow management problem for ground waters.

Now we will try to see, what is there in two dimensional things. So for any steady stage homogeneous confined aquifer system the equation can be written as this T is having single value because we are considering constant hydraulic conductivity over the two dimensional aquifer. So if we discretize this so discretization should be IJ - 2IJ + HI - J / Del X square + HIJ + 12 HIJ + HIJ - 1 / Del Y square = WIJ / T.

So if we consider that Del X = our Del Y, then we can derive this equation as HI + 1J - 4 HIJ + HI - 1J + HIJ + 1 + HIJ - 1 = Del X square WIJ / T. Again if we want to write two dimensional management problem two management problems for this kind of aquifers, we can write it as.

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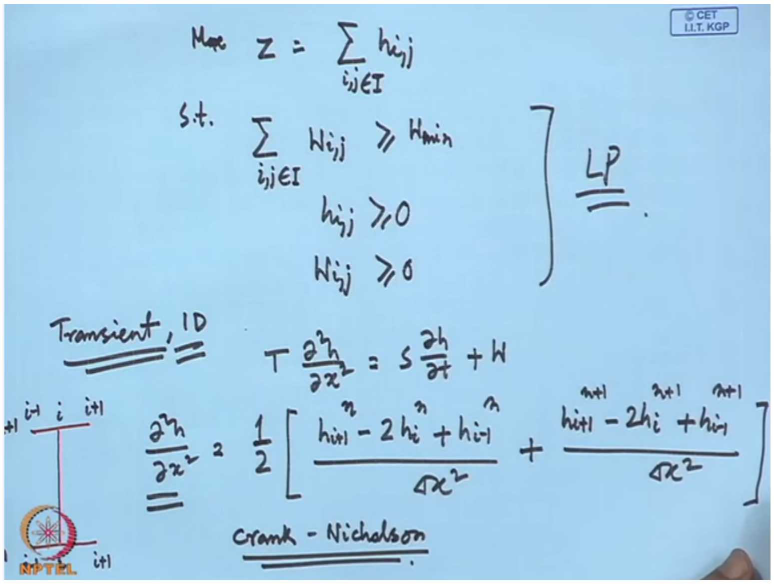

Maximized Z in this case, we should have HIJ with I comma J. This belongs to set I for wells and subject to our binding constraints that we have already derived for 2 D aquifer and IJ all I WIJ greater than W minimum and HIJ greater than 0 WIJ greater than equals to 0. So this can be solved using again by LP method or linear programming method. So if we have a transient problem, let us consider transient problem with confined aquifer system.

So transient problem with confined aquifer, so in this case we can have this Del, this is for one dimensional case and this is transient one D thing. So the equation Del 2 H by Del X 2 that can be expressed as Crank Nicholson. In Crank Nicholson half of the derivatives are evaluated at the present time step and half of the derivatives are evaluated at the future time step. Letters N denotes the time level and I + 1 denotes our space level.

So this is basically 2 HI N + HI - 1 N / Del X square + our HI + 1. This is N + 1 level 2 HI N + 1 level + HI 1 that is also at M + 1 level. So for single the second rounded derivative we are using this Crank Nicholson scheme. So in this scheme we can discretize a whole thing like this. Otherwise we can also discretize using our, any particular time strength.

So crank Nicholson scheme is basically bounded one, so we have this is Nth N + 1 level. This is nth level and this is specially I derivative or Ith location. This is I - 1 this is I + 1. this is again I - 1 I + 1. So it involves all these points within our calculation and that way it will give advantage during the solution process.

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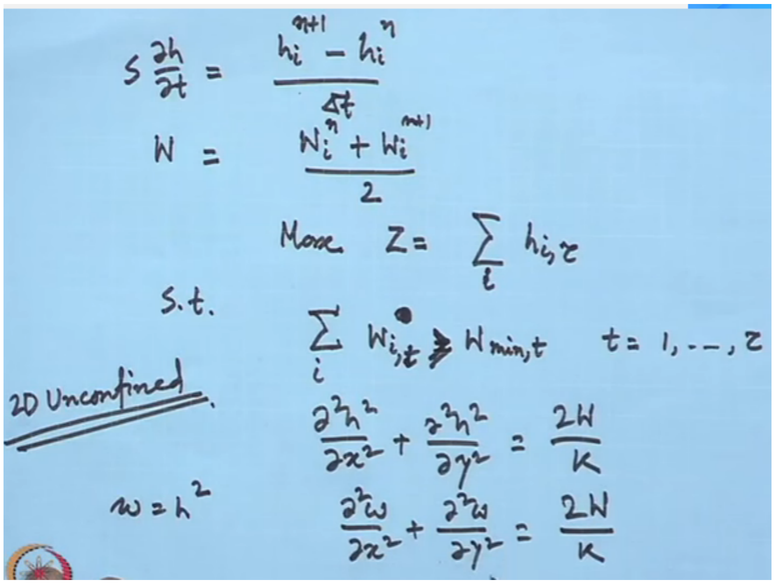

And if we discretize our this time dependent or transient term like this is IN + 1 - HI nth level and Del T. Also we can discretize our W that is average between I nth level and I N + 1 level. So if we substitute these terms in our additional equation we can get one linear equation and again if we employ our transient problem or transient values within our optimization framework. Then it will be time dependent optimization problem.

So we can write this thing as maximization of Z and our all I and this is I tow is basically our life last time step. So our subject to we can use the constraints as I W I N or we can directly use it here T level and this is greater than equal to W minimum and which is T level and this T from one to that tow.

