Advanced Foundation Engineering

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Lecture - 39
Soil - Foundation Interaction (Contd.)

In last class I have discussed about that modeling of stone column supported embankment. So before that modeling, I have discussed about what are the different load sharing mechanism if there is reinforcement used in the embankment. So, there are basically three types of load sharing mechanism. One is because of the soil arching; one is because of the use of geosynthetic layer; another is due to the stiffness difference between the stone column and the soft soil. Now today I will explain how those things are modeled by using this soil foundation interaction theory that we have already discussed.

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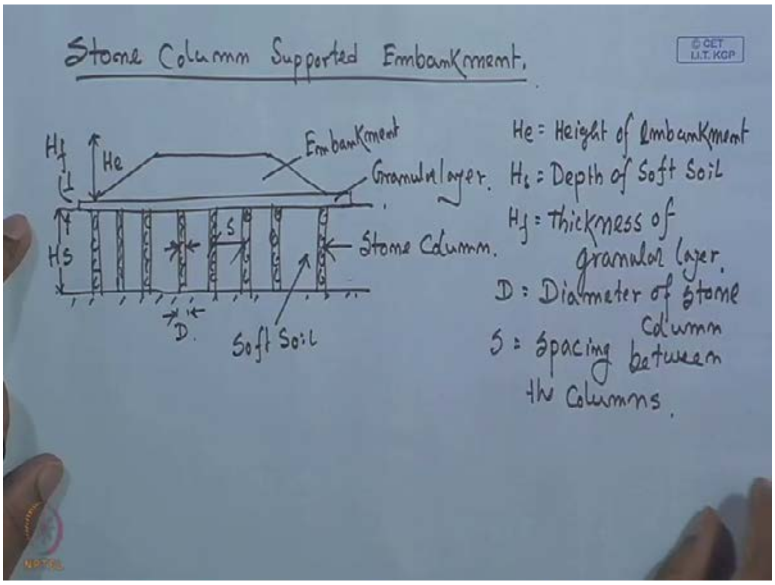

So, first if I go to that stone column supported embankment, so that thing that part, say, assume this is the embankment. Then below that we use a sand pad or granular layer; then this is the existing soil which is improved by stone columns. So, this was the problem; this is the hard stratum and this portion is improved by stone column or granular pile. So, these are equals; these are the diameter of the stone column and this is the spacing between the stone column and this is embankment.

So, these are the stones. This is granular layer and this is soft soil. So, this soft soil is improved by the stone columns. So, this is the problem and suppose this is the thickness of the soft soil, this is the height of the embankment H e and H f is the thickness of the granular field. So, that means H e is the height of embankment; this is height of embankment, then H S is depth of soft soil, H f is thickness of granular layer, D is the diameter of stone column, S is the spacing between the columns.

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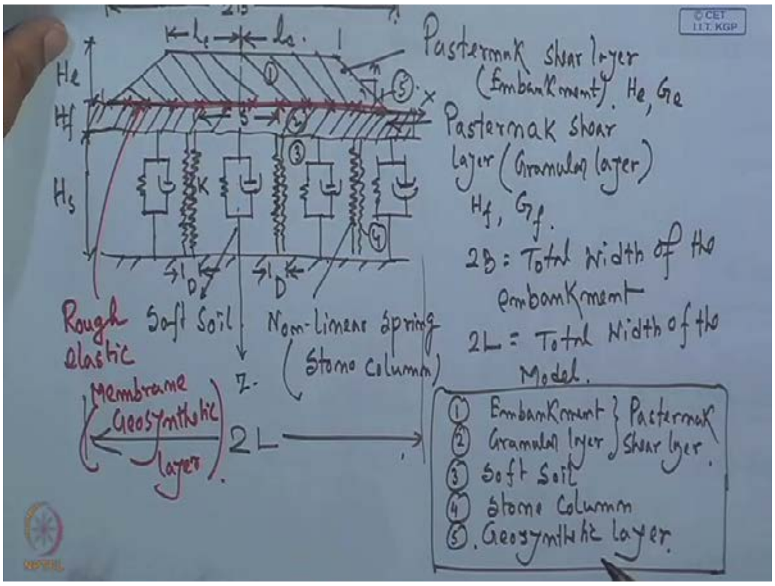

Now, these things we have to model by using this soil interaction problem. So first in this model, this is the existing soft soil. So, these soft soils are modeled by spring-dashpot system. This is spring, this is dashpot system. So, this is a spring which is stiffness is k. So, now these things again which is modeled by another spring with dashpot system. This is the hard stratum.

