Advanced Foundation Engineering
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Lecture - 36
Soil - Foundation Interaction (Contd.)

In last class I have discussed about that deflection slope, bending moment and shear force expression for infinite beams subject to the concentrated load.

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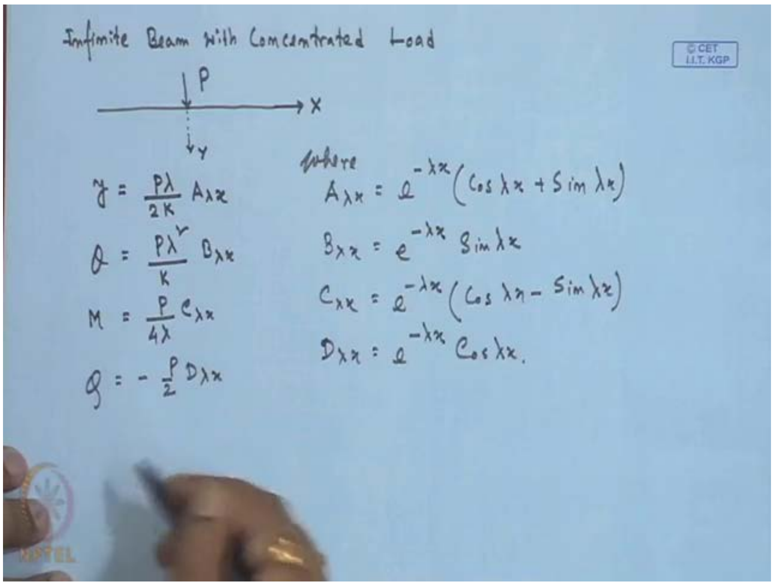

And that time it is observed that, if I put that infinite beam with concentrated load. So, this is infinite beam, we have a concentrated load here P, this is x and this direction, it is y. So, last class it is observed that the expression of y was P lambda to 2 K A lambda x, theta was P lambda square divided by K B lambda x, moment bending moment was M P divided by 4 lambda C lambda x and Q is equal to minus P by 2 D lambda x, where A lambda x was e to the power minus lambda x into cos lambda x plus sin lambda x.

Similarly, B lambda x was e to the power minus lambda x sin lambda x C lambda x that was e to the power minus lambda x cos lambda x minus sin lambda x. Similarly, D lambda x it was e to the power minus lambda x into cos lambda x. So, these four other coefficient, and we will get this type of expression for four different quantities that this is for the deflection, theta is for the slope, M is for the bending moment and Q is for the shear force. So, now today I will discuss that if the infinite beam is subjected to a concentrated load.

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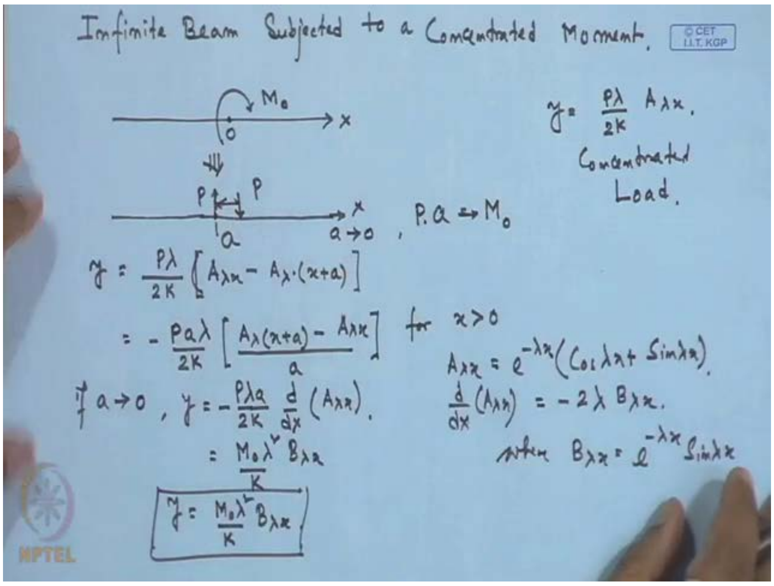

That the infinite beam subjected to a concentrated moment, so suppose if I have a beam x and at the center I have point o, this is the moment M. So, then how this expression will be looks like so that thing we will discuss today. So, the first suppose this is the x then in this thing can be converted to a load applied at a distance P a from the center that is P and then another thing that is acting upward that is P.