So at last time step our head values should be maximum or we should maximize our head values at the last time step and we have this constraint that for each time period or management period. We can have the total pumping which will be greater than our minimum value of pumping. Finally if we have any 2 D unconfined aquifer situations, then we have the equation like this and with our usual substitution for unconfined aquifers.

We can derive the equation as Del X square Del 2 W Y 2, 2 W K and we can discretize this particular equation with our final difference formulation. Now this is time independent or steady state problem for unconfined aquifer situation. Now let us talk about the simulation optimization.

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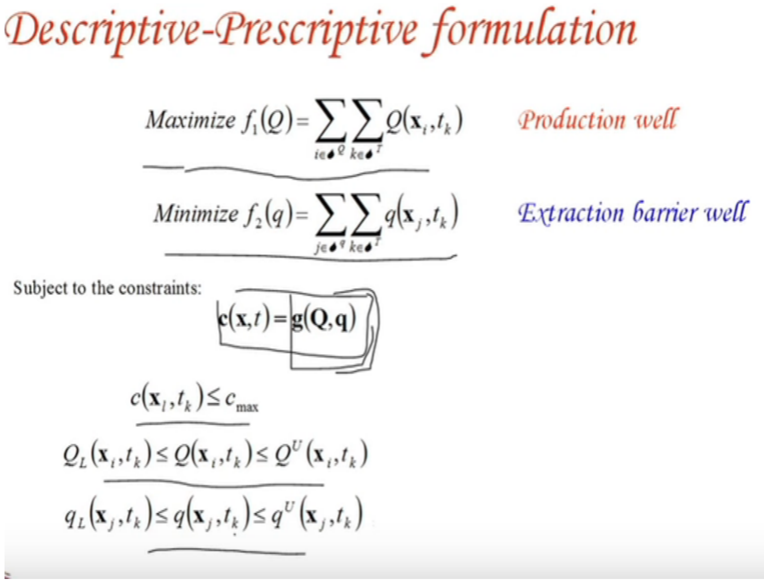

Problem in case of simulation optimization problem, we directly use the simulation model as binding constraints within our optimization model. But we link it externally using any readymade simulation model or we can write our code. But we link it externally with the optimization model for any flow and transport related problem. We can have one extra constraint that is your concentration value should be below the specified concentration limit.

So in this case this has been taken from soft water intuition management problem. So we have this maximization of pumping from production wells, then minimization of pumping from extraction or barrier well because these pump water cannot be directly used for our water supply systems. We need to have some kind of reverse osmosis or any other kind of plan to treat these water. So this capital Q is for any XI location and TK is the time level.

So time period at any specific special occasion at any particular time period, so summation over all spatial location and all spatial time periods. We can maximize this production function or production well pumping and minimize the pumping from extraction barrier well. So that is for location J and for time period K and this is subject to this external linked simulation model C.

We can directly get from this external chief function which is not exactly one proper expression but it is some kind of linked thing and this is concentration related constraint that it should not be greater than any specified concentration limit. These are the limit for our production well and barrier well.

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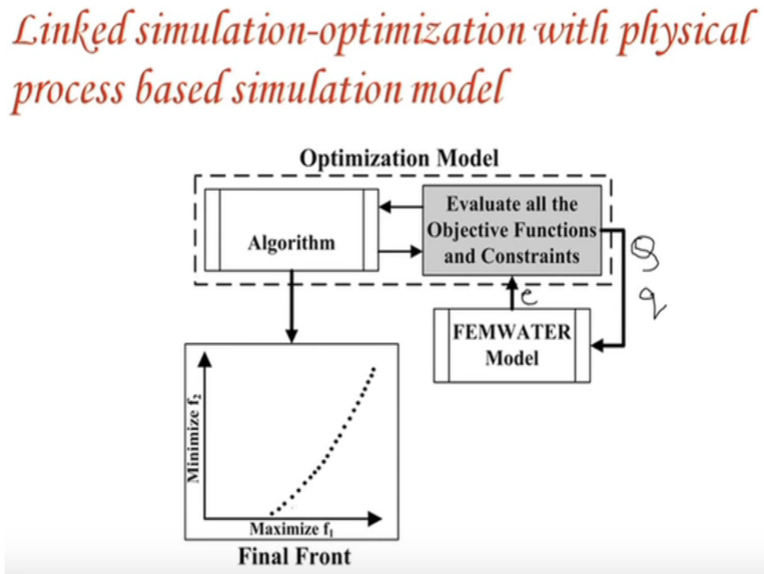

So we can have this kind of situation where this is our optimization model, so during evaluation of objective and constraints. We can go to fem water, is density dependent flow and transport simulation model. So during this objective function and constraint calculation we need the information about concentration. So what we are passing through are passing the information related to Q and small Q that is pumping values for production wells, pumping values for barrier wells.

And what we are getting out of this fem water model, we are getting the concentration values that way it is externally linked. Now with this algorithm we can have this final solution that this is one solution set that is for number of objective function values. So one objective is maximization of F1 is the production one is minimize the F2. So that way there is conflict, so if we maximize one thing there will be increase in other value. So there is trade of between first objective and second objective.

