Advanced Foundation Engineering
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Lecture - 37
Soil - Foundation Interaction (Contd.)

In last class I have discussed about various loading condition for infinite beam resting on elastic foundation and infinite beam. So, what are that various, what will happen if it is subjected to a concentrated load, concentrated moment, then uniformly distributed load or triangular load, then what would be the expression for expression for the deflection, slope, bending moment and shear force. Now, today's class I will discuss about semi-infinite beam. So, last class I have discussed an infinite beam, on this class the semi infinite beam.

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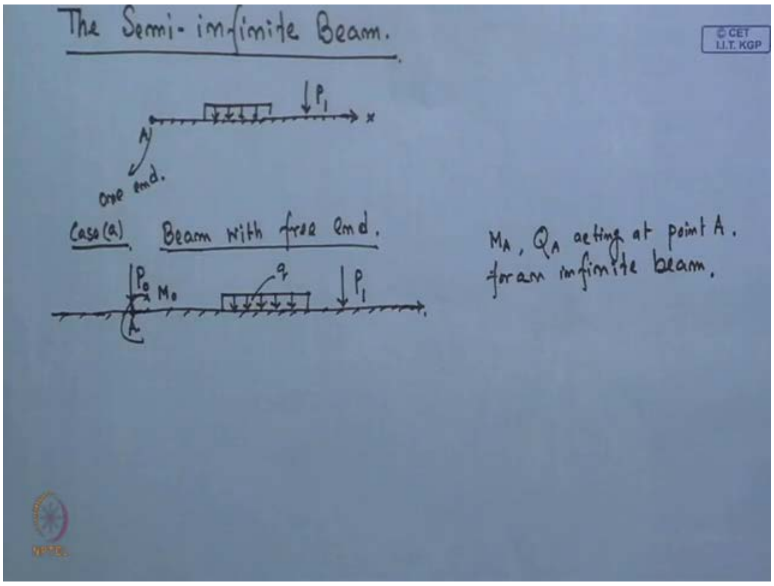

Now, first the, now first what is semi-infinite beam? That semi-infinite beam is basically beam where one end is extended infinite direction and another end is a particular fixed end or or point. Suppose, if this is a beam x direction. So, this is the particular end A. So, this particular end is fixed so that means so from here it will start. So, in infinite beam that is extended from this side and that side. So, here it will start from this side and it will extend from another direction and this direction is infinite. So, that means here we can apply the loading, here we can apply UDL, we can apply concentrated load. Then how to analyze this?

So, that means it is the beam which is extended only one direction, one direction having a point A as a fixed end. So, that means this is one end of the beam. So, this is one end. Now, depending upon the type of end condition it can be free end, this can be fixed end, this can be hinged end. So, that is basically one end of a beam and then it is extended in one particular direction. Now, how to solve this type of problem?

So, now if first case, we consider case a that is beam with free end. That means this end A is free. So, that is a free end beam. So, then how we will solve this problem say if we consider that a particular beam and then we apply this load, say one concentrated load and q UDL and this point is A. Now, consider one then in this condition this point is free end beam. Now, how to solve this type of problem? So, first we will consider this is a infinite beam that means this end is also extended in other direction this beam. Now, if we consider this is an infinite beam and the same time it is assumed that this point a have been a moment M A and Q A due to the application of this external load.

So, we consider this is an infinite beam where we have two moments M A one moment and one shear force Q A acting at point A for an infinite beam. So, this is one condition that means these things is converted to a, to an infinite beam and it is also considered that a moment M A and Q A will be acting the point A on this infinite beam due to this application of this external load here this q UDL and the concentrated load. But you have to keep in mind that this is actually a semi-infinite beam where A end is free. So, that means A end is free is only possible if the moment and the shear force both are 0 for this particular end and that is that condition we can achieve by applying so that means we have to vanish this M A and Q A such that we can make this end A as a free end.

So, that for that purpose we have to apply say one force P P 0 and moment M 0 such that at point A minus A moment and minus Q shear force is developed. So, first M A and Q A is acting on this infinite beam because of this external load. Now, we apply a P 0 and M 0 such that we can develop a minus M A and minus Q force which is acting on it on this end due to the application of this two, this one moment and force. So, ultimately so M A and Q A is developed due to the external load and minus M A and minus Q A is developed due to the application of P 0 and M 0. So, ultimately the net moment at point A will be 0 and net shear force at point A will be also 0.

