Advanced Foundation Engineering
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Lecture - 34
Soil - Foundation Interaction (Contd.)

In the last class I have discussed about what is soil structure or soil foundation interaction, and then I have discussed about various mechanical model where the soil is idealized by spring for a, and then for two parameter model soil is ideally by a spring and the shear layer or spring or by tension membrane or spring and then by beam. Because in the first soil model that I have discussed is Winkler model, where the soils are idealized by spring and those springs are linear spring, discreet and there is lack of continuity in this those spring.

So, to remove those limitations then the improved models are suggested where those continuities are removed, where then the springs are connected by tension membrane or by shear layer or by beam. So, the next model of that among those improved model is the Kerr model.

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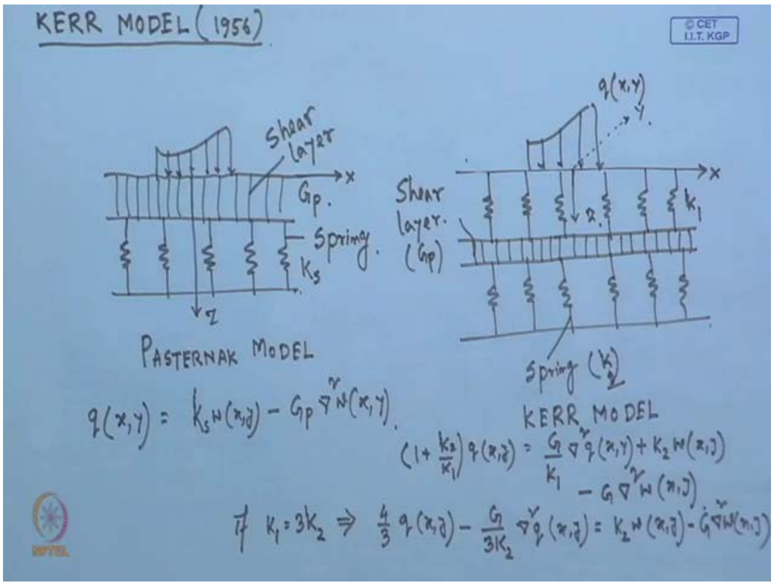

Then the Kerr model in the last class I have discussed about the Pasternak model, where suppose this is the surface and these are the shear layer and these springs are connected by this shear layer. So, this is that shear layer and load are applied on this layer. So, this is the model which is proposed by Pasternak.

So, this is shear layer and this is spring and the expression was was the shear modulus of this shear layers is G p and the spring constant of the pring are K s. So, the expression that was available for this case that q x y is K s into w x y minus G p del square w x y. So, these are the model that I have explained in the last class about the Pasternak model. In the Kerr model that suppose if this is x, in this direction this is z. So, similar in the Kerr model we will get for this is x direction and perpendicular direction this is z and this normal downward direction this is z and in this direction this is y.

Then first soil is idealized by once these springs and then this shear layer is placed in between two springs, then springs are again placed to idealize the soil. So, that mean shear this is the shear layer with shear modulus G p, this is and here again the load is applied with q x y or concentrative load can be also applied. Now, here we will get so this is springs with spring constant with spring constant say K K 1 K 2 and this is spring constant K 1.

So, this is this is produce by the Kerr. So, this is this is the Kerr model. So, these are the two, where both the cases shear layer is used by in the first case Pasternak model shear layer is used of of this springs to connect this springs, here the shear layers are used, shear layer is used in between two spring to connect this springs. So, with different spring constants so here it is K 1 it is K 2. K 1 and K 2 are the spring constant and G is the shear modulus of the shear layer. Now, in that case the response function will be for the Kerr model 1 plus if I consider this is K 2 by K 1 to p or q, this is q x y that is equal to G by K 1 del square q x y plus K 2 w x y minus G del square w x and y.

So, here another term that here the this is q x y K s w G p del square w, here this terms K 1 and K 2 that will be introduce here. Now, if that K 1 is three times of K 2 then this expression will be four-third q x y if i put K 1 is three times of K 2 and this will be four-third q x y minus G 3 K 2 del square q x y that is equal to K 2 w x y minus G del square w x y. So, we will get this type of expression so which is similar to the Pasternak model.

