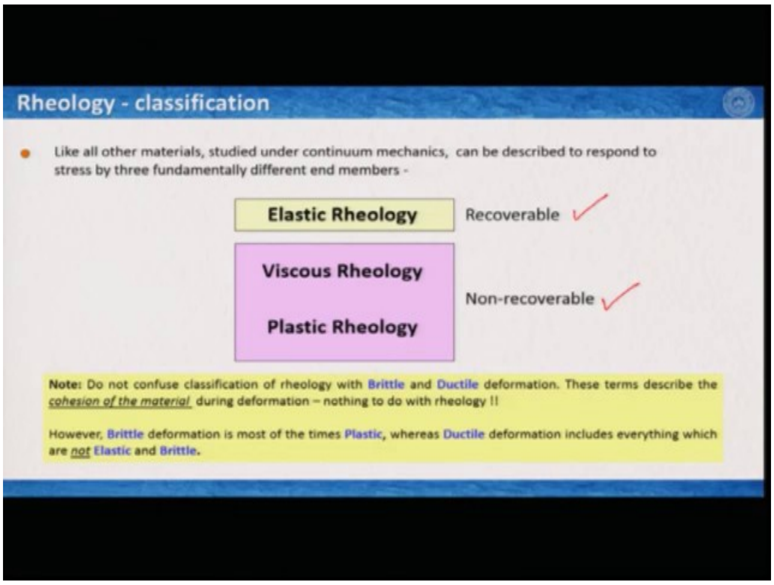
Structural Geology
Professor Santanu Misra
Department of Earth Science
Indian Institute of Technology Kanpur
Lecture No 10.2
Rheology – 1 (Basics of Rheology)

Okay, so with the basic understandings of what is rheology and whether it is applicable to our art systems or not. Now we are in this slide to classify the different types of rheologies that we commonly see in all materials and also within the rocks, so we can have 3 different kinds of rheology one is elastic rheology, then the second one is viscous rheology and the third one is plastic rheology.

(Refer Slide Time: 1:02)



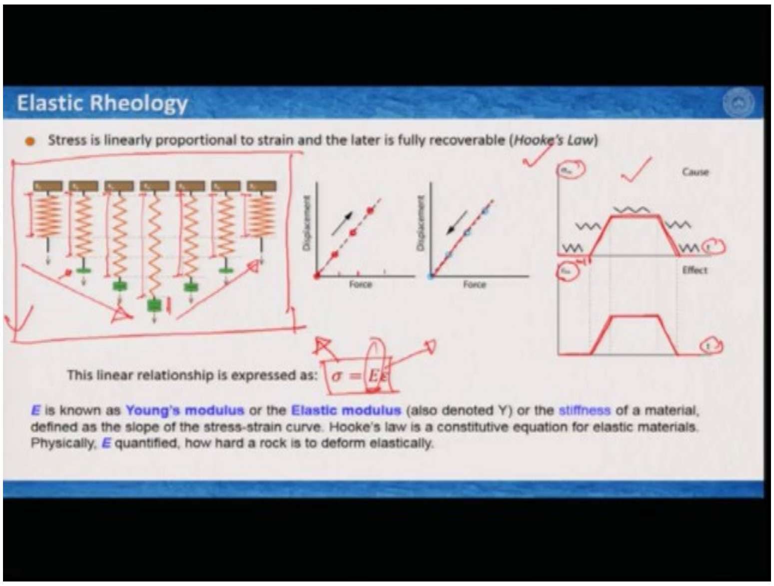
With the course of time we will see that elastic rheology when a material is deforming under elastic manner, the strain or deformation is recoverable and when they are deforming the following Viscous or plastic manners they are non-recoverable. Now, I would like to remind you one very important thing that we commonly mistake in describing structural geological deformation features and with these 3 rheological terms elastic, viscous and plastic we sometimes use more or less similarly the 2 terms brittle, ductile and sometimes brittle ductile.

Now it is very-very important to remember that the classification of rheology has nothing to do with the brittle and ductile deformation. Now brittle and ductile deformation only considers whether the cohesion of the materials are maintained or not during the deformation, that is. It does not consider the rheological part, if the cohesion is maintained then it is ductile, if the cohesion is not maintained then it is brittle.

