Advanced Foundation Engineering

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Lecture - 40
Soil - Foundation Interaction (Contd.)

In the last class, we have discussed about that modeling of stone column-supported embankment, and then how the different components of that total system can be modeled by using this mechanical element like spring, dashpot, shear layer, those things have been discussed. Today, I will discuss further how these things can be developed in equation form and then how those equations can be solved.

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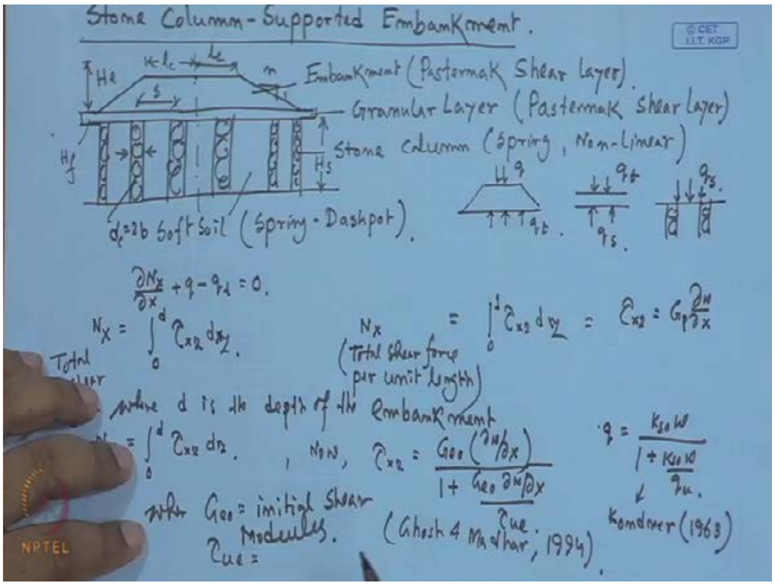

In the last class, in this embankment – if this is the embankment and this one is resting on a granular layer; and then this granular layer is sand pad or sand cushion, and then this is resting over the stone column improved soft soil. So, these are the stone columns. This embankment is modeled by Pasternak shear layer. This is by spring – stone column, which is non-linear; then soft soil by spring dashpot, then granular layer also by Pasternak shear layer. So, these are the different components, which are modeled. Then if it is central lined and this is l c; this one is also l c; where H E – height of the embankment; H f – height of the granular field or thickness of the granular field; H s is the depth of the soft soil; s is the spacing between stone column; d c or equal to 2 b is the diameter or width of the spring stone column. So, this is the plane-strain analysis.

Last class, it was derived that, for the shear layer in delta N x by del x plus q minus q t – that was 0. q t means in the embankment, if at the top, this is q; then at the bottom, reaction is q t. Similarly, for the granular layer, on the top, it is q t and in the bottom, it is q s. Similarly, for the soft soil over stone column, this will act as q s over this stone column-reinforced soft soil. Now, these things been already discussed that the stress acting on the stone column and stress acting on the soft soil are not same. So, now, in this expression, q t and this total expression, we have to write in this form that, we know that N x is the total shear force and that is, we can derive 0 to d into tau x z d x, because last class, it was derived that N x – that here N x – if it is total shear force per unit length, then N x will be 0 to 1 tau x z d z; here also d z, because it is in the unit length. So, we will get this is simply tau x z, which is nothing but G p into del w by del x. But here it is unit length; but here we will consider, this is in terms of d; where, d is the depth of the embankment.

Now, this d is not constant (( )) this length, because d is varying; up to l c distance, it is uniform H E; then it is decreasing – 1 is to n component. Now, this d value is 1 is to n. Now, next one; if I get this N x value, this will be 0 to d tau x z into x z into d z. So, now, here we consider a non-linear relationship of tau x z. So, in the similar to the k s relationship that, K s non-linear relationship that was given that, q is equal to K s 0 W by 1 plus K s 0 W into q u. So, that expression is already given for the non-linear form. So, similar expression was suggested by Ghosh and Madhur in 1994 to express this… This expression is originally proposed by Kondner, 1963. Similar expression was proposed by G e 0 – Ghosh and Madhur – d by d x divided by 1 plus G e 0 d w by d x divided by tau u e. So, this is proposed by Ghosh and Madhur, 1994. Now, in this expression… If we look at this expression, this is similar to the non-linear expression, where G e 0 is initial shear modulus and tau u e is the ultimate shear strength of the embankment material.

