Advanced Foundation Engineering
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Lecture - 20
Tension and Lateral Loaded Piles

Hello. So, in the last few classes I have discussed about the various methodology of soil structure interaction or soil foundation interaction. Then I have explained about the modulus of sub grade reactions. Now, the modulus of sub grade reaction the how it is determined by the lab test or it is also explained then why it is used, how it is used those things also explained. So, today I will just explain that based on this modulus of sub grade reaction application that how the lateral loaded pile has been analyzed, and then how the deflection of the pile then the slope bending moment, inertia force, all these thing can be done determined by using this modulus of sub grade reaction techniques. Then I will also explain the tension pile or the uplift capacity of the pile.

So, now in all the other classes of pile foundation of explain only the compression piles are subjected by compressive load. Now, today I will explain that a tension pile; the tension pile while piles are not only subjected to compressive load, piles are sometimes subjected to tension load also depending upon the situation. For example, if it is a very tall structure or it is a tower tall it tower tower then chimney, then the piles are in that case are subjected to both tension as well as compressible also.

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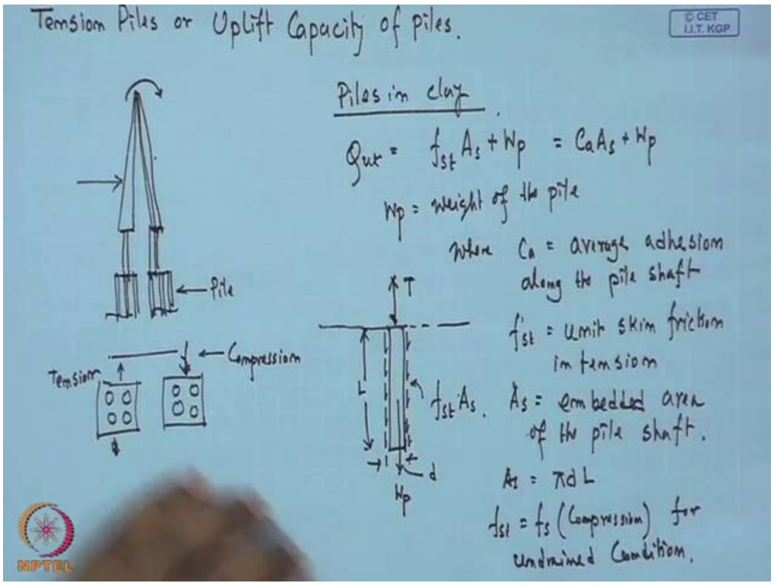

For example, if we can write we can explain in this form that the tension pile so the tension or uplift capacity of the piles for example, if it is a very, it is a tower then the, that the foundation say these are the foundation these are piles.

Now, if these are the pile foundation then if I draw the 6 plane, then suppose this is one pile cap or this is another pile cap, then these piles when they are will be a wind load subjected to a lateral direction, then there is a possibility there this thing will rooted in this form. Then these piles will be subjected by compressive load and this pile will be subjected by tension load. So, that mean this piles are subjected by tension and this pile is subjected to by compression and if the wind direction changes in this opposite direction then this things will reverse. Now, this there is a possibility it will rotated in the opposite direction. So, these piles will be subjected to by compressive load and this piles will be subjected to by tension load.

So, that means in that case piles we have to design the piles for both it can take the compressive load as well as it can take the tension load also. Now for this, how to calculate the the tension capacity or uplift capacity of a pile. Now, the first that the piles in clay. So, in the piles with the uplift capacity u l that will be friction for the tension f s in tension into A s plus W p; now where W p is the weight of the pile. Now, here that if we are considering a single pile or group of pile. Suppose, if this is one pile which is subjected to tension then the resistance this pile we will get that here there will be no tip resistance because piles are piles is subjected to tension.

So, that mean the weight of the pile that will act in downward direction and when this pile is going upward direction, then this friction will act in the opposite direction. So, this is the friction, this is f s t into the area, of the surrounding area of the pile. So, this friction so, that is why this friction that is acting in the opposite direction of the loading. So, loading in the upward direction so, this will act in the downward direction and the weight of the pile that will also act in the downward direction. So, the ultimate load carrying capacity of the pile in tension condition that is the Q ultimate in tension so that will be f s t friction force in the tension and area of the surrounding area of the pile and the weight of the pile.