So, but here the advantage of these model over this Pasternak model then state of two parameter model here we can use the another parameter or you can use the two different types of soil, it is modulus of separate reaction K 1 and K 2 so and then this is the advantage of this module so that we can use that additional bondage condition is also available for this model Kerr model. So, that means here we will get two spring constant K 1 K 2. So, we will get another parameter. So, we can represent the soil behavior more correctly.

So, now in this case so this is other two, other difference model for where the soils idealize by spring or shear layer or to connect those springs you can use beam or tension membrane. Now, in the another limitation of the, so that means in the in the last class I have mention that there is basic two limitations in the Winkler model, one is lack of continuity. So, by this improved model you can remove this limitation because here the continuity is provided by using different components and then the next one is that in the Winkler soil model we that is a linear spring is used, but if the soil behavior is non-linear then how this linear things can be converted to non-linear things. So, that is another limitation that we have to remove there.

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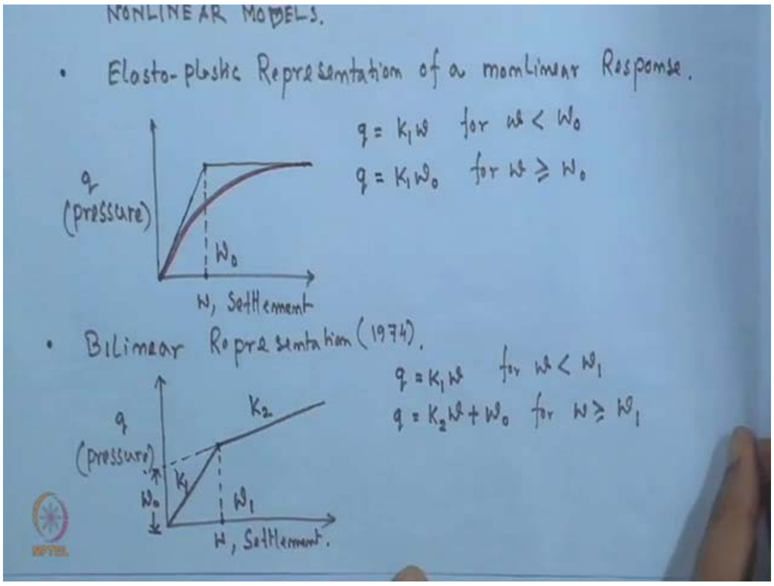

Now, in the, for that part if I consider the non-linear behavior of the soil then we will consider soil or we can consider that is a non-linear model. Now, first model that we will use that for the non-linear part, that model that the elastic plastic representation of the soil. So, that is elastic plastic, so the elastic plastic representation of a non-linear response. So, suppose we have the pressure here, this is w the settlement and q is the pressure. So, we have q and the w and the soil behavior is this one. This is the load settlement curve of the soil. Now, here we will get elastic and plastic elasto-plastic response.

Then we, if I draw a tangent here initial path and then another on the final paths. So, that is the intersection point and here we will get say the settlement is w 0. So, we will get now this non-linear response of the soil, this is the actual response is now convert it to a linear thing for a elastic one initial postern, then the plastic that is the elasto-plastic response. Now, we are converting this non-linear response to a elasto-plastic response. So, though, that means for the initial part the q relation will be K 1 into w. K 1 is the initial modulus of sub grade reaction or the initial spring constant of this initial straight line. So, that mean that will valid for any w which is less than equal to w 0. So, if w is less than equal to w 0 then the response will be q equal to K 1 into w.

Now, the next one this q will be K 1 into w 0 that for w greater than equal to w 0. So, in the next part that q will be K 1 into w 0 and that w 0 is constant because this is the, this is a second part that will be q equal to K 1 into w 0 if w is greater than equal to w 0. So, this is one model non-linear model, next non-linear model, the bilinear representation. In bilinear linear representation suppose we have load settlement curve this is w is the settlement, q is the pressure then we have two straight line, this is one linear portion and this is the another linear portion of the curve. So, actually this is the again the non-linear behavior of the soil. So, this is the actual behavior of the soil is this one, red one that is the non-linear behavior of the soil. So, those things we are converted to a different form.

First one is the elasto-plastic representation and second one is the bilinear representation. This is the initial part and then the next another linear lines. So, if I exchange that second line so that value. So, that is the q 0 and then we will get deflection of the intersection point say w 1. So, we will get stiffness or the K 1 is the spring constant and K 2 is another modulus of sub grade reaction. Now, we will get the response q equal to K 1 into w that for w less than w 1 and next one q will be K 2 into w plus w 0. So, here this value is a w 0. So, this is for w greater than equal to w 1. So, we will get this second response K 2 into w plus w 0 or we can put any other values so that is is the two bilinear representation of a curve.

