Structural Geology
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Lecture 09 - Stress (Part – II)

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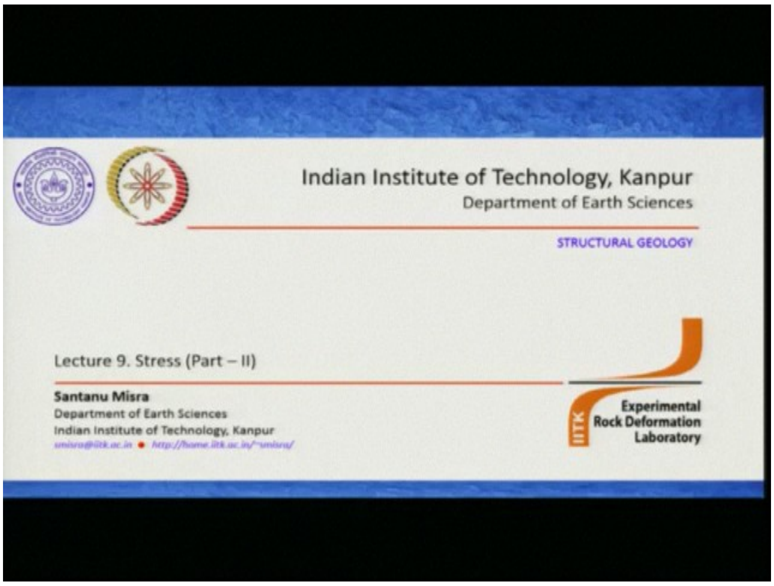

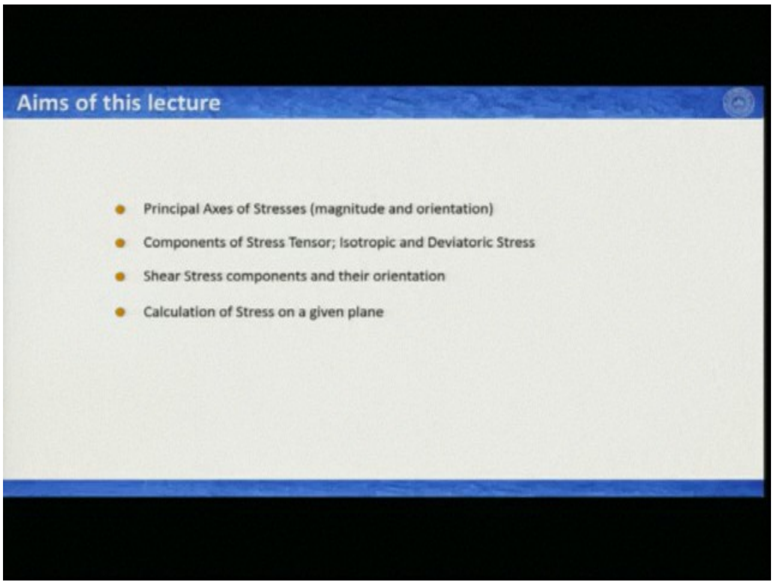

Hello everyone. Welcome back again to this online NPTEL Structural Geology course, we are learning stress. In the last lecture we have learned about the part 1 of the stress and today we are on the part 2 of this lecture and in this lecture we will particularly focus on principle axes of stresses, their magnitude and orientation, we learned about it in the last lecture. And in this lecture we will learn about it in more detail. How to derive them? How to get them? What are the different meanings of these principle axes of stresses?

Then we will move to components of stress tensor. We will also focus on isotropic and deviatoric stresses. We will also discuss after that shear stress components and their orientations along which it works and what are the different structures we produce because of the shear stresses. And then we will directly go two examples, how to calculate stresses on a given plane which we will apply in many cases, if you continue with structural geology.