Now, here we can write if it is in the clay soil that f s t is adhesion of the soil and the pile and area plus W p where C a is the average adhesion along the pile shaft. So, along the pile shaft as I have explain the addition means the, this is the interaction between the soil and the pile material. So, that is the average adhesion of the pile and this is f s t is the unit skin friction. Intension A s is the embedded area of the pile shaft. Now, for this particular pile if the d e is the diameter of the pile, so A s and A L is the length. So, A s will be pi d into L. So, pi is the cross section this periphery of the pile pi d into L is the length. So, this will be the A s value and W p is the weight of the pi.

So, f s t is equal to f s in compression. So, f s t is equal to f s in compression when for the undrained condition of the pile of the soil. So, this f s t will be equal to f s that is friction that is skin friction in compression for undrained condition of the pile. Now, sometimes pile has a enlarged base to increase this friction capacity. So, that means if pile as a enlarged base.

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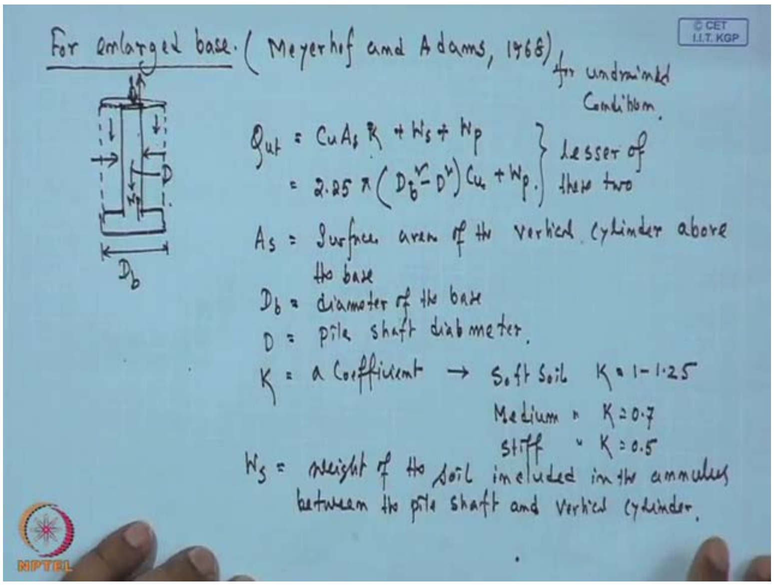

So, that means if that pile it has this is a shaft of the pile and it has a enlarged base to increase this tension. So, in that case so this will be in the tension opposite direction. So, in that case the ultimate load carrying capacity will be C u again A s into k plus weight of the soil plus weight of the pile. So, that means here this is given by the Meyerhof and Adams that is for undrained condition. So, here again so, that means again this will be another expression is available the 2.25 into pi D b square minus D square into C u plus weight of the pile. So, from here the lesser of these two will give the, in this case ultimate tension carrying capacity of the pile.

So, that means we can say that here that A s that is the surface area of the vertical cylinder, above the base. Suppose, this is the base so, A s will be from here to here the outside area of this outside cylinder of the pile. So, that means here and then D b is diameter of the base D is pile shaft diameter. So, that means here this is D b diameter of the base and this is the D pile shaft diameter and k is one coefficient. Coefficient that depends on for depends on the soil type, then for the soft soil this k is equal to 1 to 1.25 and for the medium clay soil that is k is 0.7 and for the stiff soil k is 0.5 and again weight of the soil that is W s is weight of the soil included included the annulus between the pile shaft and vertical cylinder.

So, that means here we can write the weight of the soil, here the vertical cylinder that is considering above the base. This is the vertical cylinder so, this portion this is weight of the pile and these portion the weight of the soil included this annulus between the pile shaft and the vertical cylinder. So, this is the this is the vertical cylinder above the pile and this is pile shaft within this soil. This will give the weight of the soil. So, why this expression? So, we have this two expression. So, lesser of these two will give the uplift capacity of the pile, if it has a enlarged base.