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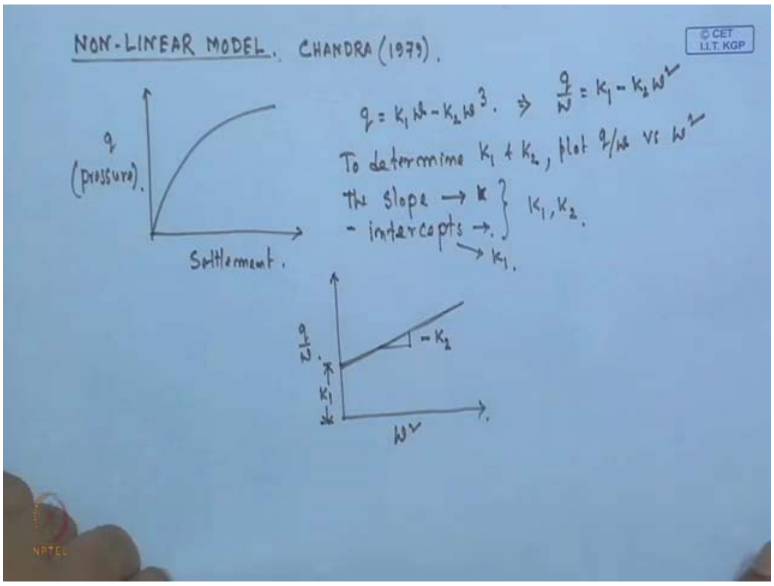

In third model that model which is defined that is non-linear model that is one thing is proposed by Chandra 1979 where this is soil behavior, this is settlement and q is the pressure. So, we will get a polynomial relationship that we can use q equal to K 1 w minus K 2 w to the power cube, now to determine K 1 and K 2 up to plot q by w versus 2 square. Now, if I put that is q by w this will be K 1 minus K 2 w square. Now, if I plot this q by w versus w square then the slope of this line will give the K 1 value and the intersection intercept will give this K 1 and K 2 value. So, that means form here will this K 1 and K 2 value. So, suppose we will get this type of curve. So, if I polo plot w square here and q by w then we will get this type of curve.

So, that slope and the intercept that will give us, so this will give give us the K 1. So, intercept will give us the K 1 and slope will give us minus K 2. So, this slope that will give us minus K 2 so that means slope and intercept from these two things we will get the K 1 and K 2 value. So, this is another representation of the non-linear response of the soil and then and here in the previous responds that here we have use this values q 0 w 0 and then use this expression though that part will be so this is a two response, one elasto-plastic representation, another is bilinear representation and then third one is the non-linear representation.

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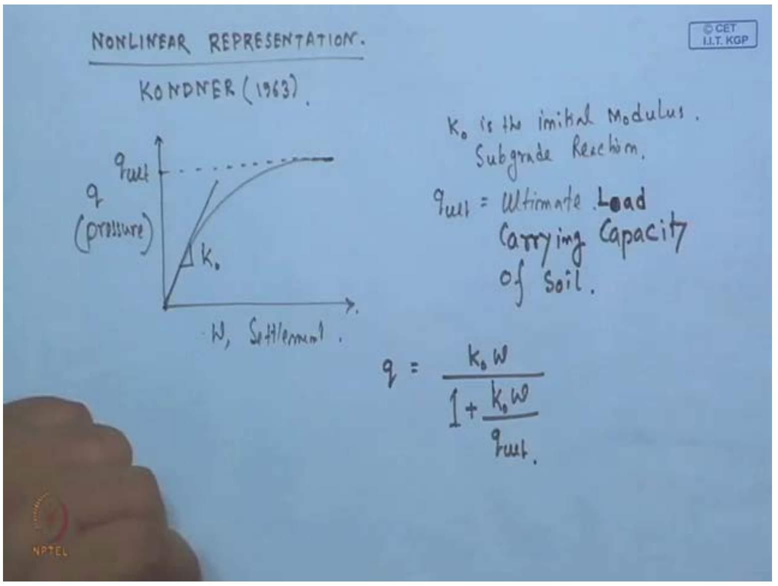

So, another non-linear representation representation you can, that is provide Kondner 1963 were also though this is a non-linear behavior of the soil and initial part that slope is a q 0 K 0. So, K 0 is the initial modular coefficient of modulus sub grade reaction or initial spring constant and then so suppose this is again q is the pressure, w is the settlement and if I draw this parallel line from this ultimate load point then we will get this point is q ultimate.

