Structural Geology
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Lecture 08 - Stress (Part – I)

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Hello everyone. Welcome back again to this online NPTEL Structural Geology course. We have delivered so far almost 7-8 lectures including lab part to measure deep and strike. I hope you are enjoying these lectures and it is not too difficult to follow. I repeat again, if you have any issues with any of these lectures or any of these points, or slides, you are most welcome to right back to us to teaching assistance or directly to me to get clarifications of your questions or whatever confusions you have about the subject.

Today, we will discuss a new topic very much related to the topics we covered before; strain and this lecture will focus stress. And I must say that in structural geology stress is one of the very important components or pillars, without understanding of this particular topic stress it is extremely difficult to understand or to comprehend the structures that we see in the field.

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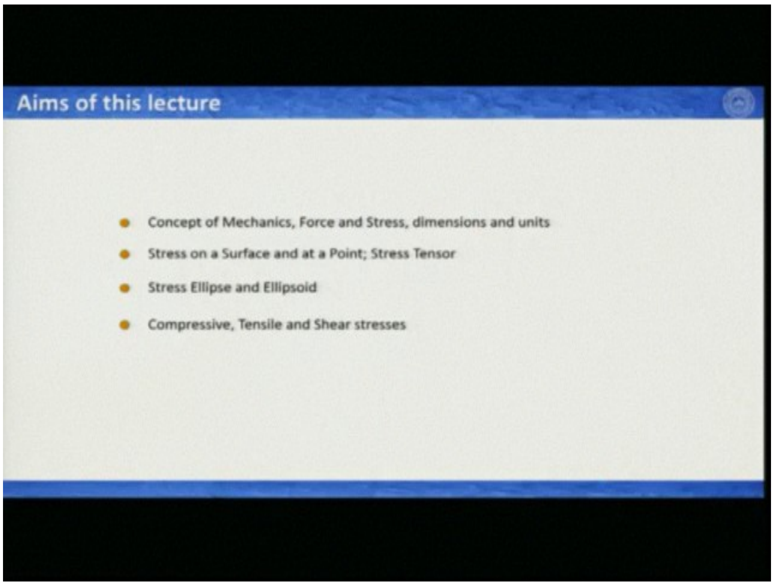

Subject stress includes concept of mechanics. Forces and stress, their dimensions, their units. Then we will slowly move to some sort of very basic mathematical operations to understand what is the stress on a surface, what is the stress at a point and then we slowly move to a concept that is called stress tensor. After that we will look at stress ellipse and stress ellipsoids and we will conclude this lecture with a concept of compressive tensile and shear stresses. We will also look at what are positive stresses and what are negative stresses that we consider in particularly structural geology and how do these concepts that we particularly apply in structural geology do vary from other subjects?

At this time it is very important to feel that stress is something that we do not see but we feel its effect. In the context of geology or to be very specific structural geology we are convinced now so far the lectures we have that rocks do get strained and if you have strain in your rock that you are considering with, that means it must have moved from its original place and at the same time it also has changed its shape and dimensions that means it got distorted.

Now to achieve this strain in this rock masses, who does it or who is a driving mechanism responsible for the strain in the rock? And the answer is that this strain material when it was getting strained it must have experienced some sort of natural forces or pressures or it was subjected to some sort of natural forces and pressures. And I repeat that we do not see these pressures or forces, but we feel it. And this is what we would like to continue in these lectures, but before that let us have some basic ideas of this general topic overall.

First of all I want to tell you that you must have heard this word stress from your school life. You must also have heard this word force again from your high school levels. And we know that the forces or stresses whatever we consider these two things they have to act on some materials. Now rocks as we can figure out, that is essentially some sort of materials. Special kind of materials, you can think so. But, so the study of deformation of rocks under forces fall certainly, falls under the subject mechanics.

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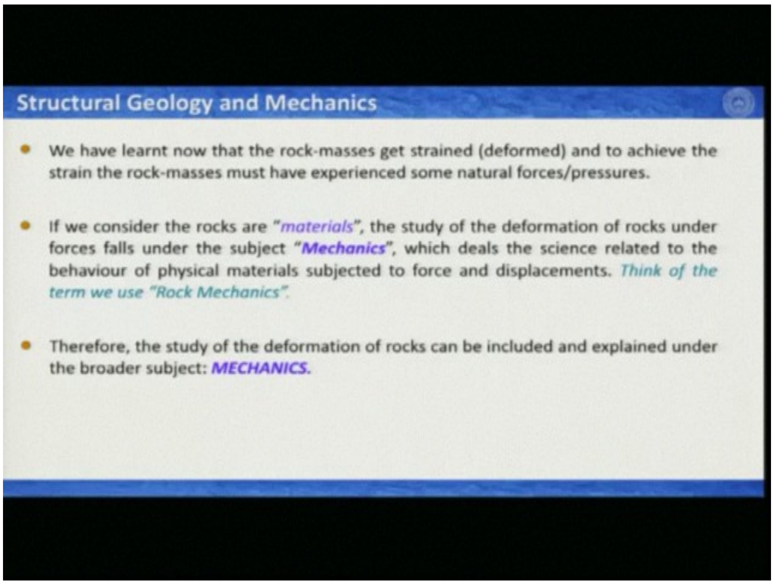

Now mechanics is something that, it is a topic. It is a sort of topic that we cover in physics mostly that deals the science related to the behaviour of physical materials. That means that you can visualise, you can feel, you can touch it subjected to force and displacements. So therefore, if we consider rocks as our materials then we have learned that these materials, rock materials do get strained bcause of some sort of forces and pressures and if so, then we can study the effect of forces and pressures on the rock materials to achieve some sort of strain. We can study it under the broader umbrella of this subject mechanics. And we generally call it, I am sure you have heard this term rock mechanics.

So it is very interesting that we will see how rocks as well like all other materials we can study under this subject of mechanics and at the same time, we will look at that there are many branches of mechanics. So you must have heard quantum mechanics, you must have heard continuum mechanics, solid-state physics is also type of mechanics that people do study. And we will see that in rock formation or in structural geology we will consider a particular type of mechanics and this is known as continuum mechanics. So, before that, let us get the idea that what is a continua or what you consider continuous in the context of structural geology or in the context of general behaviour of materials.

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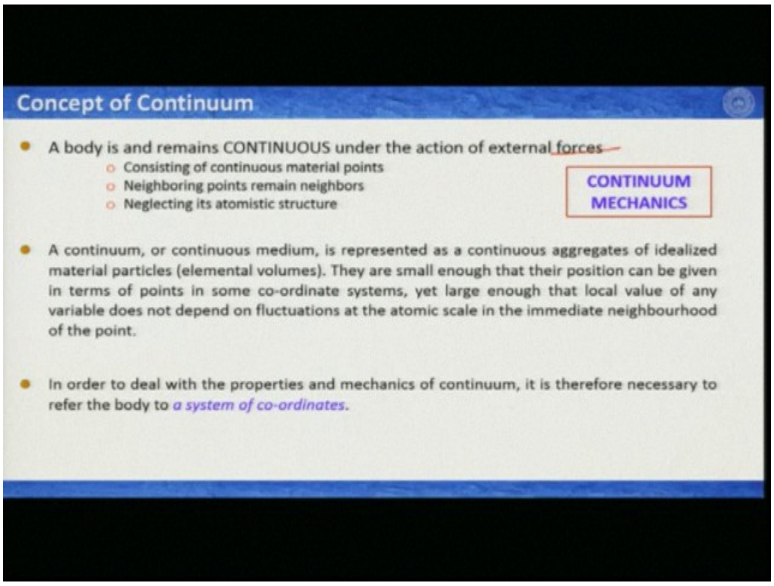

As it is written in the slide that body is and remains a continuous under the action of external forces. We learn very soon what is force. So it must have some sort of criteria that only we learned what is continuous and discontinuous and that same concept more or less I can apply here as well. So it says that consisting of continuous material points, that means if you consider a material that should not have any voids in your scale of observations, so it should have continuous materials. It can be different materials, but it has to be very much continuous.