So, if this a tends to 0 then P into a that will be equal to M 0 or M 0 which is applied here. So, this is the concentrated moment that can be equally converted to a force which is concentrated load which is acting P at a distance a from the center. So, this is the center. So, if a tends to 0 then P into a, that will be tends to M 0 then P is the concentrated load a is the distance that will be tends to M 0.

So, now in this case we can write that expression of y that will be P into lambda divided by 2 K because for the concentrated load the expression of y is P into lambda divided by 2 K into A lambda x. So, that was the concentrated load if it is, this is for the concentrated load. So similarly, here that we have two thing, one is concentrated load acting at the center, another is acting in the downward direction. So, have P into lambda and then we can write that is A lambda x then minus A lambda x plus a. Then the total one is within the bracket because here we have one concentrated load that we are applying here that at a distance of a, and then so we can write that A lambda x minus A lambda x plus x a.

So, here this concentrated load at the acting at the center. So, this will be A lambda x. Now, similarly, that expression will be minus P a lambda divided by 2 K and then we can write A lambda x plus a minus A lambda x then divided by a that is for x greater than 0. Now, if a which tends to 0 then we can write that y is equal to P lambda a divided by 2 K and then this one can be written d by d x into A lambda x. So, these one can be written d d d d x into A lambda x if a tends to 0 because now this thing is converted to a concentrated load problem where the load is applied at a distance of a from the center if a tends to 0 then P into a is close to M 0.

So, now that we know that A lambda x is equal to e to the power lambda x into cos lambda x plus sin lambda x. So, that d d x A lambda x that will be equal to minus 2 lambda B lambda x, where as we know that B lambda x is equal to e to the power lambda x sin lambda x. So, further if I put that value here so this will be P into a, that is M 0. So, P into a M 0. Then this minus 2 lambda x so there is a term y is minus. So, there is this minus P P lambda a so this will be minus, so this minus and this minus so there will be positive sign.

So, now if I put this value here, if I replace this things with this minus 2 lambda B lambda x then this will P 0 M 0 into lambda into another lambda that is lambda square, then B lambda x and then 2 2 will be cancel out divided by K. So, this will be the so this will be the deflection equation will be M 0 lambda square divided by K B lambda x. This is the expression of the deflection if the beam is subjected to a concentrated moment.

Now, next one that we will produce that is here the main thing that is this is conversion. So, that means here one for that is acting upward another is in the downward, so at a distance of a so that we can write this things in this form.

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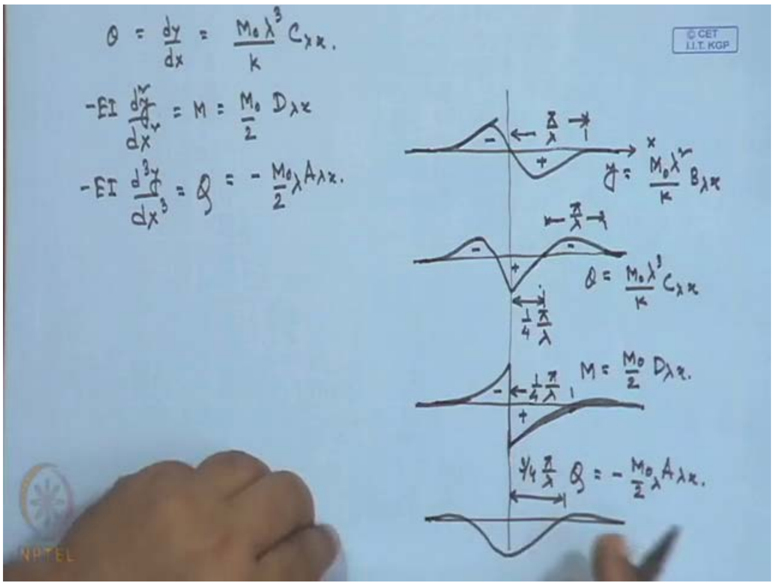

So, next one we have to determine the slope that the expression of the slope that we know that this will be theta is the slope and that is d y by d x. So, once we do that things so there will be M 0 lambda to the power cube by K into C lambda x. Similarly, E I d square y d x square that will be moment, that is equal to M 0 divided by 2 D lambda x and the shear force expression E I d 3 y d x 3 that is equal to Q that will be minus M 0 by 2 A lambda x. So, these are the four expression of the beam subjected to concentrated moment.