So, you can say very generally but that is not strictly true for all cases that brittle deformation is most of the time plastic and ductile deflation is mostly include everything which are not elastic and not brittle but remember brittle and ductile these 2 terms have nothing to do with elastic, viscous and plastic rheology.

So, we will now slowly describe the concepts of what is elastic? What is viscous and what is plastic? I will mostly show you the classic considerations of this rheological terms, their analog visualisations that what is best way to represent this rheology with some known materials we have and then we will derive some sort of different material constants or rheological constants, we will see their implications also in the study of structural geology.

(Refer Slide Time: 3:34)



So, let us start with the elastic rheology, now the definition of elastic rheology is given by the Hooke's law and it says that stress is linearly proportional to strength and the later is fully recoverable as I said in the previous slide that in elastic rheology you can recover the strain and the elastic rheology is best visualised by a spring or in other ways you can have any elastic band or rubber band and then if you stretch it, it expands and if you release it, it comes back to its original position but if you stretch it more it does not come back to its original position but that is something different.

Accordingly let us see what we can take out of it, so what we have is this particular group of images here, this is written T1, at T1 we have one spring, it has a finite length and then at T2 I have added one little load here with this green bar because I have added this little load on the spring you can understand if spring is hanging I added load at the end, so spring would expand and therefore the length has changed, if I add more load the spring would further expand and change the length and so on.

Now if I start releasing the load or taking off this green bars one after another, the first one… after the first one it would come back to the load very similar, it showed the expression with the 2 green bars. If I take one more out then I have only one then it would come back to the load that we have or to the shape or to the length that we had with the one bird at the beginning and if we remove all this loads then it would come back to its original position and original shape.

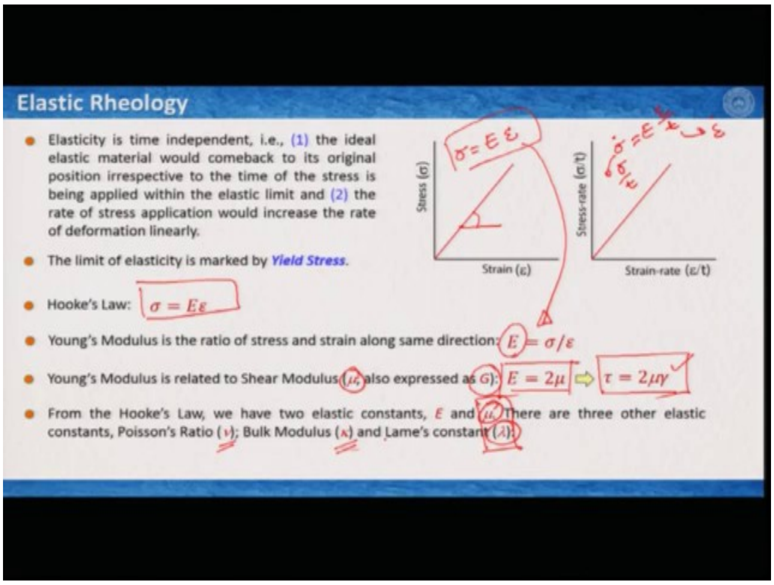
So, if I now plot them in this placement versus Force curve then with the application of force, the 3 different loads I have, when we are loading it then we can get some points and if I connect this points they generally fall in a linear pattern and when I release these loads that means this side they also fall, they come back in a very similar fashion and also maintain or linear relationship.

We can also visualise this image in a different plot what is given here, in 1 plot we have stress versus time, in another part we have strain versus time. This strain you can visualise in terms of elongations, so till this point it wars T1 where it did not have any load, any stress and the strain was also 0. Now then we slowly started applying the load with this green bars and we see this strain also or elongation also increased.

Now if we leave it with the 3 green bars here for quite some time the strain would remain constant and if we release the load by removing the green bars then it would come back to its original position, so what we see that this stress versus strain in this diagram has a linear relationship, so stress disproportional to strain and the linearity constant is defined by E which is Young’s modulus or the elastic modulus or sometimes it is known as stiffness of a material. Now in some books or texts you may find that E has written…people are writing E as Y, so you just have to see what is the common understanding.