And we can see that for rocks or we can feel that for rocks this is true, it holds good. And before and after that information the neighbouring points remain neighbours. That is one of the primary considerations of a continuous deformation. Now this may not hold true for the rock deformation. We have seen a number of examples of discontinuous, discontinuity in rocky formations, some structures we have seen where the continuity did not remain. Let us keep this in question.

And it does neglect all sorts of atomic structures. That means it does not go to the very detailed atomic scales features. So if we consider these three and of course in the rock mass looking at its scale though we do nowadays study very detailed microstructures, even we go to nano-scales, but we hardly except a few cases, few very specialised cases, we deal with atomic structures and their distributions for studying deformation of rocks.

But in general, we can figure out that if these three conditions hold good for any material and we are studying its deformation under the action of forces than we call that it falls under the subject of continuum mechanics. So therefore continuum or continuous medium, you can consider as it is written here, is represented as a continuous aggregates of idealized material particles. That means it must have some sort of elemental volumes.

Now these elemental volumes they are small enough that their position can be given in terms of points, in a set of coordinate systems, whatever coordinate you can consider. But it is large enough that local value of any variable does not depend on fluctuations at the atomic scale in the immediate neighbourhood of the points. So this subject is being studied under a sweet spot. That it is not that small that you cannot plot it in a coordinate system. But at the same time, it is not that big that some small fluctuations at the atomic scales can affect your system.

So again, this is little confusing for studying the earth because earth is really big. And little fluctuations somewhere can cause large fluctuations in other places. So again there are some sort of approximations and what is most important? That when you model something or when you study something, you actually ignore some very microscale features. What I try to say here, that if you are studying called a grain or grain scale processes then you do not take care probably what is happening at the top of the mountain. Or you do not carry if there is a lot of deglaciation or there was a huge rainfall. These things probably do not affect your scale of study.

Now if you go to little larger scales, for example if you are studying a rock specimen, then you do not take care of these internal features inside a grain. For example, dislocations or some crystal or some crystal vacancies or some interstitial impurities inside the crystal. When you study a rock specimen you do not consider all these little things, you ignore them, you believe that these things do not affect this. If you are doing, studying a scale of Platictonics, then you do not consider the different layers that we see in rock systems. For example, you do not consider a shear layer or sandstone layers and then limestone layers and so on. These things we ignore when you study the scale of Platictonics.

At the moment if you consider the scale of geodynamics, then we consider crust, mantle and core. We hardly consider the fact that the crust has different layers, mantle has two different layers or maybe we consider but the scale increases. We do not really consider that whether in the crust we have carbonates or we have silicates or whether in silicates we have shear layer somewhere or sandstone layers somewhere. These things do not influence the research of your study.

If you would like to study the solar system entirely, then you consider all the planets just as a sphere. So there you do not consider what is your core, what is your mantle, what is your crust and so on. So the bottom-line is based on the scale you are observing, so scale term is very important as I said in the first lecture. Based on the scale you are observing or your area or your interest, you somehow approximate some features and you somehow neglect features by approximating that these features which are, which do not really fit to your scale carrying out or can influence your measurements or your analysis.

So this is a concept of continuum mechanics and we will see that stress and most and strain as well that you have learned, these are being studied and the next, after a few lectures we will study rheology. These are subjects which are being studied under the broad umbrella of continuum mechanics.

I hope I will be able to convey the message to you of why we should or what is justification of studying stress, strain and rheology like subjects, particularly important for structural geology under the umbrella of continuum mechanics. There is also one important point that the approximations, the assumptions, the equations everything in continuum mechanics is little simpler compared to other topics. And it is easy to understand, easy to employ, easy to apply in the problems that we generally face in structural geology and tectonics in general.

So we move on to first get the very basic idea of what is force. Now if I ask you what is force, then generally the common answer we get that force is pull or push of a body or that you apply to a body. Say you are pulling a body that means there is a force, you are pushing a body that means there is a force. Now this is true, but this is not absolutely true in defining the force. For example, I am standing here, apparently no one is pulling me or no one is pushing me. But there are some forces, there is at least one force acting on me.

So that is the gravitational force, which is going down and then there is an opposite and equal force, which is acting from the ground to me and that is why I am more or less balanced here. So it is an interaction of the material to some quantity.

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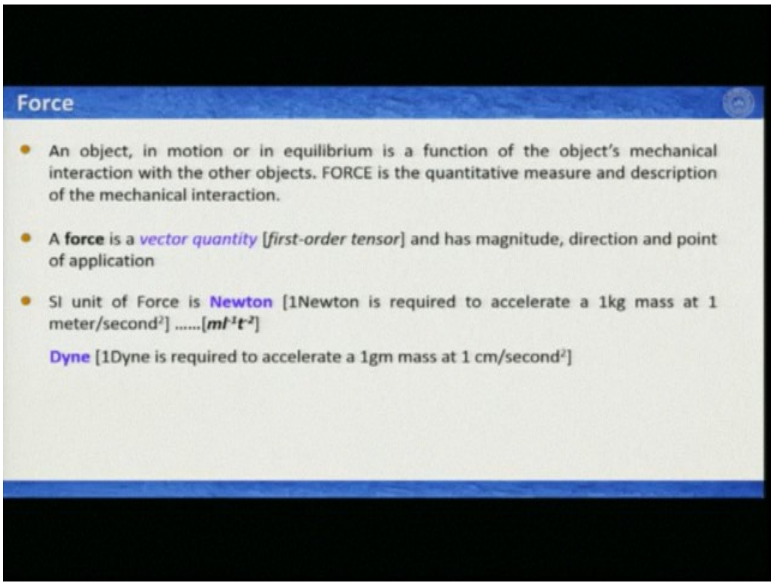

Accordingly we can define that an object in motion or in equilibrium is a function of the objects mechanical interaction with the other objects. Now force is the quantitative measure and description of this particular mechanical interaction. So that means if you have a material then how this material is interacting with its neighbouring or other objects, is the study of force or it is defined as force if you can measure it quantitatively. And that means not necessarily all the bodies have to be in motion to identify or to measure force. Now force we know from high school level is a vector quantity. That means it is a first order tensor.

Now this is something a new term, maybe you are hearing. We will not go into the very detail of what is tensor. But any scale quantity we call it zeroth order tensor. That means it has zero order. That means it has only magnitude, no directions nothing. So it is zeroth order tensor. Vectors it has magnitude and directions. So therefore it is first order tensor and there are possibilities of some second and third order tensors and these are not typically vectors or even high order tensors. We learn about it soon, but not in detail, but we will just describe what is this. So a vector is generally first-order tense. Let us get into that only. So force as it is a vector quantity, it has a magnitude and direction and point of application.

The SI unit of force, you know it is Newton. 1 Newton is required to accelerate 1 kg mass at 1 meter per second square and if we see its dimensions then it is mlt^{-2}. That means mass, length and time. So this is how mlt are considered, so would vary when we consider under materials. Then there is also 1 unit of force that people commonly use is called Dyne. So 1 Dyne is required to accelerate 1 gram mass at 1 centimetre per second square.

To get the concept of Newton or just to see that what is 1 Newton, you can actually do it very simply. I just tell you that it is about 102 grams weight if you keep it and leave it on the earth surface. Then you are generating actually 1 Newton force on the earth surface. So you just have acceleration of gravity of earth and then you add it to this weight and then you figure it out that it would come to almost 102 grams.

