

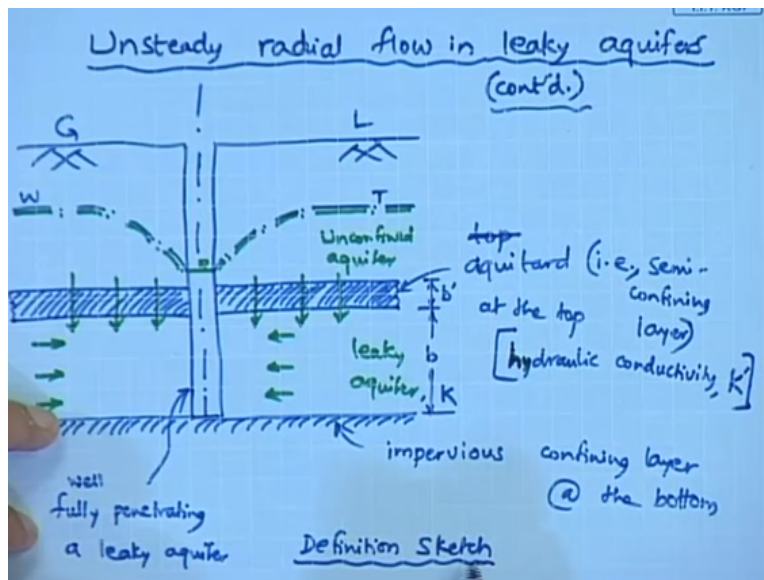
Ground Water Hydrology
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Module No # 03
Lecture No # 15

Unsteady Radial Flow in Leaky Aquifers (Contd.); Well Flow near Aquifer Boundaries

Welcome to this lecture number 15 in which we will continue with the previous lecture that is the unsteady radial flow in leaky aquifer.

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And here let us start with the sketch actually the diagrammatically sketch in which so there is a well it is fully penetrating into the leaky aquifer. And so this is the water table and here for simplicity we are considering this aquitard that is the semi confining layer only at the top although actually the aquitard can be at the bottom also instead of the top or it can be both at the top as well as bottom which was briefly listed in the previous lecture but for the simplicity.

We are considering only at the top aquitard or the top semi confining layer so this is and the bottom confining layer is fully impervious. So this is the aquitard art that is semi confining layer so it is a top or a aquitard at the top and this is the impervious confining layer at the bottom and this well is fully penetrated here let me so this is the water table and here so there is a radially inward flow into this well which is fully penetrating into the leaky aquifer.

So here let me mention here so this is the well fully penetrating a leaky aquifer so here this is the unconfined aquifer and this is the leaky aquifer. And this unconfined aquifer and this aquitard through this aquitard so there is some contribution of groundwater into the leaky aquifer. Here so this aquitard at the top so it has so the hydraulic conductivity there is K dash similarly this leaky aquifer.

So with the hydraulic conductivity K and this one the thickness of the aquitard is B dash and similarly the thickness of the leaky aquifer is B . So with this that means K and B represents the hydraulic conductivity and the thickness of the leaky aquifer while K dash and B dash represents the hydraulic conductivity and thickness of aquitard which is situated at the top. So with this definition sketch so this is the definition sketch and in the previous lecture.

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Leaky aquifer well function is $W\left(u, \frac{r}{B}\right)$

$$s = \frac{Q}{4\pi T} \cdot W\left(u, \frac{r}{B}\right)$$

$$\frac{r^2 s}{4Tt} = r \sqrt{\frac{K'}{k \cdot b \cdot b'}}$$

$$= \frac{r}{\sqrt{\frac{T}{(k/b')}}}$$

for a fully confined aquifer,
 $k' = 0 \Rightarrow B \rightarrow \infty$

$\therefore W\left(u, \frac{r}{B}\right)$ becomes $W(u)$

i.e., Leaky aquifer well function \Rightarrow Confined aquifer well function
 i.e., well function

So it is a the expression for the leaky aquifer well function was written as follows so this is denoted by W of U comma R / B where this B represents this equivalent aquifer thickness that means it is a thickness which represents the thickness of leaky aquifer at the bottom having a top aquitard and here so this is a the the Theis modified Theis equation is a the equate aquifer well function is a $W U$ comma R / B .