Now in between that, there is stone column. So, that those columns are modeled by non-linear springs. So, these columns are modeled by non-linear springs. So, this is the non-linear spring which is represented in the stone column. So, this one is the spring-dashpot system which is represented with the soft soil. So, this is the spacing between the stone column and these springs are up to the region of diameter D.

So here also, this is also within the D region; D is the diameter of the stone column. This way soft soil and the stone column are modeled. Then this granular layer is basically modeled by Pasternak shear layer. So, this is used by the Pasternak shear layer. This layer is represented by this granular layer. Now as thickness of this layer is H f and the shear modulus is G f; G f is the shear modulus of this granular layer. Next one is the embankment. This is the embankment that is also modeled by shear layer. This one is also modeled by Pasternak shear layer. Now the thickness of this depth of this embankment is H e and G e is the shear modulus of the granular material.

This is H f, this is H s. So one thing we have to keep in mind, that in case of H f which is uniform throughout the length; so that means if we consider this is x direction and then the center of this thing is z direction, then according to the z direction with respect to the j x, this H f is same. But if I consider this is symmetric, this is zero zero point; then this H e is not same. H e is the maximum depth of the embankment or maximum height of the embankment.

So, but it is up to from center up to l c point. From the center that height of the embankment is same. But after this, say this is 1 is to n slope and total width of the embankment is say twice of the B. So, this is total width of the embankment; half width is the B and lengths of the embankment say we can extend this granular layer. So, another spring we can as spring dashpot we can consider. So, this will be twice L; that twice L is the total width of the model and twice B is the width of the embankment.

So, twice B is the total width of the embankment; 2 L is the total width of the model or if we consider reinforcement layer here and that we have to place at the interface. So, this is the reinforcement layer if it is reinforced embankment. So, geosynthetic layer; so this layer is modeled by rough elastic membrane. So, that is geosynthetic layer. So, that means these are the components generally we use. So, first the soft soil is used by spring-dashpot. Here the reason of behind this spring and the dashpot, here the spring is non-linear spring and dashpot because soft soil as we are using in the stone columns; so we have to incorporate the time effect or the consolidation effect into the model.

So, that is why this time effect to incorporate the time effect, we consider the dashpot and the settlement behavior to study that thing we consider the spring. So, the time effect and the settlement, both things we are incorporated; we have incorporated by using this spring-dashpot and a non-linear springs are used to idealize the stone column and Pasternak shear layer is used to model the embankment and also Pasternak shear layer is used to model the granular layer or the sand pad which is used.

The difference between these things for the granular layer, this Pasternak shear, thickness is uniform throughout the length. But in case of embankment, the thickness or the depth of this Pasternak layer is not same with respect to the distance and the elastic rough membrane is used to model the geosynthetic layer. Now we have to model the different components of this thing. So, there are basically four components. One is the embankment is one component, shear layer is another component, soft soil is another component and the springs for the stone column is another components. So, there are four components basically and this geosynthetic layer that is also fifth component.

So, first component that is embankment which is modeled by Pasternak shear layer. Second component is granular layer that is also both things are modeled by Pasternak shear layer. Then the third component is soft soil that is modeled by spring-dashpot. Fourth is the stone column that is modeled by the non-linear spring, and fifth one is the geosynthetic layer which is modeled by rough elastic membrane. So, this five things we have model by using various component or mechanical elements. So, these are the mechanical elements; Pasternak shear layer, spring-dashpot, non-linear spring, then the rough elastic membrane.

