**Structural Geology**
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**Lecture 05 - Concept of Strain and Deformation (Part-1)**

Hello everyone! Welcome back again to this online Structural Geology course and today we are at our lecture number 5.

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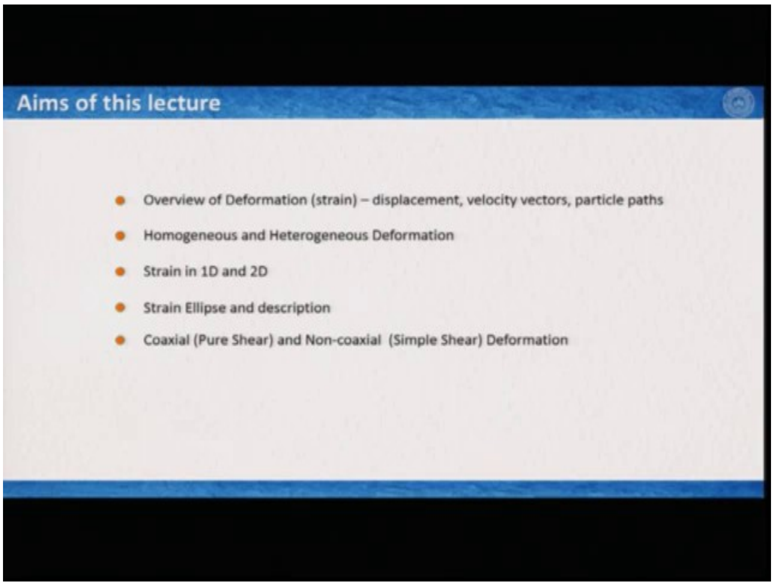


Today we will cover concept of strain and deformation, part-1. The topics we will learn today are mostly of overview of deformation or what is strain. Then, we will look at this displacement, velocity, vectors, particle paths etc. We will have some basic understandings on homogeneous and heterogeneous deformation. We have learned it before but we will learn it in a different way today. Then we will look at strain in one dimension and two dimensions. We will look at strain ellipse and its descriptions and finally we will conclude this lecture with coaxial or pure shear and non-coaxial or simple shear deformation.

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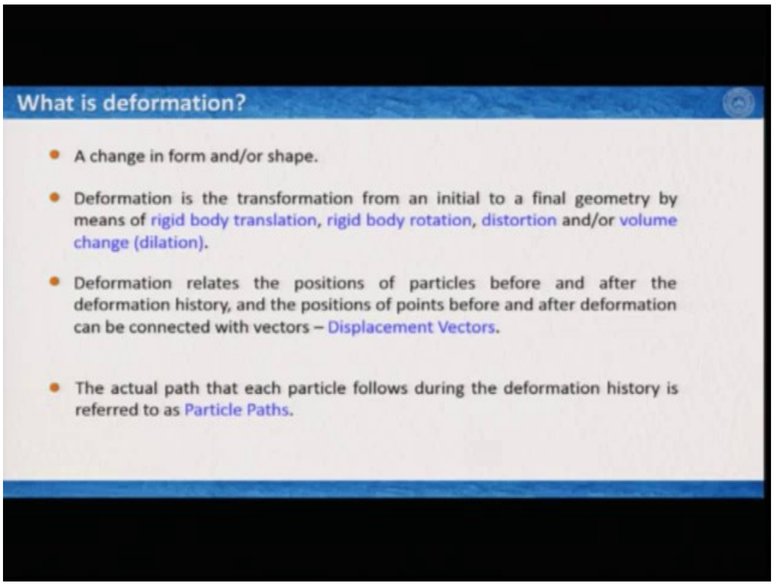


So, what is strain or what is deformation? In a very simple way, it is defined as a change in form or shape. That means I have a form; I have a shape. Due to some applications of external force this form or shape can change and that change if someone describes quantitatively is strain or deformation. Now deformation is the transformation from an initial to a final geometry. So, form or shape change is essentially change of the geometry and this can happen in various processes.

One is rigid body translation, it can happen via rigid body rotation, it can happen via distortion and it can happen also with volume change or dilation. Now all these four individual processes can happen individually or they can club together. For example, you can have the combination of the rigid body translation and rotation together, you can have rigid body rotation and distortion together and so on.

As it is an analysis of geometry, quantitative analysis of geometry, it relates the positions of the points or particles of the body, initial body before and after the deformation history, and the positions of points before and after deformation can be connected with a victor and this victor is known as displacement vector. We will learn about it soon.

Very similarly, from one point to another point when it moves and when it just connect one point to another point by a vector but not necessarily it moves from here to here following a vector, it may move in a different way following a different path. Not necessarily along the victors. So, the actual path that each particle followed during the deformation history is referred as particle path. So, in the following slides what we are trying to learn that what would be the displacement vectors and what would be the particle paths for rigid body translation, rigid body rotation, distortion and volume change.

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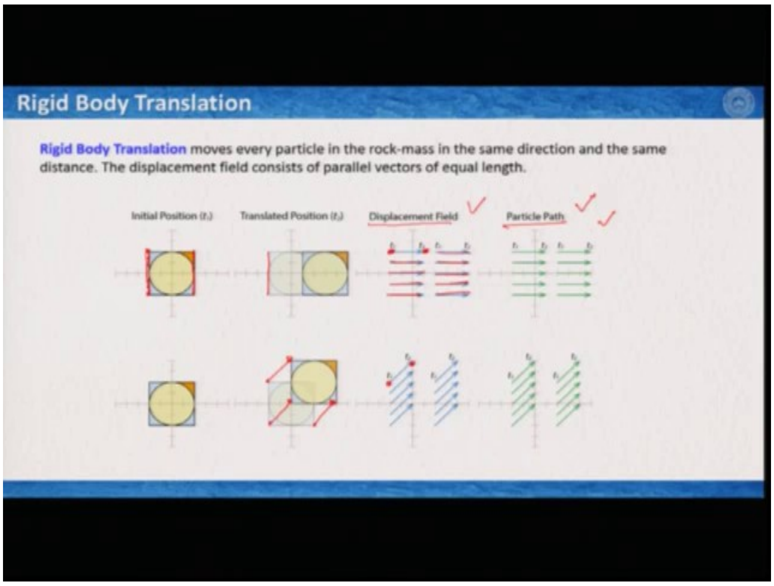


Rigid body translation is defined as it moves every particle in the rock mass in the same direction and in the same distance. So the displacement field consists of parallel vectors of equal length. I have two examples here; in the first column we have initial position or initial form or initial shape of this object. What I have here, I have a coordinate frame and then I have a circle inscribed and I have a square. To mark the orientation I have made one end of this square as orange. Now I translated position which is column two is at time 2.

So position 1 was at time 1 which is T1 and position 2 was T2. The translated position is given by this and the initial position is somehow made transparent. Now what I have done in the next two columns, the first one is displacement field, second one is particle path. I have considered, let me show it here these two sides of this form and then due to translation I have taken some points here. And how these points moved with respect to the reference frame?

So as per the definition of the displacement vector or the displacement field, initial point and end point if I connect them through a line and at a vector it is displacement field, and particle path again from the definition it is the path it followed from T1 to T2. What do you see here as is the definition of rigid body translations, this was your point at T1 and this is a point at

T2. So each and every case it moved very similarly, similar distance and they are parallel to each other.

