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**Lecture - 31**
# Seismic Design of Retaining Wall

In my previous classes, I have discussed about the analysis of retaining wall and then the reinforced retaining wall; so those analysis was based on the static loading condition. Now, today I will discuss about the seismic analysis of retaining wall, then later on I will discuss about the seismic analysis of a reinforced retaining wall. What are the additional components that we have to add during the design of seismic condition for the retaining wall, traditional retaining wall as well as the reinforced retaining wall. So those things I will discuss in couple of classes.

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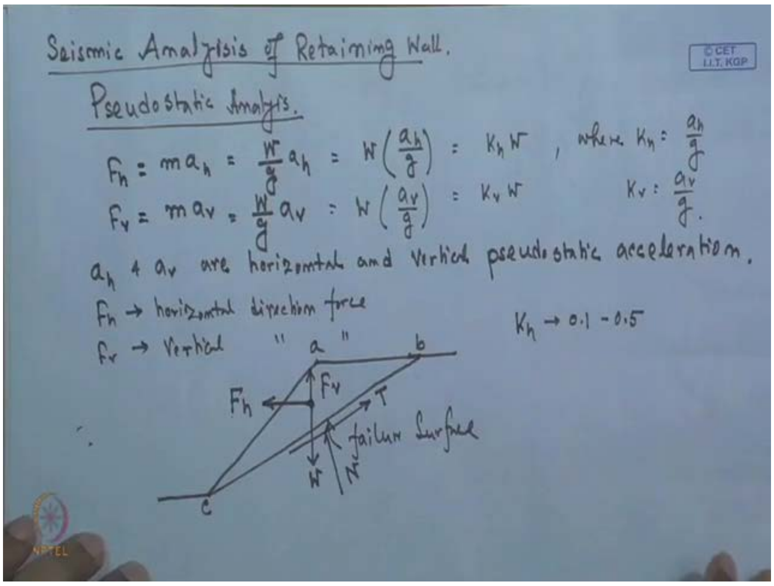


So first, I will discuss about the seismic analysis or design of retaining wall. Now in this analysis I will basically concentrate on the pseudo static analysis. Now, pseudo static analysis means that during the earthquake motion, where the two ground motion one is in horizontal direction and one is in the vertical direction.

So, the detail discussion on the various type of motion and the waves it is not the scope of this study. So, here I will just concentrate on the how this seismic force are incorporated in the design of this reinforced or in the traditional retaining wall. Suppose if we have two components, one is our force in the horizontal direction, another in the force in the vertical direction.

So, this force has the mass and the acceleration in the horizontal direction. This is vertical also the mass in the acceleration, so a h and a v are horizontal and vertical pseudo static acceleration, acceleration due to the seismic force. So, we have as we have I have already mentioned that there is a two motion, one is in the horizontal direction, another in the vertical direction, so a h and a v are the acceleration of the motion due to this earthquake force, acceleration of this any mass due to the earthquake condition. So, one acceleration in the a h is in the horizontal direction, a v is in the vertical direction and m is the mass.

So, on this mass m this a h and a v acceleration will act and they will create a force F h in the horizontal direction and a v in the vertical direction. So, this one is in the horizontal direction, this is in the horizontal direction Fh, so Fh is in the horizontal direction, force and F v in the vertical direction force. So, now we can convert these things into weight, this is mass. So W is the weight divided by g into a h. Similarly, this mass weight divided by g into a v. So, now we can write W and within bracket a h divided by g, similarly W into a v divided by g. So, these things we can write K h into W and K v into W.

So, where K h is equal to h divided by g and K v is equal to a v divided by g, where g is the acceleration due to gravity. So K h and K v are the horizontal and vertical pseudo static coefficient of acceleration. So this K h and K v are the coefficient of acceleration in pseudo static, pseudo static that means we are basically applying static force which is that means seismic force are converted to a static condition and this static force are applied here, as if and this acceleration due to the seismic condition, that is incorporated in this static force condition.

So, now if I draw a say free body diagram of a any mass. So, suppose this is any slope and we can this is a failure surface. So, this is slope and this is failure surface surface and if failure surface weight W is acting in the vertical direction, so this is the static condition, so where only the W weight of this failure weight of this zone, that means this failure zone or we can say a, b, c the weight of this a, b, c triangle soil is W. so that W is acting in the vertical direction.