So, this value will give q ultimate. So, q ultimate is the ultimate q ultimate is the ultimate load load carrying capacity of soil. Then according to Kondner the representation is q K 0 or K s 0 w 1 plus K 0 w q ultimate. So, this is 1. So, this is another represent non-linear representation of the soil behavior. So, these are the few non-linear model that we can use slow in this Winklers (( )) also this springs are, assume as a linear model. So, using this non-linear model we can remove that limitation also. So, in the next thing that is the another thing that will start that is elastic continuum model.

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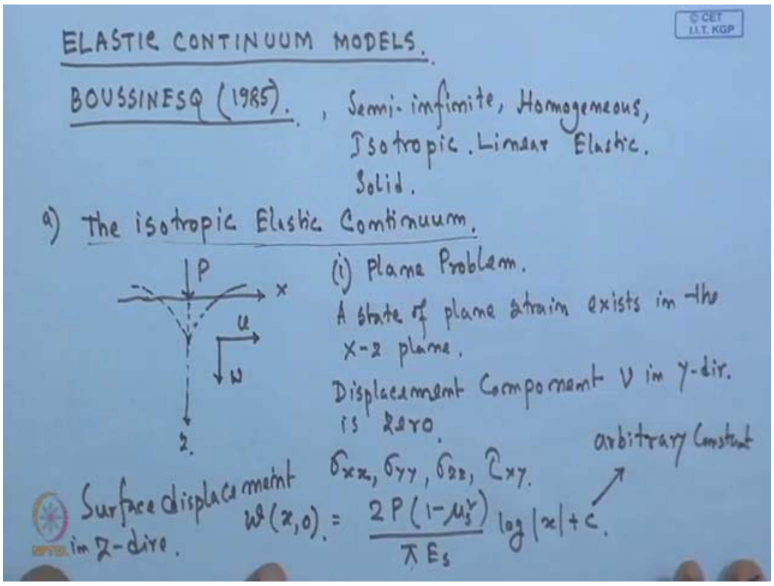

So, in the elastic continuum models that is continue response of the soil medium seems from the work of Boussinesq. So, that is first Boussinesq propose this work on this area. So, that year is 1985. So, according to the Boussinesq that we analyze the problem as a semi-infinite so there is the semi-infinite homogeneous isotropic linear elastic solid. So, where the soil is idealized as semi-infinite homogeneous isotropic linear elastic soil solid which is subjected to a concentrated force which act normal to the plane boundary. So, this is subjected to a concentrated load which act normal to the plane boundary.

Now, first that we will consider that for the isotropic elastic continuum, so suppose we have one this is soil surface, concentrated load is applied here, this is x direction, this is z direction. So, they will be one deformation will be in this form. So, that means here we will get this is u, this is w are the two different deformation component. So, in the plane strain problem or in the plane problem a state of plane strain exists in the x z plane. So, plane problem a state of plane strain exist. So, plane strain exist in the x and z plane. So, displacement component v in the y direction is 0.

So, displacement component v in y direction is 0. So, first we are doing that elastic continuum models. Your Boussinesq first propose that so here the first module that we are taking the isotropic elastic continuum where in the plane strain problem that will exist suppose we apply a load P and plane strain problem exist in x z plane and displacement component v in y direction is 0. So, the non zero stresses that is sigma x x sigma y y sigma z z and tau x y. So, these are the non zero stresses. So, that is the surface deflection w at x 0.

So, w x at 0 point in the z direction can be, so the surface deformation, surface displacement in z direction w x that will give 2 P 1 minus mu s square divided by pi E s into log x plus c. So, the surface displacement is z direction, in this plane problem that will be 2 P 1 minus mu s square pi E s log x plus c where c is an arbitrary constant. So, c is an constant. So, c is an arbitrary constant. Now, we will get this expression if it is a plane problem is a concentrated load applied on the surface.