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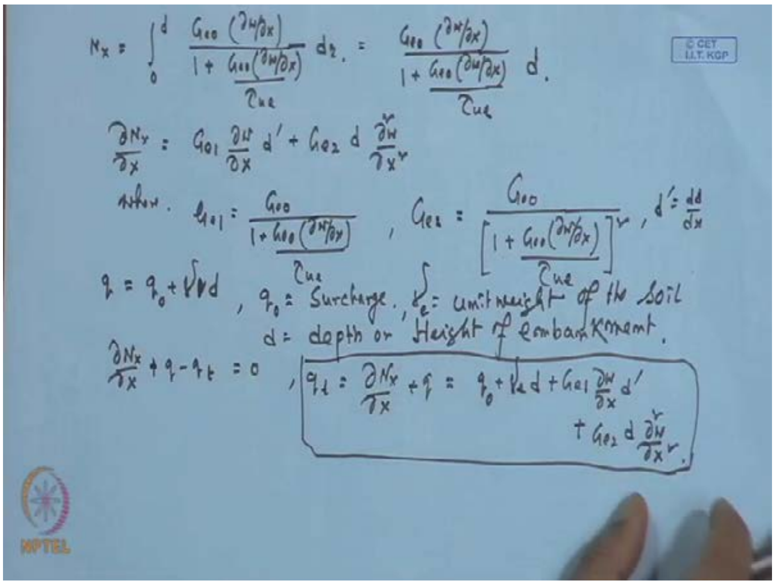

Now, this is the non-linear expression that we are using. Once we use this expression, final expression that we will get that, N x – that will be equal to 0 to d G e 0; then del w by del x divided by 1 plus G e 0 del w by del x by tau u e into d z. So, once we derive this expression, after the derivation and the integration, putting this d value, we will get this is equal to… Now, this is here. So, this will be equal to G e 0 del w by del x divided by 1 plus G e 0 del w by del x divided by tau u e into d; just you integrate 0 to d; you will get d z after integration d.

Now, again, del N x by del x – that will give a G e 1 del w by del x d dash plus G e 2 into d del 2 w del x square; where… So, just derive this expression; we will get in this form; where, G e 1 is G e 0 divided by 1 plus G e 0 into del w by del x into tau u e; and, G e 2 is equal to G e 0 by 1 plus G e 0 del w by del x divided by tau u e into whole square. So, we will get in this format, d dash is nothing that d d by dx; the rate of change of this d with respect to x. Now, this q as a tau, we can write, that is, q 0 plus gamma into d; where, q 0 is the surcharge; gamma is equal to unit weight of the soil; and, d is equal to depth or height of the embankment. So, once we write this expression, then the final form we can write, because the final expression is del N x by del x plus, that is, the final form of the expression – that plus q minus q t is equal to 0. Once we put this value, q t value will be equal to del N x by del x plus q; that is equal to q g plus gamma e d – unit weight of the soil means embankment – plus G e 1 del w by del x d dash plus G e 2 d del square w del x square. So, similarly, this is the expression, where this is the q t; this q t is acting over the embankment layer.

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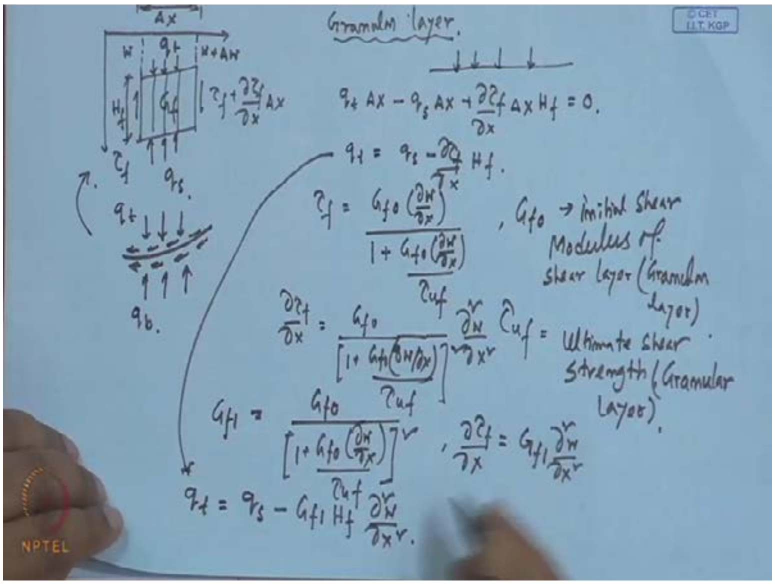