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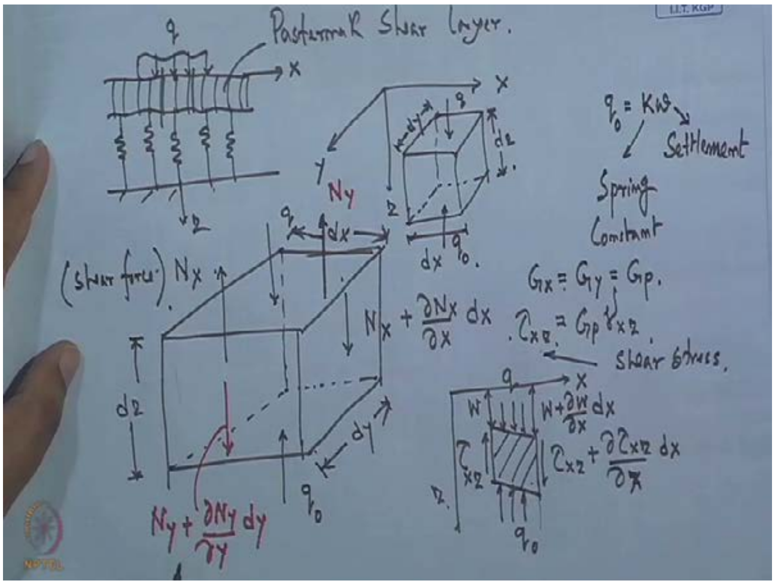

Now, we will go and derive the expression which is available for the other case. Suppose we are using the Pasternak shear layer. So, we know that for the Pasternak shear layer if we consider; so suppose this is the Pasternak shear layer and uniformly distributed load q is applied and this is the Pasternak shear layer, then springs are attached. It is in the general Pasternak shear layer model, so that we are considering here. Suppose this is z and this one is x. Now if I draw the x, y, z; this is z, this is x, and this is y direction and we consider one particular element within the Pasternak shear layer. So, this is Pasternak shear layer.

So, if I consider one particular element. So, this is one particular element of this Pasternak shear layer. So, above that we are applying q. So, if I consider this portion; so above that we are applying q and then we will get another reaction from the bottom. So that is, say, for example, that reaction is q 0. So, q is applying here on the Pasternak shear layer and then there is getting a reaction q 0. So, basically this q 0 if it is a spring on resting on which q 0 will be k into w where k is the spring constant or the modulus of subgrade reaction and w is the settlement. So, that is the settlement in the downward direction. So, these are the ones.

Now if I draw the other part of this shear layer. So, that thickness of this portion is d y. So, this portion is d x and this is d z. Now if I consider this cube in a bigger form. So, this is our q which is acting from the top and then from the bottom, this is q 0 and the thickness of this or the side this is d z, this is D y, and this one is d x. So, volume of this q is d x, d y, d z. Now what are the forces that are acting here? So, in the upward direction for this d z and d y side, there is this force that is acting N x. Similarly in the downward direction of this side that is N x plus del N x divided by del x into d x.

Similarly, in the other side of this d x and d z side, the force that is acting that is the other side; that one is N y and the downward direction of this side this is N y plus del N y divided by del y into dy. So, these are the total force which is acting on this shear layer. So, this is the side where this shear layer is acting. So, and considering this the deformation is only in the vertical direction. So, I have not considered in this analysis any lateral deformation; although it is assumed that there will be a bulging in the stone column. So, those things are not modeled here. We have modeled only the vertical deformation neglecting the lateral deformation in the model.

So now, if we get the net force from this x; before that, we consider the G x shear modulus in the x direction, shear modulus in the y direction are same; that is equal to G p or G of the Pasternak layer. So, similarly the tau x z and if I consider a two-dimensional view of this figure x and z, this is a three-dimensional view of the model. If it is a three-dimensional view, then for the two-dimensional case we will consider this shear layer is like this and the deformation at this point is say w. Similarly, the deformation in this space will be w plus del w divided by del x into d x and shear stress this point is tau x z.

Similarly this point, it will be tau x z plus del tau x z divided by del z into d x. So, this will also be del x into d x. So, similarly here also on this top the q will act; here also from the bottom q 0 that will act. So, this is the shear layer that we have considered. So, now if I consider that G x equal to G y equal to G p, then tau x z that will be equal to the G p, shear stress into the strain in x z direction; so this tau is the shear stress and gamma is the shear strain.