And so now when we apply the seismic condition and the additional force that will act and that will in the vertical surface. So this is the shear stress and this is the normal force. So this is the sheer force T and this is normal force that will act, so this is the force which is acting due to static condition. Now, if we act this pseudo static that means in the horizontal and the vertical condition, two force will act, so we will consent at this one force and the horizontal condition. So that force is equal to F h and another force in the vertical condition that is equal to F v. So, this F h and F v are the two force that is acting on this weight W in the horizontal one is at the vertical condition.

So, this is suppose example of a slope if we add this this pseudo static forces on this weight and this weight is acting on the centroid of this triangle. So, that means this force F h and F v which are acting in the one is in the horizontal direction, another in the vertical direction. So, this condition now the F h range we can apply K h that range if 0.1 to 0.5. So, that means this K h range we can apply to 0.1 to 0.5. Now, in this pseudo static force, now we will apply on a particular retaining wall and then we will do the analysis for the retaining wall in seismic condition.

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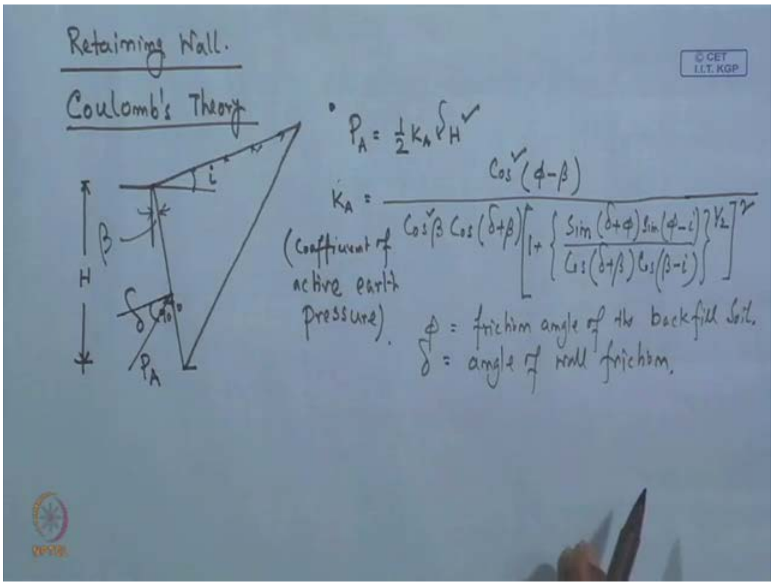


Now, in the static analysis or the retaining wall, now in retaining wall static condition analysis and we know that we have two theories, one is Rankine's theory, another is

Coulomb's theory. So here this Coulomb's theory are converted and it is used and modified for the seismic condition. Now, this shortly if I write the coulombs theory, so Coulomb's, Coulomb's theory and this is particular retaining wall, say this is the back fill this is retaining wall I is the angle of the back fill and this is the failure surface you can consider for the retaining wall.

Now, here this back this retaining wall is making an angle beta with vertical, i is the back fill which is making an angle i is the horizontal. Now say H is the height of the retaining wall and now in this condition. So now that means here for the coulomb's theory we know that this soil should be dry, homogeneous, isotopic and cohesion less if we consider. Then back fill surface can be planner or can be inclined, here the back fill surface is inclined, I can can be 0 also. Now, the back fill back of the wall can be inclined to the vertical, so back of the wall is inclined to the vertical, we can angle beta.

Now P, that force P active force, which can act with an say suppose this is active force, suppose this is the angle. Now this is the normal this line is the normal to the back of the wall. So that means this is the normal, so this angle is 90 degree. Now, this P or active pressure, P active of this active force that will act with an angle delta with the normal of the back fill; so that means first we draw a normal to the back of the wall and this P force which is acting with an angle delta with this normal. Now, this delta is the friction angle between the soil and the wall. So, now we can write that P A, P active is half into K A into gamma into H square. Where gamma is the unit weight of the soil and H is the height of the retaining wall, K A is the coefficient of active earth pressure.