Now, for the embankment layer, we have to derive similar type of expression. So, for the embankment layer, if I consider a similar type of expression that, for embankment, if I consider this shear layer; because both are granular layer also used… Granular layer – we have also used this Pasternak shear layer. So, now, for the granular layer, if I consider a particular stiff of thickness H, because H f is the thickness of the granular layer; and, here also, the shear force that will act – tau f into tau f plus del tau f d x by del x, because this distance thickness is del x. Similarly, this one will be w; and, this one – w plus del w. And this is G f, is the shear modulus. And here on the top, q t will act; and, in the bottom, q s will act. If it is a reinforcement layer; then here in the reinforcement layer also, there will be shear stress and there will be normal stress – top and the bottom. So, then this will be q t and this will be q b. And if we placed say reinforcement layer on the top or interface between granular layer and the embankment, then this will come first; then this expression will come later on. But here in place of q t, we have to use q b; this is q s.

Now, in this expression, for a small segment, if we consider the… Take this expression; then we can write q t del x minus q s del x. And from here, tau (( )) cancel out; plus del tau f by del x and this is del delta x now. Here this is H f. So, this will be H f. So, this one will be equal to 0. So, now, the q t q s (( )) If del s del s cancel out, this q t is equal to q s minus del tau f by del x into H f. Now, similar to for the del tau f… Similar to for the tau f also, if I consider the f 0 del w by del x non-linear expression, 1 plus G f 0 del w by del x divided by tau u f; where, G f 0 is the initial modulus of shear layer or granular layer; and, tau u f is the ultimate shear strength and this is for the granular layer. So, finally, once we write these expressions; then after the derivation, we will get this expression that, del tau f by tau x – then we have to derive this expression, so we will get this form, that is, G f 0; then 1 plus G f 0 del w by del x divided by tau u f into square del square w by del x square. So, once we derived this expression, we will get in this form; that is, del tau f del x we will get in this form.

Now, if I put this expression here, then we will get that expression that, G f 1 is equal to G f 0 divided by 1 plus G f 0 del w by del x divided by tau u f whole square. Then this del tau f by del x – that is equal to G f 1 into del square w by del x square. So, these are the expressions that we can use. So, finally, this we can write; this is q t equal to q s minus G f 1 H f into del square w by del x square. If I put this value in this expression, then this expression is we will write in this form. So, finally, we can write q t value in this way also. So, q t, q f and this value.

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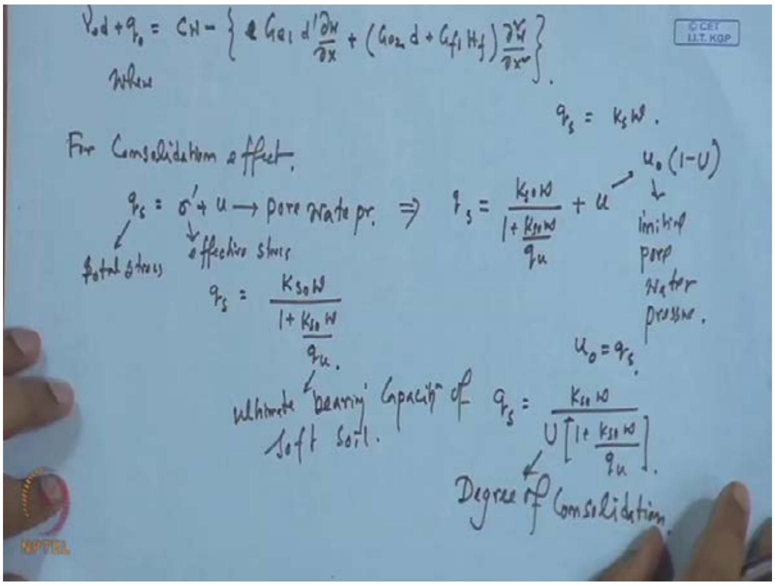

Final form of the expression will be in terms of q t and total force. That we can write that, gamma e d plus q 0 is equal to c w minus G e 1 d dash del w by del x plus G e 2 d plus G f 1 H f del square w by del x square; where… because c q s value is equal to K s into W if it is a linear spring. Now, if it is a non-linear spring and if you want to incorporate the consolidation effect, then we can write that – for consolidation effect, we can write that, q s will be equal to effective stress and the pore water pressure. q s is the total stress that is equal to effective stress and pore water pressure. And this effective stress is taken by the spring itself and then pore water pressure. So, this in case of q s, if effective stress is taking by the pore water spring; and, the spring if I consider this as a non-linear; then this q s sigma bar is dash equal to del q – 1 plus K s 0 W by q u. This q u is the ultimate bearing capacity soft soil.