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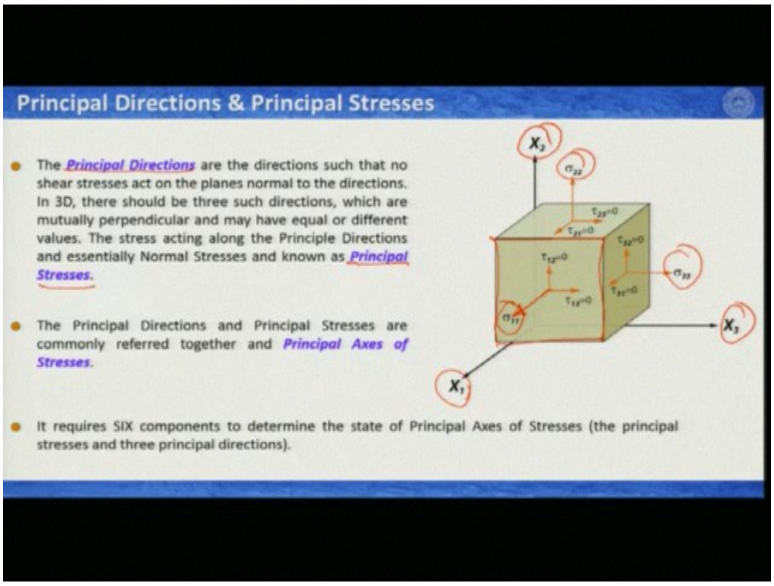

So a little bit of review of this last lecture. We learned about principal stresses and principal directions. So I just read this text. The principal directions are the directions such that no shear stresses act on the planes normal to this direction. As we have seen that if I consider the plane perpendicular to the X1, again this plane, then if I consider these stress components Sigma 11 and along this direction because this is the normal stress, no shear stress acting along Sigma 11.

So therefore if I consider a plane perpendicular to X1, Sigma 11 is a principal stress perpendicular to that plane. And because X1, X2 and X3, these 3 are my coordinate frames in a Cartesian system, so Sigma 11, Sigma 22 and Sigma 33 are my principal directions as well, along the direction it is working.

In 3 dimensions, there should be three such directions as we have seen which are mutually perpendicular and may have equal or different values. We learned what would be the meaning if they are equal and what is the meaning if they are different. So stress acting along the principal directions, that means their magnitude along the principal directions which are essentially normal stress components are known as principal stresses. So we learned principal stresses which other magnitudes and we also learned principal directions which are the directions along which the principal stresses do act or do work.

Now altogether principal stresses and principal directions they are known as principal axes of stresses. The principal directions and principal stresses are commonly referred together as it is written here as principal axes of stresses. Now if I consider a three-dimensional system, then it is quite obvious that I would have three principal stresses and I would have three principal directions. So altogether, it requires six components to determine the state of principal axes of stresses acting on a body.

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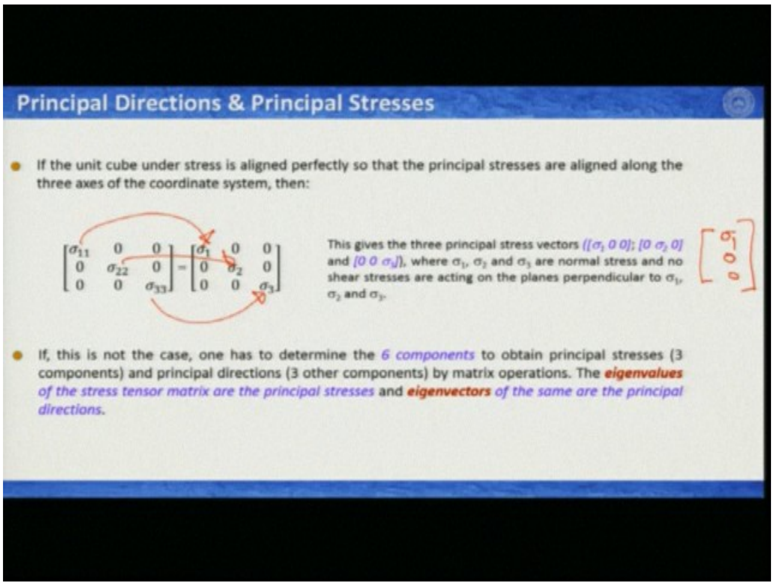

Now as we have seen in the previous diagram, that if this Sigma 11, Sigma 22 and Sigma 33, which are the normal components aligned along the principal axes of stresses, if that happens, that means if the unit cube under stress is aligned perfectly so that the principal stresses are aligned along the 3 axes of the coordinate system, then we can define this Sigma 11 as Sigma 1 which is the magnitude of principal axes of stress along X1 direction, Sigma 22 as Sigma 2 and Sigma 33 as Sigma 3.