So, this is coefficient of active earth pressure. This K A is given by this cos square phi minus beta divided by cos square beta, cos delta plus beta,1 plus sin delta plus phi, sin phi minus I, cos delta plus beta, cos beta minus i that to the power half and this total is square. So, this is the coefficient to where where phi is the friction angle of the soil or angle friction of the back fill soil, back fill soil. So, now here as I mentioned delta is the angle of wall friction. So this is the angle between the soil and the wall. So this is the active force for the static condition. Now we will exchange these things for the, we will exchange these things for the seismic conditions. Now, how we will exchange this part? So, we will consider the same retaining wall.

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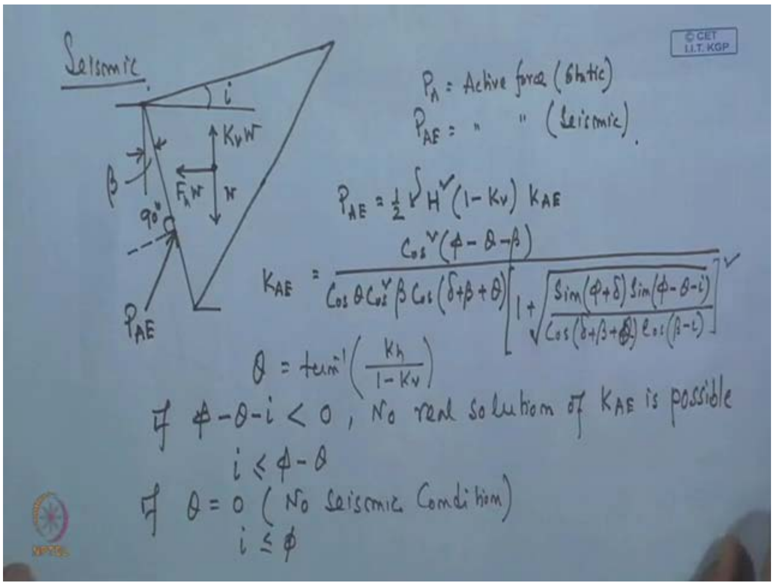


So, suppose for the seismic condition we consider same retaining wall which is making an angle beta with vertical and this is the failure surface. This one i, i is the angle of the back fill with horizontal. Now, here suppose in the static condition the weight of this failure zone is W, W is the weight of the failure zone and similarly, here the same this is the normal to the back of the wall, this is 90 degree and here P AE, P A is the active force in static condition and P AE is the active force in the earthquake condition or seismic condition.

So, now this force W will act here and then we will apply two additional force that is K vW and one is the horizontal condition K hW for the pseudo static analysis under seismic condition. Now, here P AE will be half into gamma into H square into 1 minus K V into K AE. So, this derivation is not presented here, this is the final expression is given. Now, the active force and the seismic condition is half gamma H square 1 minus K V into K AE, where this K v and K h are the coefficient of the acceleration, pseudo static acceleration. Now here K AE is given the similar form the cos square cos square phi minus theta minus beta, total divided by cos theta, cos square beta, cos delta plus beta plus theta in the total 1 plus root over sin phi plus delta, sin phi minus theta minus i divided by cos delta plus beta plus phi plus delta pl us beta plus theta, cos beta minus i to the power square.

So, all the terms are in the same as the static condition, one additional term which is added which is theta. Theta is tan inverse K h 1 minus K v. So, this is the seismic analysis so K h 1 minus theta is tan inverse K h 1 minus K v. Now, by this expression we can determine the active force of the retaining wall in seismic condition. Now similarly, because we have done the analysis for the static condition, here I will concentrate only how to calculate the passive and active earth pressure under seismic condition. Other design process will be same as it is described in the previous classes and the static condition, how to design or how to analyze the retaining wall and the reinforced retaining wall; so only the how to calculate the active and passive earth pressure that I will explain in this class.