And then finally, we can write here that, q s that is equal to K s 0 W divided by 1 plus K s 0 W by q u plus u, which can be written in this form, that is, u 0 1 plus degree of consolidation. So, q e is the effective pore water pressure at any stage; and, u 0 is the initial pore water pressure. So, finally, once we put this expression in this form and then initially, pore water pressure u 0 – that is also equal to q s. So, finally, once we get this expression, the final expression of q s will be K s 0 W divided by U 1 plus K s 0 W by q u; where, this U is the degree of consolidation.

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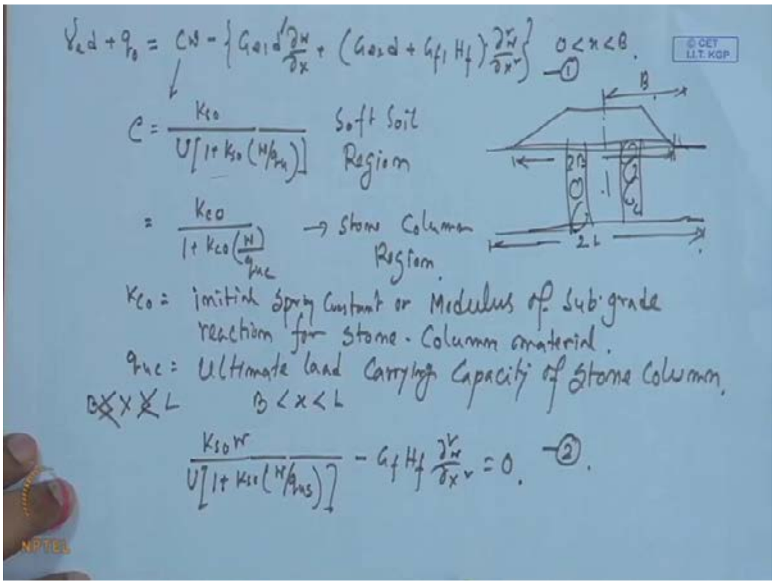

Once we write in this expression, then if I put this expression in the final form; then this expression gamma e d plus q 0 – that is equal to C w minus G one 1 d del w by del x plus G e 2 d plus G f 1 H f into del square w by del x square. So, that we can write that, if embankment is this one and this total width is B, then this expression is valid up to… Total width is 2B; then if this is the B, then this expression is valid between 0, x and B. It is greater than 0 and less than B. And C; where, C is equal to K s 0 U 1 plus K s 0 into W by q u. This is for the soft soil region. And in the stone column regions, K c 1 plus K c 0 into W by q u c. That is for the stone column region, because here we have two separate regions: one is within the stone column region and another in between this stone column, this soft soil region. So, soft soil region – the degree of consolidation will play very major issue. So, this degree of consolidation we have to use here; and, this stone column region we will get.

This K c 0 is the initial spring constant or modulus of stone column material. And this is q u c, is the ultimate load carrying capacity of the stone column. Then we will get… And for the within the... If it is x is get greater than B and less than L; L is the width of the total model from the center; then we will get expression, that is… because in this expression, this total force is 0. So, we will get expression that K s 0 W divided by U 1 plus K s 0 W by q u s; then minus G f H f del square w by del x square that is equal to 0, because here this part will not be present; this part will also not be present, because this one is also d dash – because in this part, we will get because embankment is not present here. Beyond this point – this is B to L – this region, embankment is not present. So, we will get this expression. So, this is expression number 1 and expression number 2. These are very major two expression and we have to solve these two expressions.

Now, first, we will express this expression. We can express this expression in non-dimensional form. And we can non-dimensional this expression in terms of K s and the width of the embankment. So, these things – this total derivation and this expression – how to non-dimensional this part we explain is explained by Deb in 2010. So, these things can be explained, which is can be find. This is 2010 – the paper which is… Now, once we non-dimensional this part, now we have to put the d value, because d value is varying from place to place and d dash also. d dash is the rate of change of the depth and d is the depth of the embankment at any condition.