So therefore it gives three principal stress vectors: Sigma 100, Sigma 020 and 00 Sigma 3. Now I wrote it as sort of here as row matrix, but generally they appear as column matrix. So if I write Sigma 100 and so on, so three stress vectors working on three different planes, where Sigma 1, Sigma 2 and Sigma 3 are normal stresses and along these directions there is no shear stress acting on the planes perpendicular to Sigma 1, Sigma 2 and Sigma 3.

Now if this unit cube that we are considering is not aligned, that means the edges of this unit cubes are not aligned to the coordinate system, then we cannot simply replace Sigma 11 by Sigma 1 and so on. We have to calculate what would be the Sigma 1, what would be the Sigma 2, what would be Sigma 3 first to get the principal axes, principal stresses. And then we have to get the principal directions. That means first the directions, first this direction, the magnitudes and then the directions.

Now in matrix algebra, this is an interesting eigenvalue problem. But it is very simple, we learn about it soon. So the eigenvalues of the stress tensor matrix or the principal stresses and the eigenvectors of the same are the principal directions. Now we will learn how to derive it. We will learn how to get it. Nowadays, of course, we have to learn it, how to do it manually? But there are many computer programs where you can actually apply and you just give your input. This basic stress tensor matrix and then it would automatically give you what would be your eigenvalues. That means your principal stresses and eigenvectors, what are the principal directions?

So before we jump into that particular part, that what would happen for the stress let us have some very basic idea, idea is one, what are eigenvalues and what are eigenvectors? What do you mean actually by this? And this is a very classic operation that not necessarily for this case but for many deformation related issues, people do use eigenvalues and eigenvectors.

Now this Eigen the term means the characteristics of something. So eigenvalues or eigenvectors certainly describes that some values which are characteristics, some vectors. That means it has some directions, these are also characteristic for that particular object or particular matrix you are dealing with. Now we will try to see it in a very simple way that what are eigenvalues, what are eigenvectors and what are their meanings.

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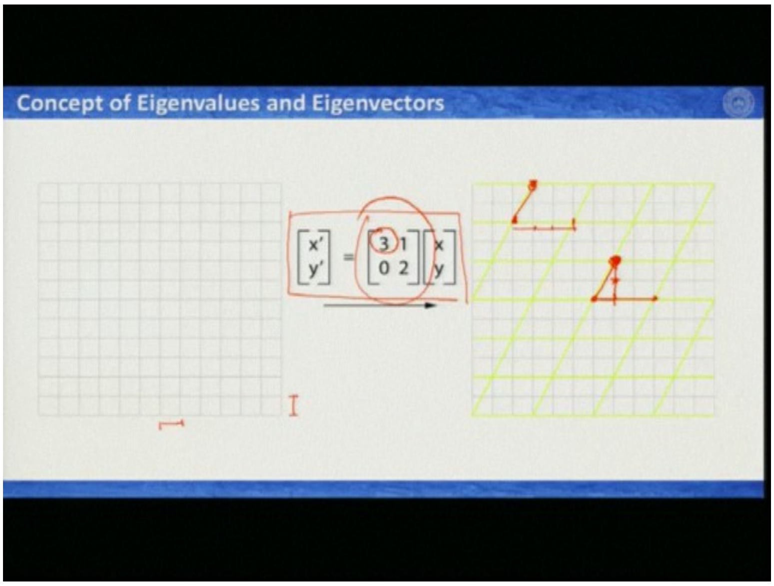