Now, some comments on that, that we can see that if phi minus theta minus i, that is this angle phi minus theta minus i is less than equal to 0, then no real solution is possible, of K AE is possible. So, no real solution is possible, so that means to get an real solution the condition is that I should be less than equal to phi minus theta. So, to get a real solution this condition we have to satisfy that i should be less than equal to phi minus theta. Now, if another condition if theta is equal to 0, that means no earthquake condition or no seismic condition then for stability i should be less than equal to phi. Another condition that, so these are the two very important conditions and third one is that for if, i is equal to 0, that we know that i should be less than equal to phi minus theta.

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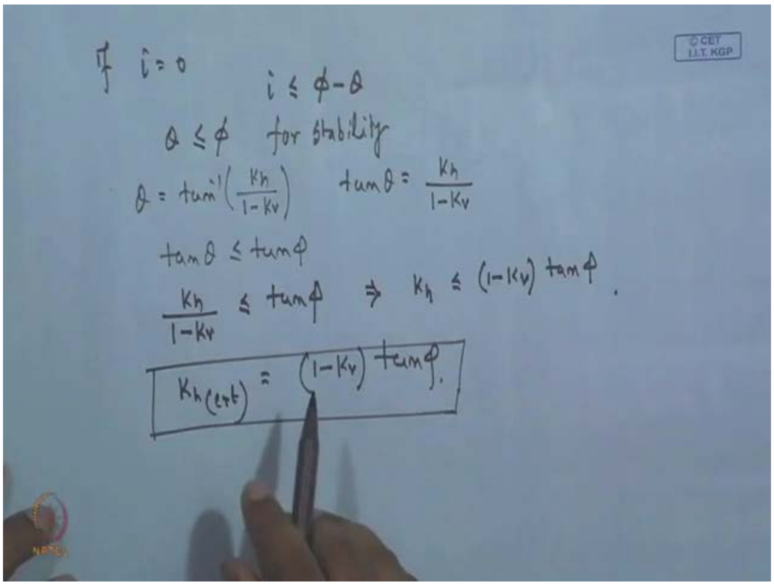

Now, if i equal to 0 then for the stability that theta should be less than equal to phi. This is for stability. Now we know that that theta is equal to tan inverse K h 1 minus K v, so we can write that tan theta is equal to K h 1 minus K v. So, if we put this value then we can write that tan theta is less than equal to tan phi. So, if i put the theta value then we can write that K h 1 minus K v that should be less than equal to tan phi and finally, we can write that K h should be equal to 1 minus K v into tan phi.

So we can write that K h is critical is equal to 1 minus K v into tan phi. So, to get an critical K h value, so when we choose the K h value for the design the critical K h value because that should be equal to or that is 1 minus K v into tan phi. So, we cannot use the K h value greater than that for a stability condition, we have to get the stability and the equilibrium condition the K h should be less than equal to 1 minus K v and tan phi. So the critical value of K h is 1 minus Kv into tan phi. So, these are the a process by which we can determine the seismic earth earth pressure for the retaining wall.

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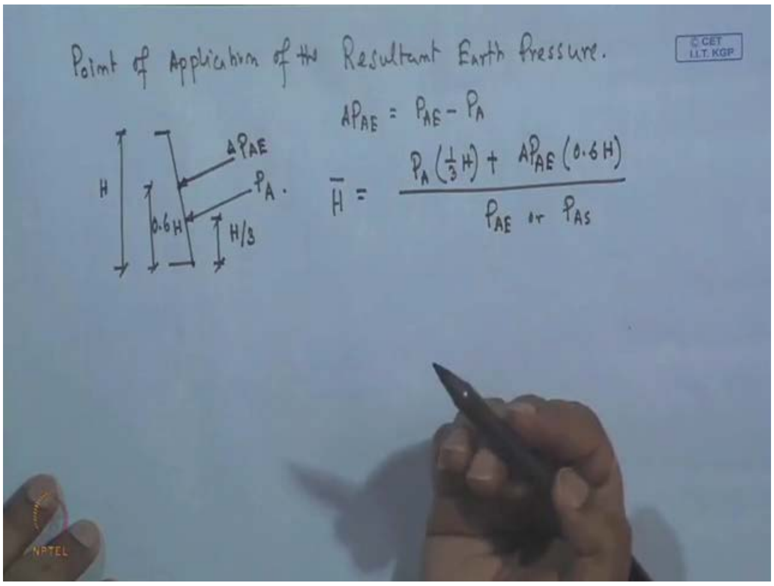


So, now we will discuss about the point of application of the resultant force, of the resultant pressure. So as I mentioned previously that if it is a retaining wall then the static condition the P A static, that is P A static condition that if this is the height of the retaining wall, so this will act at a height of H by 3.