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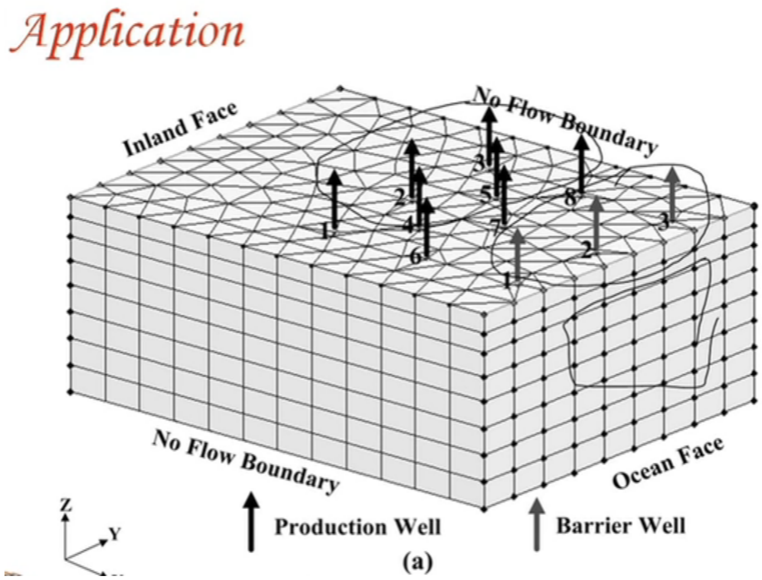

So let us consider one hypothetical example, this particular phase let us say this is our ocean face or sea face. These three wells are extraction or barrier wells and these wells are our production wells and these two boundaries are no flow boundaries and this is our inland face.

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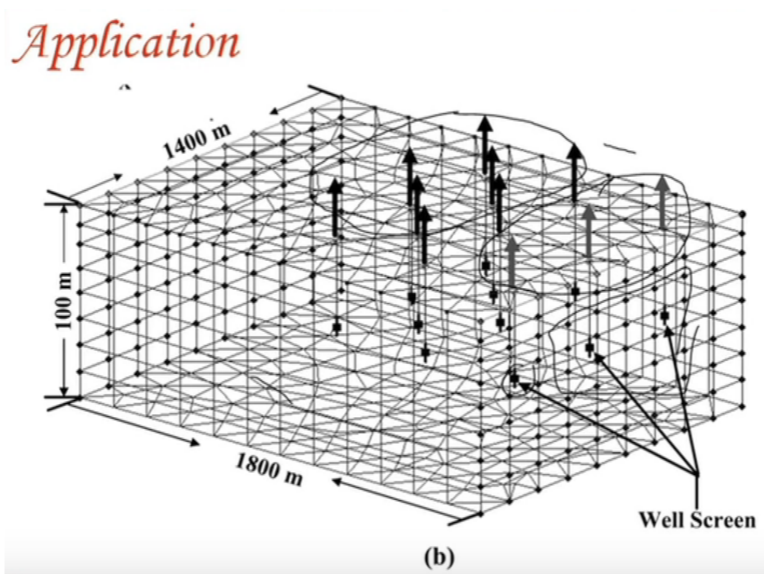

So this is the size 800 meter, 100 meter thickness of the aquifer and 1400 meters and this has been modeled using fem water.

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Different Parameters

Table 3.1: Different Aquifer and Discretization Parameters

Parameter	Unit	Value
Hydraulic conductivity in x-direction, K_{xx}	m/d	25.00
Hydraulic conductivity in y-direction, K_{yy}	m/d	25.00
Hydraulic conductivity in z-direction, K_{zz}	m/d	0.25
Longitudinal dispersivity, a_L	m	50.00
Lateral dispersivity, a_T	m	20.00
Molecular diffusion coefficient, a_m	m^2/d	0.69
Soil porosity, n	-	0.20
Density reference ratio, ε	-	8.40E-07
Vertical recharge, q	m/d	5.47E-04

$$\frac{\rho}{\rho_0} = (1 + \varepsilon c) \qquad \propto \frac{C}{C_S}$$

So these are the wells screen levels so these are the values for the hydraulic conductivity 25 meters per day for X and Y direction and .25 per day in z direction longitudinal dispersivity. Alpha L is 50 meters, Alpha T that is 20 meters molecular diffusion coefficient that is met .69 meter square per day soil porosity.

That is .2 and density reference ratio interestingly previously, we have used this Alpha C divided by CS. Now this Alpha by CS is basically this particular epsilon. So this is density reference ratio this is our vertical discharge although it is having that small Q notation but it is different from or pumping from extraction barrier thing.

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Different Parameters (Contd.)

- The U.S. EPA (1992) standard for Secondary Maximum Contaminant Level (SMCL) is 500 mg/l

C_{max}

So different parameters, let us say we have that C max value is 500 mg per liter. So this is standard for secondary maximum contaminant level.