The Young’s modulus is defined as the slope of the stress strength curve, we will see later. So this is also known as, this equation is also known as Hooke’s law and it is a constitutive equation because we have dynamic parameter on one side, kinematic parameter on one site and they are related by constant, so physically E is quantified how hard a rock is to deform elastically and that is why the term stiffness came in the picture.

(Refer Slide Time: 9:01)



Now to talk more about elastic rheology as we have seen or we have understand that elasticity is time independent that means does not matter how long you keep the green bars it would stay at its position and then you will remove the green weights or green bars it would come back to its original position, it is not a function of time there for the ideal elastic material would come back to its original position irrespective to the time of the stress it is being applied within the elastic limit and of course the rate of stress application would increase the rate of deformation linearly.

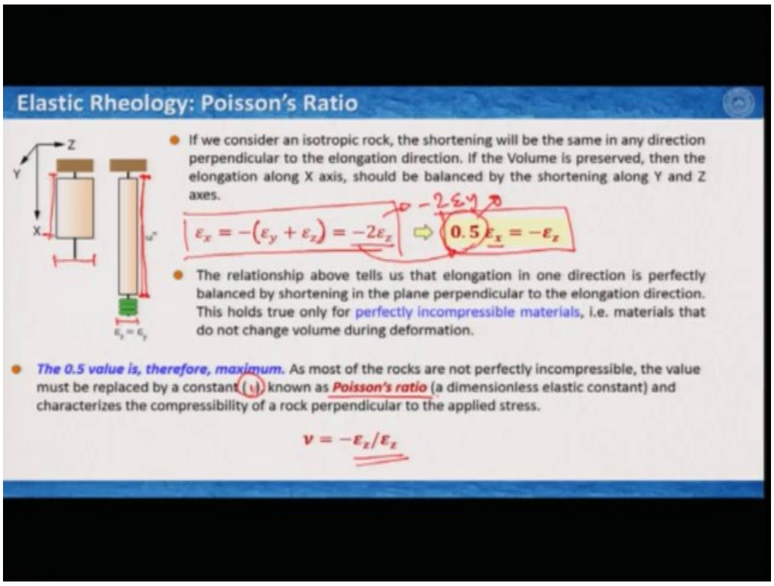
At the present if I add more green weights or green bars and I do it quickly then the elongation of the spring would happen also very quickly, so if I would like to see then stress versus strain is a linear curve, this is what the relationship we got, so Sigma equal to E multiplied by strain, so therefore the slope is your E or Young's modulus and then if I increaser the stress rate that mean if I increase the loading rate then strain rate would also increase linearly, so we can write Sigma dot which is actually Sigma by t equals to Young's modulus Epsilon by t or you can write it Epsilon dot.

So this is again Hooke's law Young's modulus therefore is the ratio if I can come here from this equation we can get here that it is a ratio of stress versus strain along the same direction that is important that you cannot measure stress in one direction and measuring stress in other direction you divide them, you get a ratio and you say this is my Young's modulus that is wrong, you have to measure them along the same direction.

Now you can replace this Young's modulus by another constant which is called shear modulus, so if you shear this elastic material instead of extending it, then this is known as mu and sometimes it is also expressed as G, so you can equate Young's modulus equal to 2 of shear modulus or twice of shear modulus and then this equation takes the shape, the Hooke's law takes the shape of Tau equal to 2mu gamma where Tau is your shear stress, mu as you have explained this car shear modulus and gamma is your shear strength.

At this time from these we have already found 2 constants one is Young's modulus and another is shear modulus, however from the folks law we can also get 3 other elastic constants, so one is poisson's ratio another is a bulk modulus and the 3rd one is lame's constants. In fact in some textbooks this mu and lambda these 2 terms they are together referred as lame's constant. Poisson's ratio is generally referred by Greek letter mu and bulk modulus is generally referred by Greek letter kappa.