And here obviously this U is R square into the storage coefficient divided by 4 transmissivity into the time since pumping and then R / B . So this R / B so this R and B is it is R into under square root K dash which is the hydraulic conductivity of the top aquitard divided by K into B

into B . So this can also be written as R divided by \sqrt{T} that is the transmissivity of the leaky aquifer divided K' / B which is the ratio of hydraulic conductivity of the top aquitard as well as thickness of top aquitard.

So this is this represents this bottom the denominator that is square root of $T / K' / B$ that is equivalent to B here and with this definition sketch as well as parameter. So now the equation for the draw down is given this $Q / 4 \pi$ into the transmissivity of the leaky aquifer multiplied by leaky aquifer well function that is $WU, R / B$.

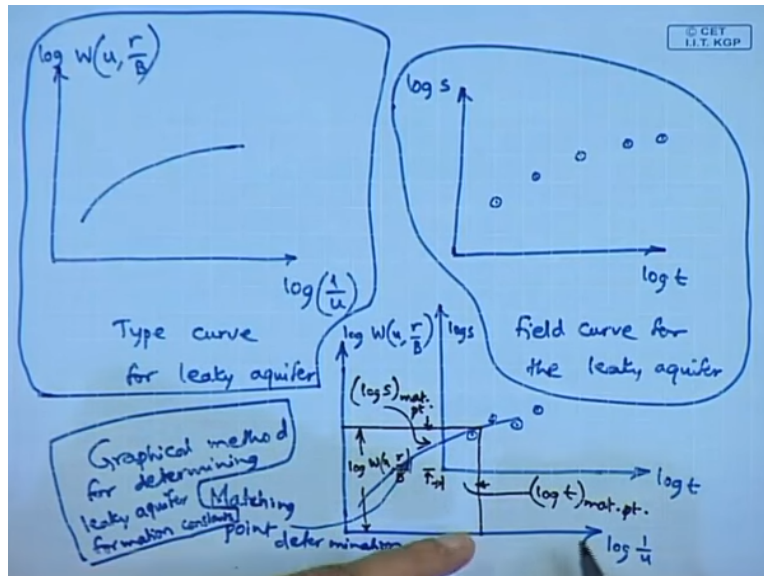
B is the capital it is capital B it represents the equivalent thickness of the leaky aquifer as well as in it as well which is confined by a top aquitard or top confining layer and here. So this one and here let us consider in case of a fully confined aquifer so what happens is this in case of fully confined aquifer so this K' that is the hydraulic conductivity of the aquifer becomes 0.

Because it is fully confined at the top as well as the bottom so here for a fully confined aquifer $K' = 0$. So this implies that B tends to infinity and therefore this $WU, R / B$ becomes WU that is leaky aquifer well function. So this becomes this gets transformed into this confined aquifer well function which is simply known as well function.

So therefore so this is a so this relationship for the drawdown in terms of pumping discharge as well as the transmissivity of the leaky aquifer and this U which is the well function parameter which is R^2 into storage storativity divided by $4\pi T$ into T^4 times transmissivity into time since pumping R / B . So this R is the radius of the well as well as B is the equivalent thickness of the leaky aquifer as well as the top aquitard.

So this one and here also we can force this is a graphical method similar to the type curve which is followed in the confined aquifer.

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So here also that similar method can be used here also so what is done here is so there is a log-log plot of log of $W(u, r/B)$ versus log of $1/u$ then another so this is the type curve for leaky aquifer. Similarly there is a plot the drawdown that is log of S versus log of T this is the field curve for the leaky aquifer. And both this will have a similar shape of this one.

And here what is done is so in this case this may be here this field curve so where will be different points depending upon the and here this is the leaky aquifer well function which we already we discussed in the previous lecture. And here so this will be something like this. So if we match the field curve over the type curve. So that most of this data points on the field curve will overlap or will lie more or less on the almost on the type curve.

So in that case while at the same time maintaining the parallel nature of the axis so this is a so here I am just and next is this one the field curve and then so this is the matching that is a matching point determination. So this is the graphical method for determining leaky aquifer parameters formation constants. So here what is done is so the firstly there is the type curve having this log of $W(u, r/B)$ versus log of $1/u$.

So it will have a this kind of nature and then here so it is matched with so this is log of T and this is log of S . So wherein we have number of data points like this so in this case this is the matching point so this is the and once we do that for any using any point we can get the we can determine the leaky aquifer formation function such as the transmissivity and storativity.