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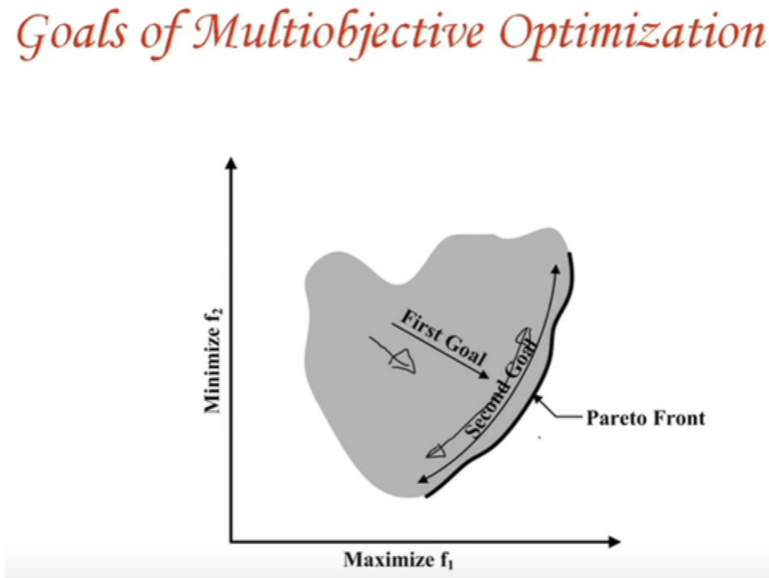

So this is 500 mg for any multiple objective optimization because we have two objectives. So we can designate it as multi objective optimization, so this is called as Pareto front. So one objective is spreading in these two direction, spreading of final solutions and second this is the second goal and first goal is movement of solutions and the final front should be almost coinciding without Pareto front or it is nearing to our Pareto front.

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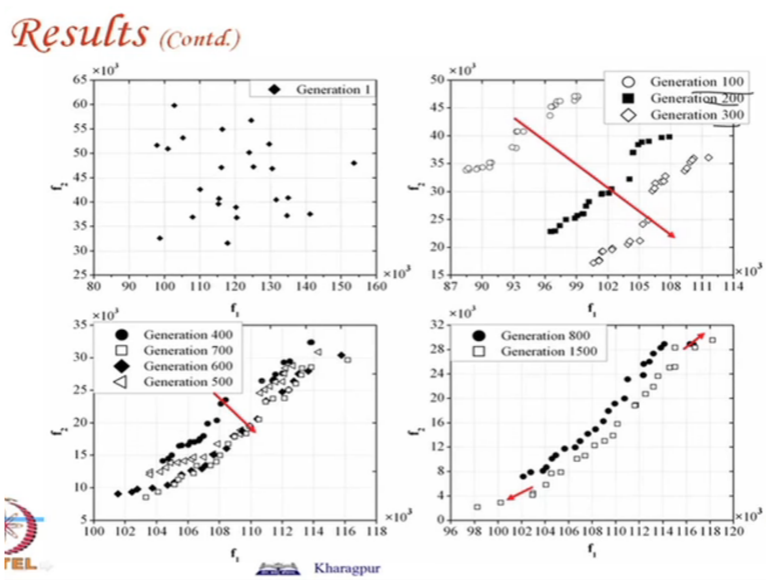

So with different number of alteration or generations, let us say this is generation number one there is first objective can be realized because it is 100 generation, 2000, 3000. So it is moving towards 300 number, in three hundred number generation moving towards Pareto front and with increase in generation number it is also spreading in both the direction.

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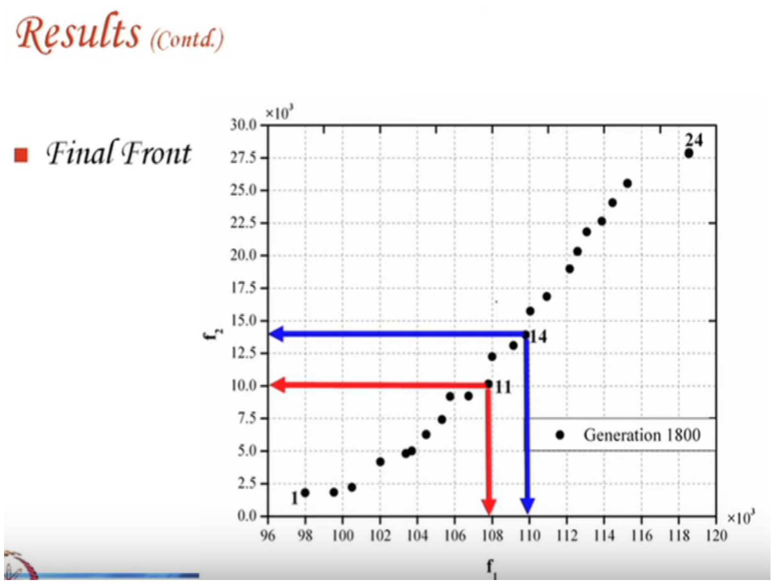

So let us say this is our final front, so in this particular front 11 and 14 are two points and these two points are showing two different results, two different means 11 and 14 will give you two sides of capital Q and small Q values and with that you can explain the process. One thing is that for point number 11 we have F1 value which is less than our F1 value compared to 14 but it is having a better value for F2 compared to solution number 14. So we can see that there is some kind of trade of between the solutions.

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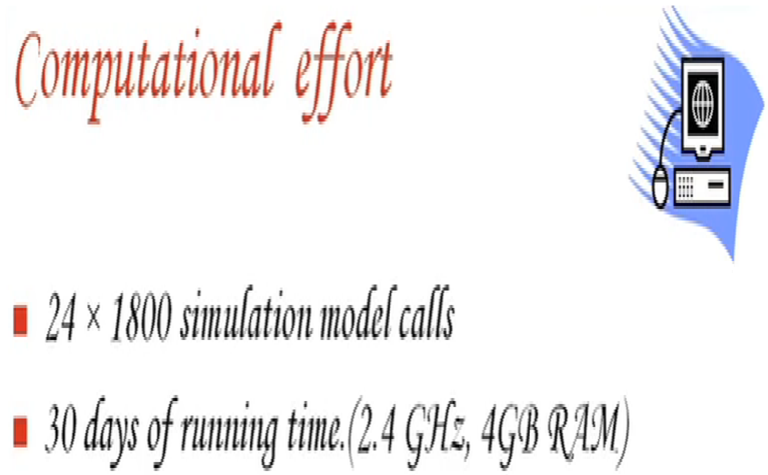

So it took around 20 for into 800 simulation, it took 30 days of running time. So the problem is that the running time is a problem for linked simulation optimization.