The particle path is also very similar. The second example, it moved along the north, east direction or it moved towards the first quadrant, so this point has moved to here, this point has to moved here and this point has moved to here and so on. And if you represent it in the displacement field it appears this way, so this was the initial point at T1 and this is your end point at T2. So this is how in rigid body translation the displacement field or displacement vectors and particles paths look like.

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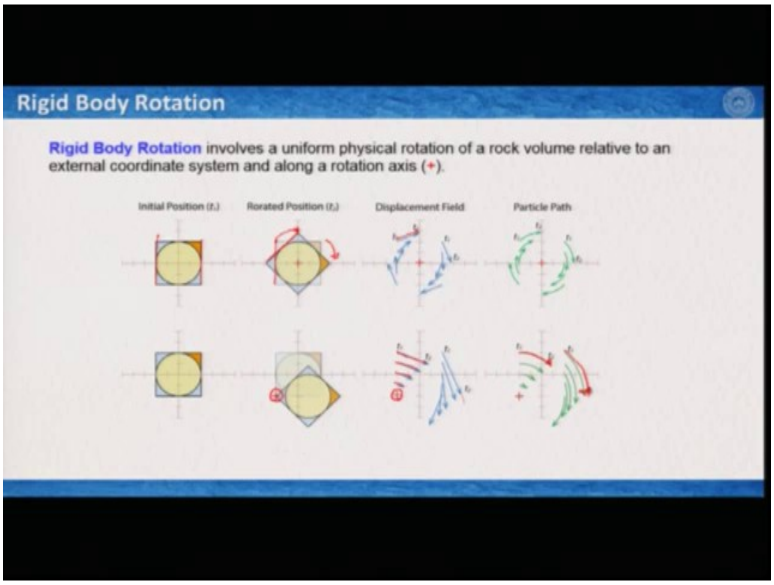


Let us have a look how does it work with the rigid body rotation. To do a rotation you need an axis of rotation. Here I have again cited two examples and in rigid body rotation it involves the uniform physical rotation of rock volume relative to an external coordinate system and along a rotation axis. So in the second column which is your rotated position at T2, time 2, the rotation axis is marked by this red cross. So in the first one it rotated keeping the origin of this coordinate system and it rotated 45 degree clockwise.

So again if I consider this line and this line of this initial position and I try to see that how this line has moved to this line and then I connect the individual points, the displacement field that is point 1 to point 2, point 1 at T1 and point 2 at T2 then it would look like this. So this is how it shows and you can see that it is showing a clockwise rotation. But particle path not necessarily it has to follow a straight line because this point moved to this point not necessarily by a straight line, it actually moved following a curve line.

Therefore particle path would not be a straight line but a curve line. In a very similar way in the second illustration you have here your rotation axis at the one end of this object and then it rotated 45 degree clockwise. So what do we see here? The rotation axis is here and it moved this way. When we consider the displacement field all are straight lines and as you can see we are approaching to the rotation axis. The magnitude of these vectors are reducing towards the center of the rotation axis. On the other hand the particle path essentially would be curved and they should look like this.

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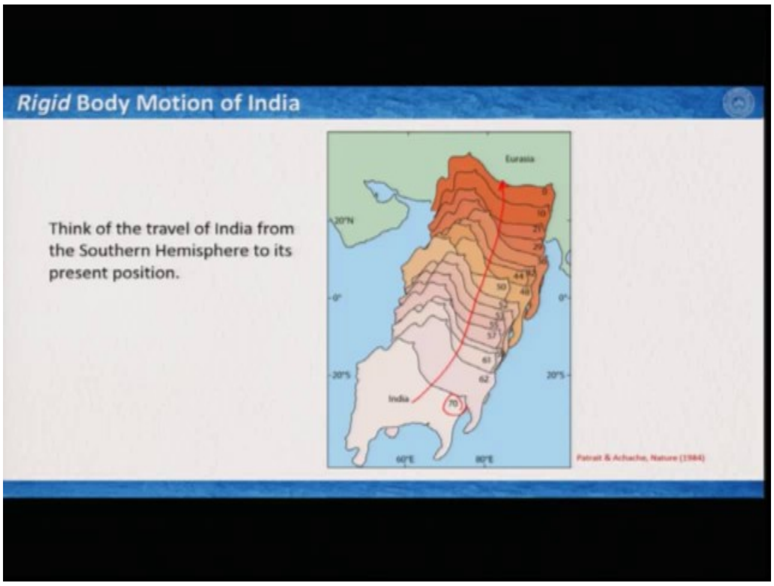


Now if we consider that what could be the examples of rigid body translation and rigid body rotation, I would request you to think the great movement or great travel of our Indian plate from the southern hemisphere to where it is now. So Indian plate was initially positioned at 70 million years ago, somehow like this in the southern hemisphere. With time it moved this way here. So it is actually if you considered Indian plate was rigid, one can explains its movement through rigid body rotation and rigid body translation, it is a very simple example. But let us have a look what happens when we talk about deformation or distortion and volume change.

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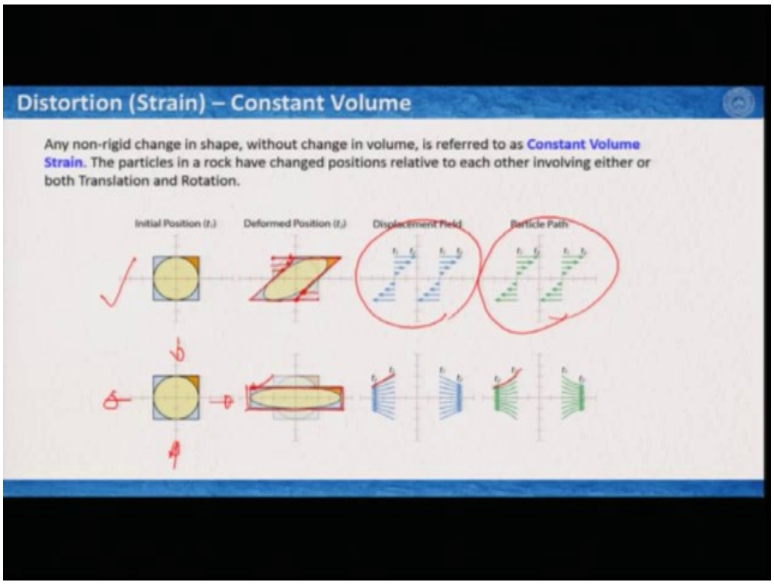


So distortion or strain at constant volume is defined as any non-rigid change in shape without change in volume is referred to as constant volume strain. The particles in the rocks should have changed positions relative to each other involving either or both translations or rotations. There are series of end members, here I have given, the are 2 end members one is simple shear, another is a pure shear, we will learn about it later. But let us have a look how the displacement field and particle path look like.

So again the concepts or initial considerations are very similar. This is your initial position or initial shape or form and then if I make this object like this without changing the volume, that means it is a constant volume strain, then the particle paths, I am sorry the displacement field would look like this, that means from this side it move this way, from this side it moved in the opposite way and this is how the particle path would look like, I am sorry displacement field would look like.

The particle path would look like in a very similar way. For another type of distortion or strain in this case what we see that if I compress this shape and extend it towards this side then it takes this rectangular shape, or the circle becomes an ellipse keeping the volume constant and in this case the displacement field would look like this, that it is converging towards the extensional side and particle paths are very similar but they are curved because this point did not move following a straight line but it moved along a curve line. And therefore particle paths for this type of formations are little curved.