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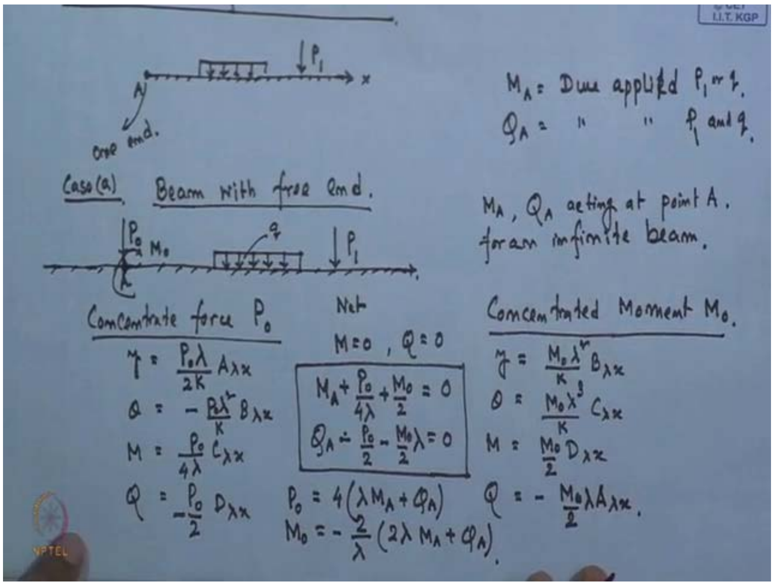

So, now so that means here we consider that now we are acting here on this concentrate infinite beam, we are acting one concentrated force P 0. So, concentrated force P 0 and concentrated moment M 0. Now, for the concentrated force you know the deflection expression for this infinite beam that will be P 0 lambda by 2 K into A lambda x for the deflection equation for the concentrated moment y will be M 0 lambda square by K into B lambda x.

Similarly, slope for concentrated moment will be minus P lambda square divided by K B lambda x slope will be for the concentrated moment will be M 0 lambda cube by K C lambda x, then bending moment for the concentrated force will be P by 4 lambda into C lambda x and moment for the concentrated moment will be M 0 by 2 D lambda x and shear force for the concentrated force, this will be P 0, this will be also P 0. P 0 divided by 2 into D lambda x.

Similarly, shear force for the concentrated moment will be minus M 0 by 2 A lambda x. So, these are the expression so these two expression, these expressions for these two conditions have already been derived. So, we are just using these expressions. Now, as we know that the due to this application of this P 0 and M 0 the net moment is so that means net moment M at will be 0 and net shear force that will also be 0. Now, for this two expressions so that means net moment one is due to do this concentrated force P 0 another so that means the, what are the moments that are developed?

One moment will develop because of this external loads that is q q and P 1 so that moment value is M A, another moment will developed at a point due to the application of this P 0 and the another moment is due to application of M 0. So, there is a three components of the net moment. So, that net moment is 0. So, first moment that we will consider that is M A that is due to the application of the external force, that means M A we can write due to applied P 1 or q. So, M A that will be plus so the moment due to this concentrated force that will be P 0 by 4 lambda.

So, that will be P 0 by 4 lambda then another moment due to the concentrated moment that is M 0 plus 2 so that will be plus M 0 plus 2. So, that net moment is 0 and another equation that net force shear force Q is also 0. So, that means the Q A is also due to applied P 1 and q or q. So, here that is Q A then the shear force due to this concentrated force P 0 is this is actually this force is minus P 0. So, this will be P 0 minus P 0 by 2 then another one due to this concentrated moment and that is minus M 0 by 2 lambda. So, there will be M 0 by 2 lambda also. This will be minus M 0 by 2 lambda that is also equal to 0.

So, this expressions are already been derived. So, this will be minus and this is minus M 0 by 2 lambda. Now, if I put these two expressions, we will get so where the ultimately net moment is 0. So, after solving this two expression we will get P 0 is equal to 4 lambda M A plus Q A and similarly, M 0 will be equal to minus 2 by lambda 2 lambda M A plus Q A. So, these two expressions, that will we get for these two conditions. Now, the net moment at point A will be 0, if we apply this P 0 and M 0 based from these two expressions. So, from these two expressions first we have to calculate this P zero and M 0. Now, if we apply this P 0 at M 0 that this point A otherwise very close to this point A, then we will get a condition for where we will get this is a infinite semi-infinite beam with free end.