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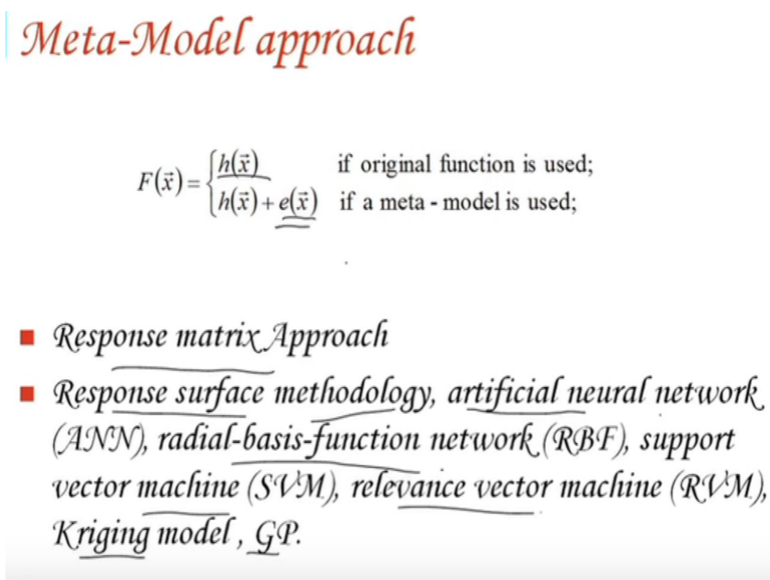

So what we can do, we can use our meta model based approach for management purpose. First thing is that response matrix approach, other thing response surface methodology or other meta model based approaches, artificial neural network, radial basis function, support vector machine ,relevance vector machine and Kriging model and GP. So what is the difference between our original simulation model and meta model.

So original simulation model will give you the exact value of H or C, but in case of meta model there will be some amount of error involved with it although we can gain in terms of solution time or gain in terms of solving a particular problem in management related issues. So first thing is this response matrix thing.

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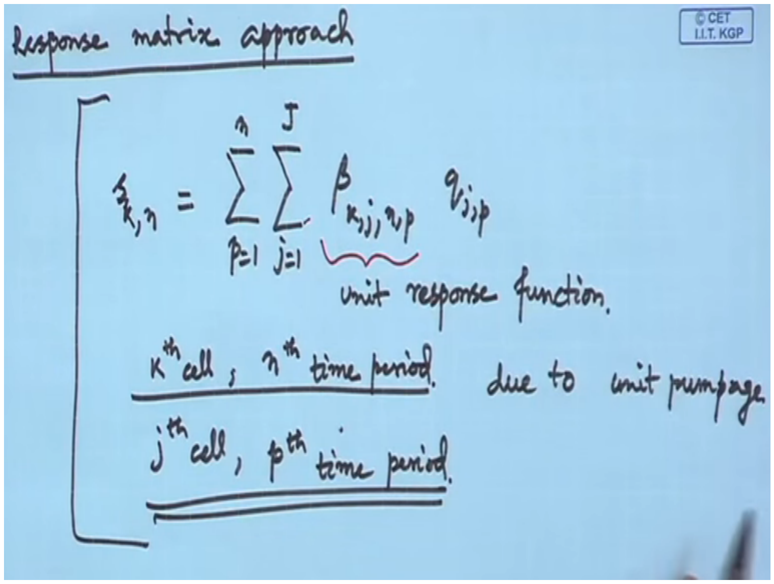

Response matrix approach in this approach let us say draw down for any location is K N, K is the special location or cell end at the end of Nth time period can be written in terms of this expression. So in this case unit response function it tells something that is in the Kth cell at the end of Nth time period due to unit.

So it is basically denoting the change in draw down for Nth cell Kth cell at Nth time period due to unit pumpage from Jth cell and Pth time period. So this is some kind of unit response for Kth cell at the end of Nth time period due to the unit pumpage or injection in the Jth cell during Pth time period.

So for Kth cell at the end of Nth time period, we need to consider all time periods which are less than Nth time periods starting from one to N and J. We need to consider all possible cell locations, so this is all possible cell locations. So we can replace our binding constraints with this particular expression. This unit response function can be obtained by simulation of the original simulation model. Now

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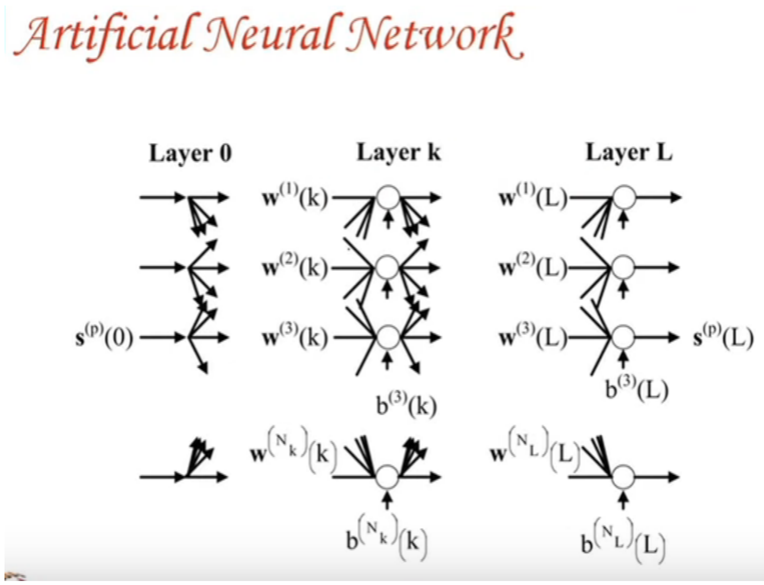

We can use ANN it has got architecture of something. So we have some input and we can get some output.