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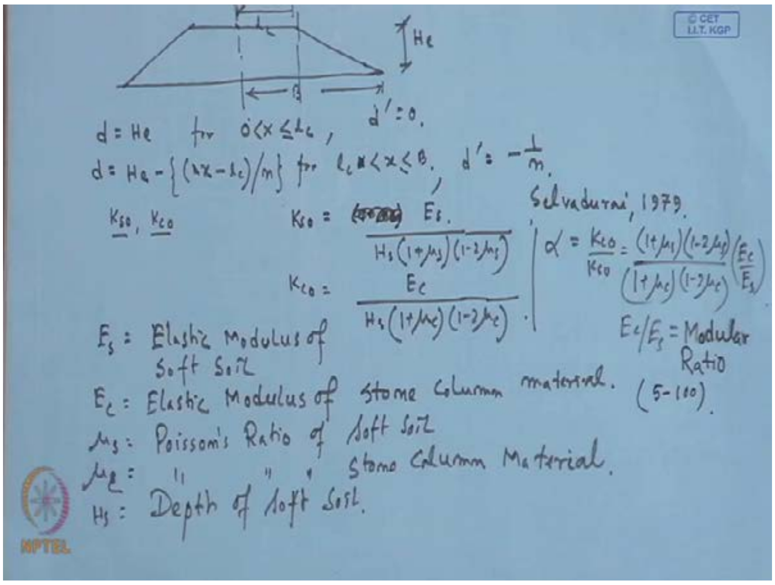

Now, if I put that part; if it is l c. So, we can write d is equal to H e – height of embankment for x less than equal to l c and greater than 0. So, this is 0. Similarly, d will be equal to H e minus x minus l c divided by n for l c less than x less than equal to B. So, that means if x is within this region, then d is equal to H e and d dash is equal to 0, because the rate of change of this depth is 0, because the depth is not changing. But in this region, this will be the d; and, d dash value will be minus 1 by n. So, now, we will get d and d dash values for different conditions.

Now, this expression – this K s term is present – K s 0 and K c 0. These two are the initial subgrade modulus of the stone column material and the soft soil material. So, these two things we can convert into in terms of E and mu. So, that we can run. That is proposed that, given the relationship between K s and e is that, K s 0 or subgrade modulus – that is equal to 1 plus mu 0 1 minus. So, this expression this K s 0 is E s divided by H s 1 plus mu s 1 minus 2 mu s. So, this is also proposed by Selvadurai; where, E s… And similarly, K c 0 is E c H s 1 plus mu c 1 minus 2 mu c. So, that is… Now, here E s is the elastic modulus of soft soil; E c is the elastic modulus of stone column material; then mu s is the Poisson ratio of soft soil; mu c is the Poisson ratio of stone column material. So, once we…

Now, if I express this alpha; where, alpha is equal to K c 0 by K s 0; then we will get… where H s is the depth of soft soil, and here we assume that, depth of soft soil and the depth of stone column are same – depth of soft soil. So, K c 0, K c alpha – we can write in this form that, 1 plus mu c 1 minus 2 mu s divided by 1 plus mu c 1 minus 2 mu c into E c by E s. So, this E c by E s is called modular ratio. Generally, this range varies from 5 to 100. So, now, these are the expressions – that expression which is… From this main expression, now, we have to solve or determine the settlement. So, once we get this expression, now, we have to solve this expression by using finite difference technique. And then we can solve this expression and then we can determine the settlement value. So, once we get this expression, then we have to solve this expression in terms of settlement; and then we have to use the boundary condition also.

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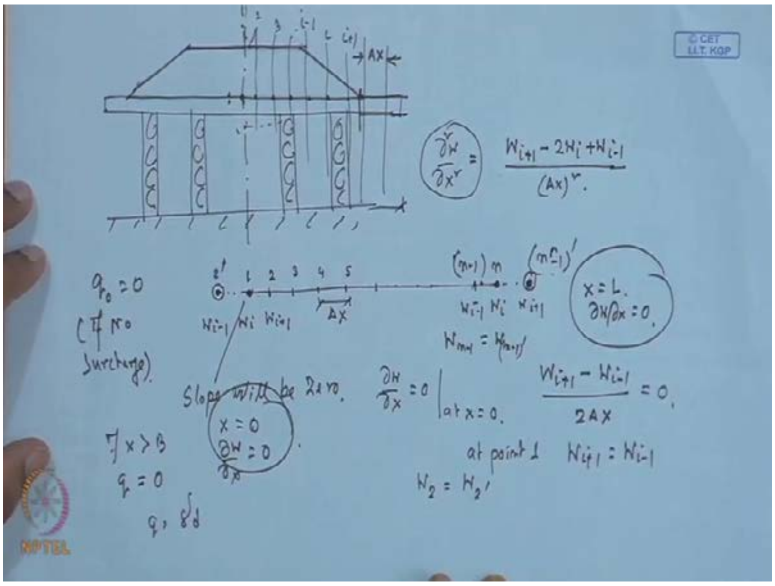