So, now next situation or case that will change, that if the end is a fixed end or if end is a hinge end then how we will do that. So, next one is that that this beam with hinged end.

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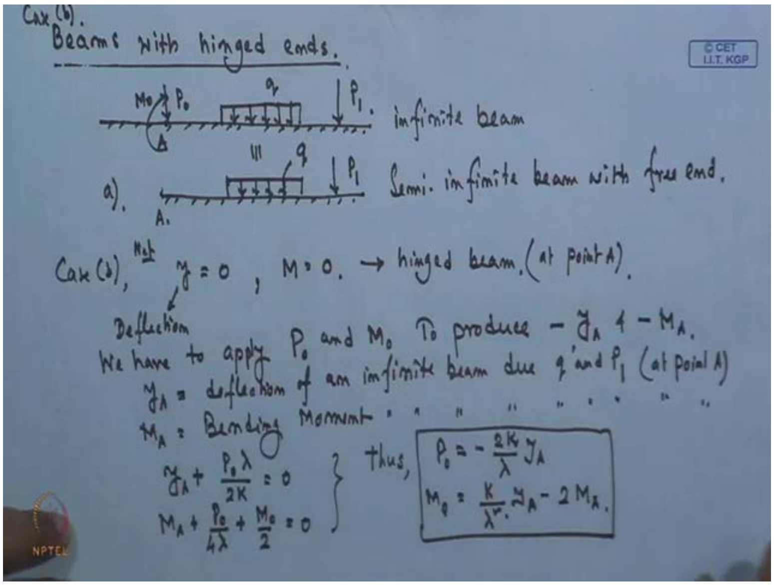

So, first if I draw that infinite beam suppose this is infinite beam. So, this is point A and we applying this UDL q or this concentrated load P 1 then we are applying this P 0 and M 0. Now, that will be equivalent to, this is also P 1 this is q, equivalent to this condition this is for case a where this is the infinite beam and this is the same condition that is the semi-infinite beam with free end. So, that is why this is the, if this P 0 and M 0 are applied based on the previous two expression that I have derived.

So, if we use based on those expression P 0 and M 0 that condition will be similar to this condition. So, next one that we have to calculate if it is the hinged end being so A A end is hinged. So, that means condition for this is case two or case b where this is condition for the this case b condition that as it is a hinged beam then the slope will be 0 as sorry deflection will be 0.

So, as it is a hinged beam so deflection will be 0 and the moment that will also be 0. For the first case when it is a free end beam we consider that moment is 0 shear force is 0. Here, it is a hinged beam so we will consider the deflection is 0 and moment is also 0. So, if deflection is 0. Now, we have to apply same concentrated load and the moment on this condition. So, we have to apply P 0 and M 0 to produce minus y A and minus M A. So, that means the y A, y A is deflection of an infinite beam due to q and P 1 similarly, M A at point A. Similarly, this is bending moment of an infinite beam due to q and P at point A. So, we have to apply this P 0 and M 0 such that to produce minus y A and minus M A so that the net deflection at point is 0 and bending moment is also 0 that is at point A.

So, now the net force that net deflection y A plus deflection expression P 0 lambda by 2 K that is equal to 0 this is the deflection due to the application of the concentrated load. Similarly, M A plus P 0 by 4 lambda plus M 0 by 2 that is also equal to 0. Now, just solve this x two expression we will get that P 0 is equal to minus 2 K by lambda y A and M 0 that will be K by lambda square y A minus 2 M A. So, K lambda square y A minus. So, now if we apply these two forces to one P 0 and M 0 from this two according to this two expression, then we will get a condition at point A that is deflection net deflection will be 0 and net moment that is will be 0 and then this condition will be equivalent to a condition.