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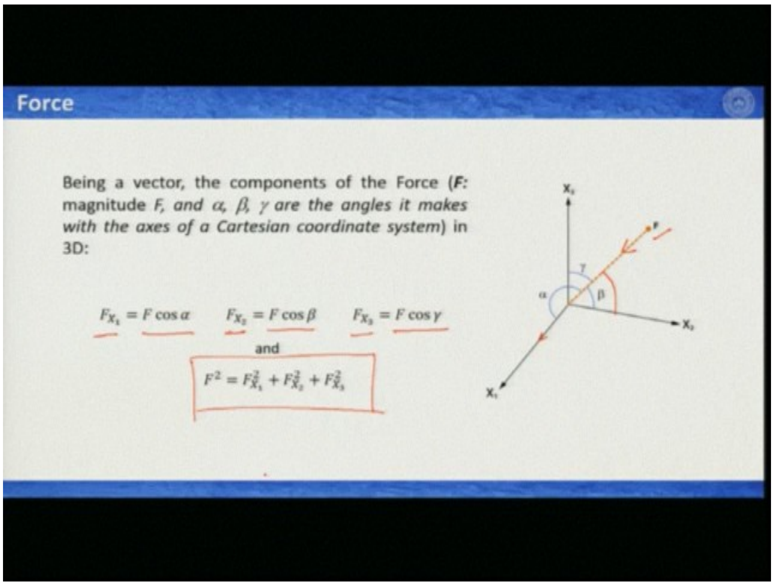

Now because force is vector, so you can represent it through a coordinate system in two-dimensional and four dimension and three dimensions I am sorry. What do you see here in this illustration? So this is a force, a point which is say being applied to this direction, to the centre of this coordinate system X1, X2 and X3. And we can resolve this force factor in three different components, acting parallel to X1, X2 and X3. And if we assign it Fx1 which is acting parallel to X1 direction then it is Fcoσα, where α is the angle between the force factor and the X1 axis.

And similarly you can have Fx2 where you can represent by Fcosβ, where β is the angle between the force vector and the X2 axis of this coordinate system and Fx3 where Fcosγ where γ is the angle between the vector F and the axis X3 of this Cartesian coordinate system. And because these are related to each other with this equation, that means square of this force vector would be sum of the squares of Fx1, Fx2 and Fx3. This relationship holds good, you know it from your high school level.

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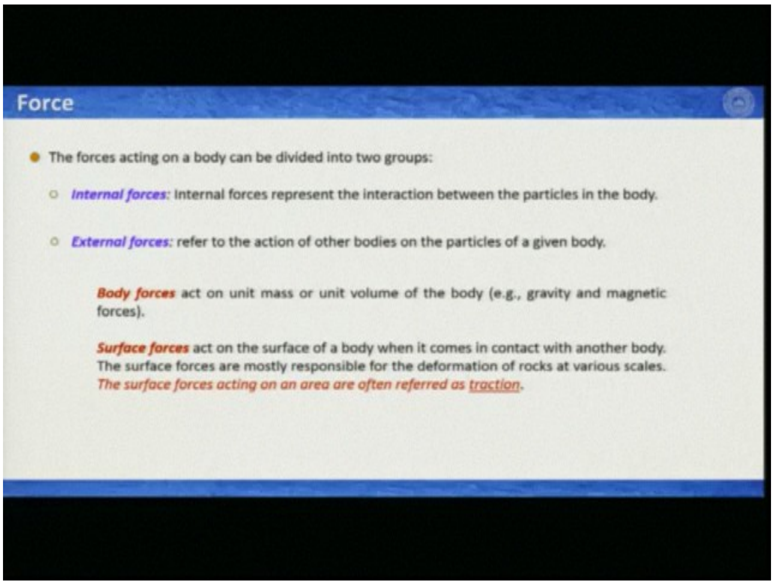

Now there are classifications of forces. There are different classifications but in general forces when it act on a body we can basically divide in two groups, one group is internal forces and another group is external forces. Now internal forces is something that you can define it that it represents the interaction between the particles in the body. So triatomic forces, interatomic forces, inter or intermolecular forces, these are generally considered as internal forces.

External forces on the other hand that is something when you have interactions of your concerned material on the particles. Other particles of the given body or what I mean by that it refers to the action of other bodies on the particles of a given body. That means it has to come some sort of externally. And this is something we generally consider. And external forces are also classified in two different ways or in two different subsets.

One set is body force and another set is surface force. At first I will discuss regarding body force and then surface force. Body force is when it acts on unit mass or unit volume of the body. And in the context of geology or structural geology in particular gravity or gravitational force is a force that you can consider as body forces. Then there could be the magnetic forces that also as a type of body force.

Now I will focus on surface force. surface force that is something very important to us is defined as, is the surface forces do act on the surface of a body when it comes in contact with another body. The surface forces are mostly responsible for the deformation of rocks at various scales. So you need a contact to have some sort of surface forces. Otherwise, your body will not deform and if there is no distortion, no deformation, then you are not dealing with surface forces, you probably are dealing with bodies forces or some other forces.

Now surface forces because it has to act on an area of the object, you are considering with, so this is often referred as traction. So we will see what traction is. That when a force acting on an area sometimes we call it traction, you remember this term we will use it soon.

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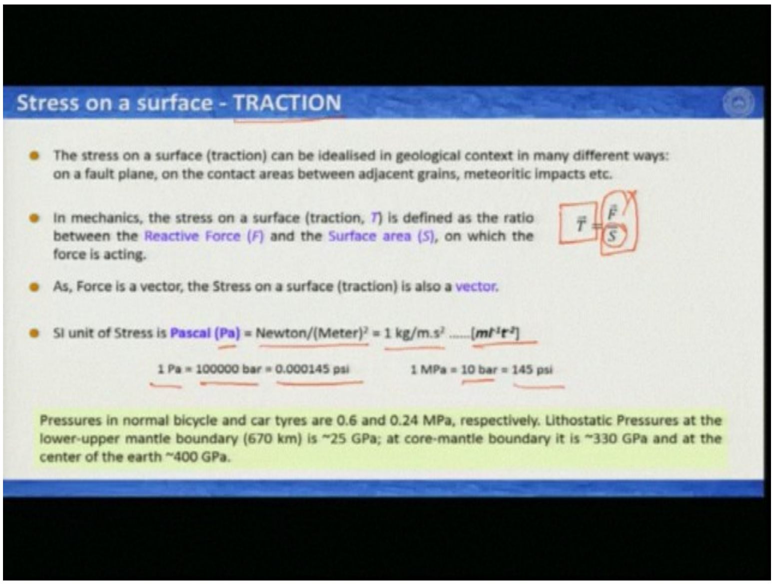

So let us have a look first on the stress term when it is acting on a surface or we define it as traction. The stress on a surface can be idealised in geological context in many different ways; for example when you have a fault plane, then two planes are moving past each other keeping a contact or you can imagine that two grains are in contact in a very micro scale and one grain is transferring its stress to the neighbouring grain. That means it is generating traction at a grain boundary. You can consider of meteoritic impact and so on. These are some very visual examples, but wherever there is a deformation there is traction. You can think this way.

In this context of giving the definition of stress, generally we say or you commonly say that what is stress? Force per unit area, you are right. But that is not absolutely true. It is actually the reactive force per unit area. That means if I apply a force here and if I calculate, if I know the magnitude of this force that I am pushing my left palm with this right punch, right-hand punch, then if I apply, take this force and divide it by the area of my left palm, I will certainly get a value.

But that is not stress. Stress is I am applying this force, and my left palm is working or pushing with opposite and negative force, to the force is being applied by my right hand. And this force is reactive force, it acts following the Newton's third law. And this force divided by the surface area is the stress generally defined.

So the stress on a surface or you can say traction we will define it as T, as defined as the ratio between the reactive force F and the surface area S on which the force is acting. So therefore we define that this is what is your reactive force. It is a vector force. Then this is area you would like to work with. So this T is a stress or the traction. Now because force is a vector, so the stress on a surface traction must be a vector as well.

Now because we are adding some area or dividing the Newton by area, then unit of stress should be Newton per metre square. Now this Newton per metre square has a name and the name is Pascal. And sometimes it is written as Pa. Now 1Newton metre, 1 Newton per metre square is generally you can sort of expand it. So it is 1 kg per metre per square.