Suppose if this is the embankment; this is granular layer; and, this is the base; and then this is the stone column. Once we get these stone columns and this granular layer, now, what will be the boundary condition? First… So, once we solve this expression, first, we can derive this expression by using this soil structure or mechanical element; then we have to non-dimensional this form of expression by using the K s 0 and width of the beam. Then once we non-dimensional this expression, then we have to solve this expression by using the finite difference scheme.

Here we have this d 2 w by d x 2. So, that expression we can solve by using the central difference scheme. So, that central difference scheme we can use by W 1 plus i minus 2 W i plus W i minus 1 divided by del x square. So, that means we have to divide these points into n number of… So, this is one point, another point… So, divide this total thing into n number of segments. So, each segment has a thickness of delta x. If it is 1, 2, 3; then this will be i; this is i plus 1; this is i minus 1. So, once we get that, then we can use this every point the same expression or we can use this equation at every point.

Now, one thing is that, as it is this point is symmetric, we will analyze this half portion. So, 0 will start from here. So, if we apply this equation, expression in this 0, then we need one point, i minus 1. So, that is here. But we do not have any node here. For example, if we are applying here; where, this is 1; this is 2, number 2 node; this is number 3 node; this is number 4, number 5 and so on. So, if this is W 1 and we are using this expression, the first node; then this will be W i plus 1; this is W i; this is W i minus 1. So, 1 will be W i; 2 will be W i plus 1 and this distance is uniform layer taking del x. But we do not have any node here; but we have to consider one node. So, this node is called imaginary node or fictitious node.

Now, we have to determine this node here. And then for that purpose, we need a boundary condition. So, that boundary condition is that, at this point that will consider the slope of the settlement, that is, 0. So, here that is symmetric. So, at this point, the slope will be 0. Once the slope will be 0, that is, del w by del x is 0 at x equal to 0; then what will happen? Then again, we have to use the central difference scheme. If we use the central difference scheme for this one, then we will get W i plus 1 minus W i minus 1 divided by 2 del x. So, that is the central difference scheme for the slope. And at this x equal to 0 point, it is 0. So, then at point 1, W i plus 1 is equal to W i minus 1. So, that means at the deflection at point 2, W at point 2 and W of this fictitious node 2 dash say 2 dash is same. So, this way by using the boundary condition, we will get (( ))..

Next point, we can use this expression – central difference scheme, all the points. Then the problem will arise in the same way in the last point. There also, we have – if we use this expression, there will be W i, W i minus 1 and W i plus 1. Then we need another fictitious point. If it is n, then we need… This is n minus 1 dash. So, this will be n minus 1. So, now, we have to use another boundary condition here to solve this expression. So, that boundary condition is… We can use that the… Here we can consider (( )) such that the deflection of this point… Here also slope will be 0. So, once and so… Where we can use this boundary condition that, the slope at x equal to 0, that is also 0; and, slope at x equal to L… So, that means slope will be 0 at x equal to 0. Similarly, at x equal to L, here also, del w by del x equal to 0. Now, this is one boundary condition. This is another boundary condition. x equal to 0 and del w by del x is equal to 0.

If we apply this boundary condition, we can determine the value of this 2 dash fictitious node or imaginary node. If I use this boundary condition, we will determine the value of this n minus 1 dash fictitious node. So, now, here also, the slope is 0; then the settlement of n minus 1 node will be equal to the settlement of n minus 1 node dash. Here also, if we apply these things, we will get W n minus 1 will be W n minus 1 dash by using this boundary condition. So, once we get this boundary condition, then the loading condition is that, if I consider, there will not be any surcharge. So, q dash will be 0 if no surcharge. And then beyond this, if x is greater than B, then the q value or the loading patterns – they will be 0 if q is equal to 0. So, beyond this point, there is no loading is applied. Within this point… This is the q, is gamma into d; d is changing depending upon the x value. So, once we determine these total values, then what we will get actually from this expression?