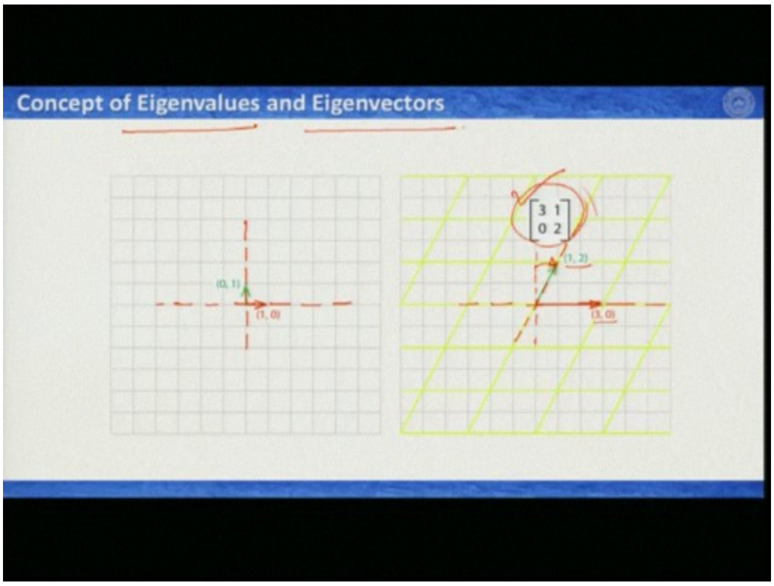

What do we see here? We see a square grid and we can consider that these are of unit lengths. Now I can deform this square grid applying some sort of deformation tensor. We did not learn about it. I said that in a strain lecture that there are lot of mathematics involved in the strain. We did not go through that, but I am sure that you read some books or online materials.

So if I apply in this grid this deformation, so this is the deformation. So that means if I have a point, what this matrix tells us or what this some sort of equation written in a matrix form tell us? It says that X' and Y' are column matrix. Then there is a 2 by 2 matrix; 3, 0, 1, 2 and then XY is in another column matrix. So this XY is the original coordinate and X'Y' prime are the transformed coordinate, deformed coordinate. And this 2 by 2 matrix is the matrix which characterises, or which describes, quantifies the transformation from XY to X' Y'.

Now this is just an example, this 3, 0, 1, 2. And if I apply this deformation matrix to this square grid, it would look like this. So your original state was this grey square grids and after if I apply the deformation, then what is coming here is the yellow, now it is not square, but yellow grids. Now what did it do?

We see that if I consider any point, for example here X, let us consider here, then this point along the X direction it moved here. Okay, so it moved three units along the X direction. So therefore this is defining it moved three units along the X direction and no movement along the Y direction. So therefore it is somewhere like this.

For the Y, if we consider that this point has moved to here, and this is applicable for anywhere, if you see this has moved here and for Y direction, it has moved along X direction 1 unit and along Y direction 2 units. So these 2 units are coming here. So this point has moved here and here as well. You can like any point of this square grid and you would see this feature.

So this is the meaning of this matrix or we call it D-matrix or deformation matrix. So this defines everything, it also defines your strain ellipse in two dimensions and if there are nine components, then it defines a strain ellipsoid in 3 dimensions. In that case, you would have X, Y and Z. So if I try to look at in a vector form, so that means I have this red vector, unit vector which is (1,0), the coordinate here and then I have the green vector which is along the Y direction. The coordinate is (0,1).

And if I apply this transformation matrix, or the de-matrix, then this 3 comes, then the red matrix (1, 0) changes to (3, 0) and (0, 1) changes to (1, 2) which actually is defining your D-matrix, deformation matrix. Now from here, we would move to these two terms, eigenvalues and eigenvectors. Now if we see this transformation, we have just taken two unit vectors aligned along the X and Y directions of this square grid. But there should be N number of vectors.

That means you can draw vectors like this any direction, is not it? And they would also deform following this matrix. To define the eigenvalues and eigenvectors because these are characteristics, you would consider some sort of feature which is known as span. What I mean by this? That means if I have, if I consider this vector 1, 0, the red one, then it has a span like this and after the transformation, I see the span remains same. It did not change.

But this green vector if I consider the span here like this and if I just import this it was here. But after the application of this deformation matrix, this span of this green matrix has rotated from its previous position.

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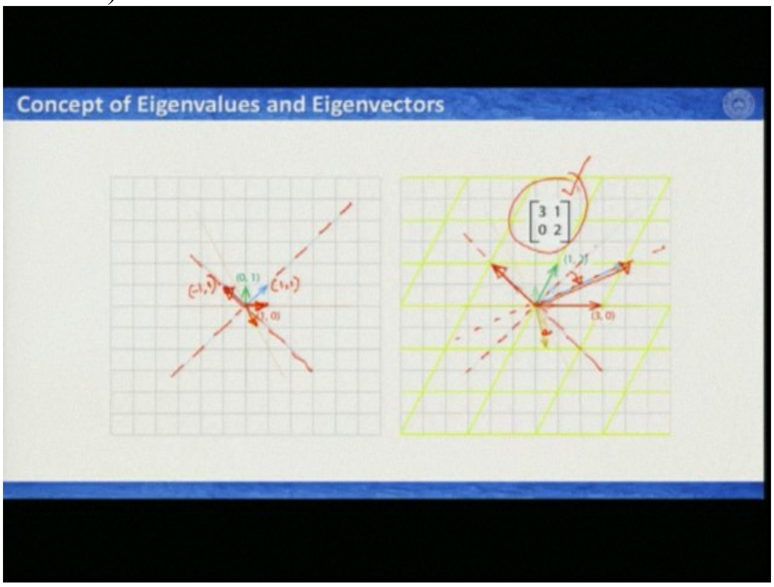