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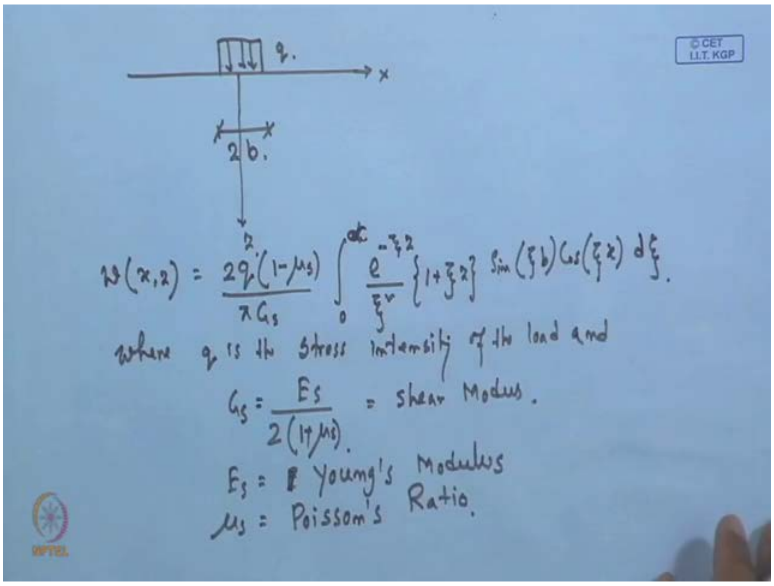

In the next problem that if the U D L is applied on the surface. Suppose, a same thing this is x direction here this is z direction. So, one uniformly distributed load which is applied, whose intensity is q and width of this loaded regions 2 b. So, in the in that case we will get another expression of the displacement of the surface of the half been subjected to uniform distributed load u, in that case w x z will give 2 q 1 minus mu s divided by pi G s 0 to the power infinity e to the power zeta z by zeta square 1 plus zeta z sin zeta b cos zeta x d zeta. Where q is the stress intensity of the load and G s is E s divided 2 1 plus mu s that is shear modulus and E s is elastic modulus or Young’s modulus, mu s is the Poisson’s ratio. So, this is the expression for the, if it is uniformly loaded region. So, the next one is isotropic elastic half space.

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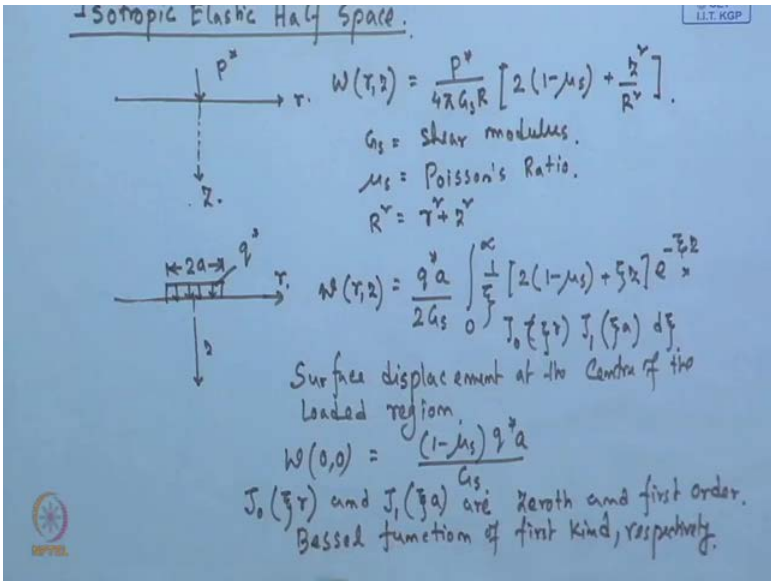

So, that is if I go for the isotropic then the displacement if a consider that in the surface of this loaded region. So, that means if the surface applied a concentered load, this is in the direction of r, this is in the z and another one applied the U D L with 2 a is the loaded region, this is z, this one is r. So, we will get asymmetric condition. So, that means here for the isotropic elastic half space, then we will get the surface deflection for the concentrated load P dash and here the intensity in q dash.

So, we will get w r into z r z is equal to P dash 4 pi G s R 2 1 minus mu s plus z z square by R square. So, here similarly, G s is the shear modulus and mu is is the Poisson ratio and we will get R square is equal to small r square to z square that is capital R square is small r square plus z square. So, if is the uniform loaded is applied then the deformation will be r z q star into a divided by 2 G s 0 to infinity 1 by eta 2 1 minus mu s plus time to z a to the power minus to z into J 0 zeta r to J 1 zeta a into d zeta.