Now similarly, the P delta P AE, so where that resultant force or P AE will act here, so how we will calculate this delta P AE that will act at a height of 0.6 H; so where delta P

AE is equal to P AE minus P A; so P AE under seismic condition minus P active, active pressure and this static condition. So P AE static force will act as a height of H by 3 from the base and the delta P AE it is the additional force due to the seismic condition that will act at a height of 0.6 H from the base. So, the resultant H that will act, P A into one third H plus delta P AE the 0.6 H this total is P AE or P AS earthquake or seismic, whatever it is.

So P AE, so by this way we can determine the resultant force, so that is acting. So suppose this force H is acting from the base of the wall, so we can if we know the first step we calculate the P A, then we calculate the P AE or AS then we calculate delta P AE and then we put all these value here we will get the H bar value. So, this is the calculation of earth pressure under seismic condition and the resultant force.

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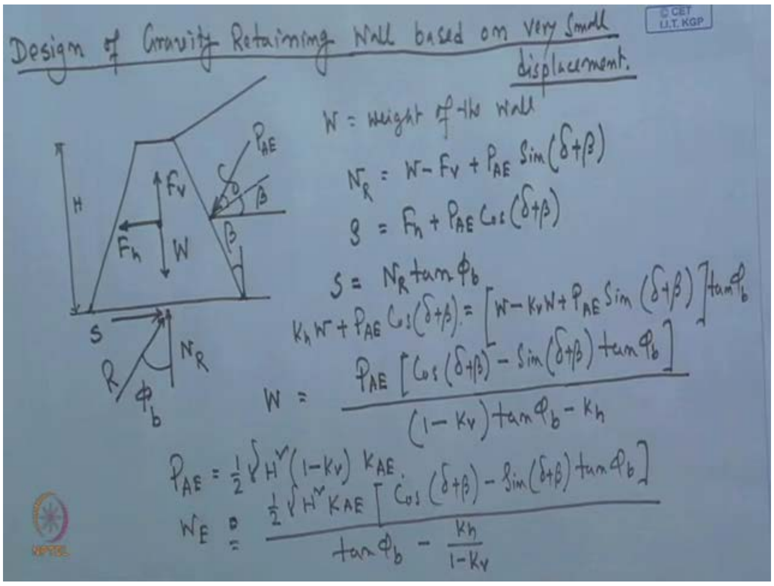


Now, next thing that we will analysis or the design of gravity retaining wall based on very small displacement. So design of gravity retaining wall or based on very small displacement. So, here if it is a cantilever retaining wall then it is not a very huge retaining wall, so that part we consider we determine the earth pressure based on the previous described methodology.

Now, if the gravity it is a gravity retaining wall then how will this weight, because we know the stability of the wall is is coming due to the weight of the retaining wall. So, we have to make this weight of the retaining wall such that it can be at stable structure during the seismic condition. So, how we will calculate that part for the gravity retaining wall? So first suppose for the gravity retaining wall, if it is a gravity retaining wall and this is back fill, so here because as I have mentioned that the stability of this wall we are getting from the weight of the wall itself. So we have to make this weight, I have to increase the weight of the wall such that it can prevent the seismic force.

So, now this is the weight of the wall, W is the weight of the wall, W is the, previously W was the weight of the soil for the failure zone. So that failure zone soil where we apply the pseudo static K h and K v, but here on the weight of the wall itself we are applying two forces, one is K F h another is F v.

So, this is the weight which is acting and this wall is making an angle beta with the horizontal. Now, here we can write that this is the normal force and this is shear force which is acting and this one may be the resultant force and that is acting as a angle of phi b, where phi is the friction angle at the base of the retaining wall. Similarly, another one we can write that this is making an angle 90 degree, then with this angle delta is P AE is acting. Now if this is beta then with horizontal this angle will also be beta. Now, if this angle is beta then with horizontal this angle will also be beta because total this one is 90 degree with this wall.