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Performance Evaluation

- Total error (E): $E = \frac{1}{PN_L}\sum_{p=1}^{P}\sum_{l=1}^{N_L}\left(d_l^p - y_l^p\right)^2$
- Average Absolute Relative Error (AARE):

$$AARE\ (\%) = \frac{\sum_{p=1}^{P}\sum_{l=1}^{N_L}\left|d_l^p - y_l^p\right|}{\sum_{p=1}^{P}\sum_{l=1}^{N_L} d_l^p} \times 100$$

- Correlation Coefficient (R):

$$R = \frac{\sum_{p=1}^{P}\sum_{l=1}^{N_L}\left(d_l^p - \frac{1}{N_L}\sum_{l=1}^{N_L} d_l^p\right)\left(y_l^p - \frac{1}{N_L}\sum_{l=1}^{N_L} y_l^p\right)}{\sqrt{\sum_{p=1}^{P}\sum_{l=1}^{N_L}\left(d_l^p - \frac{1}{N_L}\sum_{l=1}^{N_L} d_l^p\right)^2}\sqrt{\sum_{p=1}^{P}\sum_{l=1}^{N_L}\left(y_l^p - \frac{1}{N_L}\sum_{l=1}^{N_L} y_l^p\right)^2}}$$

And we can evaluate the performance using total error it is desired and actual one and again desired and actual one. Average absolute relative error can be computed using this, so this is basically number of layers and we can also point of this correlation coefficient for this one to check the accuracy of the ANN models.

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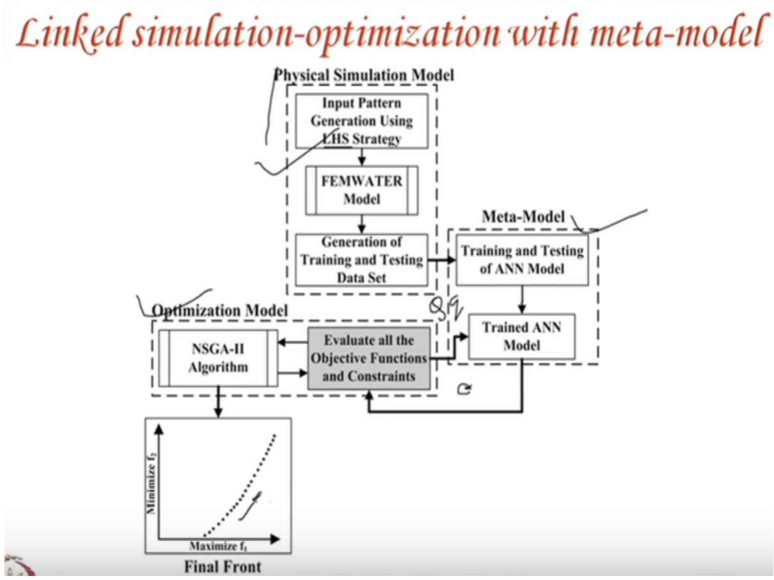

So what is there in linked simulation optimization with Meta model? We have physical simulation model here and we have Meta model here and we have optimization model here. So from simulation model we can generate input patterns using that this is Latin hypercube sampling.

This is one sampling strategy with which we can generate our input pumping values and we can input that into fem water model and we can generate training and training data set. We can train that ANN model or any SPA GP model Then we have that train model in we will pass that capital Q and small Q. Again we will get C value out of this and we can use this C value for optimization and again we can get this multi objective solutions.

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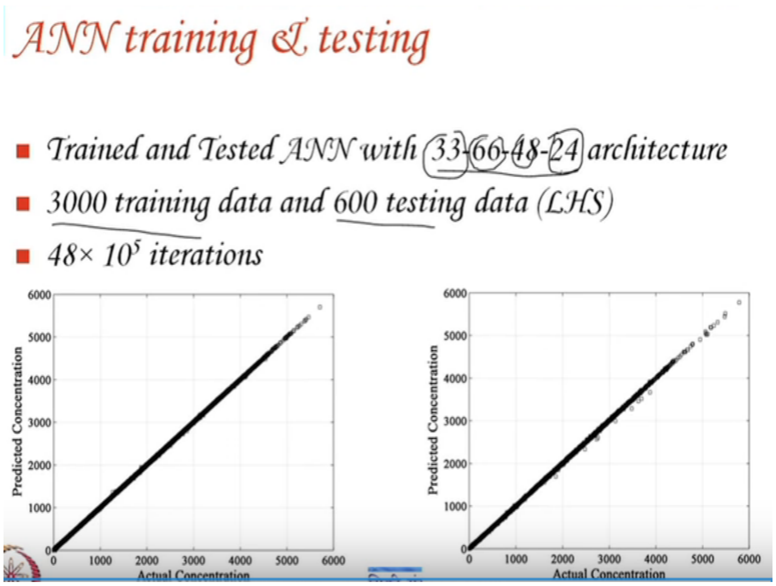

So this many alterations are required for the testing part and the 3000 training data set was used and 600 testing data set and this was the architecture 33 input these are two hidden layers and this is the output thing.