So for any point so here let us say typical matching point in this case so this represent log of T so this is log S for the matching point. Similarly this represents log T for the matching point and this represents that is log WU comma R / B and this represents log W. And so therefore here what we can do is using this matching technique we can determine the formation constants of the leaky aquifer.

So by this matching technique so that is we know this S versus T so in that case the corresponding this one is there.

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The image shows handwritten mathematical derivations on a blue background. At the top right, there is a small logo for 'I.I.T. KGP'. The equations are as follows:

$$u = \frac{r^2 S}{4Tt}$$

$$s = \frac{Q}{4\pi T} W(u, \frac{r}{B})$$

$$\therefore T = \frac{Q}{4\pi s} \cdot W(u, \frac{r}{B}) \quad \text{--- (1)}$$

for the matching pt.

$$\therefore S = \frac{u \cdot 4T \cdot t}{r^2} \quad \text{--- (2)}$$

So therefore this is $U = R^2 S / 4Tt$ so here this based on the matching point $1 / U$ is known so therefore U is known log of $1 / U$ is known. So therefore the UE this law 1 by is known and then this time is known and then this R which is the radius of the leaky well fully penetrating through this that is known. So therefore we should be able to calculate this one they so this is one equation and then secondly we have the other equation this is $S = Q / 4 \pi T$ into $W U$ comma R / B .

So here this is known this is known based on this so this is known so therefore $T =$ that is $Q / 4 \pi$ into S into $W U$ comma R / B . So this is a for the so this is for the matching point once this T is known so then so this is here it can call this as equation so this you can call equation one so

therefore using this equation one and the expression for the well function leaky aquifer well function parameter that is U .

So we can determine this storativity of the well function storativity of the leaky aquifer = U into $4Tt/R^2$ so this is for the matching point. So this is also the matching point this also for the matching point so this is equation two so like this once we match the both the curves similar to the one which we did in the confined aquifer.

Similar to the one we did in the confined aquifer so here we are matching the individual points on this pumping test based on this pumping test data where in the drawdown as well as the time since pumping that is plotted in this curve and we match that with this the plot of that is the like is the WALTON's theoretical curve for the leaky aquifer. So this is the type curve so this is also the WALTON's theoretical curve.

So using this and matching maintaining the parallel nature of the vertical as well as the horizontal axis. So you get the matching point and then for any point we will get four different values of log of the leaky well function parameter as well as the log of $1/U$ log of leaky well function as well as log of reciprocal of the well function leaky well function parameter and the log of drawdown as well as the log of time since pumping.

So then simply substitute all values and then obtain the the transmissivity and firstly the transmissivity using this expression that is the Theis the modified Theis equation for the leaky aquifer as well as from that the the storativity of leaky aquifer using as per the equation two. So like this we can solve leaky aquifer and of course imagine and here we can imagine how involved it is.

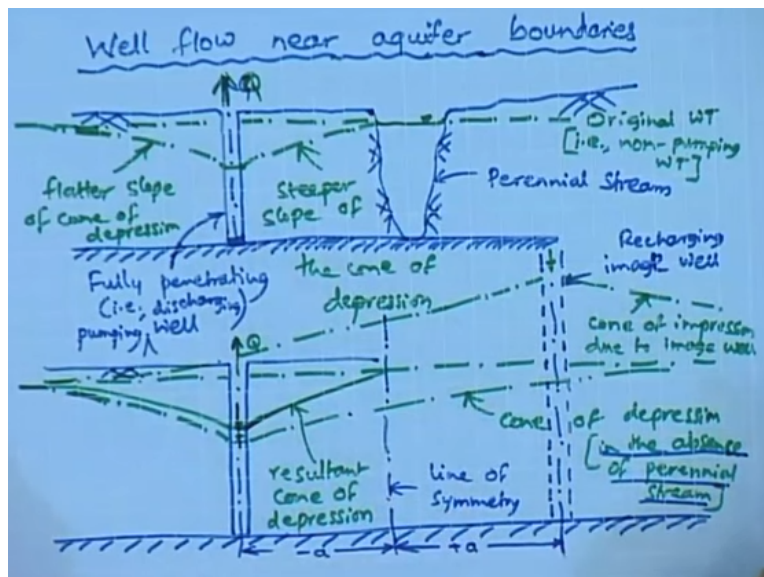
So here considering things which is simply we have simplified it to a great extent so that means although in case of a leaky theoretically or there can be it can have semi confining layer at the top as well as bottom and more than that so the even between these two semi confining top as well as bottom one may have less hydraulic conductivity one may have more hydraulic conductivity.