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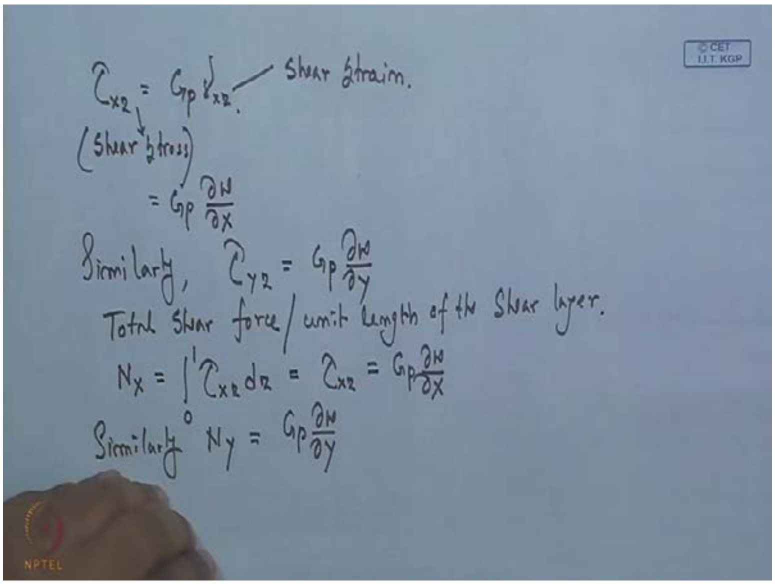

So finally, if I write that expression the tau x z that will be equal to G p and strain in x z direction; so this is shear stress and this one is shear strength. So, similarly you can further write that G p is equal to del w divided by del x. So, del w by del x is the shear strength that is equal to gamma x z. So, similarly tau y z is equal to G p del w del y. So, total shear force per unit length of the Pasternak shear layer that is equal to N x in the x direction.

So, we can see that this is for a particular portion, this is the stress which is acting every. So, this side this stress is acting. So, N x is the shear force; this is shear force and tau is the shear stress. So, that shear stress is acting this side. So, total shear force if I consider per unit weight that will be N x 0 to 1 unit weight tau x z into d z. So, this tau is the shear stress which is acting along the side or along this side d x and d y and d z. So, on this side these forces are acting. So, you can write this is 0 to one 1 d z according to the depth. So, that is equal to tau x z into this z z that is one because 0 to 1 unit length.

So, z is one we have considered because this is shear force per unit length; this one also shear force per unit length. So, now here tau that will be equal to G p del w del x. Similarly N y that is also G p into del w del y. So, G p also del w del y. So now, if we consider the free body diagram of this total figure; so N x we know, so the net force in the z direction. So, this is acting upward, this one downward, this is acting upward this one also acting in the upward direction; this is q in the top and q 0 on the bottom.

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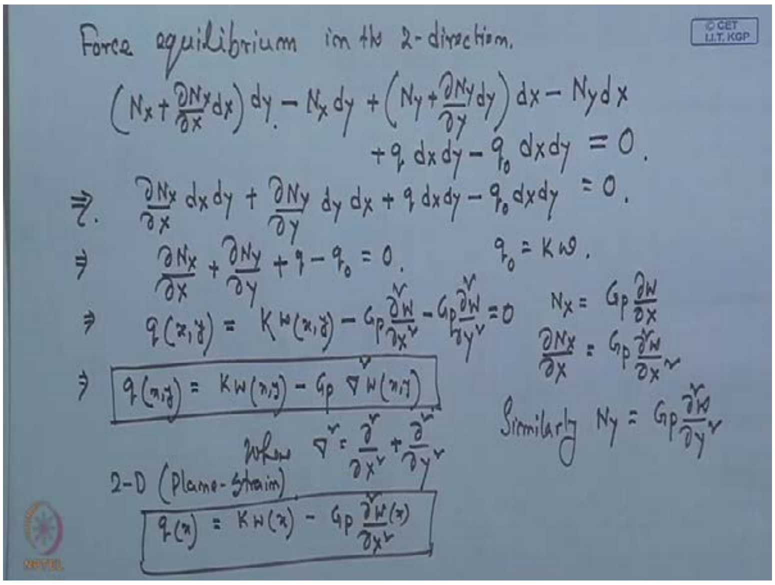