(Refer Slide Time: 12:33)



Let us see what is poisson's ratio, now we have talked about volume constant deformation and so on, so if I have increased the length of the spring, in the previous example then if this volume of the spring I have to keep it constant, then it has to shorten in some other directions to keep the volume of the spring constant. Now this is the example, this little drawing, so what we are visualising it on X Z plane and why is perpendicular to the board or parallel to our view direction, so this was initially the width of this bar say this is the spring and this is the length of this spring.

Now upon applying this load this green bars then I can figure out the strain which is along X direction which is epsilon X and if I consider this is cylindrical and a perfectly isotropic body then on this direction it has to shorten to keep the volume constant and therefore epsilon Z and epsilon Y should be equal, so this is only possible if the material that we are considering the rock we are considering is isotropic. The shortening therefor will be the same in any direction perpendicular to the elongation direction and if the volume is preserve then the elongation along X axis should be balanced by the shortening along Y and Z axis and therefor you maintain your volume.

So we can write therefor this equation that EX that means the elongation you happen along x direction should be equal to the sum of EY and EZ, now these are happening in different directions so therefor I have a negative signs. Now EY and EZ because this is an isotropic material they are equal so I can write them minus 2 EZ or I can also write it as minus 2 EY, whatever be the case EZ or EY we can now further summarise this equation as EX multiplied by 0.5 because this 2 can come to this side equal to minus epsilon Z.

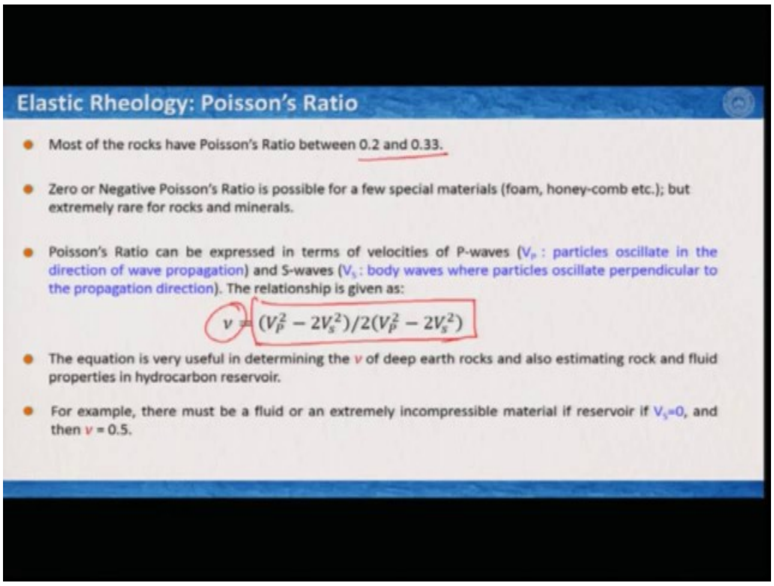
This minus sign further signifies that if you extend along X direction have to shorten along the Z direction and then it is related by a numerical value 0.5 and 0.5 you can consider that it happens when the volume is remaining constant, so this relationship 0.5 epsilon x equal to minus epsilon Z. This tells us that elongation in one direction is perfectly balanced by shortening in the length perpendicular to the elongation direction and when that happens then we call it perfectly incompressible material, that is the materials that do not change there volume during deformation.

So therefore if you have your volume constant that 0.5 value is there for maximum, so most of the rocks we know or we will see later that they are not perfectly incompressible, all sorts of volume changes or compressibility are involved where the volume shrinks okay, so to account this changing volume or compressible volume of the rock mass that we are considering, instead of writing at 0.5, people do replace it with a new constant which is constant for a particular material and this constant is known as poisson's ratio and represented by this Greek letter mu and as you can see poisson's ratio must be a dimensionless quantity because it is a ratio of 2 strain parameters, so poisson's ratio essentially characterises the compressibility of a rock perpendicular to the applied stress.

Now I give you a very simple example of poisson's ratio that you may have some glass bottles where instead of the caps we use to seal the glass bottles using some cork. Now the cork are very interesting material in the sense that because if you have the bottle and then you have to press cork inside because you are pressing and if the cork hash to maintain its volume constant say you are compressing it this side, so length is shortening on your compression direction, so it has to expand on the other direction that means the cork is now expanding and it cannot go inside the bottle's mouth but cork is such a material that this expansion is very less and therefore we use sometimes to seal the mouth of a bottle using a cock.