So, and the surface deflection at the center of the loaded region so this is at the any point so at the center, displacement at the center of the loaded region we will get w 0 0 that will be 1 minus mu s q dash a divided by G s. Now, here this J 0 term and J 1 term are zeroth and first order Bassel function of first kind respectively. So, these are the two different conditions. One is concentrated load, another is U D L for isotropic elastic half space medium. So, the next one that we will consider, that is the third type of condition that we will consider that is the orthotropic elastic continuum.

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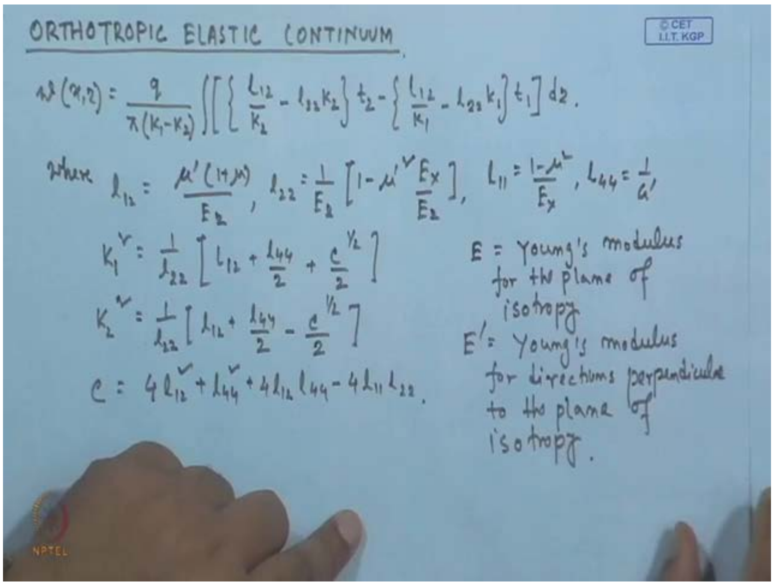

So, that is your orthotropic elastic continuum where we will get the expression that is available that q x z that is equal to q if it is a U D L pi K 1 minus K 2 integration l 1 2 by K 2 minus l 2 2 K 2 t 2 l 1 2 K 1 l 2 2 K 1 t 1 to d z. So, there will get the deformation for this conditions so where l 1 2 is mu dash 1 plus mu divided by E z l 2 2 is equal to 1 by E z 1 minus mu dash square E x by E z l 1 1 is equal to 1 minus mu square by E x and l 4 4 is equal to 1 by G dash.

Now, K 1 dash K 1 square is equal to 1 by l 2 2 into l 1 2 plus l 4 4 by 2 plus c to the power half by 2 and K 2 square that is equal to 1 by l 2 2 l 1 2 plus l 4 4 divided by 2 minus c to the power half by 2 where c is equal to 4 l 1 2 square plus l 4 4 square plus 4 l 1 2 l 4 4 minus 4 l 1 1 l 2 2. So, these are the expression by which we can determine for the orthotropic elastic continuum case what will be deformation, surface deformation and then here we will get that E is the, here we will get the E in somewhere we are using that E dash. So, we will get the E will be the Young's modulus for the plane of isotropy. So, E will be the Young's modulus for the plane of isotropy and E dash will be the again Young's modulus for directions perpendicular perpendicular to the plane of isotropy.

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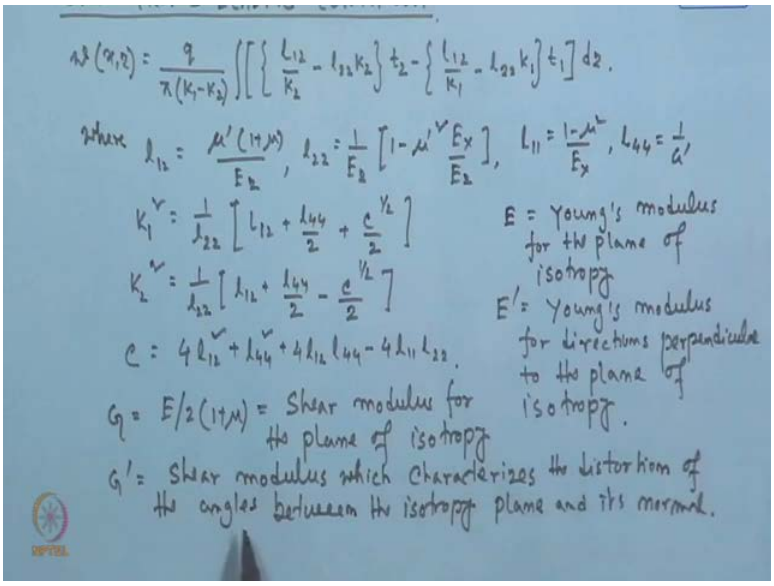