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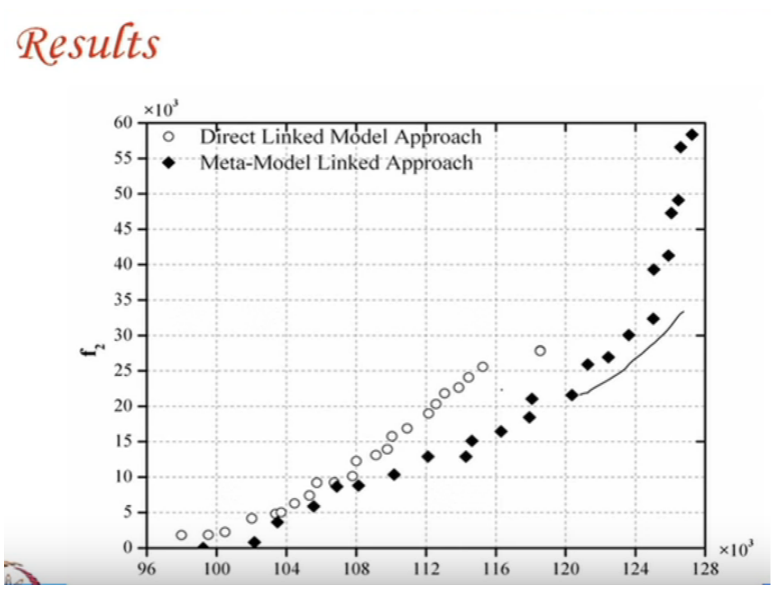

So we can see that meta model based approach is giving almost equal or better results compared to our direct link simulation optimization model. There can be another approach where we can use this screening model thing.

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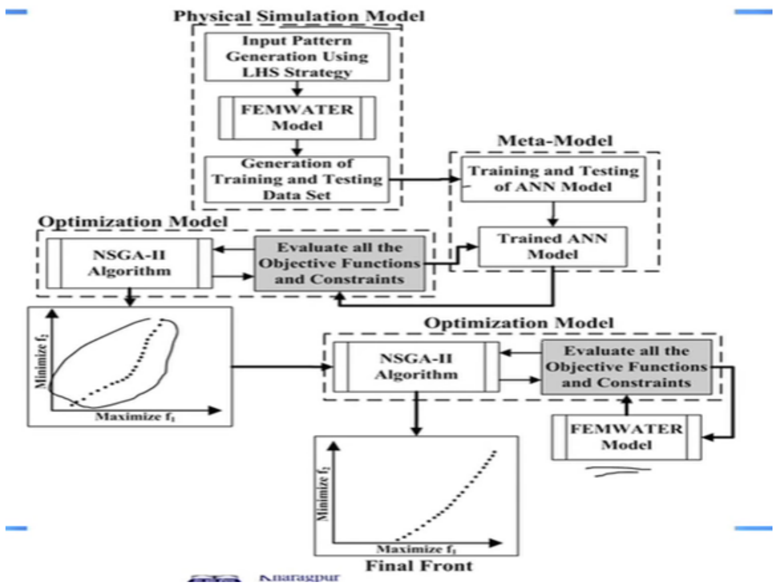

So we have this physical simulation model from physical simulation model we can have our meta model from meta model we can run our optimization model and we can get intermediate perotal front or intermediate solutions and we can pass that particular solution to the final objective or final optimization model where we have original optimization.

Original simulation model is linked with the optimization model, so that way we can get a good accuracy in terms of results and also we can reduce the number of alteration that will be required for simulation of original femwater model during management optimization simulation process.

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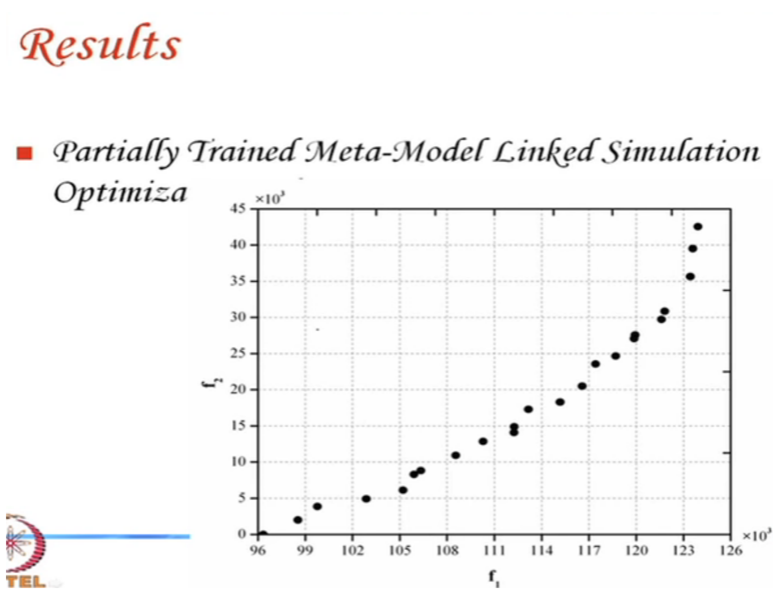