So, this is the total free body diagram of the retaining structure. Now we can write that N R or normal reaction that is equal to W of the wall minus F v plus P AE. So we will take P AE is acting here, so we will take the sin component of this P AE, so P AE into sin delta plus beta. So, here and similarly, is shear force is F h plus P AE cos delta plus beta. So, here two components that one is the reaction force, normal force and the this is normal force and the this will be the reaction. So this is normal force in R at the base and S is the shear force, we will get from this expression.

So, now we can write that, as S is equal to N R into tan phi b, so we can write S is equal to N R into tan phi b, so we can finally write that F h ,in case of F h we can write K hW plus P AE, cos delta plus beta that is equal to, W minus K v W plus P AE sin delta plus beta into tan phi b; so that is equal to tan phi b; so here F v we have replaced this K v into W. So finally, if I take the expression after simplifying this one or we can take the W expression is P AE into cos delta plus beta minus sin delta plus beta into tan phi b divided by 1 minus K v tan phi v b minus K h. So we can take W common, then we will get 1 minus K v into tan phi b minus K h from here, so this will give you the expression of the W.

Now, we know that P AE is equal to half gamma H square 1 minus K v into K AE, where this if this is the weight of the gravity retaining wall. So finally, for the earthquake condition we can write that W E is equal to half into gamma H square K AE because this K v if we put this expression in place of P AE, then this K v and K v will be cancelled out. So up K AE into cos delta plus beta minus sin delta plus beta into tan phi b, that whole divided by tan phi b minus K h and then 1 minus K v. So this will give us the total expression of the weight of the W in this form.

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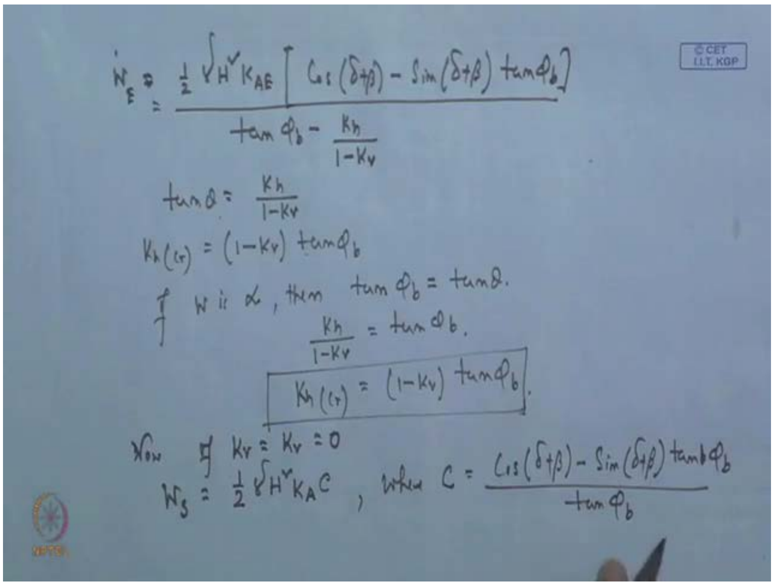


So, now if I again write the final expression of the W, that W is equal to half into gamma H square K AE, K AE expression is already given in the previous section.

So, that is equal to cos delta plus b minus sin delta plus b into tan phi b, then tan phi b minus K h 1 minus K v. So, now this expression will give you the final expression of the W, W means weight of the gravity retaining wall. Now we know that that tan theta is equal to K h 1 minus K v, so we can replace this K h 1 minus K v by tan theta. So, now that for the K h critical that value you know, that is 1 minus K v and tan phi b, from this expression also we will get that critical value of K h will be 1 minus K v into tan phi b.

Now, one thing that that to make this retaining wall stable we have to increase the weight and it is properly stable if you can make it infinitely. So, now if W is infinity then we can write tan phi b is equal to tan theta, so we can write K h 1 minus K v that is equal to tan phi b. So again from that expression this K h critical is 1 minus K v into tan phi b, so the again this critical value of K h is coming out to be 1 minus K v into tan phi b.