And the dimension here changes to ml power minus 1 t power minus 2. Now there are many other units of stress, sometimes we also call it pressure in the context. So 1 Pascal is equivalent to 10^{-5} bars. Now this 10^{-5} bars that means 5 zeros after 1 and this is about 0.000145 psi pound per square inch. And 1 mega Pascal therefore, 10^{6} Pascal, equivalent to 10 bars and 145 psi.

Now what is 1 Paschal or what is 1 mega Paschal? How much it is? I give you an idea or it is written here we can read. The normal bicycle that we write everyday, the tyre of these bicycles have some pressures, we have to pump it. So the general range is mostly 0.5 to 0.7 MPa in a normal bicycle tyre. In a car tyre it goes to 0.24 mega Pascal or 0.3 mega Pascal maximum. But that is in a very rare case.

So the pressure on the bicycle tyre is 0.6 mega Pascal and the Lithostatic pressure at the lower upper mantle boundary which is around 670 kilometres down from the surface is about 25 Giga Pascal. That means 1 MPa multiplied by 10 to the power 6 multiplied by 25 gives you the Giga Pascal. At core mantle boundary it is about 330 Giga Pascal and at the centre of the earth it goes close to 400 Giga Pascal or the temperature is close to 6000 or 7000 degree Centigrade.

So this is the range of pressure or this is the magnitude of the pressures that structural geology tectonics or geodynamic people think and work with. So we will look at now that how to mathematically describe the stress on the surface element. But before that we will take a break, we will continue this lecture in the next segment.

Welcome back. Now we rercaptulate what we learned? That what is stress on a surface, what is its dimension, what are the units and more or less we have an idea that what is mean by stress on a surface. Now in this segment, we learn mathematically how to express what is stress on a surface. And for that we consider some sort of approximations under the broader umbrella of continuum mechanics.

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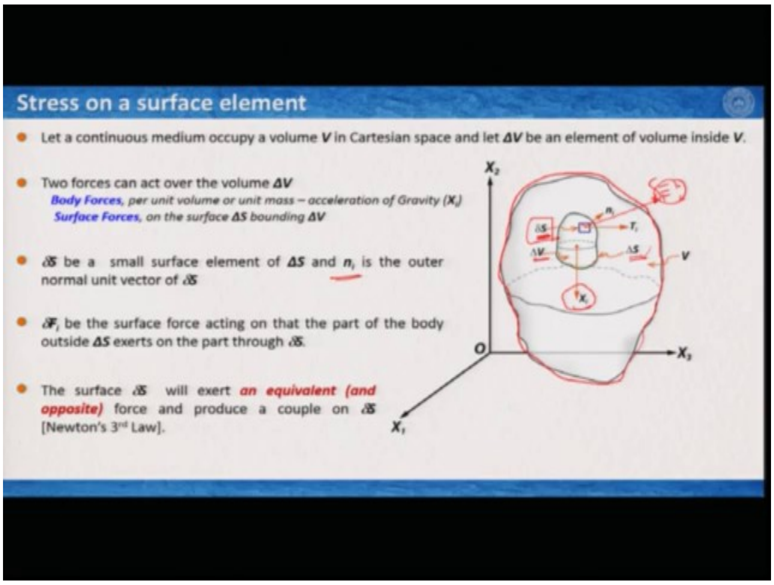

So stress on a surface element to the consider that, let us consider a continuous medium which is occupying volume V which is this larger area that you see this one. Okay. So this is volume V in Cartesian space and within this you have a very little element, volume element which is Delta V inscribed in the volume V. Now if we consider this volume V, now why I am considering this? I am considering this entire V is a very large body and it is a continua and within this continua I am considering a very small volume.

Okay, now if I consider this very small volume Del V then as we have learned the two different forces would apply on this. One is body force, which is acceleration of gravity, which is X i and then surface forces that must act on a very small surface. So let us consider that this is Del S, which is this entire area. On the surface of this volume, a force is acting.

Okay, now we can consider that this very small surface within this volume Del V, which is Del S and on this Del S, this course which is acting say some sort of F or whatever or Del F i.

Now what is this i hear? Like as we said this N i, N i is a unit normal vectors. So if I consider any surface or any plane then it must have three unit normal vectors parallel to the coordinate axis, right.

So if I have this very small area and then this Del F i is the surface force acting on that very small area Del S of this body, then this little green, I am sorry, this little blue surface area would react to this force Del F i, right. So it would exert and equivalent and opposite force, and it would produce a couple.

Now I repeat that I have a large volume V, within this large volume I have small volume, which is Delta V, which has a surface area Delta S. And within this small volume, small area Delta S we have a very small area Del S where a force is acting. And because the force is acting on that, on that little surface, it would exert an equivalent and opposite force and produce a couple on the surface Del S from inside.

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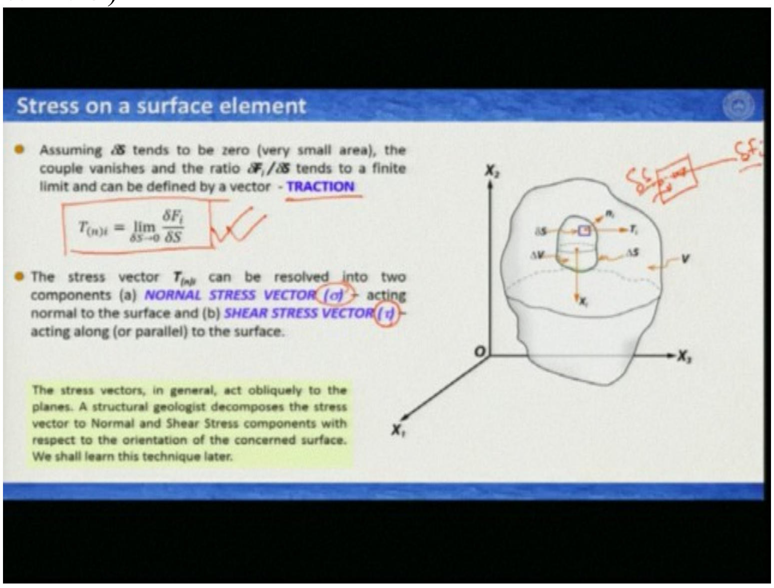

So now if we consider Del S tends to be 0. That means it is a very, very small area then the couple vanishes and the ratio, that means Del F i and Del S tends to be a finite limit and can be defined by a vector which is traction. Now by doing this, that means I have a very small area. I am applying a force, so we are getting an opposite force on this same point from the other side. And this is Del S, this is Del F i.

So if this area goes very small, therefore I have an area which is Del S and I have a force acting on this Del F i and if I reduce this area that means the area tends to 0, then I get some sort of a very, in a very small area the stress or the traction on a surface. So mathematically we can define it this way, even if you not consider these strange mathematical notations n and I, so traction is equal to the limit where Del S tends to 0, is if Del F i divided by Del S.

So this is the mathematical expression of stress on a surface area, now this is not really very straightforward expression as force per unit area or reactive force per unit area. So in stress, in considering stress this is how we apply the definition of stress on a surface element. Now we clearly see that if we have a surface and I have a force acting on this surface, say this is how it is working.

Now because this is a vector we have understood before that traction is a vector, I can resolve this vector one way that is perpendicular to this plane and another way parallel to the plane. The force vector or the component of this force, which is perpendicular to the plane is known as normal stress vector or we define it or mathematically write as Sigma. That acts normal to the surface and there is a force that acts parallel force component we can resolve that acts parallel to the surface, and this is known as shear stress vector or mathematically generally written as Tau which acts parallel or along the surface.

We can say in most of the geological cases the stress vectors in general act obliquely to the planes. So structural geologists have to decompose these stresses, I repeat again. A structural geologist therefore has to decompose the stress vector to normal and shear stress components, with respect to the orientation of the concerned surface. Now we learn about this technique later. But what I try to convey with this statement that if we have a force acting on this surface, we can immediately resolve it to the normal and stress component by simple vectors.