So, now if I write the force equilibrium in the z direction then we have N x plus del N x divided by del x into d x into y; because this is the in the z direction total force which is acting is force per unit length. So now, the total force is acting; we have to multiply it by the length and this length is d y. So, this is then acting in this surface, the dimension of N x is shear force per unit length. So, total force we have to multiply it by the length that length is d y. Similarly that is minus N x into d y, then plus N y plus del N y into del y d y; here the length is d x, then minus N y into d x.

Now, these two forces the top one and the bottom one both are acting. So, top one this will be q into this total upper surface and that surface area is d x into d y and similarly q

0 d x into d y. So now, if I write that form that is plus q into d x into d y minus q 0 d x into d y, that is equal to 0. So, we take all the forces in z direction and then now final form will be, if this is N x d y minus N x d y those things will be cancelled out. So, from here this will be del N x del x into d x d y, then plus here also N y d x minus N y d x that will cancel. So, plus N del N y divided by del y into d y into d x, then plus q d x d y minus q 0 d x d y equal to 0

So, from here d x d y is not equal to 0. So, we can write del N x divided by del x plus del N y divided by del y plus q minus q 0; that is equal to zero. So, this is the final expression and then finally in the further we can write that this is q x and y because in the two-dimensionally; that is equal to we know q 0 is k into q 0 we have written that is k into w previously. So, that will be k into w x y minus. So, now if I put this N x is equal to we have written N x N y that is G p and del w del y and N x is del w del x and N y is G p del w del y. So, now, N x is G p del w del x. So, here del N x by del x that will be equal to G p del two w del x square. So, if I take in this side. So, this will be G p.

Similarly N y is equal to G p del two w del y square. So, that is equal to G p del square w into del x square minus G p del square w into del y square that is equal to 0. So finally, we will write q x y is equal to k w x y minus if we take G p common, then this will be delta square w x y where delta square is del square del x square plus del square del y square. So, this will be the final expression for the Pasternak shear layer in the two-dimensional form or three-dimensional form x and y both.

Now if we convert this expression into a 2-D form, then for the 2-D form or plane strain condition, then we can write q into x equal to k into w x minus G p del square w del x square w in x. So, this is further simplified in for the plane strain condition, this expression and in this model we have considered the loading and the geometry is the plane strain. Because the as the embankment is in the plane strain condition, we generally analyze the embankment in the plane strain condition. But the problem here that will be associated is that when we convert the stone column in the plane strain condition because the stone columns are generally placed in circular diameter and it is in the triangular or rectangular grids or square grids.

So, if we place it in the square grid or the triangular grid and below the embankment, then the problem is not actually the plane strain problem. If it is only embankment or reinforced embankment without stone column, then easily we can analyze this thing by plane strain; considering it is a plane strain problem. But if it is in the stone column supported embankment, then we have to convert these things. So, generally we can use the axisymmetry unit cell approach and then but you have to convert these things for the plane strain condition.

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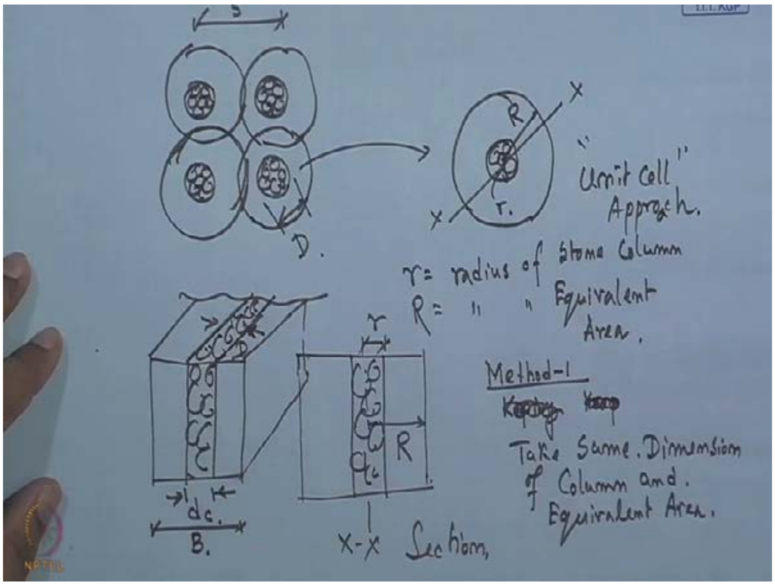