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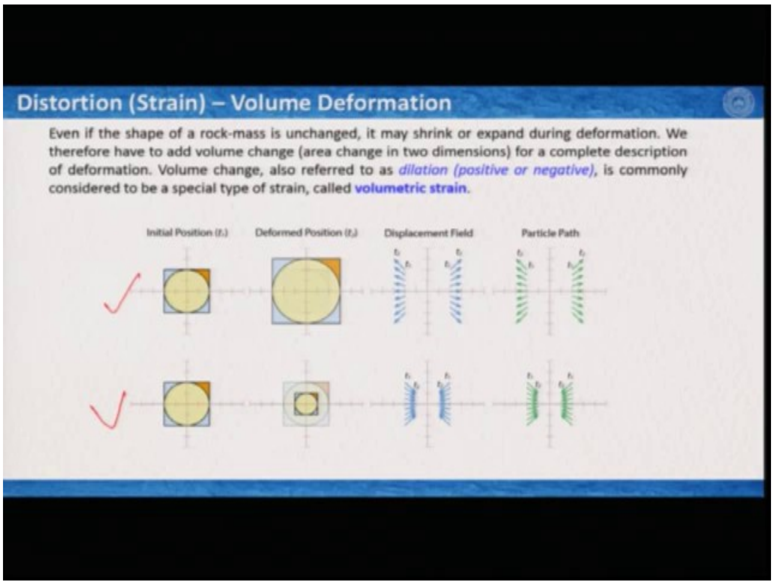


Then volume deformation is something where you do not keep the volumes constant during deformation. So you can keep the shape similar but not the volume. So even if the shape of a rock mass is unchanged it may shrink or expand during deformation. We therefore have to add volume change that means area change into dimensions for a complete description of the deformation. So volume change generally refers or also it is referred as dilation. So if the volume increases then we call it positive dilation or positive volume change and if it is, if the volume decreases or shrinks then it is negative dilation or negative volume change and together it is referred as volumetric strain.

And here we have a very similar examples, I have the very similar configurations, two configurations. In the first case if I look at here the volume expanded, therefore the displacement fields and the particle paths are outwards. In the second case the volume shrink and therefore both displacement field and particle path are towards the center of this object. Now given these concepts of rigid body translation, rigid body rotation, distortion and volumetric strain, you may think that do we apply it in geological contexts? The answer is yes. So here we will see few examples that where and how we can apply these concepts of translation, rotation this to a rigid body and distortion and volume changes.

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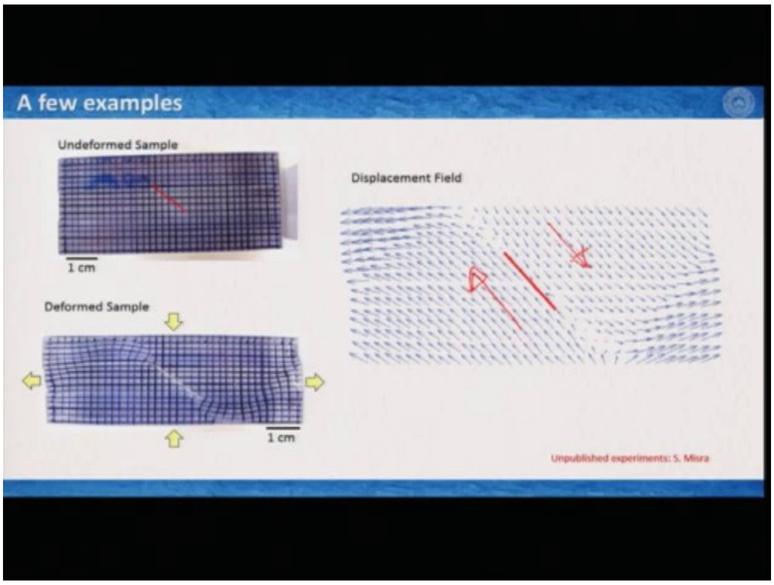


So here is an example of experiments that I performed in the laboratory. What do you see in this undeformed sample? This is a polymer (PMMA) polymethyl methacrylate, it has a cut in the middle like this and then I made some grids, square grids, each grid is about 2 millimeter squares, and then I compressed it. And when I compressed because of this cut in the middle there are some distortions in the deformed sample.

Now it is possible because I have the initial shape and I have the deformed shape and considering each of these grids, I can actually determine the displacement vectors of this deform sample and if I do it then it looks like this. As you can see along this cut all these materials on the top side of this cut moved downwards, the field at the bottom of this cut moved upwards and this is how we can interpret what was deformation at different points of this sample.

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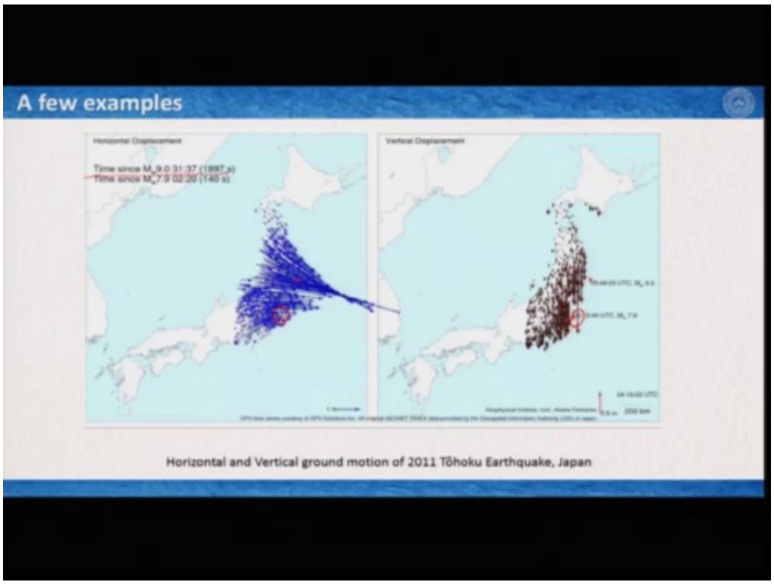


Now this is an example of an earthquake. You know when this earthquake happens the psychologists generally describe these earthquakes on the surface in 2 vectors, one is vertical ground motion, another is horizontal ground motion. So they actually try to look because of the earthquake how much vibrations you have vertically, how much vibrations you have horizontally. So this is an example from 2011, the Tohoku-Oki earthquake in Japan, this is a movie of about one minute.

And you can see here that all these arrows which you do not see right now are there in stationary mode, we will see the main earthquake and then aftershock and we will see how vertical and horizontal ground displacements would happen during the earthquake. So on this side you have horizontal displacement and on this side you have vertical displacement, let us have a look. So the earthquake happened, these arrow heads are exaggerated and there is a time laps, you can read the time here.

So this is due to main shock and then you had a second shock somewhere here with the magnitude of 7.9, the first earthquake had magnitude of 9 and this is how one can interpret by this rigid body translation that how much vertical and how much horizontal displacement you can have during an earthquake.