But for stress or for traction it is not that straightforward because you have to also consider the orientation, the area and its orientation where the traction is working with. Now, once we have more or less a steady idea that what is stress at a point, now we would like to see a stress on a surface, I am sorry and then we would like to switch to another part and which is even more relevant that is stress at a point.

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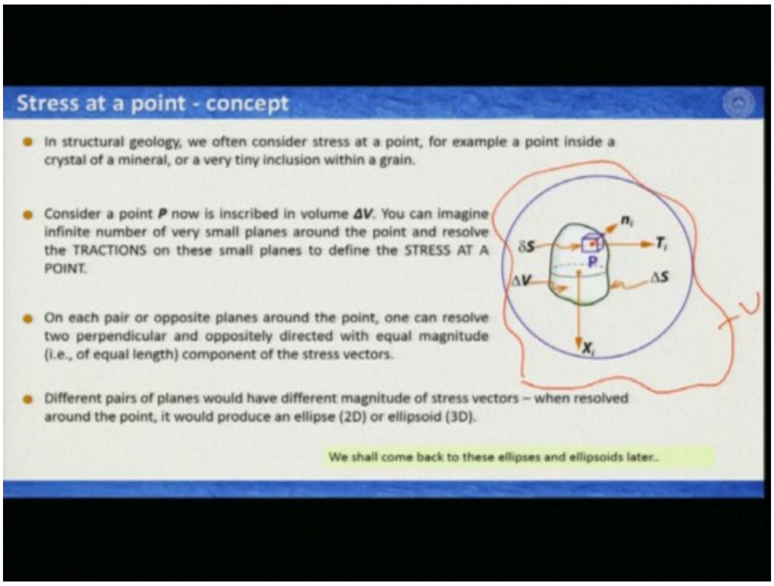

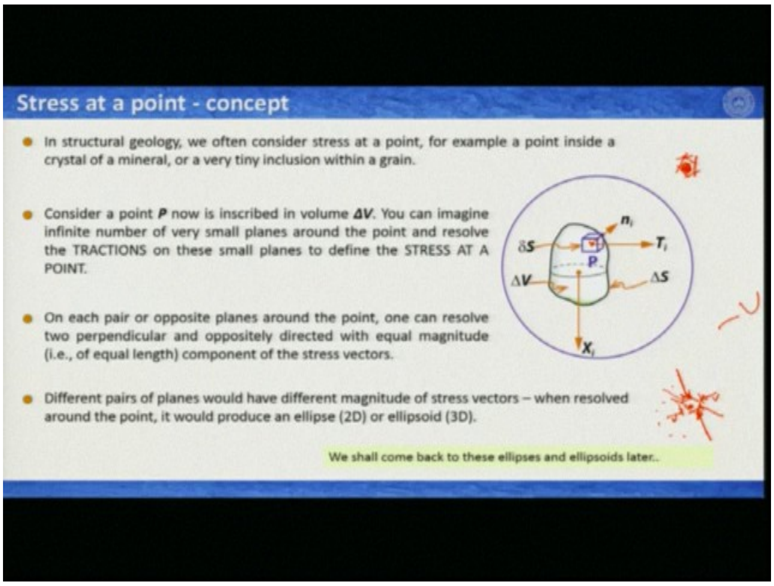

So let us talk first about the concept. As I said in structural geology, we often consider stress at a point, for example a point inside a crystal or mineral, or a very tiny inclusion within a grain and we would like to know what is the stress acting or what is a stresses in this point. Now we will follow the same process. So we have the same drawing again, we have this volume V somewhere outside and I just highlighted the area here, where you have Del V. Then you have Delta V, Delta S and then Del S small surface. And we have again the same point P along which or on which we have already derived, we have generated a very small surface and we looked at how we can get the stress on a surface resolved.

Now to get the stress at this particular point, if we consider a point is now inscribed in the volume Del V. And if I considered this as a point P then it is possible actually, I can draw infinite number of surfaces around it. What I mean by this? If I have this point P then I can draw infinite number of surfaces and so on. And these surfaces are very small because these are around a point and on each surface, you can calculate what is the stress because stress classically is defined as reactive force per unit area.

So in short that you can imagine infinite number of very small planes around the point and resolve the tractions on these small planes and defined the stress at a point. Now this is some sort of approximation or consideration. And these areas have to be very, very small. Now on, each pair of opposite planes around the point, we can resolve to perpendicular and oppositely directed with equal magnitude. Therefore equal length component of the stress vectors, we will learn about it very soon.

Now if I have a little point here and I have two planes oppositely directed, I have two planes oppositely directed, I have two planes oppositely directed and so on, and a force is acting on this plane, then it is highly possible that this oppositely directed magnitudes are equal. But with other pair of oppositely directed planes the magnitude may be different. That means if I have a point here. I have two surfaces it may be acting like this and I have two points here, the magnitude. In these two cases, here and here the magnitudes are same. But the magnitudes are different in the other two pairs.

So if I consider many such planes and these magnitudes may vary constantly. And this would come or this would finally yield what we would call very soon as stress ellipse. When you consider it 2D and what we call stress ellipsoid when we consider in three-dimension. But before that let us again come back to this particular topic, stress at a point.

Now what we can do actually, we can draw a very small unit cube around this point P considering six of these many surfaces, that is possible to draw around this little point. Now these six surfaces, one pair of each are aligned perpendicular to one of the principal axes of stresses.

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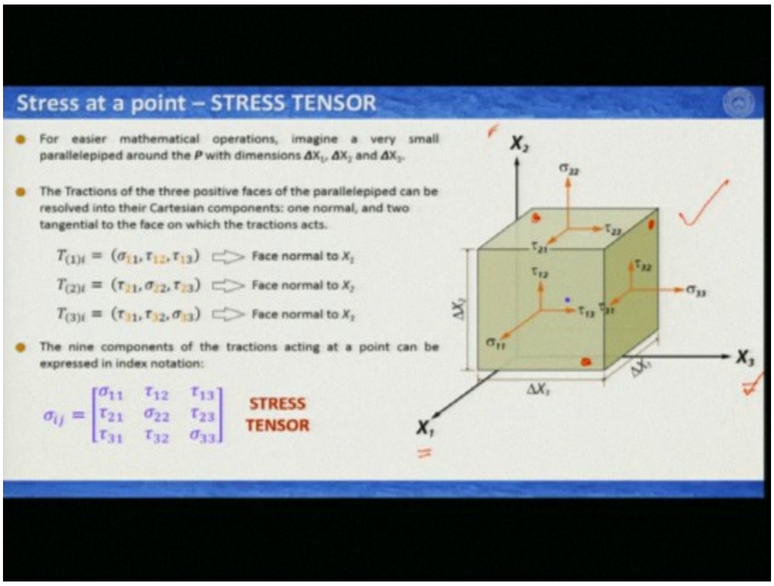

So how it would look like? It would look like something like this, what we are seeing here. We have these reference frames in Cartesian coordinates X1, X2, sorry, X2, and here is X3. This blue dot here is a point P and we have considered six planes. In the positive side, we are now seeing three planes, there are of course three planes on the other side.

In this context this plane where I am marking a little dot is perpendicular to your X3. This plane is perpendicular to your X2 and this plane is perpendicular to your X and these three planes and their opposite planes hold this point P. So this is a very magnified view of around point P.