Now, if I draw the deformation shape and the bending moment diagram of the beam. So, the deformation shape as it is subjected to moment. So, this will be so this is the deflection shape, so this direction this is o x. So, this will be minus deflection positive downward so and this value will be pi by lambda and this expression y is equal to M 0 lambda square divided by K B lambda x.

So, next one is the bending moment diagram. So, the sorry next one will be the slope diagram. So, that our slope diagram will be this is all negative. So, this will be positive. So, the expression is theta is equal to M 0 lambda cube divided by K C lambda x. Now, this distance from here to the point it is, that is pi by lambda. So, this point where it is changing the sign to the point where it is 0 and then this portion is 1 by 4 pi by lambda.

So, next one is the bending moment diagram. So, this will be going upward then it will go downward and then it will again follow this path. So, the M that is equal to M 0 by 2 into D lambda x. So, this is negative, this is positive and the point where it is just going upward direction so that point will be 1 by 4 pi by lambda.

So, now next one is the shear force diagram. So, that expression so that diagram will be in this form, it is 0. So, Q is equal to minus M 0 by 2 into A lambda x M 0 by 2 into lambda into lambda x A lambda x. So, that is minus so the value from here to here where it is changing the sign is three-fourth divided by pi by lambda.

So, here there will be a lambda term. Now, we will get this three-fourth expression. One is for the deflection, one is for the slope, one is for the bending moment and one is for the shear force. So, next one that we will go for that if other case is that if the moment is this is we have discussed for the two cases, one is for the concentrated load and concentrated force, so next one that we will discuss for the uniformly distributed load.

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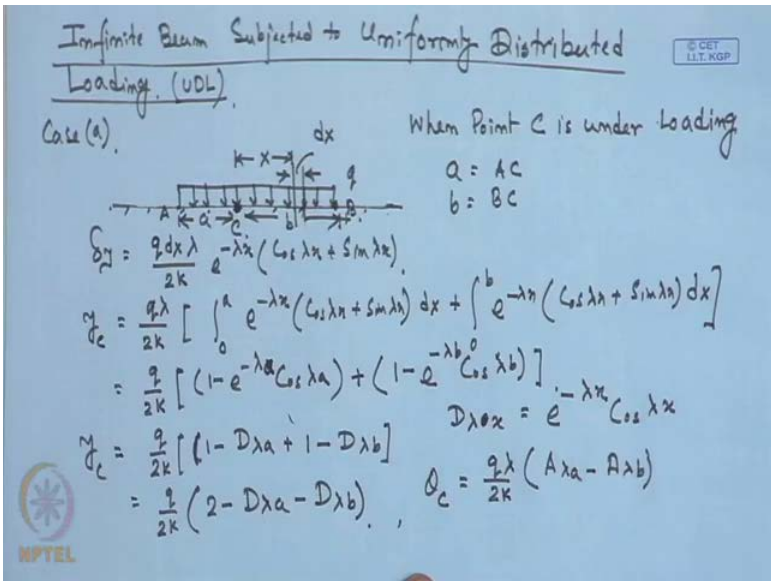

So, this things will be discuss that if beam is subjected to that infinite beam. So, if it is subjected to uniformly distributed loading or UDL. Suppose, we have infinite beam and where it is subjected to a UDL of intensity q, this is q and we have a points C say within that beam. So, first case what we have done that we can take a infinitely similarly, small segment so that is the segment with d x at distance of x from the C.

So, first case, case a that when this point C is under loading. So, that means this C can be left side of this loading, this C can be the right side of the loading or this C point can be within that loading. So, here first case where you consider the point C which is within the loaded region and then we consider one small segment of with d x which is at a distance of x from this C point.

Now, for due to this the settlement due to this small segment that we can write so that is basically these are the if I consider this is the small segment each has a one concentrated load then that one concentrated load is acting at a distance of x. So, that concentrated load value will be q into d x, so that a delta y deflection due to the concentrated load acting at a distance of x from this point C. So, this will be q into d x that is the load concentrated load, small one because d x is the small segment and q is the UDL intensity. So, this will be q into d x then lambda divided by 2 K and then that is e to the power minus lambda x into cos lambda x plus sin lambda x.