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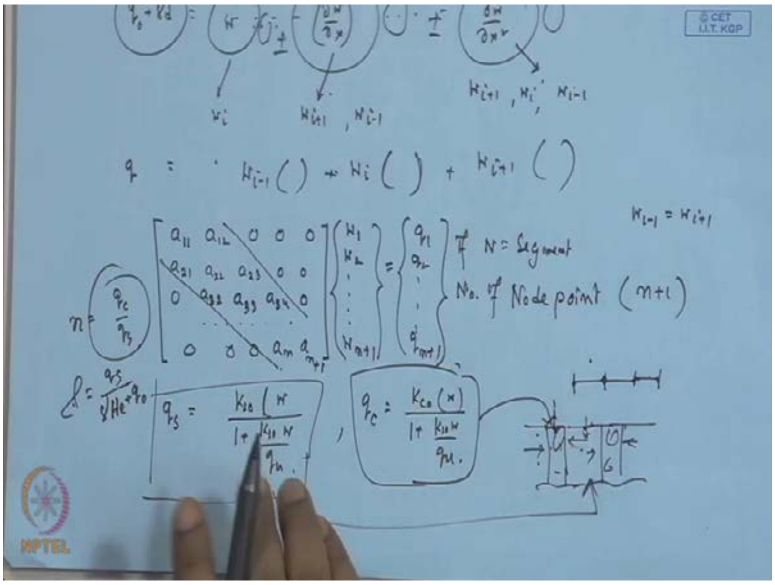

We have some expression. This is in terms of q 0 plus gamma d. Then we have this expression here in terms of del w by del x; then we have expression in terms of del square w by del x square. We have the expression in terms of w also. So, we have the expression in terms of w, in terms of del w; and then we have the expression in terms of q. So, what we will do, now, we will… So, here we will express it in terms of W i plus 1 and W i minus 1. This one also; this is W i. So, this one also we will express W i plus 1, W i and W i minus 1. So, these things we will express in this form. This thing we will express x plus 1, x minus 1 and this is W i. So, now, we… This is q force. And then we will determine, take W i; then this coefficient W i plus 1; then this coefficient and W i minus 1; and, this coefficient. So, we will get… Once we express these things… So, there must be some constant here, some constant here, some constant here. This is plus plus; and, maybe plus minus, anything.

Now, we will express this term and then we will separate all the components of W 1 in one bracket, W i plus 1 in another bracket, and W i minus 1 in another bracket. So, ultimately, these are the constants. So, we will get one matrix in this form. This is a 1 1; then for the first row, you will get…………………….(End of video)

a 1 2; and, all the other values are 0, because for the first row, we have only W i minus 1 is equal to W i plus 1. So, there will be only two components: W 1 1. For the third row, we will get W 2 1, W 2 2, W 2 3. Then other things are 0. In the third one, we will get 0 W 3 2, a 3 3, a 3 4, then 0. And for the last one also. And similarly, for the last one also, we will get a n plus 1, a n, because if I take N number of nodes.

If N is the segment, then node point will be N plus 1, because we have one segment, two segment, three segment. These three segments; then the node point number is 4. So, this is… So, that is why this will be N minus 1. So, this is N minus 1. And this portion – all are 0. So, we will get a boundary-type of matrix. And then here we can write W 1, W 2 up to W N plus 1. And similarly, that is equal to q 1, q 2 up to q N plus 1. So, we will get this matrix form.

And then if we solve this matrix, then if we know boundary from the loading condition; if we put the loading values here and then determine all these coefficients: a 1 1, a 1 2, a 2 1, a 2 2, a 2 3. And then if we solve this one – inverse this matrix, then we will get the W 1, W 2, W N plus 1. So, these different solution techniques are available to determine this W 1, W 2 and up to W N plus 1 from this type of expression. So, that means first, we will get the expression; then we apply the… Then we express it in terms of finite difference scheme. So, once we express this in terms of finite difference scheme, then we determine all the coefficients of W 1, W i plus 1, W i minus 1. Then we will get a matrix.

And then we will solve the expression; we will get W 1, W 2, W 3. So, we have to apply the boundary condition also. To determine the values of the fictitious node, we have to apply the loading condition; and then we will get the W 1, W 2 and W N minus 1. So, settlement or the deflection of each point.