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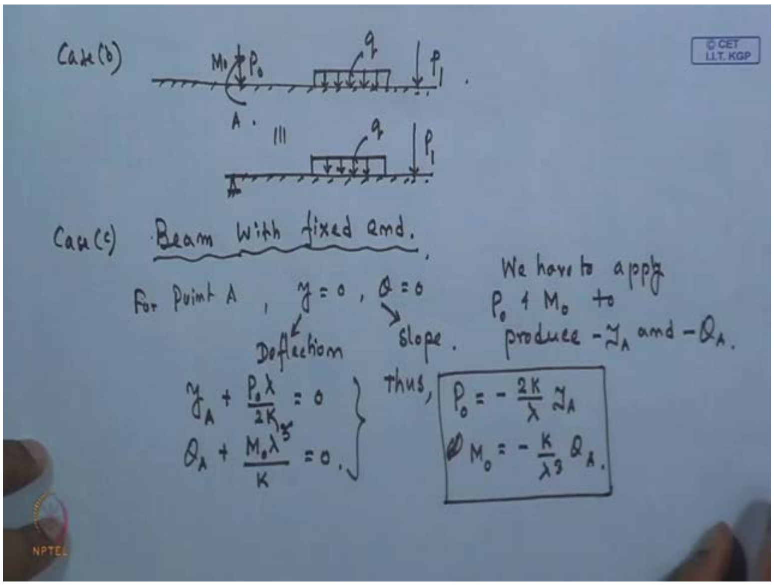

So now similarly, for this case two this is infinite beam and at point A M 0 and P 0 is acting, this is q, P 1 is the load and that is equivalent to a hinged beam with q and P 1 provided that this is infinite beam, this is semi-infinite beam provided we applied P 0 and M 0 according to the calculation of case b. Now similarly, for the case c we have a beam which is fixed end. So, beam with with fixed end so that means for this condition, for the fixed end, for point A of the infinite beam, the condition will be net deflection is 0 and slope is also 0. So, this is deflection, this one slope because this is a fixed end beam.

So, at the that end, fixed end the deflection will be 0 and slope will also be 0. Now, if now we have to apply with, we have to apply again P 0 and M 0 to produce minus y A and minus theta A so that the net deflection and slope that will be 0. So, now for this condition that will be so net deflection that is y A and for the concentrated load P 0 delta by 2 K that will be equal to 0 and theta A plus M 0 lambda square by K that is equal to 0. Now, thus for this after solving these two expression we will get P 0 will be equal to minus 2 K by lambda y A and M 0 will be equal to minus K by lambda cube into theta A.

So, these are the two expression that we are talking about. So, that means here this will be M 0 cube lambda cube. So, that means M 0 will be so that means we will get final two expression of this case is P 0 is equal to minus 2 K lambda y A and M 0 will be K divided by lambda Q into theta A.

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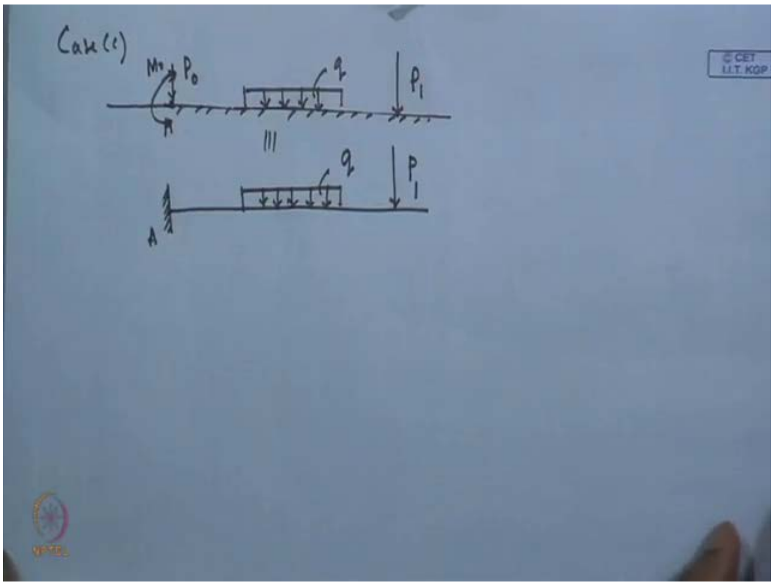

So, now if we produce this P 0 M 0 based on this two calculation, then we will get the condition or we get this case three or case c that this is infinite beam that is q and we apply a concentrated load and then at point A we are applying P 0 and M 0. So, that is equivalent to a fixed end beam a semi-infinite beam and this is q and this one is, this is A point and this is P 1. So, that means this is the third case where we will get this condition.