So, this is the expression for the horizontal coefficient critical expression. Now, if K h is equal to K v is equal to 0, then the expression of W this one is the expression under earthquake condition, now this one will be the expression under seismic condition, is half gamma H square K A into a constant C, where C is equal to cos delta plus beta minus sin delta plus beta tan phi b divided by tan phi b, where phi b is the friction angle at the base. So, now from this expression W S will get the weight in the static condition and W E will get the weight under seismic condition, so that we can once we calculate all this weight then we can understand what is the additional amount of the weight required to make this gravity retaining wall stable, under seismic condition as compared to the static condition.

Now, here we have consider there is there is no displacement is allowed, so that means when the earthquake force or seismic force will come, so this wall will try to display will try try to move along the direction of the motion. So, now here we have to make, now if we do not allow any displacement then we have to make this wall weight a huge one, so that means if we increase the wall weight then we can do that thing also. So, that means here, we in this analysis we have done that we have not considered any displacement of the wall. So and then how much amount of the weight is required to make the wall stable that we can calculate by this expression of W E. So if I make that we can provide that amount of W E we can make this wall stable under no displacement condition. Now, if we allow any displacement then definitely the amount required weight that will reduce. The next section we will discuss about the some displacement if we allow some some displacement.

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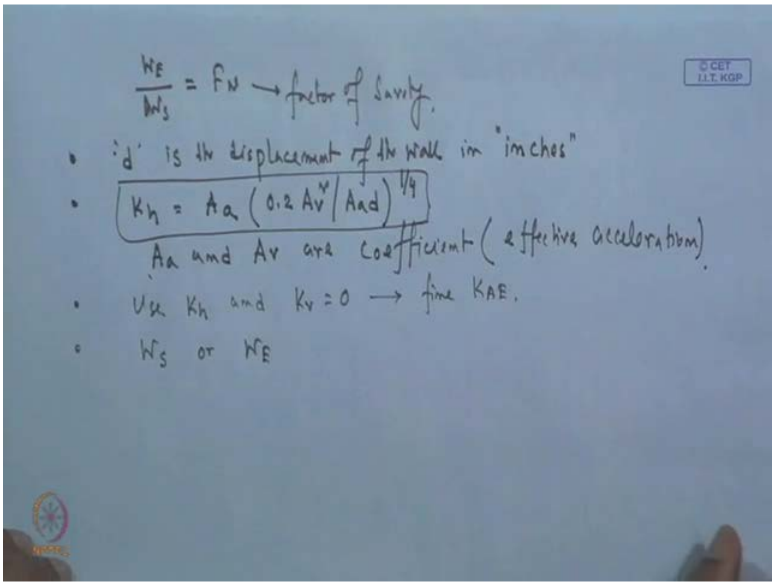

So, now here we have two weight W E is the seismic condition and W, W S in the static condition. So that is a factor of safety, so that is also called a factor of safety for the design because this is factor of safety because these additional factor of safety we have to provide to make stable this under seismic condition as compared to the static condition. So those things are on the no displacement condition. Now, if we provide suppose if we provide d, a displacement of the d is the displacement of the wall and d value is in inches. So, now allow at d displacement of the wall, it is in inches then first the step is we calculate K h value, that K h value is A a, 0.2 A v square divided by A ad to the power one fourth.

So, where A a and Av are two coefficient are coefficient, these are called effective acceleration coefficient. Now we use this K h and make K v equal to 0 and then we find K AE. Then putting this expression we will get W S and then we multiply or we put we can W S static condition or W E in the seismic condition.

So, that means the steps are, first we consider a displacement of the wall d inches, then we consider this K h expression with the help of this equation and then where A v and A a are the two effective acceleration coefficient. Then what we will do we will calculate the K h from this expression after knowing this K A a and A v values and d is the displacement in inches and then we will put K v equal to 0 then we calculate K AE.

Once we calculate K AE then we can calculate W E, what is the amount of weight required if we apply a very small amount of deformation, if we allow the deformation. So these are the analysis of the retaining wall under seismic condition.