For example the general approach for the, if it is the stone column is placed in square or triangular pattern. So, generally we consider one unit cell or equivalent area. So, this is the equivalent area or cover area for this stone column. Similarly, we can consider another equivalent area for this stone column and for another equivalent area for this stone column and finally, we can consider; so if this is the spacing S and this one is the diameter of the column. So, these analyses; so that means when we do the analysis this is we will consider only this one, this is stone column and then surrounding equivalent area. So, this equivalent diameter say radius is R and this one is small r. So, r is the radius of stone column and capital R is the radius of equivalent area.

So, generally we analyze these things in this form. So, if I cut the section x x or r r, then this will be the section where this is stone column, this is central line, this is x x section. So, this is stone column whose radius is r and this radius from the center is capital R, small s is the stone column radius. So, then we analyze in this form as a axisymmetric condition, but sometime it is observed that axisymmetric condition; that is called unit cell approach because this is called unit cell approach.

So, now this unit cell approach sometimes we will not get the exact result, because the stone column within the center region of the embankment and the stone column at the edges of the embankment; their behavior are not same. So, if I consider this as the unit cell approach then sometimes we will not get the exact result. So, that is why here in this model, all the stone columns are considered into the analysis; that means the total embankment is analyzed here and considering all the dimension; all the stone columns are considered, total embankment dimension is considered into the analysis; that means total things is considered into the analysis.

So, that means here actually when the consolidation expression which is available for stone column, those are derived for this axisymmetric condition. So, we have to convert these consolidation expressions for this plane strain condition. The problem is that when we do not convert these things, then what will happen? If I do the plane strain analysis the same problem, then the plane strain analysis will basically consider the diameter of the stone column here and then it is extended. So, this is plane strain model. So, that means this is the column diameter, but actual case this diameter is only up to this portion.

But if I consider the plane strain condition, then it will extend in infinite length with the same diameter D. So, that means actually we are replacing more amount of the soil. So, to get the actual behavior, there is two method of conversion from this axisymmetric condition to plane strain condition. One method is that we can keep this dimensions same. So, we can keep this dimension same and we can keep this radius r is equal to this spacing and then we can change the permeability and stiffness of the column material.

So, thus permeability of the soil we can change and the stiffness of the column also we can change. So, one method approach to convert this axisymmetric one to the plane strain one, the method one is take same dimension of column and equivalent area. So, that means that column equivalent area; that means, here in the stone column we will not use the diameter. We will use the d c, that is the width of the stone column and here also we can use the B, width of the equivalent zone.

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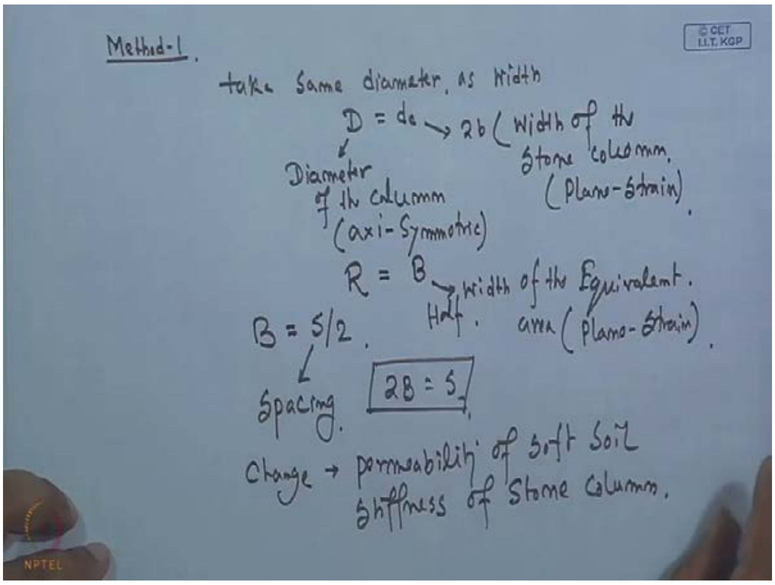