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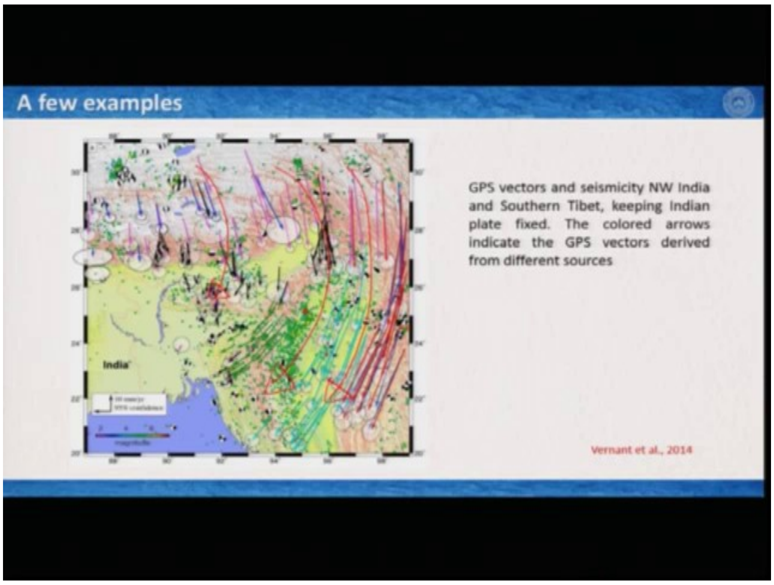


Now again an example from Indian continent. Now GPSs are something that can give you that with time how displacements are happening or how a continent is suffering or enjoying, you never know through strain. So here is an example from a very fascinating paper, what do we see with all these arrow heads that how Indian continent is moving towards the Tibetan plate. Now this is with respect to a reference point and therefore it is showing in a different way but if I stand on Indian sub-continent the arrow heads would be in a different direction.

But however let us have a look, we have rotational movement of Indian sub-continent. So it is actually with reference to an external frame, it is moving this way. So these GPS vectors and seismicity of north-western India and southern Tibet where you keep the Indian plate fixed and then you can have different vectors from the GPS data which are nothing but what we have learnt as rigid body translation or rigid body rotation or combination of these two. So with this background we will move on.

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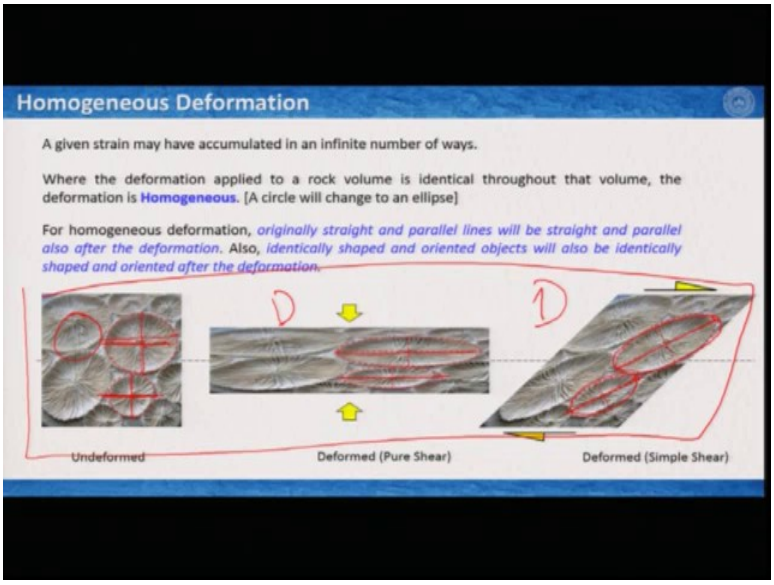


And we will look at the different concepts or different aspects of deformation. The first thing we will takeover is homogeneous and heterogeneous deformation. In one of our first lectures we have learned what is homogeneous and what is heterogeneous. Now we will describe them in detail and understand them in the context of strain. So in a body strain can be accumulated due to application of external force in many different ways. But where the deformation applied to a rock volume is identical throughout the rock volume then the deformation is known as homogenous deformation.

So for homogeneous deformation characteristically you can consider or you can appreciate a homogeneous deformation if originally straight and parallel lines remain straight and parallel after the deformation. You can also consider identically shaped and oriented objects will also be identically shaped and oriented after the deformation. So these two are the key parameters to assign a homogenous deformation or in a very general way that each point of this body have suffered or enjoyed similar ways of deformations.

So here are some examples we have an undeformed object here, I just took a photograph and then mark these circular features in this image with some circle and then made two diameters which are perpendicular to each other. Now I compressed this using a computer program and then I see that it remained deformed keeping the volume of this area constant. So it is volume constant deformation and the question is, whether it is homogeneous deformation or not?

Now for homogeneous deformation the definitions say that originally straight and parallel lines should be straight and parallel lines after the deformation. So this was originally a straight line it is also a straight line, it was parallel to this line it is also parallel to this line. So in that context it is homogeneous. The second context was identically shaped and oriented objects will also be identically shaped and oriented after deformation.

What we see here, these were circles and after the deformation all of them become ellipse. So shape has changed but they changed in a similar way. In the second example which is a simple shear example what we see here, these two were parallel lines. In this case they rotated but they still become parallel they still are parallel. And the shapes are also similar in these cases.

So from the definition we can say that these features that we see here, this is the original and then deformed two different ways, these two are homogeneous deformation or these two represent homogeneous deformation.

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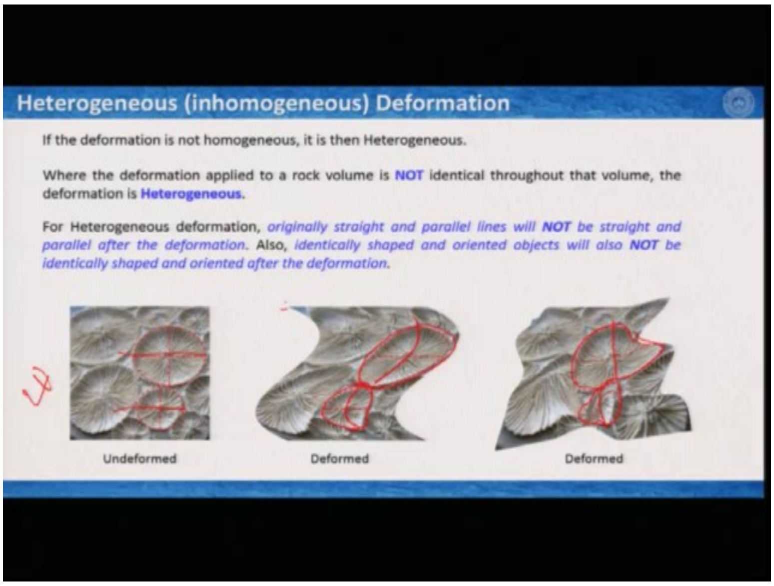


For heterogeneous formation it is as simple as you can define it if the deformation is not homogenous then it is heterogeneous deformation. People also refer it as in homogeneous deformation. So in a very similar way where the deformation applied to the rock volume is not identical throughout that volume, the deformation is heterogeneous, and you can again say that for heterogeneous deformation originally straight and parallel lines will not be straight and parallel after the deformation and that is applicable for the shapes as well.

So initially shaped and oriented objects will also not be identically shaped and oriented after the deformation. And here is the example of heterogeneous deformation. So I have a very similar object and using a computer simulation I deformed it into two different ways. What we see here that apparently this geometry looks probably it going to give you a homogeneous deformation but it is not. These two lines were parallel to each other but in this case in the deformation they are not parallel to each other. The shapes here also changed.

So therefore this is essentially heterogeneous deformation and in the next case which is sort of crumbled but the volume more or less remains same. But you can see here the parallel lines are not parallel and also the shapes are different in different objects which were initially similar to each other. So now we have a very good understanding I believe on what is homogeneous deformation and what is heterogeneous deformation. Let us have a look in a different way.