And if you try with many other such examples, for example here, apart from this red and green, we can consider this blue vector which is actually (1, 1) and this line is the span of this blue vector. And we see that after deformation because this was (1, 1), if we apply this transformation matrix, it would come to here. Therefore the span which was initially like this, now moved to this plane or this orientation and this is how it has rotated. Similarly, if we consider this orange vector, we will see that it has also rotated after we apply the de-matrix.

If we consider the opposite vectors of (1, 1), that means along X direction, it is -1 and Y direction, it is +1. Another vector, this purple one and this is the span of this purple vector. Then after this application of this D-matrix, it takes the form like this and interestingly the span remains same, it did not rotate. So we found two vectors, one is this one, this red unit vector and this purple unit vector. That did not rotate after application of this transformation.

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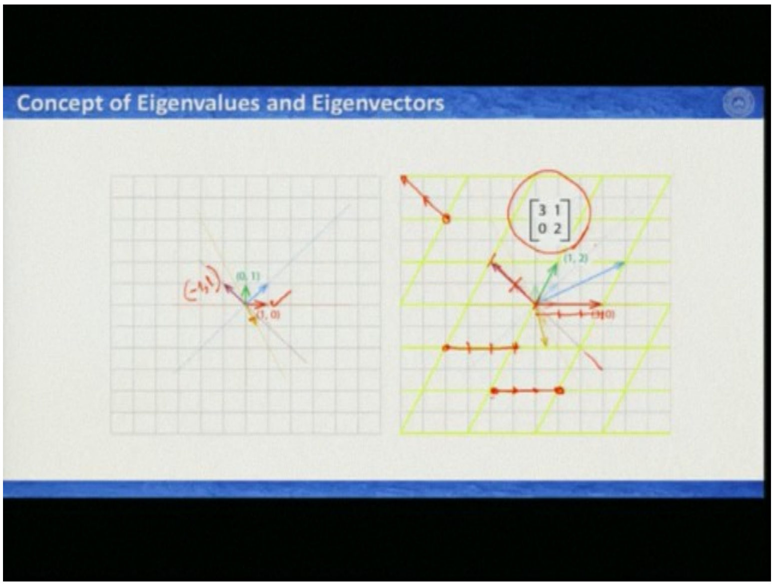

And interestingly, let me wipe this one out, all other unit vectors do not matter how they are oriented, except this red and purple, this red and purple, all other vectors oriented in any directions if you try, you would see that there are spans which were initially oriented in theta direction. And after the application of the D-matrix the theta direction is not remaining same, except these red and violet vectors.

So we can consider that these two vectors, this (1, 0) and (-1, 1), the purple one. These two vectors are characteristics for the application of this deformation matrix. What does it mean? Further, that I see that does not matter whichever point I take, this unit red vector has a stretch of unit 3 anywhere, you consider it was here and then from its initial position it took 1, 2 and 3, it came here. It was here, it came here 3 units.

Similarly, this purple vector I can see that it did not change its span. That means its direction is maintained. But along this direction, it moved two units and it is obvious for any directions. If it is was here, then it moved 1, 2.