So, only for this small segment so we are considering each as a segment so is the for the these concentrated load that deformation will be for the concentrated load deformation expression is P lambda by 2 K e to the power minus lambda x into cos lambda x plus sin lambda x. So, here this P is we can determine for q into d x. So, that is the total deformation of the center due to these UDL at C point the deformation so that will be q into lambda divided by 2 K then this is 0 to a e to the power minus lambda x cos lambda x plus sin lambda x, this is into d x another one plus 0 to b e to the power lambda x into cos lambda x plus sin lambda x into d x.

Now, what is a and b? a is basically this is point A this is point B and this is point C. So, a is distance between A to C and b is the distance between A to this point B. So, a is A C b is B C where A B are the two end points. So, now after integrating these things we will get q divided by 2 K into 1 minus e to the power lambda x cos lambda a to the power lambda a sorry. So, this will be after integrating this from 0 to a, and 0 to b so we will get 1 minus e to the power lambda a into cos lambda a plus 1 minus e to the power lambda b into cos lambda b.

So, now we know that e to the power lambda x, we know that lambda D lambda x is equal to e to the power lambda x into cos lambda x. So, here if I replace this x by a or b then we can write these expression will be y C will be q by 2 K 1 minus D lambda a then plus 1 minus D lambda b. So now finally, this will be q by 2 K into 2 minus D lambda a minus D lambda b. So, these will be the expression of the deflection for the, if it is subjected to a infinite beam subjected to a uniformly distributed load. Similarly, we will get theta C slope at C point that will be q into lambda divided by 2 K A lambda a minus A lambda b.

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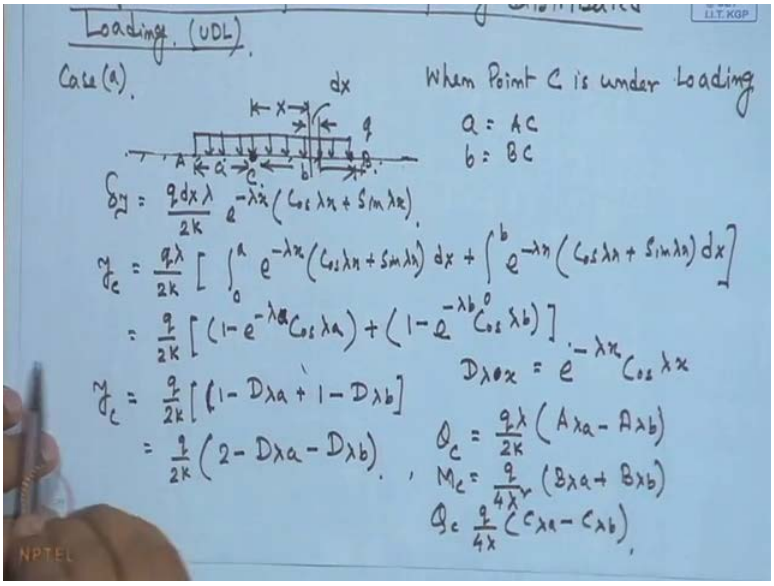

Similarly, another one that M will be q divided by 4 lambda square into B lambda a plus B lambda b. Similarly, Q at C point, this is M moment at C point, Q at C point will be q divided by 4 lambda into C lambda a minus C lambda b. So, these are the four expression of y C deflection at center slope at C points. This is deflection at C point, then moment at C point, shear force at the C point. So, next one this is the case where C point is within the loaded region.

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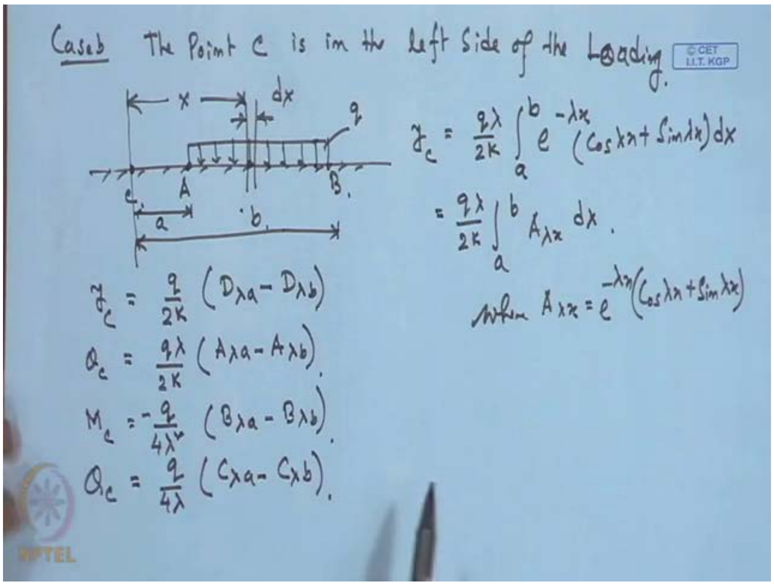

Now, the case b, if I consider the case b where the point C is in the left side of the loading. So, that means if I consider that is a uniform infinite beam where this is the uniformly distributed load so as I have mentioned that A and B point as a two end of this loading A and B and C point is at the left side of the point of this loading. So, that means C point again we have considered the, we have small segment of d x at a distance of x from the C point.