So, for the first we will consider that is our first method we will take; method one we will take same diameter as width; that means D is equal to d c, where D is the diameter of the column in axisymmetric condition and this is the 2 b which is the width of the stone column in plane strain condition. Then we have to change. So, because this method take same diameter as a width and then also we can take R this is the radius of the influence zone; that is also equal to the B, capital B that is the width of the of the equivalent area that is in the plane strain condition.

So, here in this analysis B is taken as S by 2 where S is the spacing. This is B is the half width of the equivalent area same like the 2 B. So, B is here taken. So, 2 B is equal to S. And then to keeping the dimension same, we have to change the properties of the soil and the stone column and we have to change this is to axisymmetric region; if I consider the properties in the real condition of the axisymmetric properties as the actual properties, then we have to change those properties for the plane strain condition.

So that means if I take the same diameter, then change permeability, soft soil, and stiffness of stone column. So, that means we have to change the permeability of the soft soil and the elastic modulus or the stiffness of the stone column to convert. If I take the same width, then we have to change the stiffness of the stone column, we have to change it from the axisymmetric condition to plane strain condition. So, one expression is available for this thing.

So, that thing will be in the next class when I will discuss about how to use this stiffness, then we will give those expression how to convert this stiffness of the stone column from a axisymmetric to plane strain condition, but today I will just give you the idea what are the methods. So if I take the same dimension, then you have to change the material properties; there will be permeability of the soil and stiffness of the stone column to convert axisymmetric one to plane strain condition.

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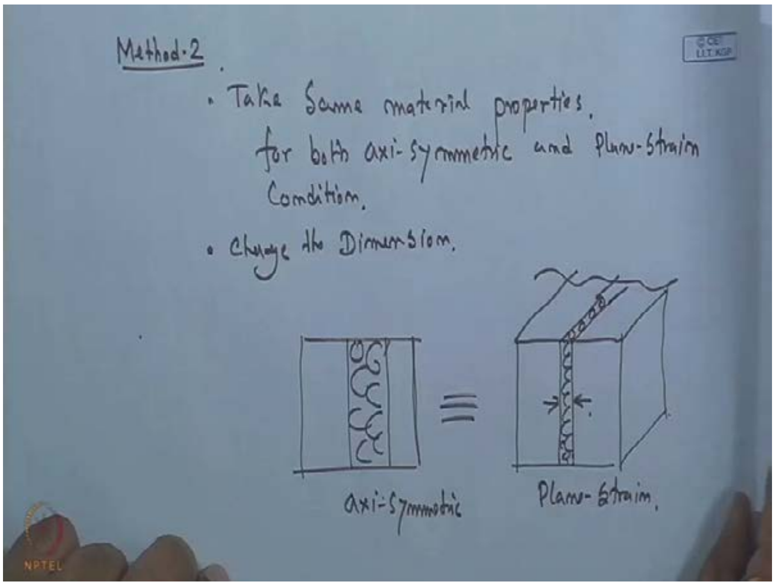

In the second method, in the method two; so here we take same material properties. That means permeability and stiffness same for both axisymmetric properties and plane strain condition but then we have to change the dimension. So, we have to change the dimension. In that case we cannot take that radius or the diameter of the stone column in axisymmetric condition is equal to the width of the stone column is plane strain condition; that we cannot change.

So, that here in first case we keep the dimension same and change the material properties and second method we have to keep the same material properties or we have to change the dimension. So, here also we have to change; so that means this case we have to reduce the width. So, suppose in that case if this is the axisymmetric condition in one section. But for that thing, we have to convert here by reducing the width of the stone column here. So, this is for the axisymmetric and this is plane strain. So, that means because we are keeping that, this is also continued here. Because here we are here reducing the width of the stone column in the plane strain condition.