Once we get the deflection, then you know that q s is K s 0 by W divided by 1 plus K s 0 W by q u. This is for the q s soil. Then q c for the stone column. So, we will get… So, now, if for the… Once we decide these things, and then we have to change the condition, because within the stone column region, this will be q c in this form. We have to use the q c. And in the soft soil region, we have to use the q s. And now, once we get the settlement of every point, then we can determine the stress at every point. If within the soft soil, then stress will use this expression; within the stone column, if it is in the soft soil, then we use this expression. So, once and for… For any point, we will determine the stress within the soft soil. But first, we will calculate the settlement; then we can calculate the stress at any point.

Now, once we get the stress and the settlement, then… Once we get the stress, then we can determine the stress concentration ratio, that is, q c by q s. And then also, we can determine the rho value, that is, the soil arching coefficient also; that is, the q s by gamma into d or H E plus q 0. So, q s also… So, all these things we can determine here. And then we can use these techniques. So, by using that we can determine the settlement at every point – differential settlement, (( )) settlement. If we know the settlement at any point, then we can know the differential settlement; that means settlement defines the center of the stone column and center of the soft soil. We will get the differential settlement.

Then, we can determine the stress concentration ratio; we can determine the soil arching ratio. So, all these things we can determine once determine the deflections. So, from this deflection, we can determine every unknown quantities. And then we can use those variations and we can use… because our aim is to determine the stress which is acting on the stone column. And then we have to check that amount of stress this column can able to carry or not. And then another check is, if we know how much stress is coming on the soft soil, then we have to check whether that amount of stress this soft soil can carry or not. So, we can check all the… Once we calculate settlement, we can calculate stress; and, from that stress, we can check whether the system can be safe or not; whether this stress, which is coming… So, once we have to check stone column and soft soil separately, so that they can able to carry this load or not. So, these are the techniques. By using these soil interaction problems, elements, we can solve field-oriented problems also.

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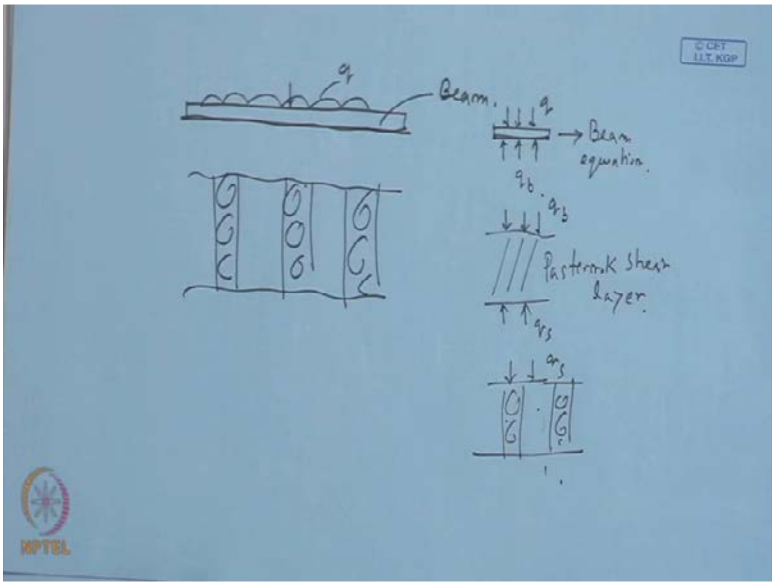

And, suppose if we again… If this type of problem, where this is the granular layer and this is stone column; we can place say one beam here. If it is a stiff footing, this is a beam. That thing also we can solve here, because here also… So, that thing that time if a u d l is acting here; this will be q; and then if we calculate, this will be q and this will be say q b. Then for the granular layer, this will be q b and this will be q s. And soft soil layer – this will be q s and this will be stone column. So, once… So, in this portion, you have to use the beam expression, beam equation. And then here this is Pasternak shear layer. So, we have to use the Pasternak shear layer expression. And here the soft soil once the… Here the stone column expression and the soft soil expression. And then if we solve this expression, then we can determine what would be the settlement, because the previous problem, we can determine the settlement.

Here from the settlements, we can determine the bending moment of the beam, shear force of the beam. But we have to apply the boundary condition here also for the beam. So, this way, we can use the beam also to… And then we can determine what is the response of the beam if the beam is resting on stone column-improved soft soil, because in the study, it is showing that, if we increase the stiffness of the stone column, then the bending moment and shear force will increase and settlement will decrease. So, we have to choose a proper stiffness, so that this can be balanced. So, those things we can solve by using this technique. So, this is one application area, where we can apply this mechanical element or such foundation interaction methodology to solve this field-oriented problem.

Thank you.