So if a top aquitard is having less hydraulic conductivity then the leaky aquifer may be losing its ground water volume whereas on the other hand if the that is the bottom confining semi confining layer is having a less hydraulic conductivity then in that case it could be a gaining leaky aquifer generally even without any pumping well.

So therefore we have even with the simplified one wherein the bottom confining layer is taken to be fully impervious and there is only the top that is the semi confining layer or a aquitard layer for the leaky aquifer and that too we are considering that to be having uniform hydraulic conductivity of K_{dash} and a uniform thickness of B_{dash} with this the we are we are in a position to determine the formation constants of the leaky aquifer such as the transmissivity and storativity using this the equations.

So that so this will complete the unconfined radial flow in a leaky aquifer and now we will go to that is the next component of this lecture that is the well flow near aquifer boundaries.

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And here let us start with the well flow near the stream and in this case say let us consider a stream and of course here I am exaggerating the depth of the stream and here there is a pumping well on one side of the stream and of course we are considering only the so this is the stream this is the perennial stream. So this is the perennial stream and then this is the pumping well fully penetrating well.

And we are considering so there is a bottom confining layer which is common for the well as well the perennial stream this is Q okay. Let me follow the color code that is green for this representing water and then so this is the fully penetrating well and here so we will once. So when there was no pumping so this is the so this is the original water table when there is no pumping and once the pumping starts so there will be a cone of depression and this is the shape of cone of depression fully penetrating this is a pumping well pumping.

So here which is also can be denoted as a discharging well and it is on one side of this perennial stream. Now here so the this case the water table will have a flatter this cone of depression will have a flatter slope away from the stream and steeper slope towards the stream and this one can be represented by that is to separate this one that is say let me redraw this one so here let me write here so this is a flatter slope of cone of depression.

Whereas this is a steeper slope of the cone of depression steeper slope will be towards the perennial stream and the flatter slope is away from the perennial stream and so these are the so this is the perennial stream. So now this can be thought of as a so let me redraw so this is Q and this is the ground level and so this is the non-pumping or the original water table that is non-pumping water table.

And here so the cone of in this case suppose we replace this upstream bank I am sorry the near bank of the perennial stream by a barrier. Here the resultant cone of depression so this can be thought of as a symmetrical that is a recharge well. So this is so here this is this is recharging image well so which is imaginary then so this is symmetrical with respect to the so this recharging image well.

So this is the line of symmetry so this is the actual discharging well which is on one side of the perennial stream and because of the perennial stream so it is getting more contribution more radial contribution on the perennial stream side on the other side for this discharging well. So therefore this cone of depression shows this unsymmetrical slope the flatter slope on the side away from the perennial stream and the steeper slope towards the perennial stream.

So this can be explained by introducing so this is an image well which is symmetrical with respect to the line of symmetry so which line of symmetry represents the nearest bank of the

perennial stream to the discharging well. And so here these distance are say if this distance is $-A$ representing because it is to the left of the line if symmetry and this will be $+A$.

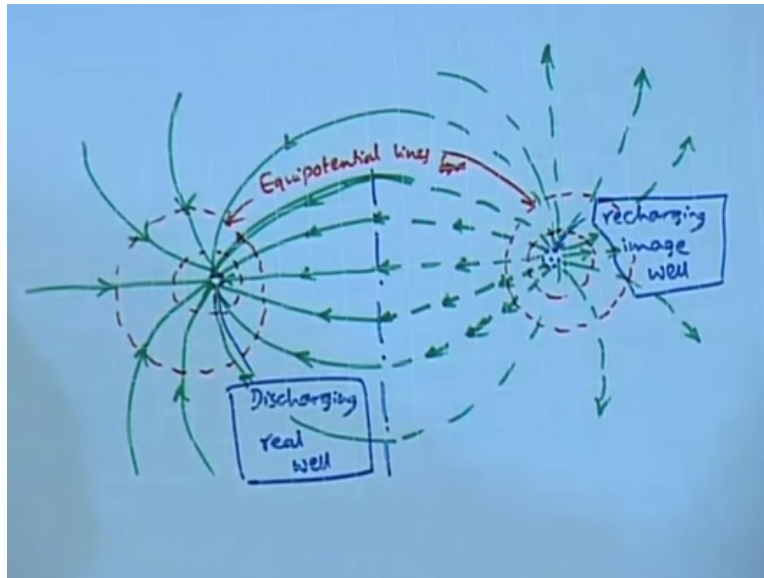
So therefore here so this is the resultant cone of depression and then this is the cone of impression due to image well. So this image well is analogous in its effect it is analogous to contribution by this contribution to the discharge by the perennial stream. So therefore this is the cone of depression and this is the cone of impression. So the resultant of the cone of depression in the absence of perennial stream.