There are some other materials that do have some sort of negative poisson's ratio and that means if I compress in the side instead of expanding in this side they can also shrink or if I extend something in this site instead of compressing in this side they actually extend, these are some complex composite materials, honeycomb is one of the examples that you can think of that to happen negative poisson's ratio.

(Refer Slide Time: 18:50)



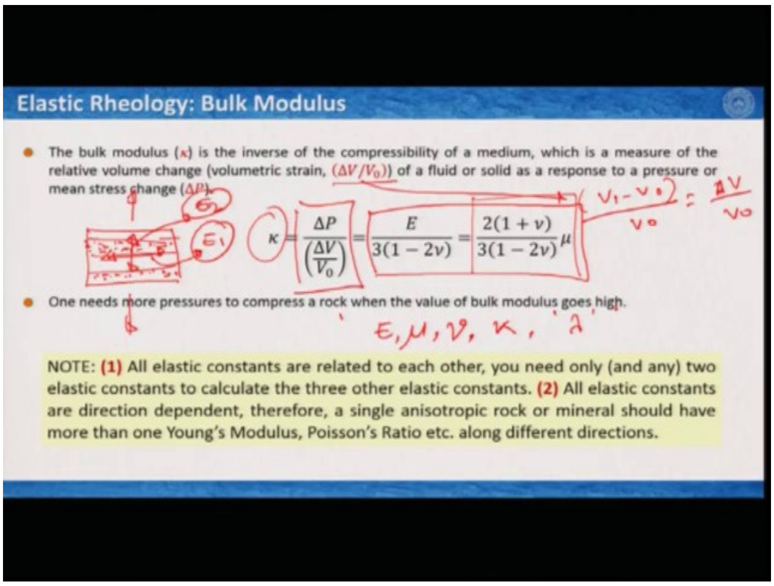
So most of the rocks that we generally consider in the wide range of conditions they do have poisson's ratio between 0.2 to 0.33 it is not a very wide range but in terms of poisson's ratio is pretty wide. Now 0 or negative poisson's ratio as I talked about is also possible for special materials like form and honeycomb but extremely rare for rocks and minerals with negative poisson's ratio for a common isotropic rocks or minerals, so you do not see but people have reported some sort of negative very little negative poisson's ratio in some particular direction of some isotropic minerals.

Now you can also express the poisson's ratio in terms of the velocity is of P waves and S waves. Now this is something little difficult to understand right at this point, but if I say you that this seismic velocity is that we consider this P waves and S waves these are elastic waves and we are dealing with elastic rheology, so there must be some sort of relationships. So what is P wave? P wave is when the particles to oscillate in the direction of wave propagation and S wave is some sort of a body waves where the particles oscillates perpendicular to the propagation direction.

You can see that these 2 terms are very important that one is propagating along the propagation direction or oscillating, not propagating, one is oscillating along the propagation direction and another is oscillating perpendicular to the propagation direction, so is not it very similar the way we can think that if we compress the side and the things should be extend, so one is perpendicular and one is parallel.

So their relationship if we write then poisson's ratio in a different way can come in this form and this is very useful because in deep earth we only receive, the signatures we get from the deep except some rare cases, most with the seismic waves and with the analysis of the seismic waves it is possible to determine the poisson's ratio of their deep Earth rocks and also this is important for the hydrocarbon industries because this gives us an estimation of fluid properties in the hydrocarbon reservoir. So, for example, you can think that if the VS is 0 that means there is a fluid and then poisson's ratio must be close to 0.5 and so on.

(Refer Slide Time: 21:48)



Now we have learned 3 elastic constants one is Young's modulus, one is shear modulus we just learned poisson's ratio, now let us talk about the bulk modulus, the bulk modulus are kappa is the inverse of the compressibility of the medium, so in general it measures the relative volume change of fluid or solid as a response to a pressure of mean stress change. Now we have learned what is mean stress in our stress lecture, so volume change is defined by this del V versus V0, del V 1 minus sorry V1 minus V0 where V1 is your changed volume. I will just write it because I think I confuse by my statements, so V1 minus V0 by V0, so this we can write del V by V0 that is your relative volume change of the material that you are considering with the change of the pressure.