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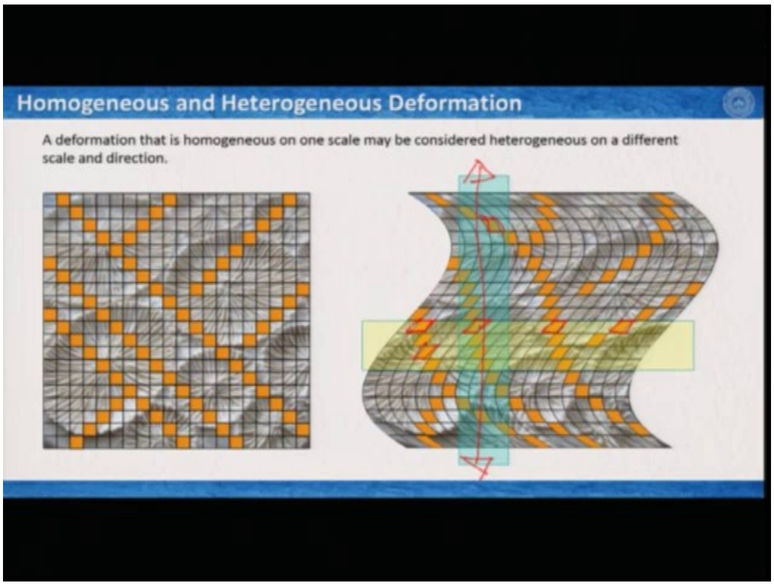


What I have done here in this drawing that I gridded this image and with some square grids and field some of the grids not randomly, with orange colors. Now what I would do, I would deform this in heterogeneous manner and if I do it then it looks like this. Now the question is, when everything or the bulk deformation appears to be heterogeneous it is possible to discretize this entire deformation in smaller areas and find if they are homogeneous or not. For example if I consider this area highlighted by this little rectangle, I can see that initially this was square.

So these two are not similar at all, so along this direction the deformation or if I consider this area the deformation is heterogeneous. However if I try to see here along this area this yellow shaded area I see here that, here the squares have transformed to this shape here to this shape here to this shape and they are all identical. The parallel lines also remain parallel and their shapes also remain very similar to each other.

So it does not matter how heterogeneous is you deformation. It is possible to discretize it in small areas and understand that in terms of homogeneous deformation. That makes life easier for analyzing the deformation and also for mathematical descriptions.

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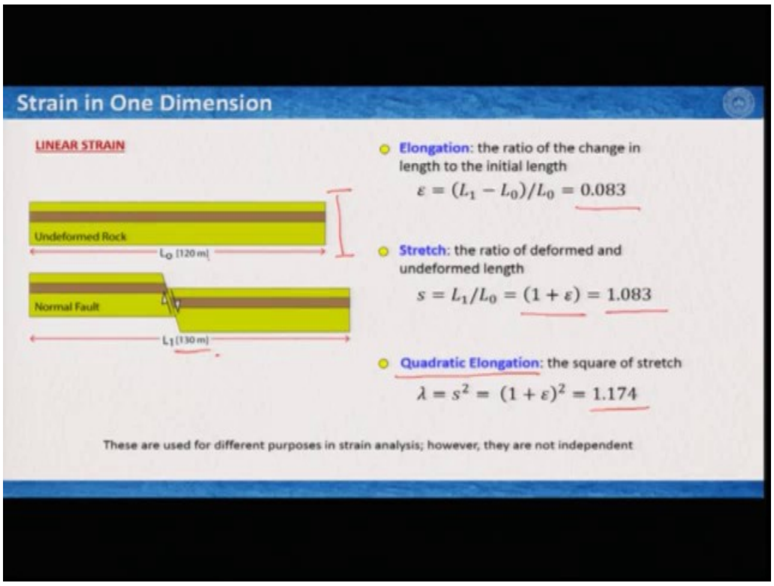


So strain in one dimension as it is one of our topics, we will first understand in terms of one dimension then we will go to two dimensions and then in the next lecture we will learn in three dimensions. There are 3 basic terms one is elongation, one is stretch and another is quadratic elongation. So here I have a representation of un-deformed rock with length say 120 meters L0 and if I make a normal fault on these layered rocks then say the length changes to 130 meters because it is, it has extended.

Now how to describe or how to quantify the strain? One parameter as I said is elongation which is the ratio of the change in length to the initial length and if we calculate this for these values which are given here it would be 0.083. As it is ratio of 2 length parameters it is dimensionless. The stretch is the ratio of deformed and un-deformed length. So L1 versus L0 or you can algebraic manipulate it to get this form 1 plus elongation and in our example this is 1.083, again it is dimensionless. And quadratic elongation is nothing but the square of the stretch and in our example this is 1.174.

As you can see that these three parameters they are not independent, if I know one then I can also get 2 other parameters very easily however they are used for different reasons and different purposes in analyzing strain.

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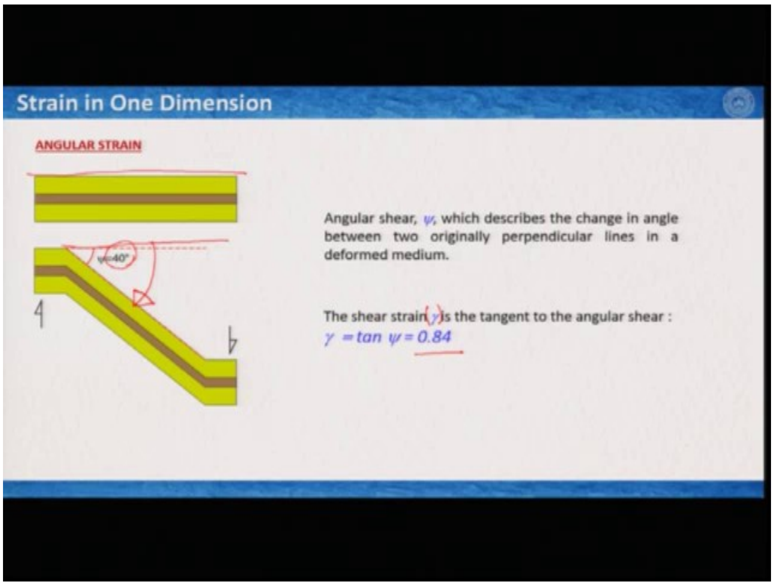


Now angular strain is defined as some sort of deformation which is not linear. So angular shear which is in this case in this diagram which is 5 and in this case this is 40 degrees is the change in angle between two originally perpendicular lines in a deformed medium. So what do we see here this was the line and then it is deviated to this way. So this angle in this case 45 degree, so strain the angular strain which is known as shear strain or we generally say it gamma is the tangent of this angular shear angle. So in this case because this is 40 degrees so it is 0.84 again it is dimensionless.