So therefore and this perennial stream can be replaced by symmetrical image well when which is recharging so therefore the contribution of this this one the perennial stream is a replaced by a symmetrical recharging image well. So finally we get this depression cone of depression as shown by this solid green line which is resultant of cone of depression in the absence of perennial stream shown by this dot and dash convention as well as the cone of impression due to the image well.

Which is imaginary and which is having the same which is providing the same effect as the perennials stream. So here like this the because of the boundary so in this case upstream that that is the near boundary of the perennial stream. So the cone of depression will show an unsymmetrical slopes steeper slope towards the stream and the flatters slope away from the stream.

Now let us consider the next case where in there is a solid boundary and so here in this case we can also draw that is for this figure we can also draw the flow net.

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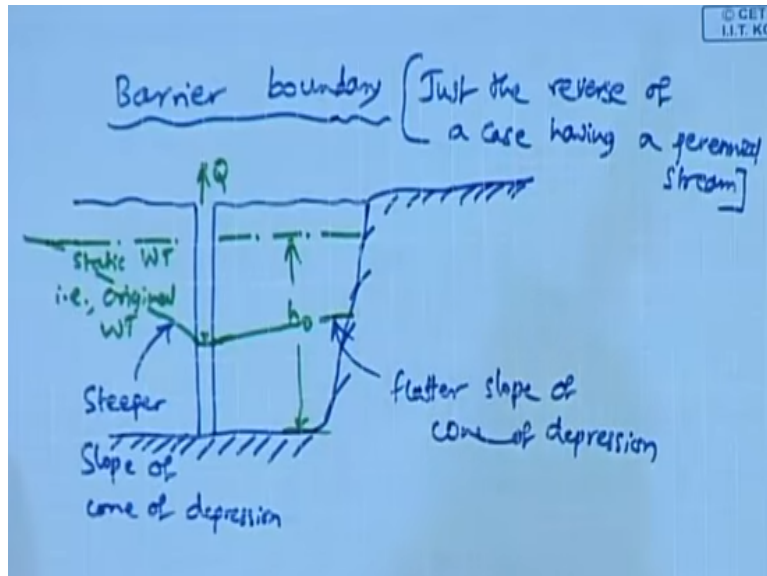


So here we will get this is the pumping well and this is the imaginary recharge well the recharge image well and here so the streamlines so there will be and then this is the boundary here and here so this is the pumping well and then this is the this is the discharging real well and this is the recharging image well and as usual. So this is and here so like this we get so these are the streamlines for the recharging image well.

And then similarly which are radially diverging radially outward as one having radially outward shape were as the discharging real well the stream lines are converging and obviously these are the streamlines and then the equipotential lines. So will be circles then similarly here the so these are the equipotential lines for the discharging real well okay anyway it is obvious and so this is equipotential line and then this is the here is the equipotential line this is the recharging image well.

So like this so this is the flow net and next we will consider the case of a barrier boundary so far we have studied we consider the case of a stream and now let us consider the case of a barrier boundary

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And in this case so this is the barrier boundary and here we have a pumping well or discharging well. So this is a Q from the discharging and then the static water table or the original water table. So this is the static water table that is original water table and let us consider so this height as H_0 from the impervious boundary and then so this is the and in this case the final the shape of this the water table.

So in this case it will be reverse of the perennial stream so in case of perennial stream so the cone of depression was having steeper slope towards the well and the flatter slope away from the steeper slope towards the stream and flatter slope away from the stream in this case it is reverse. So because of the boundary so the contribution of ground water contribution towards the boundary will be less therefore we will have a flatter slope whereas the ground water contribution away from the boundary will be more so that it will have a steeper slope.

So in this case so this is a flatter slope of cone of depression and this is a steeper slope of cone of depression so this is just the barrier boundary so this is a just the reverse of a case having a perennial stream. So we will so this is a so we considered in this first we considered the that is the well flow near well flow near perennial stream and next we will consider well we consider well flow near barrier boundary. So we will stop here and then I will continue the next lecture thank you