And distance from A to C is a and distance from A to B as C to B is b. So, here we can get that y C deflection of the C point because we have taken three condition of the C point, one if the C is within that loaded region, C is left side of the loaded region and C is the right side of the loaded region, but deflection of the C point, so here again again we can calculate the concentrated, again we can consider the each point as a concentrated load where concentrated load value is will be q into because this is again the intensity is q, q into d x that will be P.

So finally, we will get q into lambda divided by 2 K and here we will get the limit for initially because that point was within the loaded region. So, we get C as a 0 if consider so that will be limit was 0 to A and then from 0 to B, but here C is at this point and so the limit will be here from A to B. So, that means here limit will be from a to b that is the difference from the first case.

So, that means this will be e to the power minus lambda x into cos lambda x plus sin lambda x into d x. So, finally, again we know that e to the power lambda x into cos lambda x plus sin lambda x that is A lambda x so we can write q lambda divided by 2 K limit a to b then e to the power so this is A lambda x into d x. So, where A lambda x is equal to e to the power lambda x cos lambda x plus sin lambda x.

So, now after integration we will get y C is equal to q by 2 K into D lambda a minus D lambda b. So, here in case of x we are just putting a and b depending upon the requirement. So, then the slope theta C will be q into lambda divided by 2 K into A lambda a minus A lambda b. Similarly, bending moment expression for the C is q minus q by 4 lambda square into B lambda a minus B lambda b, shear force at C that is q divided by 4 lambda into C lambda a minus C lambda b. So, these are the expression of this four quantities for case b if the point C is left side of the loading.

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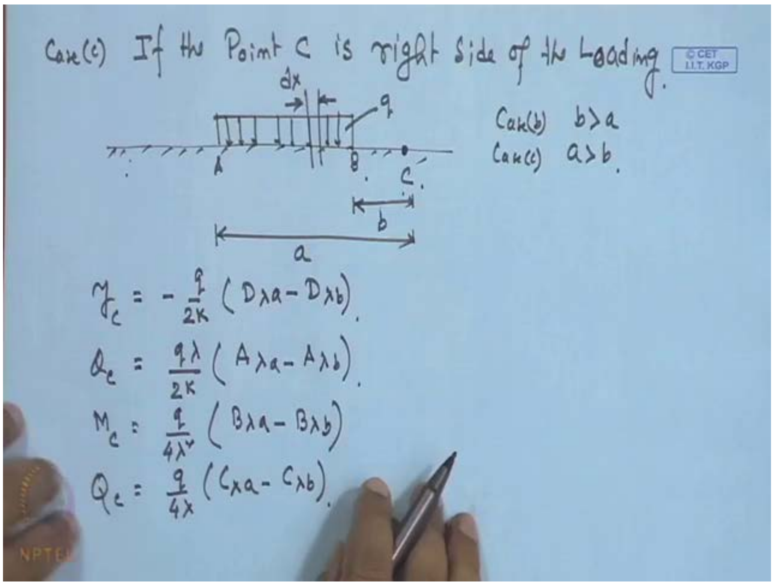

So, next case that is, if the point C is the right side of the loading. Now, the case c that if the point C is right side of the loading. In that case that right side of the loading means that we have this infinite beam, this is the UDL of intensity q and if the point C is here and similarly, this point is A this point is B. So, A to C and A to B that is b and A to C is a.

So similarly, here also we have to take the segment at a distance of this is d x. So, at a distance of x so finally, we will get the y C here it will be minus q by 2 K, then this things D lambda a minus D lambda b is same. Because in previous case, case b that b was greater than a, in case b that b was greater than case a, in case c that a is greater than b. So, that will be this point would be negative because it is, the all are in the opposite direction compared to the case b. So, that means theta theta C slope of C point that will be q lambda by 2 K into A lambda a minus A lambda b, moment at C point will be q by 4 lambda square B lambda a minus B lambda b.