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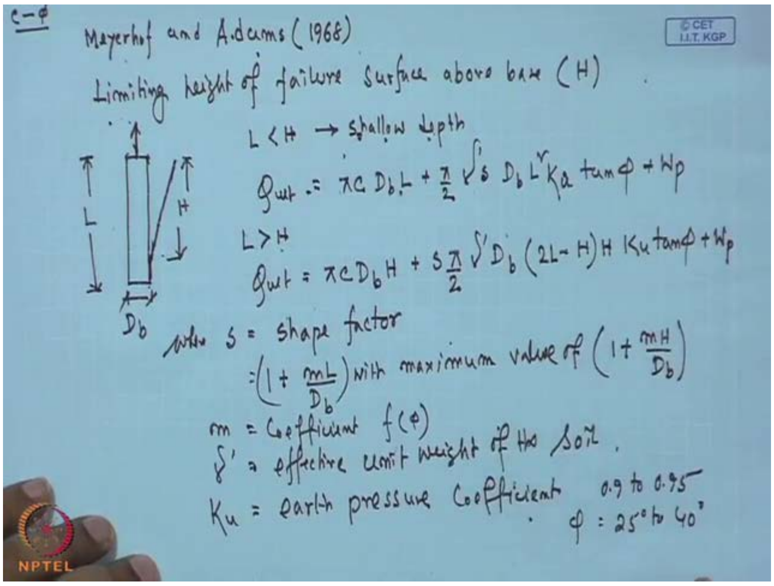

Now, the next section that if the soil has a C phi soil. If it is a C phi soil, then what would be the uplift capacity of the pile? So, here we have explain about if the soil is a totally sea soil or the cohesive soil. Now, if the soil is C phi soil then how if the uplift capacity of the pile is then we determine? In that case again it is given by the Meyerhof and Adams 1968, that for a limited limiting height of failure surface above base suppose for a limiting height surface above base say, that is H So, so here we have considered that if this is the pile, then we subjected by uplift force then if this L is the length of the pile and for a limiting failure surface that means this consider that for, that mean the failure surface of the height of failure surface in the pile is H.

So, that means here that is if the failure surface and pile will fail along this surface. So, that means here it is say intact then in this up to this portion there is a separation between the soil and from here it has been fail. So, that means the height of failure surface say H. So, that means that is the limited height of failure surface and then that case if L is less than H that means the shallow date. So, that means the the length of the pile within the height of the failure surface then Q ultimate pi C D b into L plus pi by 2 gamma s D b L square k u tan phi into plus W p. So, W p is the weight of the pile. Now, if L is greater than H then Q ultimate will be pi C D b into H plus s pi by 2 gamma D b 2 L minus H to H k u tan phi plus W p. So, that means this W p is the weight of the pile. Now, where s is equal to shape factor, which is 1 plus m L by D b with maximum value of 1 plus m H by D b. So, that means this cannot be a greater than 1 plus image by D b and m is a coefficient, which is function of phi friction angle of the soil, gamma dash is the effective unit weight of the soil, of the soil k u is the earth pressure coefficient is varies from 0.9 to 0.95. If 5 varies from 25 degree to 40 degree.

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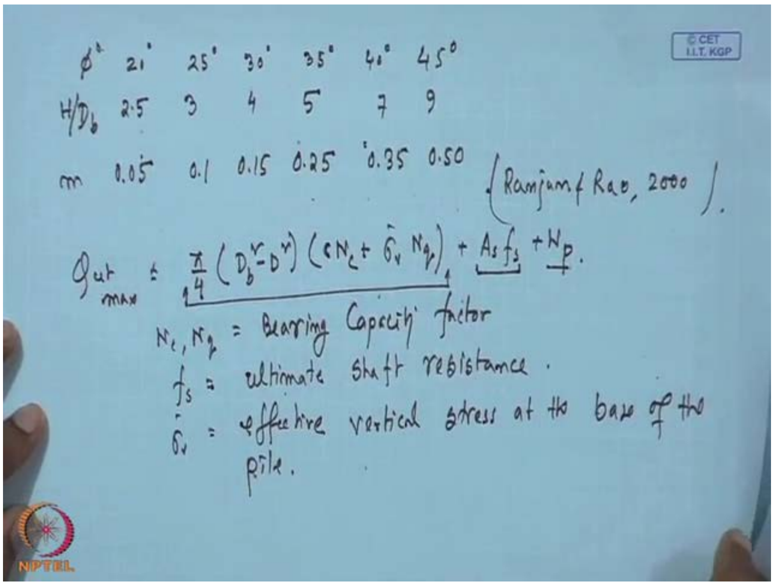

So, we can write in this expression where c is the coefficient D b is the diameter of the base so, this is the diameter of the base depending upon and s is safe factor and k is the earth pressure coefficient and W p is the weight of the pile. Now, that again this m value is given in this form that if we have a phi value say in degree is 20 degree, 25 degree, 30 degree, 35 degree, 40 degree and 45 degree, this is the phi value.