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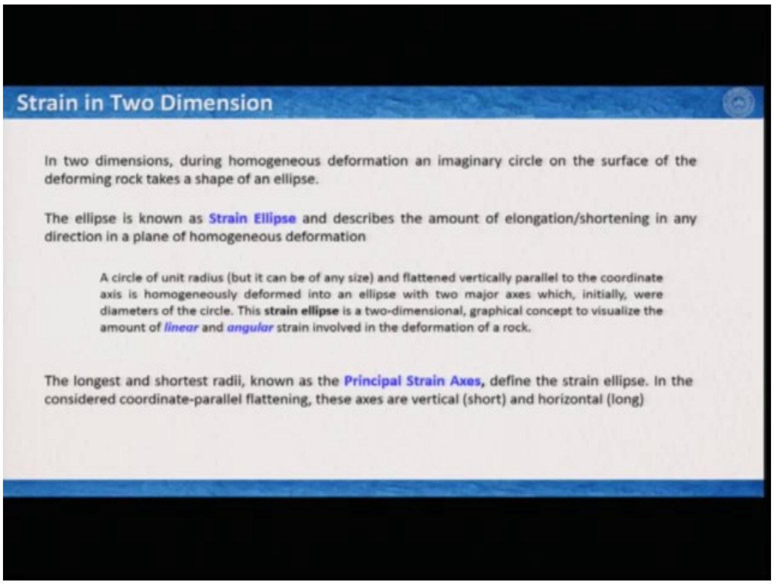


Now if we look strain in 2 dimensions, in 2 dimensions during homogeneous deformation and imaginary circle, on the surface of the deforming rock takes a shape of an ellipse. Now this ellipse is known as strain ellipse and describes the amount of elongation and shortening in any direction in a plane of homogenous deformation. Now we will see the descriptions of the strain ellipse in the following slides:

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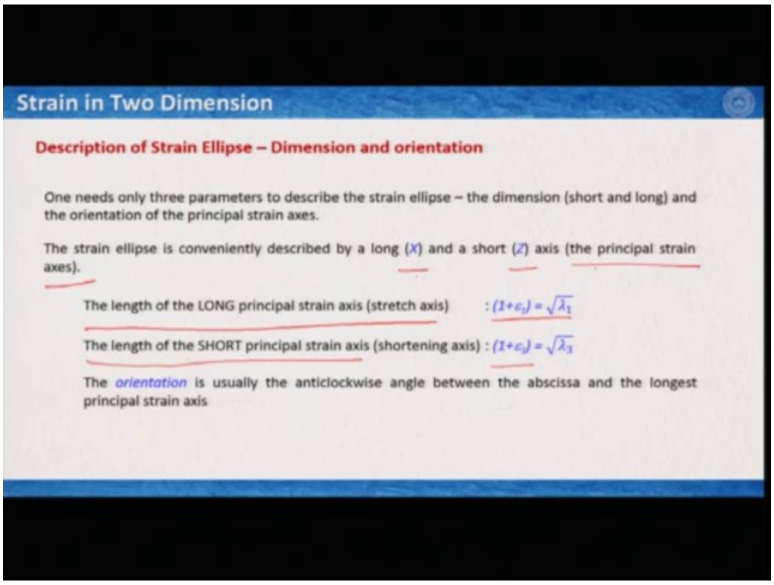


So if I consider that the 3 parameters are required to describe a strain ellipse, the dimensions which is the long axis, short axis and also the orientation of the principal strain axis, so the strain ellipse is conveniently described by a long and short axis. So in our example we will use it X and Z you can ask where is Y, now Y we kept it for 3 dimensional deformation. So and the principal and we will see later that this X and Y directions are actually your principal strain directions, I am sorry X and Z directions.

So the length of the long principal strain axis, we can call it stretch axis or elongation axis is defined by 1 plus epsilon 1 or square root of lambda 1 and the length of the short principal axis can be defined as 1 plus epsilon 3 or square root of lambda 3. So the orientation is usually the anti-clockwise angle between the abscissa and the longest principal axis.

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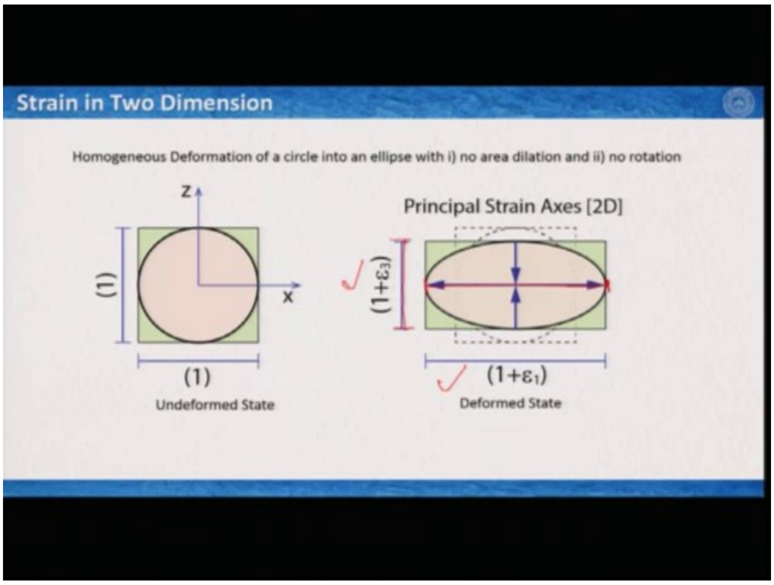


So this is how the pictorial description of the principal axis in 2 dimension of a deformation of a circle. Now if this is the circle and then if I deform it then keeping the volume constant it takes the shape of this ellipse, now this is the long axis of this ellipse which is 1 plus epsilon 1 and this is the short axis of this strain ellipse which is 1 plus Epsilon 3. And these two are your principal axis of strain in 2 dimensions. So let us have a look how we can describe the strain ellipse with respect to the principal strain axis.

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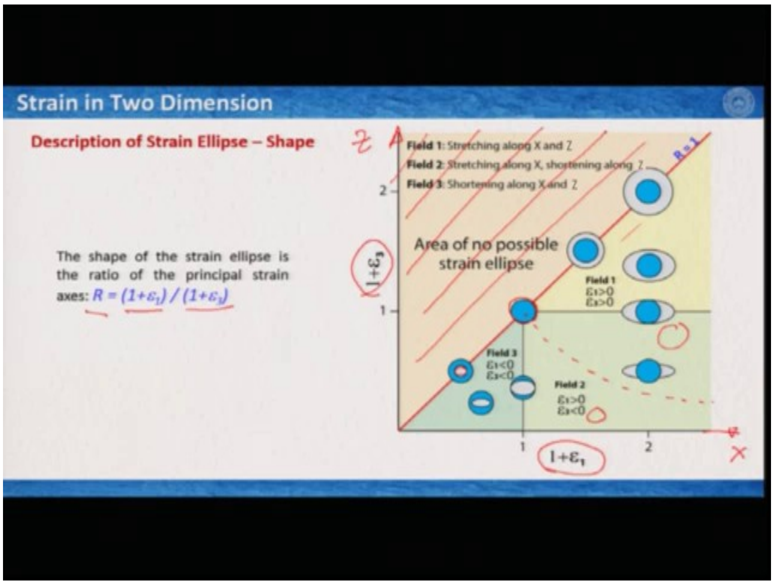


So it is very important that how do you define the shape of this ellipse, now the shape of this ellipse as it is done in mathematics or in geometry it is defined by the ratio of the long and short axis. So in this case we define it as R which is the ratio of long axis 1 plus E1 of the strain ellipse with the short axis which is 1 plus E3 and if we plot them, here 1 plus E1 along the X direction and here 1 plus E3 short axis along the Z direction.