Similarly, shear force at C point that will be q by 4 lambda C lambda a minus C lambda b. So, these are the four expression and three different cases of the UDL and the three case means if the point particular we have chosen a point, one first case it is within the loaded region, second case it is the left side of the loaded region and third case it is in the right side of the loaded region. So, when we got this three different expression. Now, say now next one is that here, the we have used the UDL then what will be the deflection slope bending moment shear force expression if the infinite beam is subjected to a triangular loading. So, first in the third one in the next case, it is the triangular loading.

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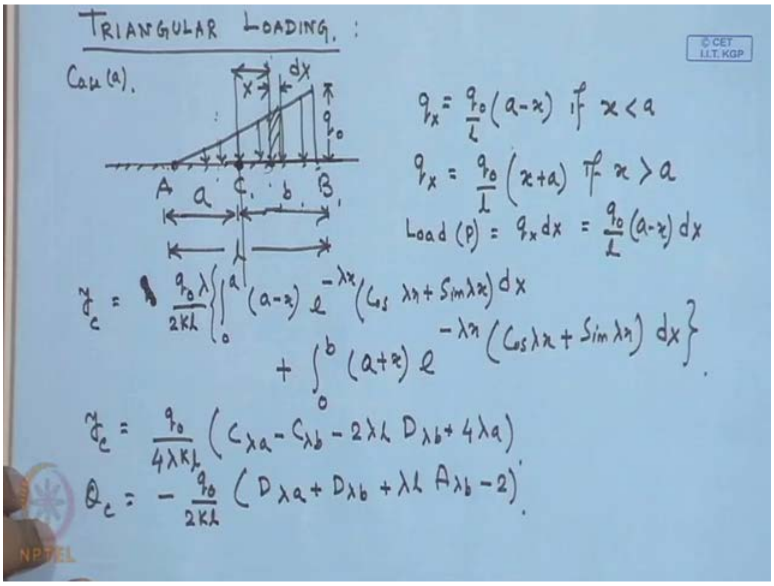

Triangular loading this is also case a similar to the UDL here also will case three case one is the point C within the loaded region, one is in the left side, another in the right side of the loaded region. So, suppose first one this is the infinite beam which is subjected to a triangular type of loading, where intensity is varying with distance and the maximum one is a q 0. So, here the maximum intensity is q 0 and then we consider one point C within the loaded region and one end is A another end is B. So, distance from A to C is a, distance from C to B is b, distance from A to B is say l distance from A to B is l.

Again from this point C that one segment is taken with thickness d x at a distance of x from the C point. Now, the q x the intensity of this, loading intensity of at a distance of x from the C that will be equal to, so this q x that will be equal to q 0 divided by l into a minus x if x is less than a. So, if a is within this zone, if the x is within this zone from A to C then if this is x then the intensity at any point x that will be q 0 a minus x and similarly, delta q x is q 0 divided by l into x plus a if x is greater than a.

So, now in this case x is greater than a. So, this point the loaded point then for this case this will be, q will be the distance is a plus x. So, this will be q x will be q 0 divided by l, l is the total length to x plus a. So, now the deflection at the C point that will be q 0 again here the intensity into that small segment d x will give the load which is acting at this point. So, that means the load will be q x into the d x. So, the load which is acting at a point so let us say P that will be q x into d x.

Now, q x is q 0 by l into x plus a and x plus b now here. So, first case we will we will get that q 0 is here that means the total load so this will be q 0 divided by 2 K l into lambda and then d x is the loading. So and if it is from 0 to a so that means from 0 to a, and that is within this point this zone A to C so we have to use this expression. So, 0 to a so that will be so l q 0 then next part a minus x then as usual e to the power lambda x into cos lambda x plus sin lambda x into d x. Then another part plus and that is will be 0 to b 0 to b and then we have to use the next expression, so q 0 we have already used, l also we have taken, then only the x plus b part will be here.

So, a plus x or x plus a then e to the power minus lambda x into cos lambda x plus sin lambda x and this is d x within the bracket. That is why we have taken two different one q because in previous case it is UDL so if I take it is in the right side of the C or left side of the C value will be same that is q 0, but here the q 0 value will change depend q value will change depending upon the position or with region where we are taking the limit.