So, we have this H by D b ratio is 2.5 say and here 3, 4, 5, 7 and 9 then m that value has 0.5, 0.1, 0.15, 0.25, 0.35, 0.50 So, this as a m value that coefficient that will get so that is safe factors similarly, we can determine the s max, then what will be the s max value? If it is taken from Ranjan and Rao 2000, now similarly, that Q ultimate max that will be pi by 4 D b square minus D square into c N c plus sigma v into N q plus A s f s plus W p. So, that mean Q ultimate max will be this value as N c N q this is the bearing capacity factor are determined by the table which is proposed earlier. So, f s will be the ultimate shaft resistance and sigma v bar will be the effective vertical stress at the base of the pile. So, that means the Q ultimate x that the maximum ultimate force that is the W p is the weight of the pile.

So, this is this portion, is the weight of the pile. This is the friction, resistance and this is the bearing or the tip resistance or this C N c value. So, that means you have Q ultimate of the maximum force that is given this friction, ultimate shaft resistance and bearing capacity factor C N c and this is given for the total maximum ultimate load of the pile.

So, this this way we can determine the tension capacity of the pile. Now, this so pile is again can be subjected to lateral load also. Depending upon if it is subjected to wind load or if it is subjected to seismic load, it can be subjected to water pressure or water or if it is a water found structure so, this way and if it is a retaining wall a below the retaining wall if pile, you have placed then the piles can be subjected to lateral load also. We have to know how to calculate this lateral load capacity of the pile. So, then that means process here now one method is been explained.

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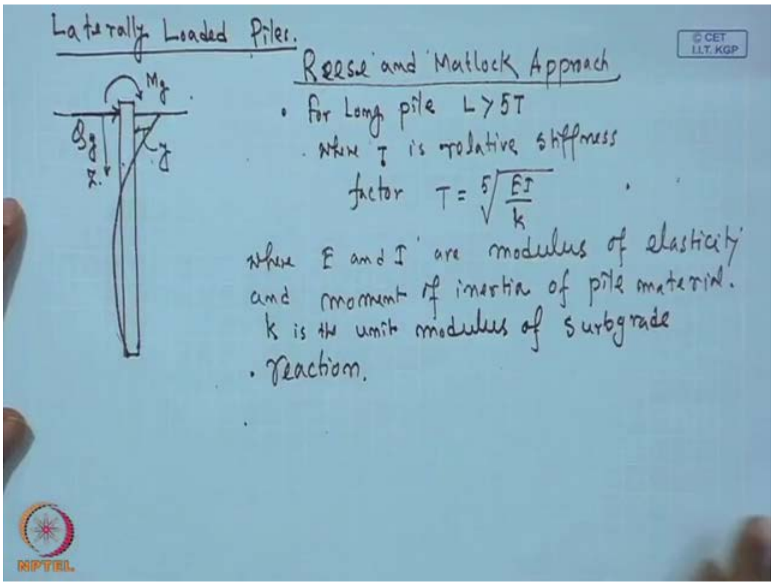

So, that is for the lateral loaded pile. So, once piles are subjected to lateral load or moment so, that means here we can say say suppose this is one pile. So, this is ground surface so, that can be subjected to a lateral load which is applied Q at the ground and that can be subjected moment at the ground level or the top so, that means from because of these two moment and this lateral load pile has some deflection at any depth Z So, suppose this is the deflection y of the pile at any depth Z. Now, these deflection of the pile is combination of two thing, one is due to the moment another is due to the lateral load.

So, that means here the process the method this is proposed by Resse and Matlock which is applicable for long pile. So, first condition and this theory is applicable for long pile. When length of the pile is greater than 5 times of the T, where T is the relative stiffness factor can be determine, the using this expression that E I by small k, where E and I are modulus of elasticity and moment of inertia of pile material. Now, small k is the unit modulus of sub grade reaction. So, this is modulus of sub grade reaction. So, that means here during the analysis this is this soils are replaced by spring and then based on that got the basic differential equation.