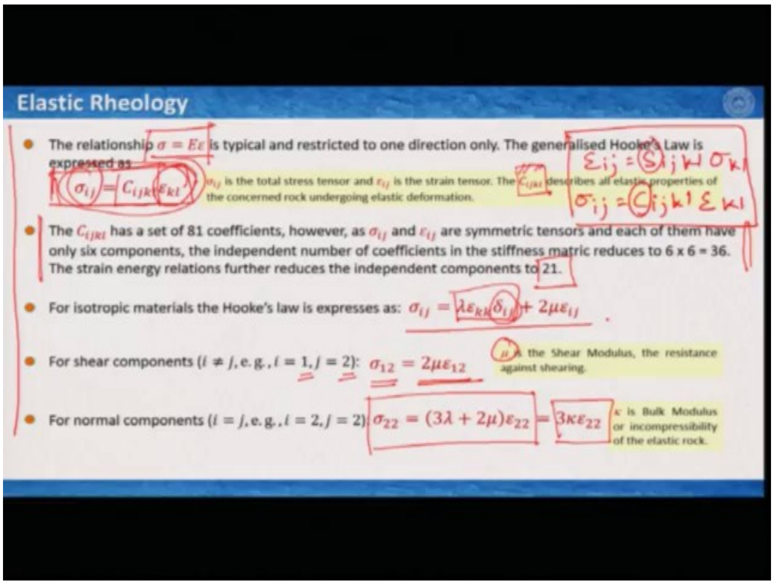
So if I am increasing the pressure or a decreasing the pressure how much volume change I am experiencing or the rock is experiencing within the elastic domain, so that is your kappa or bulk modulus, so this you can write it this way, so this is your pressure change and this is your volume change and then with some calculations and relationship you can figure out that you can express your kappa in terms of shear modulus poisson's ratio and Young's modulus and poisson's ratio. And these equations says that you need more pressure to compress a rock when the value of the bulk modulus goes high.

Now we have now learned Young's modulus, we have learned shear modulus, we have learned poisson's ratio and we have learned bulk modulus. Now there is also one left lam is constant, we will learn it later but not right now but I would like to give you at this point few considerations, so all elastic constants are related to each other because you are measuring it from same material, so they have to be related to each other. So you need out of these 5 you need only 2, so if you have only 2 then you can calculate all 3 other elastic constants and this is most important all elastic constants are direction dependent.

So when you say Young's modulus you measure it in a particular direction, poisson's ratio you measure elongation in a particular direction and then shortening in a particular direction and so on. So if these are direction dependent, therefore a single anisotropic rock or mineral should have more than one Young's modulus, more than one poisson's ratio and so on or what I mean by that if I have a rock in 2 dimensions if I draw it like this, so the dotted areas are 2 different materials.

Now, if I deform it extend it in this direction then the Young's modulus I would get along this direction okay but if I extend it along this direction, the Young's modulus I would get is along this direction. Now because this is an isotropic we have 2 different materials, so if I consider this one as E1 and this one as E2, this is also a measure of anisotropy, elastic anisotropy in terms of Young's modulus of this rock.

(Refer Slide Time: 25:59)



Now the generalised Hooke's law is something… is little expanded from what we have learned. This equal to Young's modulus multiplied by strain but this equation is typical and you can see that it does not include any direction that we talked about, it does not include the anisotropic components, so therefore the generalised Hooke's law is written in this form, in this tensorial form, we know more or less water is tensors so Sigma IJ is equal to CIJ KL Epsilon KL, now Sigma IJ for 3 dimensions I equal to 3, J equal to 3, so you can have 9 components here and so on and for 2 dimensions you can have them I equal to 2 and J equal to 2 then you will have 4 components there, so Sigma IJ here is the total stress tensor, Epsilon KL is the strain tensor.