So, if it within the 0 to A then will case a minus x, if it is 0 to B we have to take a plus x. Suppose, load is delta x into d x so delta x if I consider q 0 for first case say so l into a minus x into d x. So, in case of P we have to put this value. So, P P this y C is P delta by 2 K. So, this will be first case if 2 k l this l and then q 0 and then a minus x and d x will be in this region, another this will be a plus x into d x.

Now, once we integrate after integrating all these expression, then the deflection at the center point this will be q 0 divided by 4 lambda K l into C lambda a minus C lambda b plus 2 lambda l D lambda b plus 4 lambda a. So, after integrating this expression the final form will be C lambda a minus C lambda b minus 2 lambda l D lambda b plus 4 lambda a. Similarly, the slope theta C that will be minus q 0 into 2 divided by 2 K l into D lambda a plus D lambda b plus lambda l A lambda b minus 2.

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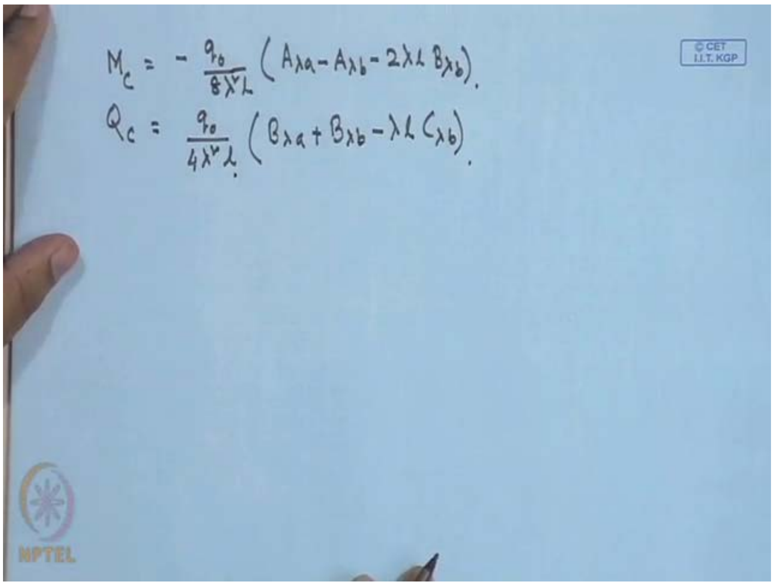

Similarly, the expression of the M, so at the C point expression of the M at the C point that will be 0 the q 0 8 lambda square l A lambda a minus A lambda b minus 2 lambda l B lambda b. Similarly, Q C that will be q 0 divided by 4 lambda square l into B lambda a plus B lambda b minus lambda l C lambda b. So, these are the four expression of the C deflections slope bending moment and shear force.

So similarly, we will get two other cases here the C point is taken within the loaded region. Similarly, we can take the point beyond the loaded region at the, may be in the left side of the loaded region or right side of the loaded region and similar to the UDL case we can change the limit here from 0 from a to b and then we can put this expression we will get the final form.

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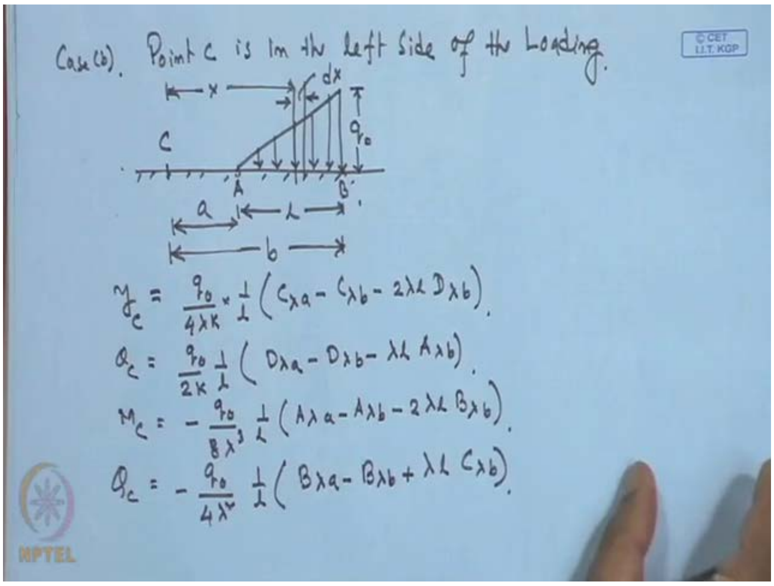

So, in the second case that case b, the second case if the point is the left side of the loaded region, if the point C is in the left side of the loading. So, similarly, this is infinite beam, we have the triangular loading here. So, this is q 0 the maximum intensity and if we consider the point C at here it is left side of the loaded region. So, externally this point is A and this point is B and from A to B it is l from C to A it is a and from C to B it is b.