Now, we have discussed that for this semi-infinite beam then how how to calculate the that means for this, first to solve this semi-infinite beam we have to consider one infinite beam where we assume that for at a point A will produce a developed a moment and shear force M A and Q A because of the application of the external loop. So similarly, it can produce a slope at that point and can produce the deformation of that point due to the application of the external load. Now, we have three different condition, one is free end condition, one is fixed end condition, another is hinged end condition.

So, for the free end condition we assume that the at A point that will be moment will be 0 and shear force will be 0. Now, we have to apply for all the cases we have to apply moment M 0 and force P 0 such that for free end moment there will be net moment will be 0 at point A, net shear force will be 0 at point A. Now, if it is a hinged end moment, hinge end, hinged beam then the net deflection of that point will be 0 and net shear force that will also be 0, net bending moment will also be 0 if it is a hinged beam and if it is a fixed end beam then the net deflection will be 0 and net slope will also be 0 at that point. So, slope will also be 0.

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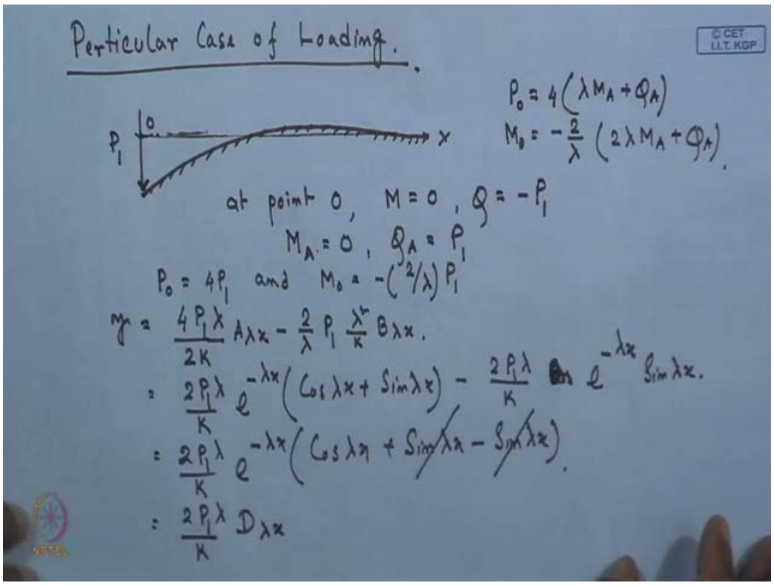

So, now in this condition we can derive all these expression, all these equation. Now, how to where we have to apply these things. Now, we will consider few cases or discuss few cases where we can apply these expressions for these theories to determine the various slope bending moments deflection and shear force.

Now, first case we will consider that is the particular case, first case so we will consider few cases. First one we will consider that this is the original or portion of the beam. Now, after the deflection application of this is x and the P is applied at point O. So, this is the deformed shape of the beam. So, this is a condition of the case one. Now, we have to determine the slope bending moment shear force and the deformation of this expression of this beam. Now, it from this loading condition we can see that this is a free end semi-infinite beam condition. So, if it is a free end semi-infinite beam so that expression of P 0 is equal to 4 lambda M A plus Q A then M 0 is minus 2 lambda 2 lambda M A plus Q A.

So, at point O that as it is a free end beam and which is subjected to a concentrated load P, so the moment will be 0 and shear force that will be minus P 1, that will be because P it is acting this side downwards so we will consider this is minus P 1. So, M A so that is the moment which is acting so M A will be 0 at this point and Q A will be P 1 at this point. So, now if we because here the net moment that means we are applying a load P so that means shear force at this point will be minus P 1. So, that means the M A will be 0 in this point and Q A will be P 1 at this point. Now, if I put M A and Q A at this two expression then we will get P 0 will be 4 P i P 1 and M 0 will be minus 2 by lambda into P 1.

Now, we have to determine the slope or deflection of this point. Now, so we have, we are applying so two cases. One is concentrated load, another one is concentrated moment, so P 0 and M 0. The deflection of the net deflection for the concentrated moment and concentrated load then the for infinite beam so we will get 4 P 1, this is P 1 so that means for P 0 we will apply 4 P 1 into lambda by 2 K then A lambda x minus 2 by lambda P 1 into lambda square by K B lambda x. So, these two are coming from the general expression of this concentrated load and also the in case of P 0 we have to put 4 P 1 and in case of M 0 we have to put minus 2 by lambda P 1.