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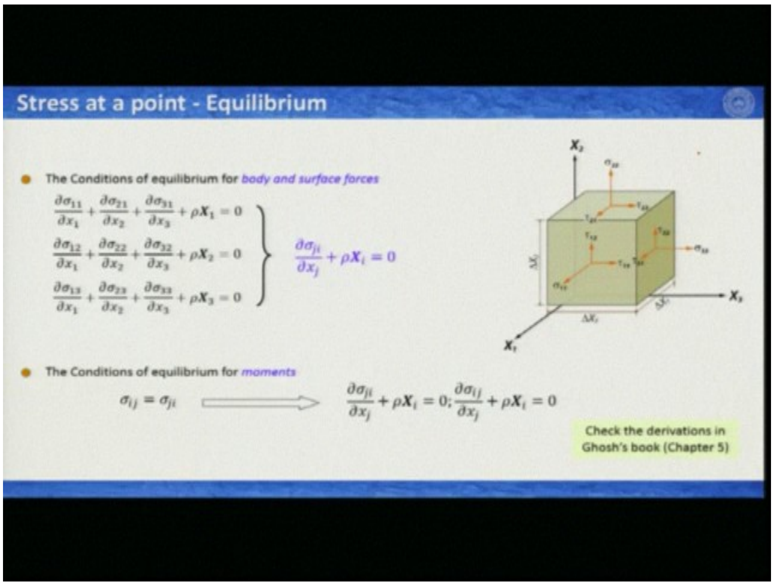

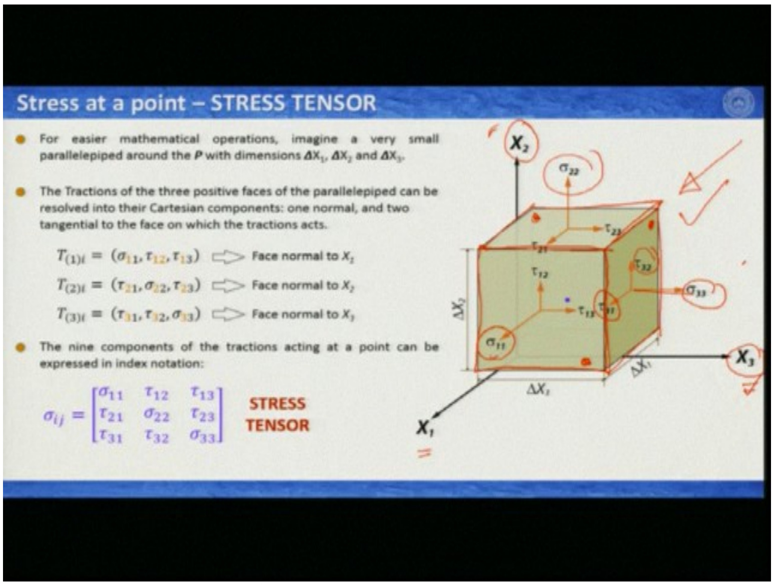

We already know that a force is acting. Now we know that a force is acting here and for each surface we have defined following a traction that there should be a normal stress and there should be a shear stress. Now normal stress, for the normal stress if we consider this plane, which is perpendicular to the X1, the normal stress on this plane which is perpendicular to X1 certainly would be directed along the S1 direction, which in this case is Sigma 11.

And the shear stress again we can resolve it in two different components. Because one would go along the X2 direction and one would go towards the X3 direction. So we actually get three mutually perpendicular set of stress components. One of them which is a normal stress component and two of them are the shear stress component. And one of these two is aligned to X2 and this one is aligned to X3.

And a similar case happen, if we consider the plane which is perpendicular to X3. That means Sigma 33 is a normal stress component working along the direction of Sigma 33. Then two shear stress component Tau 32 and then Tau 31. And then similarly on this plane, which is perpendicular to X2 direction we would have normal stress component which is acting towards X2 and then to shear stress components.

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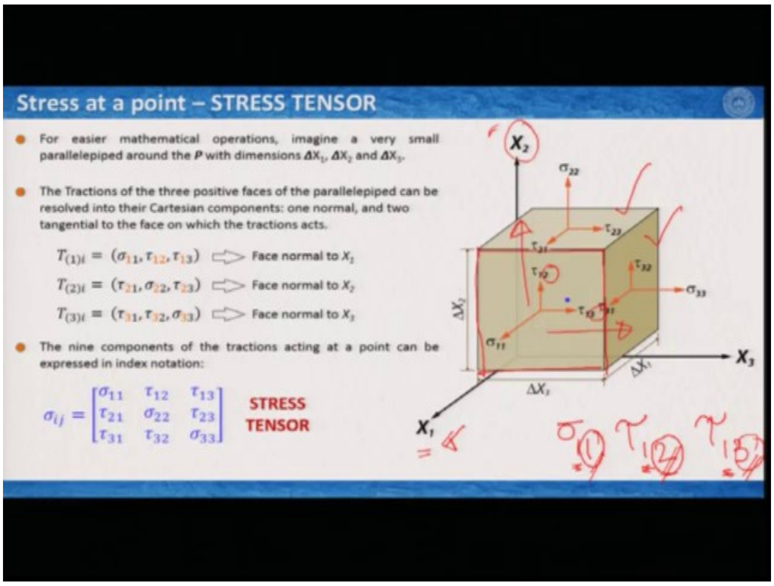

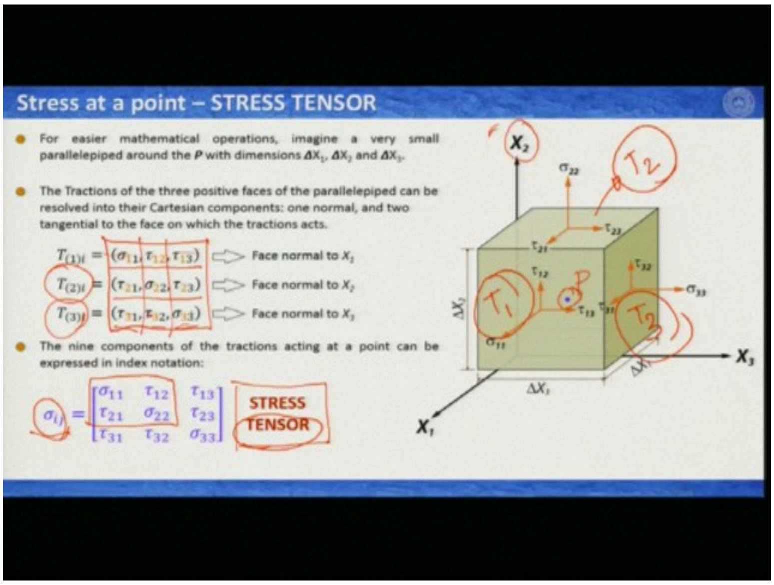

Now you have seen that three different nomenclatures or different notations are given. Let me explain what it is. What we see on this plane, I come back again to this plane, so Sigma 11, Tau 12 and Tau 13. In this case, this first index in each of these stress components indicate the plane it is acting or the plane perpendicularly it is acting. So in this case, this plane is perpendicular to X1. So therefore, in all cases, this is first component is 1. That means this

Sigma, this Tau and this Tau, these stresses are working on a plane which is perpendicular to X1 direction.

The second component 1, 2 and 3 at the direction along which the stresses are working, so in this case, the second component here is 1. So it is working along the X1 direction, Tau 12 second component is 2. So this is working along the X2 direction and this is 3, Tau 13. 3 is the last component, so it is working along the X3 direction. And this is also obvious or applicable for the other two planes.

The other three negative planes that we do not see in this image and exactly oppositely directed and. But similar magnitudes stresses would work there. So if we now come back to our traction idea or what we have learned from the traction. So on the surface we have, in this case, this is parallel to or sorry, perpendicular to X1. So here I can, I am sorry, here I can write that this is T1. That means traction working on the plane which is perpendicular to X1. Traction working perpendicular to X3 which is T3 and this one is T2.

Now for each of these T1, T2 and T3, we see that we have three components to define the stress on this plane which is perpendicular to X2. But everything we are doing to define the stress along this point P. So to define the state of stress along this point P, we first have to resolve the stresses acting along the three surfaces. So for T1, we see that this is Sigma 11, Tau 12 and Tau 13 for the face normal to X1.