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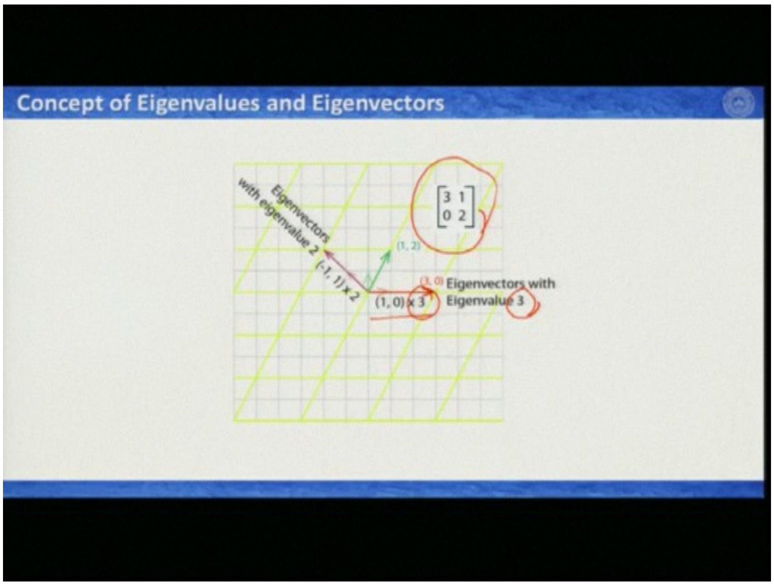

Will see that these two characteristics vectors, this red and this purple these are known as eigenvectors. That means in two dimensions, if I apply this matrix, does not matter, I take any point of my deforming body, these two directions would remain, that means remain constant, that means their orientations would remain same, and therefore they are very much characteristics of this matrix, deformation matrix.

And the red unit vector which is one of the eigenvectors of this deformation matrix has eigenvalue 3. Because if I apply this deformation then it would be always multiplied by 3 and if I take this purple vector, its next incremental position would be always multiplied by a factor of 2. So in general if I consider, in two dimension if I deform a material, there should be to characteristic directions, along which the material lines do not change their orientations. But they only get stretched and when they get stretched, they get stretched in each and every increment of deformations equally.

The first one that there are directions are maintained and these directions are known as eigenvectors. And the second one that the multiplication it does in each and every increment of strain or deformation is known as eigenvalues. And if you have eigenvalue for each and every direction, in three-dimension you would have three eigenvectors and three eigenvalues.

Now you can consider it in a different way. If you have a cube and if you stretch the cube along a particular axis, say you are stretching it along X direction, then you would get your three directions, three perpendicular directions X, Y, Z would remain constant. So in that case, X, Y and Z are your eigenvectors. And eigenvalues for X and Y, for Y and Z would be zero because it is not getting stretched. But for X each increment you are deforming it would maintain the same direction. But it would get increased, increased and increased with the same factor.

Now this concept is applied for calculating the principal stresses and also principal direction of stresses. But before that this is how we have understood geometrically that this is the red and purple, these two are my eigenvectors with some eigenvalues. But how to calculate them mathematically?

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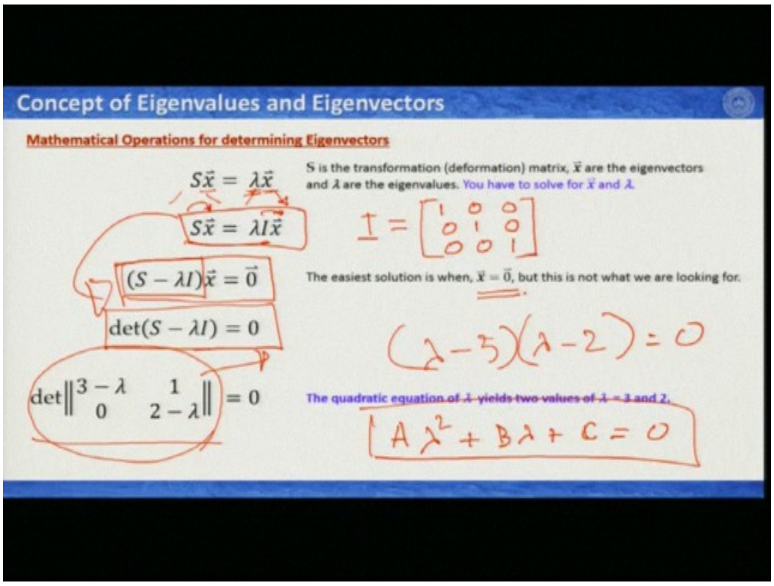

In this context eigenvalues or eigenvectors are defined in this way. So if S is a transformation matrix, then I can multiply this X with a vector which is X and then I equal it to a scaler quantity, which is your eigenvalue and then multiply it again by the same vector X. So your transformation matrix multiplied by a very unique vector which is your eigenvector should be equal to your eigenvalue which is your scalar term multiplied with the same vector.