And then similar to the here also we have to take the small segment d x and depending upon the position of these d x. So, that will be the distance x from the C point then we put the value in the form of the expression and then the final form after integrating those values or the after integrating we will get the final form of the deflection. So, this is same as for the UDL where we have to change the limit from A to B and the once we do that the final form is q 0 divided by 4 lambda K into 1 by l into C lambda a minus C lambda b minus 2 lambda l D lambda b.

For the theta C that will be q 0 divided by 2 K 1 by l into D lambda a minus D lambda b minus lambda l A lambda b. Similarly, for the M C that is minus q 0 divided by 8 lambda cube 1 by l A lambda a minus A lambda b minus 2 lambda l B lambda b. Similarly, theta

C sorry Q C is minus q 0 divided by 4 lambda square 1 by l B lambda a minus B lambda b plus lambda l C lambda b. So, next this is the third one.

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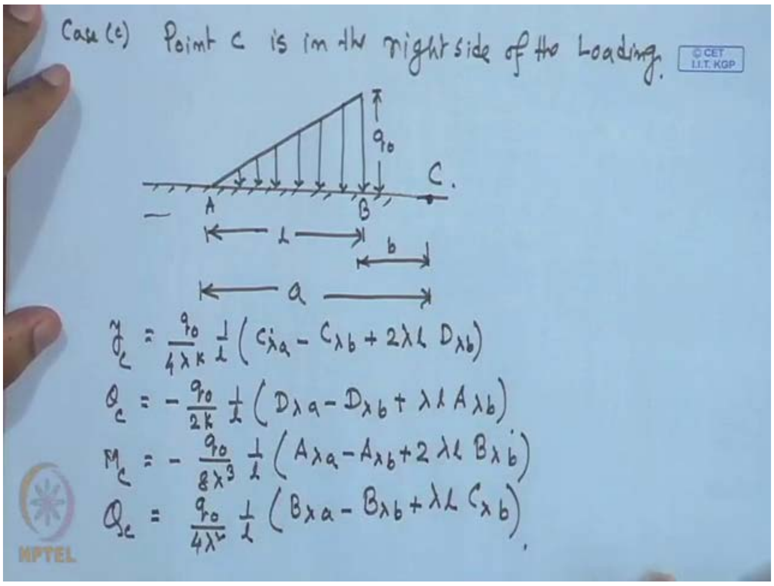

So, next one that we will get the third case that is if it is in the right side of the loaded region so point C is in the right side of the loading. So, we have this infinite beam and this is the triangular loading, this is q 0 and C point is taken here. Similarly, this is B point, this is A point, A to B distance is l, here from A to C to B is b and A to C is a.

So and finally, we will get the y C here also that q 0 divided by 4 lambda K 1 by l into C lambda a minus C lambda b plus 2 lambda l D lambda b. Theta C is minus q 0 2 K 1 by l D lambda a minus D lambda b plus lambda l A lambda b. Similarly, M C is like to the case P we will get that is q 0 by 8 lambda cube 1 by l A lambda a minus A lambda b plus 2 lambda l B lambda b and the shear force at C point q 0 by 4 lambda square 1 by l B lambda a minus B lambda b plus lambda l C lambda b.

So, the all the values are remain almost same for compared to the case b, a sign will change as the position of the point has been changed. So, these are the cases for the infinite beam where the beam are subjected to a concentrated load, beam is subjected to a concentrated moment, then beam is subjected to a UDL and the three different cases for the deflection of the within the load any point within the loaded region or any point outside that loaded region and the both side, same the we have been subjected to triangular type of loading then how the deflection slope, bending moment shear force we can calculate within the loaded region for any point outside the loaded region that we have discussed.

So, in the next class so this all the things we have discussed for the infinite beam. So, in the next class we will discuss the point if the beam is semi-infinite or infinite, it is infinite beam. So, the, what is the different between that infinite beam, semi-infinite beam or the finite beam. So, the infinite beam that part is been discussed already. In the next few classes I will discuss about the different expression, how to develop the expression for the different loading condition, for the different end condition of the beam, if beam is a semi-infinite beam or beam is a finite beam.

Thank you.