Then using the finite difference technique this equations of sault and then ultimately the shear this all the other unknowns that means the deflection the slope or shear force bending moment can be determine. So, that means, but it is applicable only for the long pile because in the, if pile is very very length is high in the long pile then it is assume that a very high depth or that means the deflection of the pile is negligible. So, in that is applicable here so, that is why it is only applicable for the long pile condition, where condition that at the higher length or depth the pile deflection of pile may be 0 or negligible.

So, now if we have consider that pile is subjected to lateral load, as well as a moment at the top and then we will get the deflection of the y at any depth Z. So, that means we have a different kind of deflection pattern depending upon the type of so, that mean this y is a combination of this load due to the Q g and due to the M g also. Now, if we can draw the figure which is due to the both deflection. So, that if we supper imposed this things suppose we have a pile which is subjected to lateral load Q g and a moment and it will have a deflection like this. Say said, any depth Z say deformation is or deflection is y.

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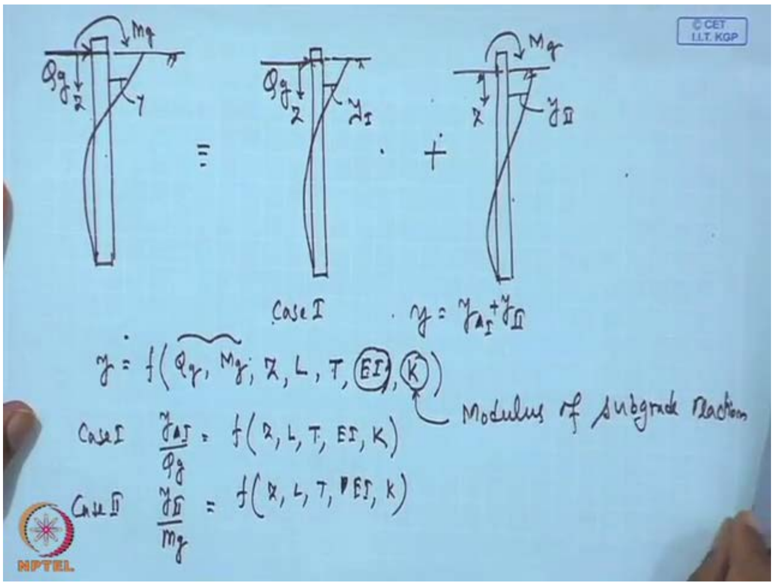

So, that can be equal to this is ground surface. So, we can separately take the two cases one is due to the lateral load due to the lateral load, another is due to the moment. So, this is due to the lateral load began we will have some deflection at the same depth said we have a deflection say y 1. So, this is case one where deflection is y 1 at the same depth Z, due to the lateral load only and then plus we have the same pile this is ground surface. So, we will have a moment applied at the top and then again at the deflection that will act here. So, that means we have a same depth we have a deflection y 2.

So, that means y 1 is the deflection of the pile at a depth Z due to the Q g all at horizontal load applied on the top and y 2 is the deflection at the same depth due to the applied moment M g. So finally, we can write that y will be y A plus y y 1 plus y 2. Then again we can say that any deflection if I case the general once any deflection, that is function of the load which is applied. Again which is function of the moment which is applied, again the deflection is function of z also because s the depth changes the deflection value will also change, it is not uniform to throughout the pile.

So, that is deflection is function of Z. Deflection is also function of L depending upon the length of the pile, deflection will change deflection will also function of T that a relative stiffness of the pile deflection is also function of E I that means the material properties are the geometry of the pile. So this six things are and again this is also the another one, there is a modulus of sub grade reaction that is k. So that k is the modulus of reaction. So, that mean there is 1, 2, 3, 4, 5, 6, 7 factors so, deflection depends on this seven factor so, it is form here this two are the loading or the moment and this Z is Z Z is the any depth of the pile L is the length T is the lead is stiffness.

So, this is material properties, the geometry of the pile and k is the soil properties. There is a modulus of sub sub grade reaction of the soil surrounding soil so, k is also the soil properties or the material properties. Now, if I consider that in the two case, where case one and case two. Then this right this case a for the case a or the one that in this form y 1 divided by Q g and then that is function of Z L T E I and k. So, that the first case so here y g's function of again Z L T E I and k similarly, for the case two y 2 by M g that is also function of Z L T E I and k so, we can see that both the cases case one and case two. So, there in general six factors per so any case take the case one.