So, now we will get this expression then we can take that is 2 P 1 lambda by K and this A lambda X. So, we know this a lambda x is e to the power minus lambda x into cos lambda x plus sin lambda x, then minus there will be 2 P 1 lambda by K B lambda x so B lambda x we can finally write that e to the power minus lambda x into sin lambda x. So, ultimately if I take common 2 P 1 lambda by K e to the power minus lambda x so this will be cos lambda x plus sin lambda x minus sin lambda x. So, sin lambda x sin lambda x will be canceled out and then e to the power minus lambda x into cos lambda x that is equal to D lambda x.

So, final expression will be 2 P 1 lambda by K into D lambda x. This will be the final expression for the deflection of this condition where this end we are applying a concentrated load P 1.

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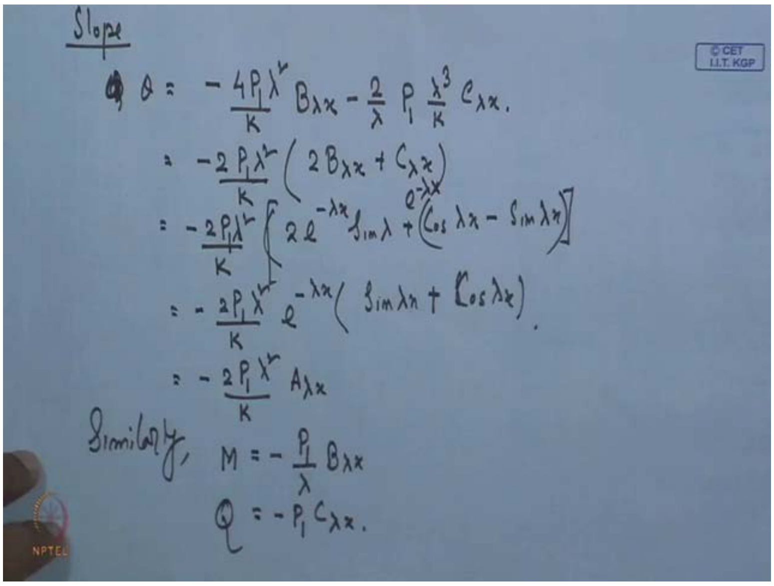

Similarly, if I want to determine the expression of the slope, then slope expression Q as theta that will be equal to minus 4 P 1 lambda square by K into B lambda x minus 2 lambda P 1 into lambda cube by K C lambda x. So, that means that that two (( )) adding two effect, one is due to the concentrated load another due to the concentrated moment. So, next one that if I take this minus 2 P 1 lambda square by K common then we will get 2 B lambda x plus C lambda x. So, this will be minus 2 P 1 lambda square by K into 2 e to the power minus lambda x, B lambda x is sin lambda x plus cos lambda x minus sin lambda x because C lambda x is cos lambda x minus sin lambda x.

So, one sin sin will cancel out so this will be minus 2 P 1 lambda square by K into e to the power there also this will be into e to the power lambda x. So, this will be e to the power minus lambda x into sin lambda x plus cos lambda x. So, this will be this e to the power minus lambda x into sin lambda x plus cos lambda cos lambda x that is equal to A lambda x. So, this will be minus 2 P 1 lambda square by K into A lambda x. So, this is the expression of the slope. Similarly, we can derive the expression of M that will be minus P 1 by lambda B lambda x. So, if I add the effect of two cases, one is concentrated load and one is concentrated moment. Then we will get and then if I simplify this expression like the deflection and the slope then we will get this final expression of the moment in this form.

Similarly, for the shear force this expression will be minus P 1 into C lambda x. So, this is the one condition particular loading condition. So, thereof we can discuss the few other loading condition also, where we can determine how to calculate the other forces, other moments and these things.