Tau 21, Tau 22 and Tau 33 for the face of normal to X2, which is Tau 2 for a traction 2 and then traction 3 is Tau 31. I am sorry, this must be, Tau 31, Tau 32 and Sigma 33, which is normal to X3. Now all these nine components that we see here are required to define the stress at this point P. And each of them as we have understood this T1, T2 and T3 it is tractions and tractions are vector.

But when we have, when we need all these nine components 1, 2, 3, 4, 5, 6, 7, 8, 9 components to define the stress at a point, it does not remain a vector anymore. It comes to a new component or new sort of dimensions or description of the parameters and this is known as tensor. So you can write these three components in a matrix form, Sigma 11, Tau 12, Tau 13, Tau 21, Sigma 22, Tau 23, Tau 31, Tau 32 and Sigma 33 and you can simply notify or note it in the form of index notation Sigma i j and this is your stress tensor which is neither a scalar quantity nor a vector quantity but a tensor. But I tell you this is a second-ranked tensor.

So force, traction was the first order tensor and stress at a point is a second-order tensor. And this is neither a vector nor a scalar. So I believe you have now the idea that from the traction which was a vector, now we arrive to a new concept, when we have to define the stress at a point. It cannot be a vector it is a tenser and in three dimensions you need nine components and in two dimension you can define it only with four components. But it would still remain a tensor.

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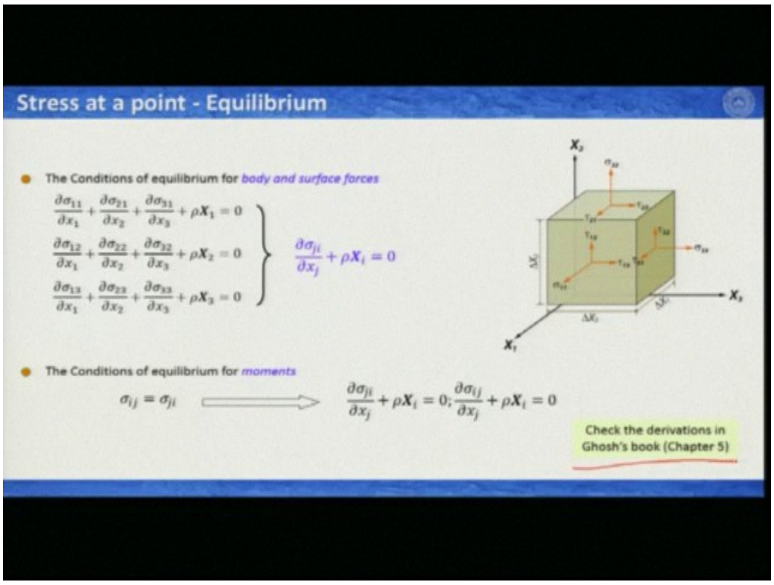

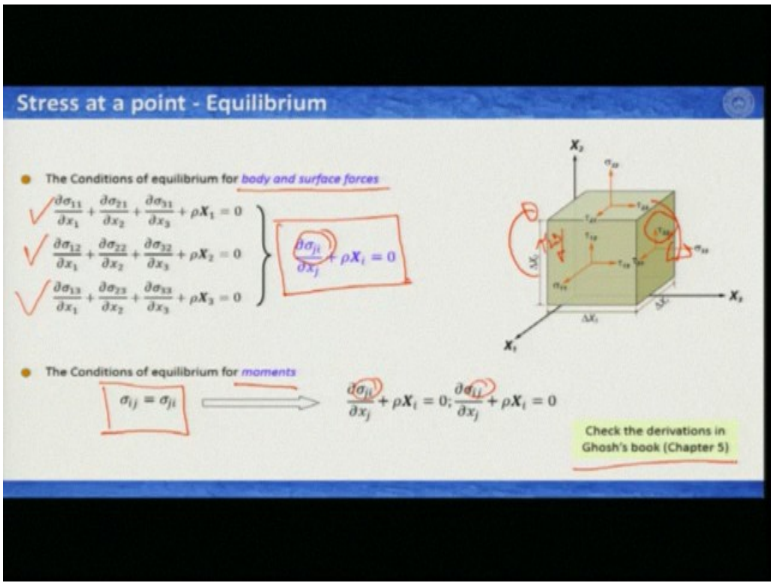

Now there are some fundamental equations that we need to know, learn and which is known as equilibrium concepts of stress tensor. I am not deriving these equations but as I have noted here you can get this from the book, one of the books I referred Professor Ghosh's book. If you go to chapter 5, you can see what is this. But the concept of this equilibrium or stress equilibrium is that this cube would stay in a static mode.

That means it does not translate along X1, X2, or X3 direction or it does not rotate. So if it has to translate then we have to consider the body and surface forces. That means body and surface forces, the sum of body and surface forces in a particular direction should be 0, that means they are balanced. So this is for X1 direction, this is for X2 direction, this is for X3 direction, you can get these equations. And in index notation form, you can write it this way. And if that this is not rotating, that means it is not rotating this way. Say for example clockwise.

Therefore this Tau 32 has to be counterbalanced by on the other side, Tau 23 and we can show that in equilibrium condition, this is possible. That is only condition that it does not rotates. So this is the equilibrium that we consider for moments and therefore it defines as I said, Tau 32 has to be equal to Tau 23. Therefore, Sigma i j should be equal to Sigma j i when your body is in equilibrium and therefore here we had Sigma j i you can simply represent Sigma j i to Sigma i j.

I did not dedicate too much time on this because you can get these derivations from other books, but I hope you have now understood the concept of stress on a surface which is a vector, traction and stress at the point which is a tensor. Now let us come back to the concept of stress ellipse and stress ellipsoid.

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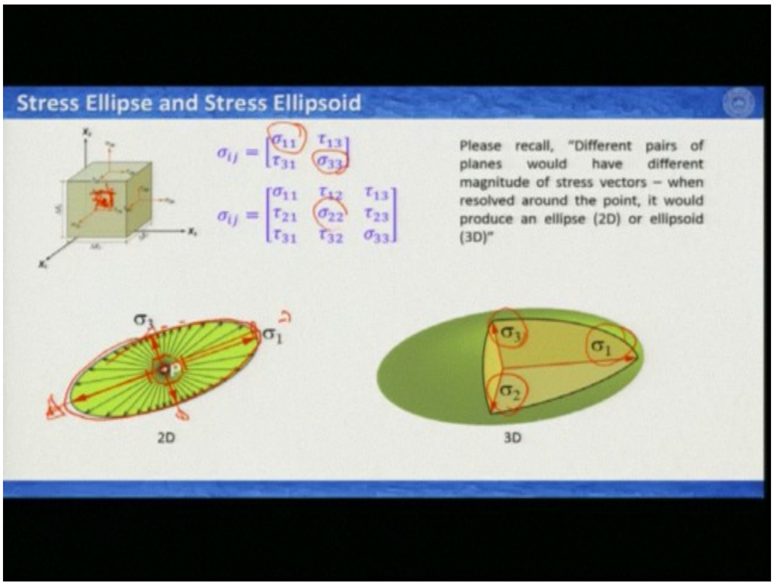

As I said that on this particular point P here, we can consider N number of planes, surrounding this point P and oppositely oriented planes would have same magnitude. That means same length, but different directions, different senses of action. So if we look at this is the point P here and if we look at in two-dimension, then we may get actually, so these are actually for example if I consider these two red arrows, so around this we had this plane and this plane around the small arrows, we had this plane.

So here it is working on this side and also it is working in this side. Their lengths are equal, that means magnitudes are equal, but their orientations or action, action directions are different and similarly here. Now of course you can have N number of planes around it and you can draw their normal components with their respective magnitudes. So if you connect all these points, you certainly would get or end up with an ellipse and this ellipse is known as stress ellipse when you consider in two-dimension.