So, an write this y A divided by Q g then that is so function of this phi parameters and in similarly, actually this y 1 is function of phi parameters or including the q g. Now, in this form you have to write and we have to some non-dimensional factor that is proposed in for different cases.

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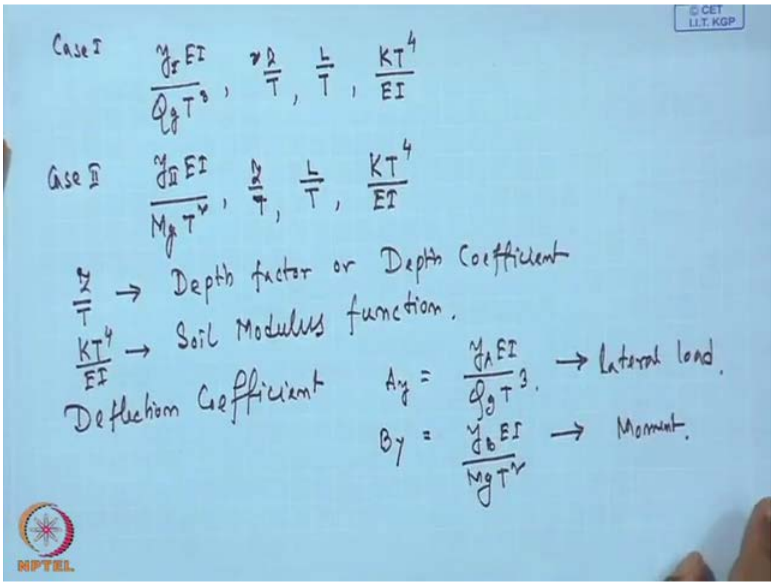

Now, for the case one; so it is represented in this form y on1e into E I by Q g T cube is one factor. Z by T another factor, L by T another factor and k to the T 4 by E I is another factor. So, that means this four thing express in this form then for case two y 2 into E I by M g into T square. So, this factor so again Z by T is one factor L by T is one factor and k T to the power 4 E I is another factor. So, we are representing in this this 4 factor of all the cases. So, here so this form here this Z by T is called as a depth factor or depth coefficient, then the soil modulus function k T to the power 4 E I is called soil modulus function.

Then another one is the deflection coefficient, which is for the load it is A y so, for the y A E I Q g into T T cube and for the this is for the lateral load and B y that is y B into E I by M g into T square that is for moment. So, these are the factor. So, this is depth factor or depth coefficient soil modulus function then deflection coefficient all the force are so, this represented in this non dimensional form. Now the finally, the deflection so once it is so after the so finally, this deflection y is given y 1 plus y 2 and that is express in this form Q g T cube divided by E I into A y plus M g into T square divided by E I into B y.

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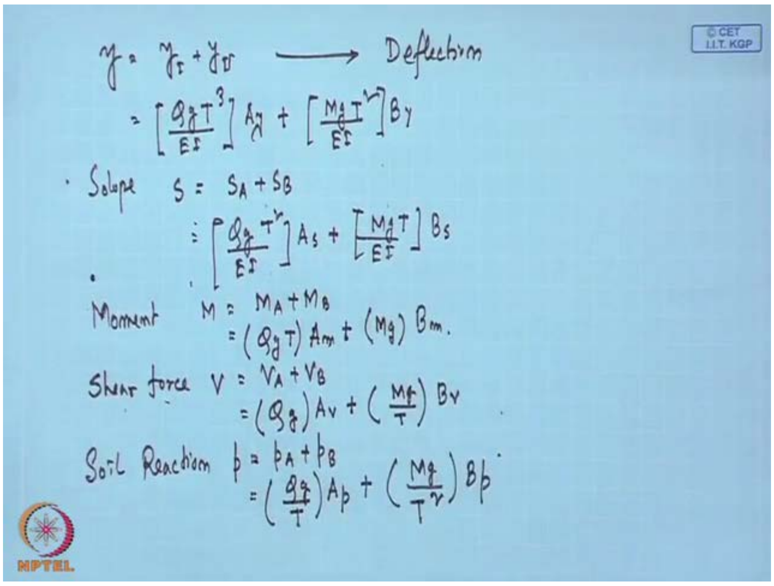

So, now the slope S is represented by S A plus S B similarly, that is Q g T square by E I into a S plus M g T divided by E I into B S. Similarly, the moment M is return as M A plus M B so A due to the lateral load and B means which is due to the moment. So, again it is a retain in this form that Q g into T to a M plus M g into B M this is moment. Similarly, shear shear force V. This V A plus V V or it is returned Q g into A V plus M g divided by T into B V and finally, the soil reaction is p is returned p A plus p B and that is Q g divided by T into A p plus M g divided by T square into B p. So, that means here this is the deflection expression.