So, these are the two methods generally we apply and the second one and the first one to convert this stone column condition from plane strain to axisymmetric condition. So, next class I will discuss this how to take this width of the stone column from different condition; for second method how to change this width and for the first method also how to change the stiffness of the stone column. Then we can go for the next part or our analysis part. So, now this is the one part and now these things once we are doing in the plane strain condition.

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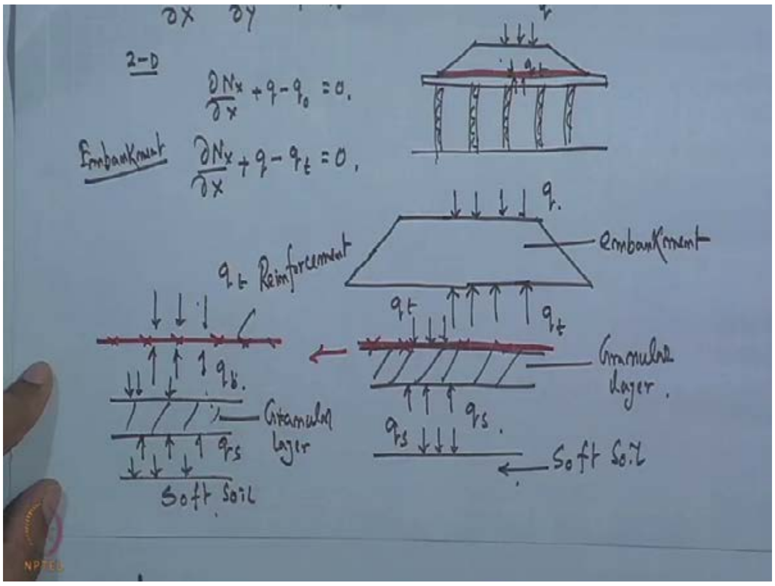

So, we know that the from the final expression part the general expression we establish that del N x plus del N y del y that is equal to q minus q 0 is equal to 0; this is general. For in the 2-D condition, this expression will be del N x by del x plus q minus q 0 is equal to 0. Now here if I consider this is the embankment and this is the granular bed, then the stone columns. So, if I consider this; above this is the total stress which is acting above the stone column, the q is the load which is acting above the stone column. This point is q and as the base of the stone column this force which is acting is q t; so that means that q at the base here, this is all the point this is acting q t and in the top it is acting q the force.

Similarly, to the granular layer also this will act q t at the top and in the bottom it will act q s. Similarly in the soft soil, it will act q S only. So, this is embankment, this is granular layer, and this is soft soil. So, in this fashion it will work. So, if it is a reinforcement layer is placed in between these two here; if reinforcement layer is placed here, then this q t will act on the top of the reinforcement layer and then the q b will act in the bottom of the reinforcement layer, same in the granular layer q B will act and q f will act in the base of the granular layer, then the q s will act in the soft soil.

So, if it is a reinforcement case, then q b q t this is soft soil; this is granular layer, and this is reinforcement. So, these are the combination of different load. So if it is on reinforced embankment, then q t at the bottom and the base of the embankment, then q t at the top of the granular layer, and q s at the base of the embankment, and q s on the soft soil. If it is reinforced embankment, then q t q b top and bottom of the reinforcement respectively; q b on the top of the granular layer and q s in the bottom of the granular layer and q s on the top of the soft soil. So, these are the loading distribution. So, now here if I write for the embankment then we can write del N x del x plus q minus q t; that is equal to 0 because q t is in the lower end. So, that thing is the expression for the embankment.

So, now in the today's class I have discussed about how these things are modeled by using this soil foundation interaction components and then in the next class. So, we have now derived the general expression for the embankment and then next class how to express these things in terms of deflection and then what are the properties, how will change the properties or stiffness of the soil to convert axisymmetric of the plane strain for the method one, and what are the width we will consider for the method two. So, those things I will discuss in the next class.

Thank you.