The maximum length you get will define it as Sigma 1 and the minimum length, we get of this stress magnitude we define it as Sigma 3. In three dimension very similar to the strain ellipsoids, we will look at we would get a very similar shape. So at the maximum we would get Sigma 1. At the minimum we get Sigma 3 and intermediate we get Sigma 2. Now what is Sigma 1, Sigma 2 and Sigma 3 and how does it differ from this Sigma 11 or Sigma 33 or Sigma 22? We will hear it in the next lecture, we will learn it on this next lecture. But before that, let us get some more ideas about the stress ellipsoid.

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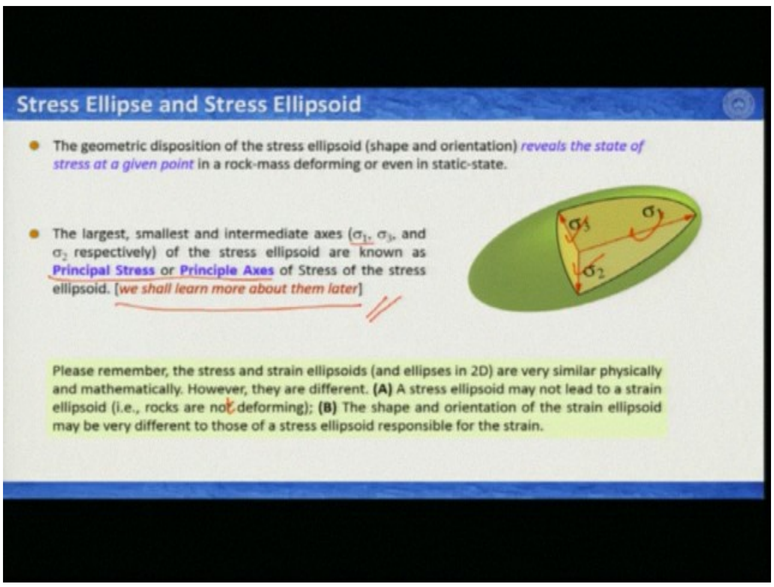

So the geometric disposition of the stress ellipsoid, its shape and orientation is generally described by the stress ellipsoid and it reveals the state of the stress at a given point in a rock mass deforming or if it is even in a static mode. Now this is something very important that we have just learned. Not necessarily you have to deform a material, that there is a stress or because there is a stress not necessarily the rock mass has to deform.

And we have just learned that the largest, smallest and intermediate axis which will define Sigma 1, Sigma 3 and Sigma 2, respectively of these stress ellipsoids, this one, this one and this one are known as principal stress or principal axes of stresses of the stress ellipsoid. This is something which is very interesting, we will again learn about it later in the next lecture. But let us have some very basic ideas.

That the stress and strain ellipsoids, they look very similar. I just changed the colours, but they look very similar. Their physical appearances are very similar, their mathematical descriptions are also very similar. However always remember that these are different. A stress ellipsoid may not lead to a strain ellipsoid, that I just said that rocks are not deforming. An important the shape and the orientation of the strain ellipsoid may be very different to those of a stress ellipsoid responsible for the strain.

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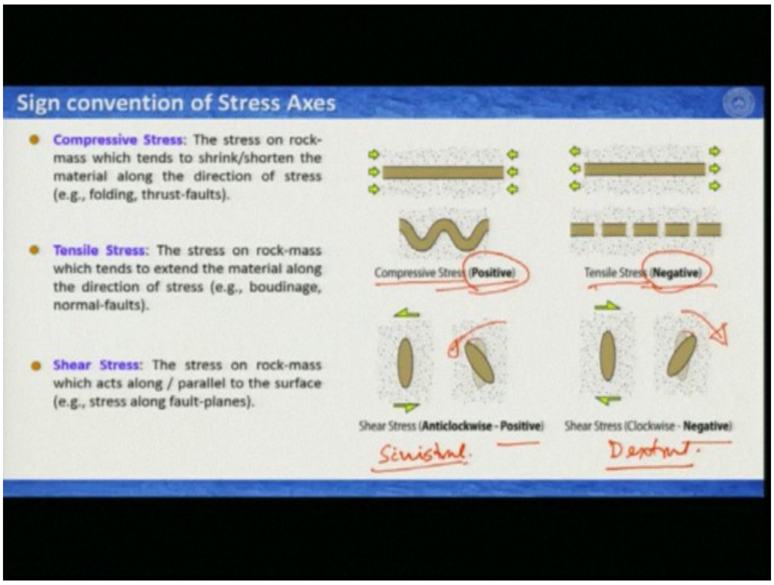

And now we see that what are compressive stresses, tensile stresses, shear stresses and what are their sign conventions. Now compressive stress is the stress on rock mass which tend to shrink or shorten the material along the direction of applications. And if you have a domain of compressive stress, you form structures like folds, buckle folds or thrust faults. So for example, he if I have a layer embedded in a body, embedded in a matrix and if I apply the stresses directing towards each other, I will produce a fold and it is a compressive stress.

If there is tension on the same layer, then I may produce a structure which we call boudinage, we will learn about it soon. Then stress is tensile stress. In structural geology interestingly, we consider compressive stress as positive and tensile stress as negative. If you read literature of any other engineering subjects or physics like mechanical engineering, material science, you will see their conventions are exactly opposite. Compressive stress is negative and tensile stress is positive and this is because we learned later that tensile strength of any material is less than its compressive strength. And because material scientists or mechanical engineers they everyday use or their applications are mostly with the practical materials, so therefore they are most concerned with the tensile stress.

And in earth most of the stresses are mostly compressive. So we deal mostly with compressive stresses. So mostly to deal with some sort of mathematical easiness we use compressive stress as positive and tensile stress as negative. Now, shear stress is the stress on rock mass, we have learned that acts along or parallel to the surface. That means you can consider the stress along the fault plane.

In this case, if I have shear stress working like this on this elliptical object and the rotation of this elliptical object due to application of shear stress is anticlockwise, then we consider is positive, if not, that means if it rotates clockwise, than we call it negative. Though we do not call it positive or negative way, general in structural geology we call this type of, that means when top part goes towards your left side we call it sinistral and this one we call it dextral, we learn about it soon.

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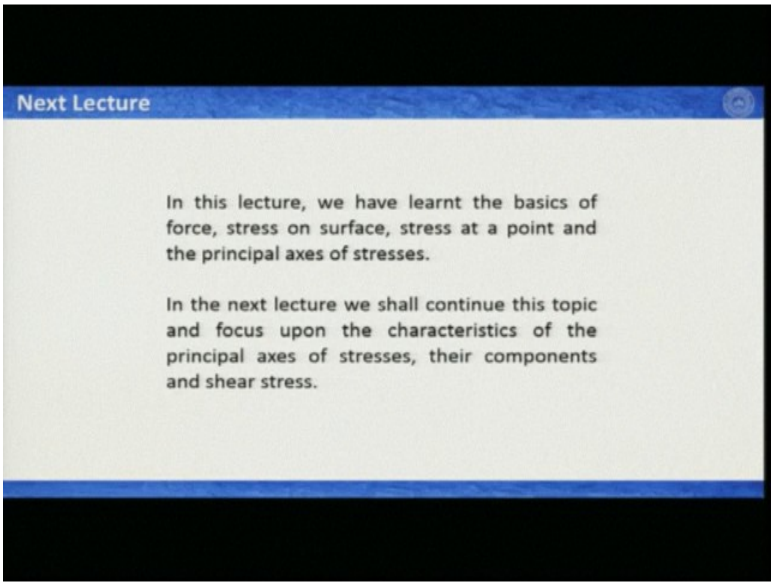

So with this note, I conclude this lecture and in this lecture we have learnt the basics of force, stress on a surface and stress at a point and we also learnt that why is stress on a surface is vector and stress at a point is a tensor. This is something confusing, but this is how it is. And we also learnt what are principal axes of stresses.

But we learn about it soon. So this is the topic of next lecture, we will focus upon the characteristic of the principal axes of the stresses, how they are derived, what are different mathematical considerations involved, their different components and we will also look at in detail the shear stress components. Thank you very much, see you in the next lecture.