Then we certainly get this 45 degree line which is R equal to 1, that means along this line no ellipse would be produced because your long axis and short axis have similar value. So this value 1, 1 plus E3 and 1 plus E1 where it is 1, it would be a circle. But if we increase them along this line both E1 and E3 would increase equally. So this originally orange I am sorry originally blue circle would take a shape of circle but with positive volume strength.

If we come towards the down side where E1 and E3 have values less than 1 then it would shrink to produce a smaller circle, now in this domain that is above this R plus, R equal to 1 line any strain ellipse is not possible and that is because the consideration that E1 is the longest axis and E3 is the shortest axis. Then the bottom side of this R equal to 1 line we can divide it in 3 domains field 1, field 2, and field 3.

In field 1 epsilon 1 and epsilon 3 both are greater than 0 so that means here you will have your volume increased. In field 3 both epsilon 1 and epsilon 3 have values less than 0 that means your volume would shrink. In field 3 E1 and E3 have opposite sensors, that means E1 is greater than 0 and E3 is less than 0. So at the top part you can have your volume increased, at the bottom part you can have your volume decreased and there should be a line along which there should not be any volume change.

This diagram is also known as Flinn diagram in 2D. We will learn about this Flinn diagram when we learn more about strain in 3 dimensions.

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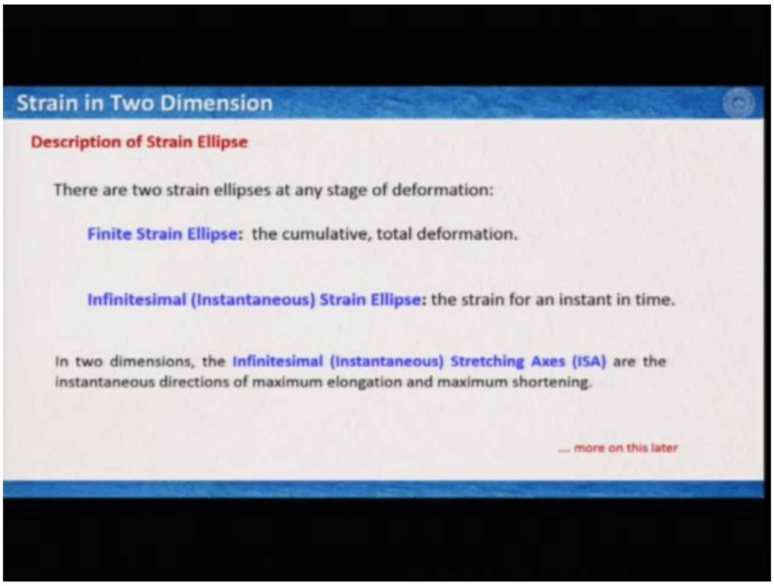


Here I try to give you some sort of description strain ellipse with 3 terminologies, we will learn about it later but I think it is important at least to introduce you with the terms. So there are two types of strain ellipses at any stage of deformation. One is finite strain ellipse that means cumulative or total deformations. So it sums all deformations and the end product or end ellipse that you get is finite strain ellipse and then there is infinitesimal or instantaneous strain ellipse.

So that is instantaneous, so the strain ellipse for any instant of time. Now in 2 dimensions there is also one term that we call infinitesimal or instantaneous stretching axis and these are the instantaneous deductions of maximum elongation and maximum shortening. We will learn more about it later.

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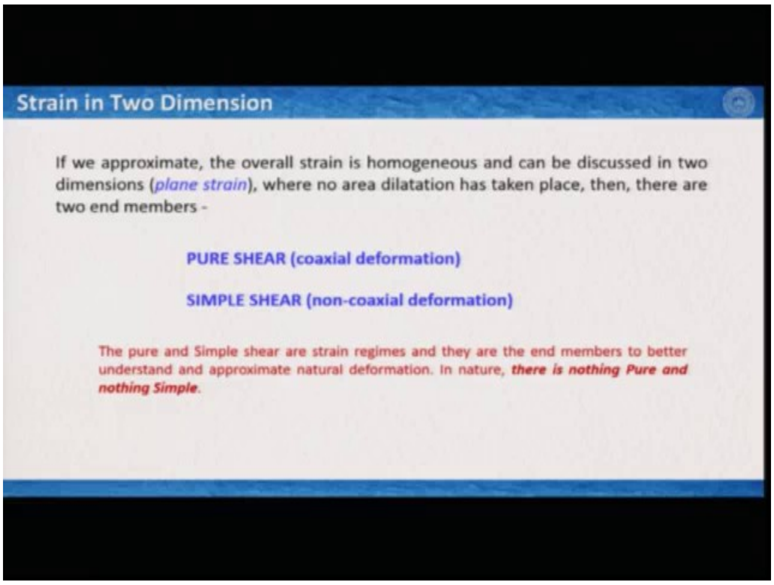


Finally we will learn about what is pure shear and what is simple shear. So if we approximate the overall strain is homogeneous and can be discussed in 2 dimensions, that means it is plane strain, your Y axis has no deformation then no area dilation has taken place that is also a consideration. Then in the inter-range of 2 dimensional homogeneous area constant deformation has 2 end members, one is pure shear which is coaxial deformations sometimes referred to, and another is simple shear or non-coaxial deformation.

I must remind you that this concept of pure and simple shear are strain regimes and they are just the end members to better understand or mathematically define or approximate the natural deformation. In nature it hardly happens in pure shear and simple shear manner, or other words you can say that there is nothing pure and nothing simple in natural deformation.

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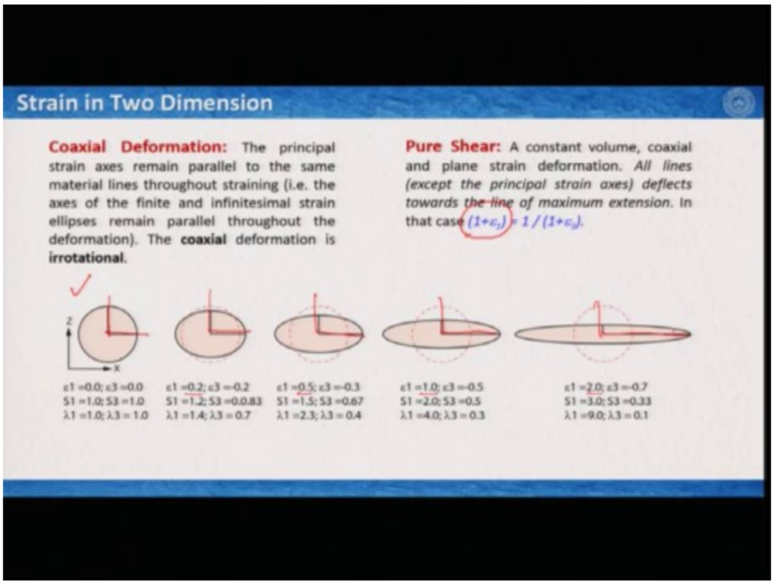


So a coaxial deformation is defined as a principal strain axis remained parallel to the same material lines throughout the straining process. That means the axes of the finite and infinitesimal strain ellipses remain parallel throughout the deformation. So the coaxial deformation is irrotational, that means the principal axes of strains do not rotate. It also can be defined as a pure shear deformation where at constant volume coaxial and pure strain and plane strain deformation, all lines except the principal strain axis deflects towards the line of maximum extension, and you can define mathematically as your principal elongation axis 1 plus E1, actual the inverse of principal shortening axes.