Similarly, we will get the expression of G that G is if I consider that is E divided by 2 1 plus mu at a shear modulus for the plane of isotropy. Similarly, G dash will be shear modulus which character, which characterizes the distortion distortion of the angles between isotropy plane and its normal.

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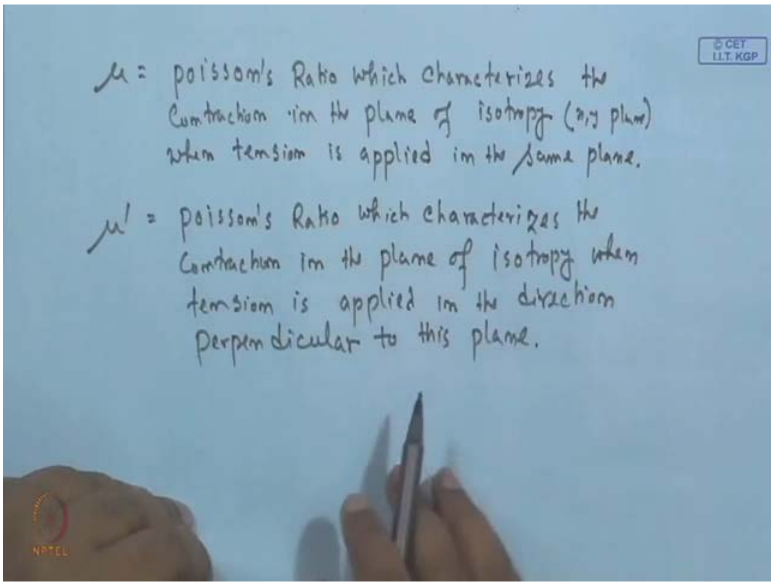

So, we will get, if we get E E dash is the Young modulus in the plane of isotropic, E dash is the Young's modulus in the direction of perpendicular to the plane of isotropy, shear modulus is the shear G is the shear modulus in the plane of isotropic and G dash is the shear modulus with characterizes the distortion of angle between the isotropic and its, isotropy plane and its normal.

Similarly, have mu and mu dash. So, similarly the value of mu is Poisson ratio ratio which which characterizes the contraction in the plane of isotropy. Basically, here x y plane when tension is applied in the same plane and mu dash mu dash will be the Poisson ratio which characterizes the contraction in the plane of isotropy when tension is applied in the direction perpendicular to this plane.

So, we have two mu, one is mu and mu dash. So, mu dash is the Poisson ratio which characterizes the contraction the plane of isotropy when tension is applied on the same plane. Another is Poisson ratio which characterizes the contraction of the plane of isotropy when tension is applied direction to the normal of this plane. So, one different direction of the tension, one is applied to the same plane other so direction to the perpendicular to the plane then this will give you the mu and mu dash. So, these are different model that we are talking about.

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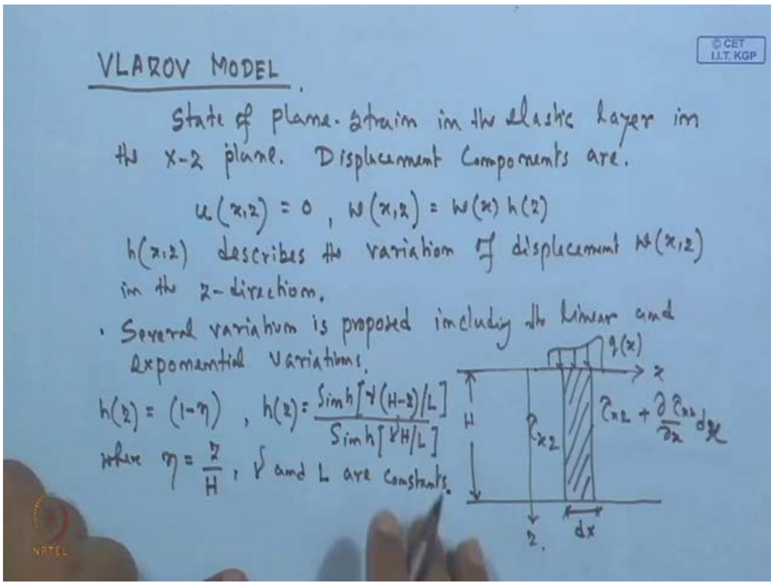