So partially trained Meta model linked simulation optimization and

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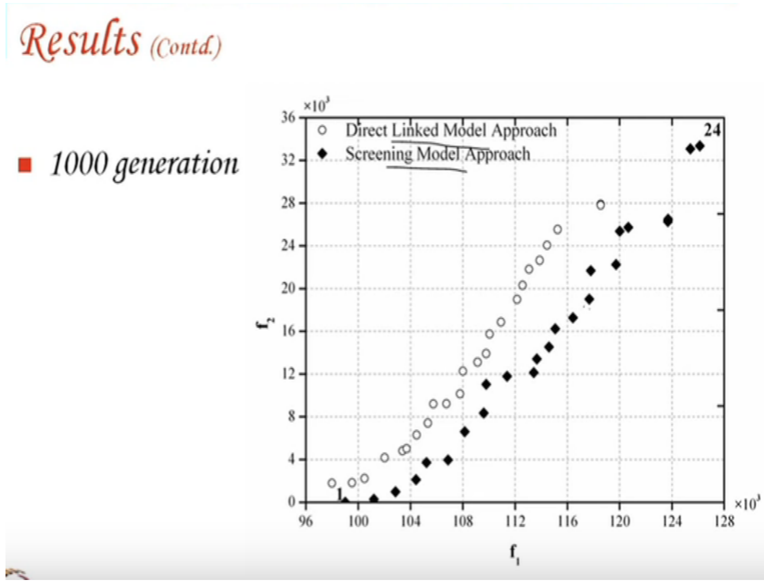

With 100 generation, 1000 generation these are giving screening model based approach. This is with linked simulation best approach direct simulation based approach it is giving always the better result because these are the correct values and it is more near to our perotal front because as it moves in the right hand direction it is more near to out perotal front.

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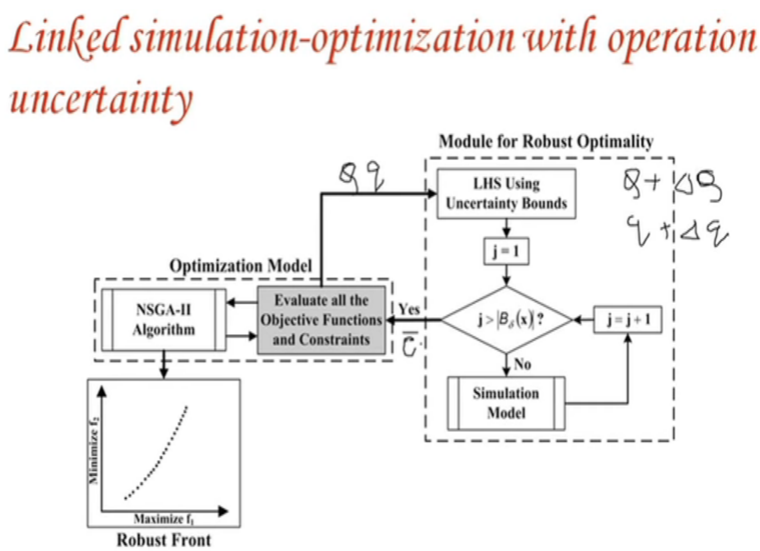

There can be linked simulation optimization with operation uncertainty. We can run this simulation model for multiple realizations. We can give Q and small Q values and we can generate Q + Del Q combinations Del Q is small variations because in reality there will be variation of the these values in filled situations.

So with these combinations we can run the model and we can get the average objective function value and standard deviation that we can utilize here and we can get some kind of C bar with some C standard deviation. C bar means some mean value for the concentration. We can pass it and we can get robust optimal solution for the management problem.

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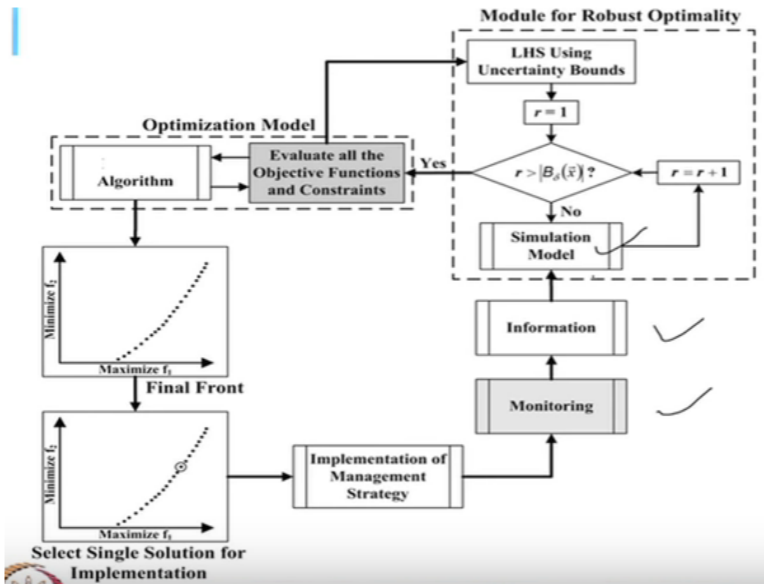

So integrated planning mechanism, so we have robust optimal solution from this multiple realization approach we can get final front or final perotal front. Out of that we will select one strategy and will implement that single strategy in the field there will be monitoring and will collect some information. Again with those information we can update the simulation model that is the actual process for any integrated planning approach. Thank you.