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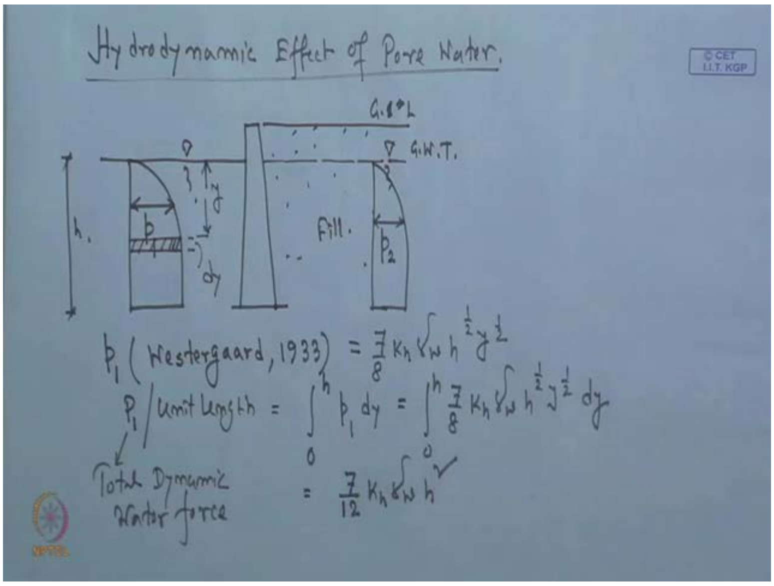


So, next step that we will discuss about the hydrodynamic effect of pore water pressure. So, in this till now we have discussed about the seismic forces in the retaining wall, how to calculate the seismic coefficient of earth pressure and then the forces on the retaining wall and then now we will discuss about the hydrodynamic pore water pressure. Because we know that if earthquake or seismic forces act acts then the voids filled with water, that pore water pressure will also provide some additional force on the structure.

So, now and there is a possibility that the if the retaining wall is one side is void and one side is soil and one side is water, then also there is water pressure will act on the retaining wall. So that water pressure, how this water pressure variation that we will we will consider here. So, now according to suppose if this is a retaining wall, so this is the retaining wall which this side is the fill, so this is filling or the fill side and this side where water surface is this side is water and here also water table is acting here say this is ground water table, this is ground surface G L ground level this is ground water table.

Now, so now here this both side water present. Here water is free and water is water is within the soil. Now here we will get variation of the water surface according to this. Suppose, this is the variation of p 1 this is water pressure variation due to this earthquake condition this is free water and this side also we will get one variation, similar type to p 1 and that variation is say at any point p 2, small p 1 and p 2. So this is the variation because according to (Westergaard,1933) this p 1this p 1 variation is 7 by 8 K h gamma W h to the power half y to the power half, where h is the height of the water level and y, suppose we consider a very small element here the thickness d y and from the height y from the top. So we consider small element d y at the level of y from the top of the water surface. So we can calculate this p 1 per unit length, this is the total p 1 is the total dynamic water force. So that we will we will get by the integration of 0 to h, p 1 into d y.

So, the total force with this water is giving on this retaining structure is by integrating this pressure at any point into this d y because this pressure on any small segment d y is p y, p 1. So, p 1 into d y is the total force on this small segment and the total force for the total structure is integration of 0 to h to p 1 into d y. So again 0 to h, 7 by 8 K h gamma W h to the power half, y to the power half d y. So, here K h is the horizontal pseudo static coefficient of acceleration, gamma W is the weight of unit weight of the water, h is the height of the water level and y is the depth of this segment from the top of the water surface and d y is the thickness of this small element.

So, once we get the up to integrating this value we will get 7 by 12 into K h, gamma W into h square. So, that is the total force of this water, this is remember that this one is the free free water, which is applying the force on the retaining wall and this surface this is the fill water within the soil. So here water is within the soil, here this is the free water, so this expression 7 by 12 K h, gamma W h square will give you give the total dynamic water force of free water giving on the retaining wall.

So, next class, I will derive or I will give you the expression of the water within the soil and the location of this force, because we know here p 1 is the total force, water force acting on the retaining wall, but where this force is acting that location and the resultant force, how it is acting on the retaining wall those things, I will explain in the next class.

Thank you.