Now this term C IJKL this term describes all elastic constant in one house, this is a matrix so this is stored in one matrix and this is known as stiffness matrix. Now this stiffness matrix you can represented by 81 coefficients simply because you have 9 components here, you have 9 components here so this matrix has to has 81 components but we know that Sigma IJ and Epsilon IJ are symmetric tensors and each of them can then only have 6 components, so therefore this 81 coefficients of C IGKL or the stiffness matrix reduces to number 36. Now you can further reduce it by using some strain energy relations then it comes to only 21 Independent coefficients.

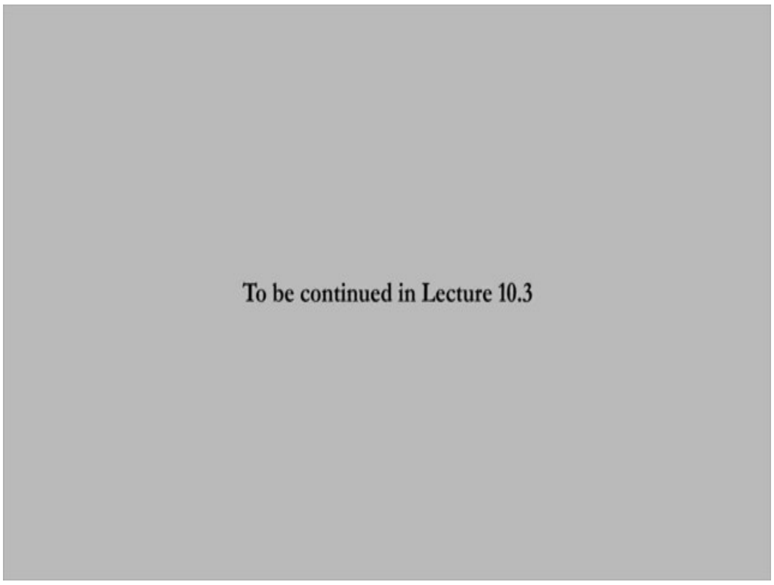
Now if this equation is written this form that Sigma IJ equal to C IJKL strain KL you can also express in terms of strain, so instead of that you can write strain IJ equal to… Then it will not be a different matrix S IJKL stress KL. Now C is known as stiffness matrix and here S is known as compliance matrix, so this is something that you may note, now this expression is that how there were 81 coefficients and then out of that we get 36 and then using strain energy we can get 21 Independent coefficients you may not have to go to the detail derivations of this but at this stage it is important that you know that from number 81 you can come down or you can reduce the independent coefficients to number 21.

Now let us straight this equation in a different way, if I have to consider this as isotropic material then this generalised Hooke’s law you can express it in this form okay where Sigma IJ equal to lambda which is one of our lame’s constants then Sigma KK chronic Delta Delta IJ plus 2 mu Epsilon IJ. Now Kronecker delta is a very interesting term if I equal to J then it becomes 1 if I naught equal to J then it becomes 0. So for shear components there for if I am applying a shear modulus that means I naught equal to J therefore if I consider I equal to 1 and J equal to 2 where they are not equal then we can write Sigma 12 equal to 2 mu Sigma 12 because then Kronecker delta I naught equal to J becomes 0.

So this term vanishes and here we get mu as the shear Modulus, so you can see how from generalised Hooke’s law we can derive the shear modulus just implying or just taking into account the shear components of the matrix and if it is for normal components that means I equal to J, so therefore Kronecker delta value should be one and this can be expressed with some algebraic calculations you can represent it further by this and therefore you get the bulk modulus from this equation.

Now this is how it is done, this slide what I recommend you that you do not have 2 go into all details of this equation and how it is derived but it is important that from a stress tensor when you applied this to elastic field we know that it has some normal components and it has some shear components. So when you apply the normal components we get the bulk modulus related to volume change and things like that you remember that mean stress and all other issues and when we are not dealing with normal components that means off diagonal components where your shear stress are acting then you can get simply by considering I naught equal to J you can get shear modulus of the elastic material.

(Refer Slide Time: 31:58)



Now with this I would stop and then they will move the next topic viscous rheology.