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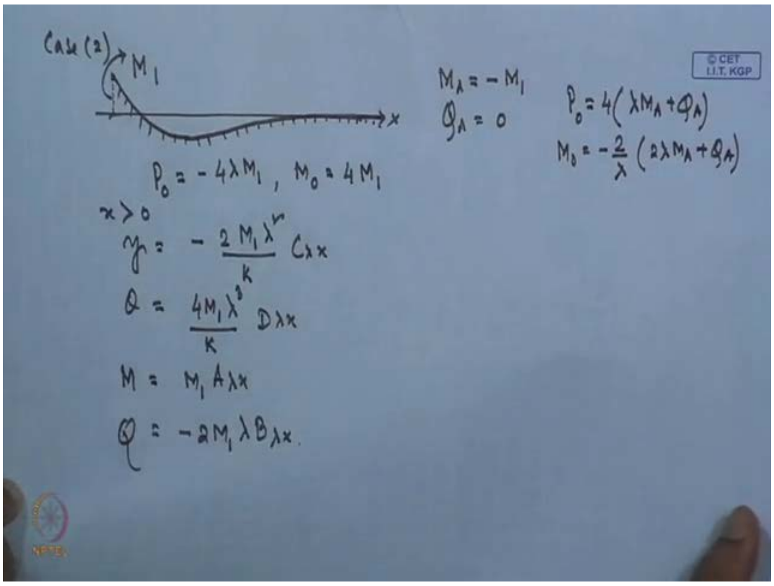

So, next condition the loading condition that we consider that is for the case two that this is the original position of the beam or the x axis. Then it is subjected to a moment at this end M 1. Now, if this subjected to a moment then the conditions will be M A will be equal to minus M 1 and Q A that will be 0 because we are not applying any force here concentrated force. Now, this is the again from here we can see this is the free end semi infinite beam expression we can use. So, that expression is P 0 again is equal to 4 lambda M A plus Q A again M 0 is equal to minus 2 lambda by 2 lambda M A plus Q A. Now, if I put Q A equal to 0 and M A equal to M minus M 1 then finally, we will get P 0 that will be minus 4 lambda M 1 and M 0 that will be 4 M 1.

So, for the expression the x greater than equal to 0 again if I, again if we add the contribution of the concentrated load and concentrated moment and then simplify these things like the previous case then we will get the final form of the deflection in this particular case two that is two M 1 lambda square by K into C lambda x. Similarly, theta slope is equal to 4 M 1 lambda cube by K into D lambda x and bending moment M will be M 1 A lambda x and shear force Q will be minus 2 M 1 lambda B lambda x. So, previous case one this is free end semi-infinite beam subjected to concentrated load and this is case two subjected to concentrated moment.

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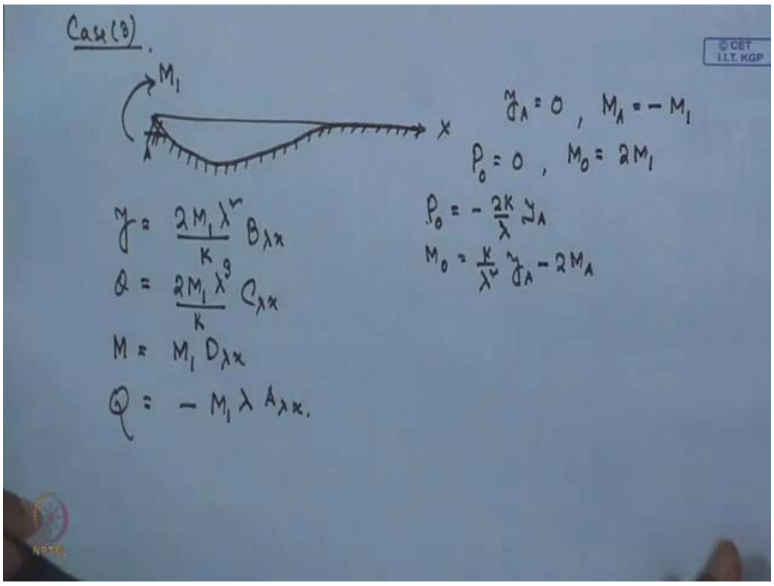

Now, case three if we consider the next case that is case three where this is a hinged semi-infinite beam, this is hinged and is subjected to a moment at this hinged end. So, there will be a deformation step, this form so this is A point. So, as it is a hinged beam y A will be 0 and here M A will be minus M 1. So, y A will be 0 and M A will be minus M 1. Now, for this case if I consider this is the hinged end so and then if we put this y A and M A value on the two derived expression that we have already derived and then finally, we will get that P 0 that value is equal to 0 and M 0 that value will be 2 M 1 because the in that case expression was P 0 is equal to minus 2 K by lambda y A and M 0 is K by lambda square y A minus 2 M A.