So, next module that we will explained that is the Vlazov model. So, in the Vlazov model so that means in the consider the state of plane strain. So, this is consider state of plane strain in the elastic layer in the x z plane. Then the displacement components are so u x z that is equal to 0 then w x z that will be w x into h z where h x z or that is describe the variation of displacement w x z in the z direction. So, we will have the h x z function which will show the variation of the displacement y x z in the z direction. Now, several variation have been proposed including the linear and exponential variation.

So, several variation is proposed including the linear and exponential variation. Now, if we have in the Vlazov model, if we have the soil layer suppose it is x and here we have a z direction here. Then if this is the thickness of the soil layer is H and we have one components, segment if I consider, if load is applied over there with intensity q x then this segment is d z then definitely we have tau x z because here is state of plane strain elastic layer in the x z, this is the x z plane where the, this is the elastic layer we consider and displacement components u x z 0 and w x z is w x and h z where h is variation of the describes the variation of displacement in the z direction.

Now, several variation of tau x if we have the tau x here then tau x z plus del tau x z divided by del x 2 d x. So, we have this variation. Now, this variation of different h v value, so h z that will be 1 minus nu where nu is equal to z by H. This is one variation and another one we can say this is h z that is equal to sin h gamma H minus z divided by L divided by sin h gamma H divided by L. So, here gamma (( )) L are constant.

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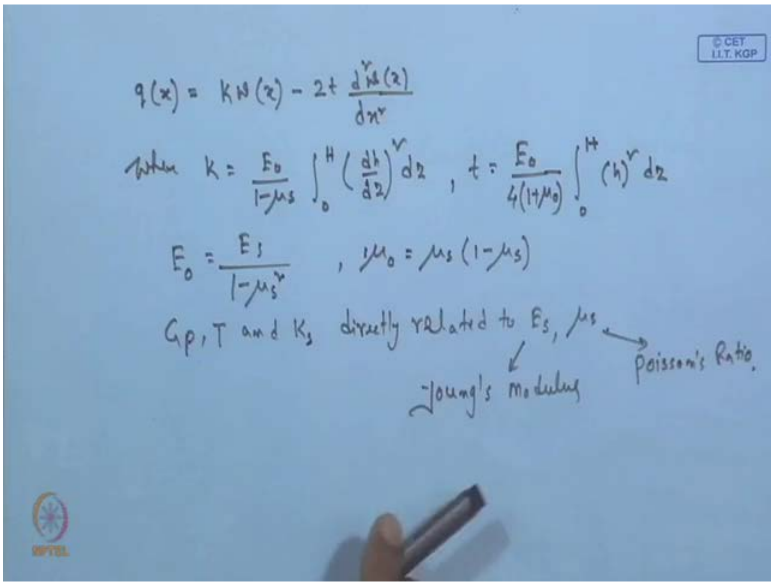

Now, the expression using the principle of virtual work, the expression of the q x the final response function that will get that is K into w x minus 2 t d square w d x square, this is w x. For this model we will get so the here also this is a 2 parameter model where we will get K is one parameter, t is another parameter. So, where K is related to E 0 1 minus mu s 0 to H d h d z d power square d z and t is equal to E 0 4 1 plus mu 0 0 to H h square z in d z, where E 0 is E s 1 minus mu s square and mu 0 is mu s 1 minus mu s.

So, it is very important to note that, that G p T and K s are directly related to E s and mu s, where E s is the Young's modulus of the soil and mu s is the Poisson ratio. So, these are the response that we will get for this Vlazov model. Now, so next class we have discussed about various other type of model improve model and then some non-linear model to overcome the limitation of the Winkler model. And then how to determine the settlement response at various different condition of the, that is also explained in this class. So, next class I will explain about the, how the beams behaves if it is resting on elastic foundation.

Thank you.