So, we have deflection slope moment shear force and soil reaction. So, here we have the oil unknowns A y, B y, A s, B s, A m, B m, A V, B V, A p, B p. So, all this unknowns are then so based on this this modulus of sub grade reaction techniques. This final basic differential equation is solved using the finite difference method and then these unknowns all this 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 this 10 unknowns are determine and then it is given your tubular form. So, that means this form this table we can determine we can pick the any value at any depth.

So, at any depth based on this any depth in non-dimensional form this values are given. So, that means so one A y B y so if we know this A y B y at any depth. Now, for different condition and A s B s so, we can draw the deflection pattern along the depth the slope of the pile along the depth and the bending moment and shear force and soil reaction of the pile along the depth also. So, now one thing that we have the different pile head condition. So, if we have we can it can be have a free hand pile it can be a fix and pile also, now if it is a free and pre free head pile of free hand pile then they will be known moment will develop due to the fixity.

But if it is a purely fix head pile then if apply the lateral load then have fix and or our the moment that will develop due to the fixity; so for the fix and pile or completely fixed head pile.

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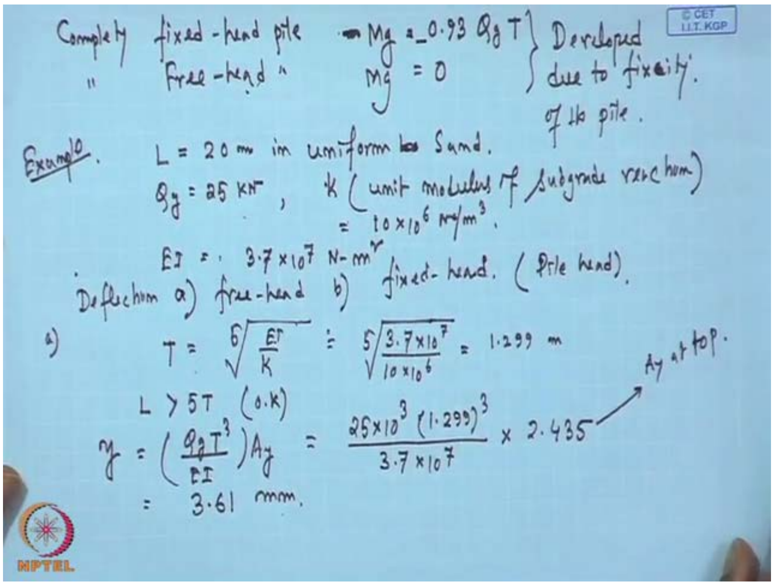

So, due to the fixity a moment that will developed minus g is point which is equivalent to 0.93 Q g into T and for completely free pile this M g or this is that means this is minus we can write here. So, that is 0 so that means free head by no moment due to develop due to the fixities. So, here this will developed fixity of the pile. So, that means if it is a free head pile so, that mean no moment you will develop due to the fixity of the pile, but if it is a fixed and pile then moment that will develop in the opposite direction of the direction of the a loading. So, that is it is given additional rigidity or the fixity of the pile.

So, that due to that is equivalent to 0.93 Q g T and if the fixity of the of the pile head is within this value 1 is 0 that means the free head other perfectly fixed and if it is within that range then, by interpolating from 0 to 0.93 and Q Q g t from 0 to this value we can determine. What will be the moment due to the fixity? If it is fixities within that range, now this is the completely how we can determine this is the load. So now, we will calculate solve a example problem that we have a length of the pile L is equal to 20 meter and it is in uniform sand now, Q y lateral load that is applied is 25 kilo Newton and the modulus unit modulus of sub grade reaction that is the, reaction of the soil is equal to is taken a 10 into 10 to the power 6 Newton per millimeter cube.