So, if y A equal to 0, P 0 will be 0 and if M A equal to minus M 1 then this will be 2 minus M 1 this will be 2 M 1. Now, again if I will use these two value P 0 equal to 0 that means there will be no contribution for the P 0. Only the contribution for this moment concentrated moment will act here and then if we consider the concentrated contribution for this moment M 0 if we place 2 M 1 in place of M 0 then we will get the deflection that is equal to 2 M 1 lambda square by K B lambda x theta is equal to 2 M 1 lambda cube by K into C lambda x. Bending moment M will be M 1 D lambda x shear force Q will be minus M 1 lambda A lambda x. So, this is case three where we consider a hinged moment or a hinged hinged end. Now, we apply a concentrated moment M 1 that is the hinged end semi-infinite beam.

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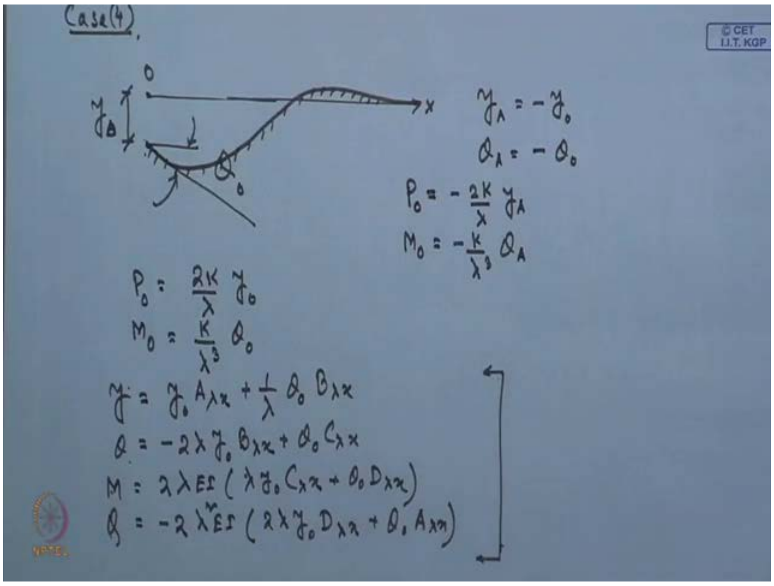

Now, case four we can consider a fixed end also. That is the case four. Case four we can consider that this is the x axis. Now, this beam at this end has deflection as well as slope. This is the pattern of the beam deflection. So, here at this point the deflection is y A and slope at this point that is theta 0 or theta y 0. So, y 0 is the deflection at this point y O or O point y 0 and this is theta 0. So, now as it is a fixed end beam we can consider so we can take at y A that will be minus y 0 because the y 0 is the deflection, it is in the downward direction. So, that will be y A will be minus y 0.

Similarly, theta A that will be minus theta 0. So, we have these two things. Now, we can because we know for this particular fixed end semi-infinite beam P 0 is equal to minus 2 K by lambda y A and M 0 is equal to minus K by lambda cube into theta A. Now, if I put theta A and y A value then we will get P 0 is equal to 2 K by lambda y 0 M 0 is equal to K y lambda cube into theta 0.

And finally, if I put the contribution of these two and then final form of the expression that will be equal to y equal to y 0 into A lambda x plus 1 by lambda theta 0 B lambda x after simplifying this so this theta is equal to minus 2 lambda y 0 B lambda x plus theta 0 C lambda x. M will be equal to 2 lambda and then that will be equal to lambda E I and then lambda y 0 C lambda x plus theta 0 D lambda x and Q is equal to minus 2 lambda square is E I which is E I then 2 lambda y 0 D lambda x plus theta 0 A lambda x. So, these two expression, this four expression that we will get for this particular case of the where we are considering the moment and the shear force. So, this will be E I.

So, in the, through this class I have discussed that about the semi-infinite beam and the previous classes infinite-beam also been discussed. So, next class I will discuss about the finite beam and what are the expression for the finite beam and where we can use this, all these theories of the beams on elastic foundation in the real field.

Thank you