So here is the example the first one is an un-deformed state, and what I have given E1 is the elongation along the X axis, E3 is the shortening along the Z axis, S1 and S3 are stretches in the corresponding X and Z axes and lambda 1 and lambda 3 are the quadratic elongations along the X and Z axis respectively.

What we see here if we consider the X axis then this is, this has experience strain 0.2, this has experience strain 0.5, this has experience strain 1 and this has experience strain 2. What is interesting here as per the definition of coaxial deformation and pure shear that these two axes they have changed their length. But they did not rotate and therefore this deformation is coaxial and irrotational.

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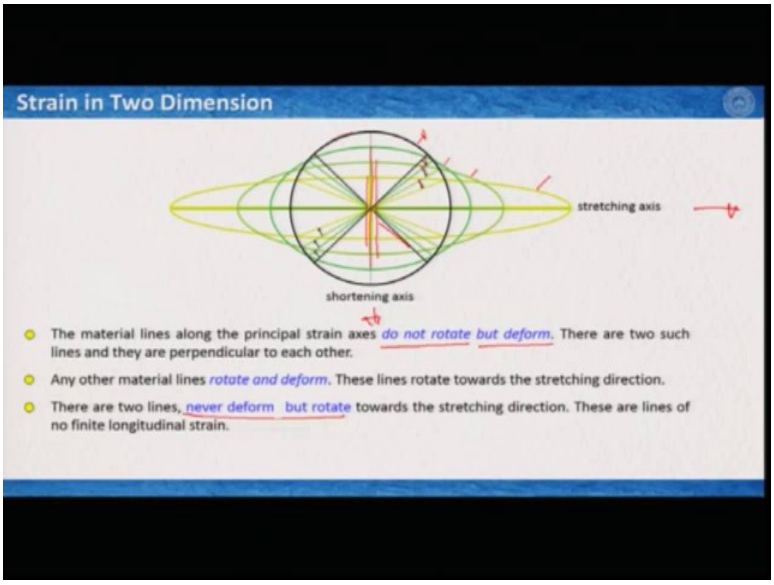


To understand this coaxial deformation or irrotational deformation or pure shear let us have a look of this illustration or this combination of all this strain ellipses in a different way. So this black circle is your initial un-deformed strain ellipse or strain circle and then slowly this green, light green and even lighter green are different progressive stages of shear deformation.

This is your stretching axis, stretching direction and this is your shortened direction. Now what we see here there are 3 different kinds of material lines possible in this description of these strain ellipses. Now certainly you have learned about the principal axes of stretches. So the material lines along the principal strain axes they do not rotate but deform. For example, this line now deform to this then in the next it deform to this and in the third it deforms to this length.

Any other material lines than these two principle axes of strain axis they rotate and they also deform and these lines rotate towards the stretching direction of the principal of the strain ellipse. Interestingly there are 2 other lines that never deforms but rotate. So therefore they always define the diameter of the primary or initial circle. In this case you can see for this circle it is at 45 degrees and what happens with progressive deformation, these lines they do not change their length.

The still are inscribed as a radius of the circle but they continuously rotate towards the extension direction. So in 2 dimensional strain ellipse you have 2 lines that never deforms but rotate, is this one.

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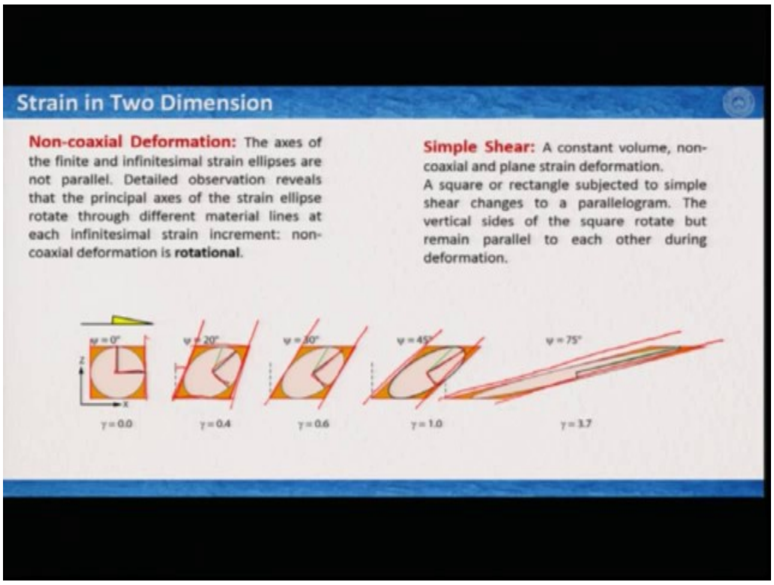


Let us have a description of the simple shear or non-coaxial deformation. Now in non-coaxial deformation the axes of the finite and infinitesimal strain ellipses are not parallel, so detailed observation reveals that the principal axis of the strain ellipse rotate through different material lines at each infinitesimal strain increment. Therefore this is a non-coaxial deformation and it is rotational and it is also known as simple shear deformation because it requires a constant volume, a non-coaxial deformation and a plane strain deformation.

So what we see here in this illustration, a square or rectangle subjected to simple shear changes to a parallelogram as you can see here this is the square and it changes its shape to a parallelogram. The vertical sides of the square rotate but remain parallel to each other during deformation and therefore it is a homogeneous deformation. What do you mean by this, these two lines are the vertical lines, so with deformation it rotates. For example, here the rotation is 20 degrees and therefore the shear strain is 0.4.

But these two lines remain parallel. Again it does not matter how much you rotate them by means of simple shear these two lines remain parallel and therefore this is a homogeneous deformation. Interestingly these two axes you can see they are also rotating with progressive simple shear, therefore this is rotational deformation. We will learn more about it later with time with particularly with when we discuss the 3 dimensional strain.

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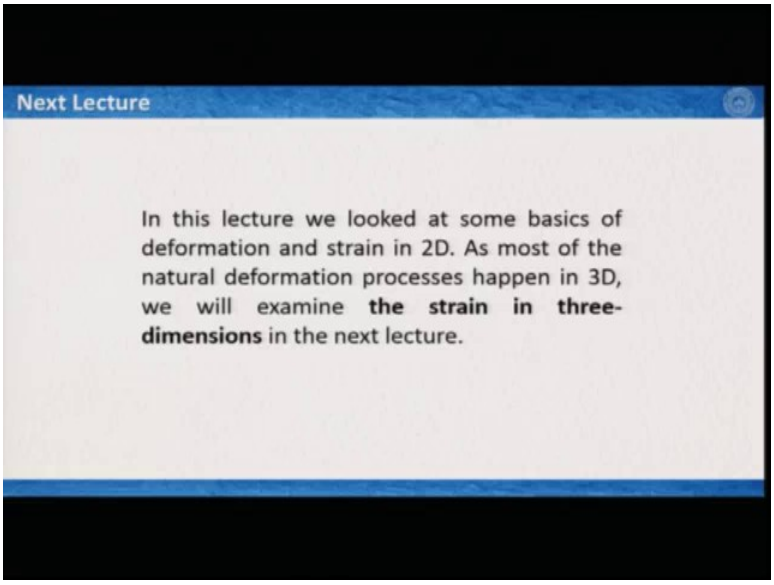


But for the time being I conclude this lecture and in summary in this lecture we looked at some basics of deformation and strain in two dimension. But in nature we know that most of the deformations do happen in three dimensions, so it is important that we describe the strain in three dimensions. And that is the topic of the next lecture. Thank you very much, have a nice time.