Then E I is given 3.7 into 10 to the power 7 Newton meter square. Now, what is the deflection? deflection one condition is is the free head and another condition completely fixed head. So, first we will calculate what is the T value of the pile that is E I by k. So, root over 5 E I is 3.7 into 10 to the power 7 and k is 10 into 10 to the power 6. So, we have a value 1.299 meter Now, here we can say L is greater than 5 times of the T. So, we can apply this things for the long pile so, we can apply this theory. Now, we can know that y is here only no moment is applied if it is a free head and only the Q g is applied.

So, the expressions for the y value is y is y 1 and y 1 is Q g into T cube divided by E I into A y. So, Q g is 25 into 3 Newton and T is 1.299 cube divided by 3.7 E I into 10 to the power 7 and it is we are determine, the deflection at top most point of the or the pile head. So, we are determine the what is the deflection at pile head condition. So, that means at Z equal to 0 at Z equal to 0 that A y value is given 2.435 so, 2 point that will get from the table. So, that is A y value at Z of this non dimensional of the depth factor is equal to 0 at top of the pile.

So, ultimately we will get deformation that is 3.61 millimeter. So, that is a deformation if so, that is the deformation if pile head is completely free. So, we will get a 3.61millimeter deformation at the pile top, but again if we have a fixed Head pile fixed head pile then one additional moment that will generate because of the fixity, if it is a completely fixed pile 0.93 Q g into T. So, this fixity will developed in the opposite directions.

So, point that values 9 3 into 25 into 10 to the power 3 to 1.299. So, ultimately this value is coming minus 0.320 into 10 to the power 3 Newton meter. So, due to this moment, which is acting in the opposite direction. So, we have some deflection and basically that will reduce the deflection of the free hand condition. So, now here the total deflection will be y 1 plus y 2. So, here y 1 is 3.61 millimeter and plus so, this will be minus because it is a minus 30 so minus M g it is due to the moment T square divided by E I into B y.

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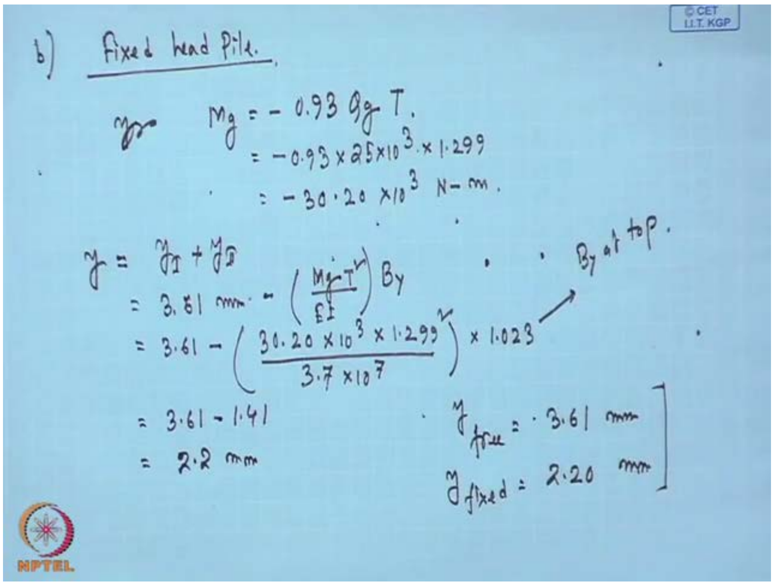

So, this is M g T square that is about the slope expression, that is given so, M g T square E I into B y. So, ultimately 3.61 minus M g is 30.20 into 10 to the power 3 T square is 1.299 square. Now, divided by 3.7 into 10 to the power 7 into B y 1.023; so this is the B y value at top of the pile. So, ultimately we will get 3.61 minus 1.41 so, that is so that that means this is 2.2millimeter. So, we have a deflection. Now, we can say for the y for free is 3.61 millimeter y fixed is 2.20 millimeter. So, we can see that because of this fixity you are getting a moment which is basically reducing the deflection of the pile head. So, that means here this is a 3.61 millimeter was the deflection for the free end pile free head pile and for the fixed pile they reduce and now the deflection is 2.20 millimeter. So, now this way the if it is a free end pile and if it is a fix end pile so friction pile due to fixity we can get the less amount of the deflection at the top.

So, similarly, for this is a deflection at the top so, from the table if we know the other coefficient values and then we can determine the any moment, deformation, deflection or information or deflection slope, bending moment, shear force, reaction at any depth for the pile for different loading condition or different pile boundary condition also. In the next class we will discuss about the foundation and then that will be the last section of this course.

Thank you.