Now here you may have a confusion that S is a matrix, X is a matrix, here lambda is a scalar and X is a vector. So matrix vector multiplication and scalar vector multiplication, you can imagine that how it happening together. To avoid this confusion, you actually can multiply this lambda to bring a vector here, to bring matrix here, I am sorry, you can write a matrix which is known as identity matrix. If I write in two dimension, I am sorry in three-dimension then it would look like this 100, 010, 001. This is identity matrix.

So if you add it here, it does not change anything but then you are convinced with the fact that in this side we are doing matrix, matrix multiplication and here we are also doing matrix, matrix multiplication. Now taking over from this equation, we certainly can write this equation. Okay, where I just changed the positions opposite to the sides. And the easiest solution, you can get out of this for X vector is when X equal to 0. But that is very easy for us and this is exactly what we are not looking for. We need a non-zero X vector solution for that.

And to do this, we have to get the determinants of this matrix, of this term, which is this, and then all its components should be vanished. That means determinant of S minus lambda multiplied by your identity matrix should be 0. Now if we rewrite or expand this equation with our transformation matrix which was 3012, then we can write this equation determinant of 3 minus lambda 102 minus lambda should be 0.

Now if we do the determinant, if we identify or calculate the determinant of this matrix, you can do it and it would yield a quadratic polynomial. So it would take a shape of, I am not doing it here, but it would take a shape of, anyway this is 0. So you actually get the equation lambda minus 3 multiplied by lambda minus 2 equal to 0. This is easier because it is 0, right. So you get the two values, lambda two real values, lambda equal to 3, lambda equal to 2.

Otherwise, you would get the equation in the form of lambda square plus some constant, lambda plus some another constant. Okay, let us try it this way, it would be much easier, say A lambda square, where A is a scalar or a constant. Then B lambda and then you would get another constant here C that would be 0. So this is your quadratic equation for two dimension and you can get the solutions. The roots of this equation, roots of this lambda and this would be your eigenvalues for this particular matrix that you are dealing with.

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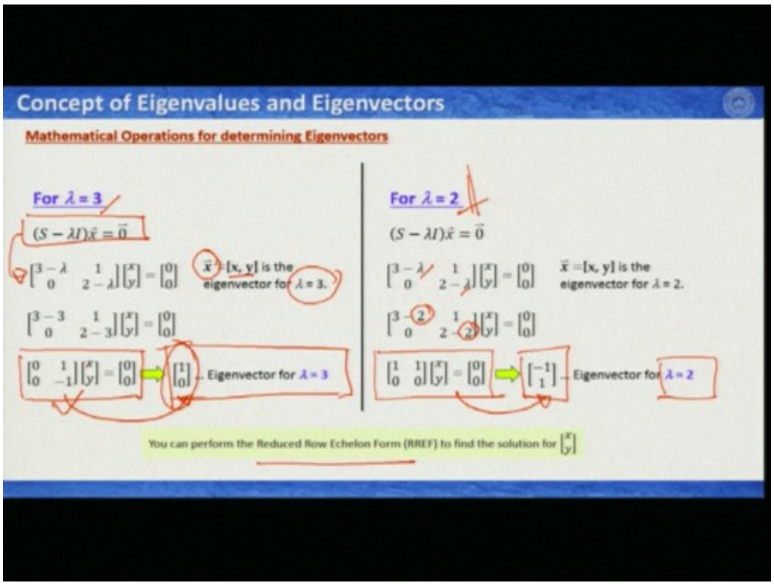

So now we know the eigenvalues for the particular matrix we are concerned with. So one eigenvalue we found was 3 and another was 2. Now the task is to determine the eigenvectors for corresponding eigenvalues. Now the operations as you can imagine this is very straightforward, so we again see this same equation, where S is your concerned matrix, lambda is your eigenvalue, I is identity matrix. And here X is eigenvector for lambda equal to 3. This is what we have to determine that what is the value of this vector X in terms of X and Y coordinates.

Now you can expand this equation in this form, very simply can put the value of lambda and then it takes the shape of the equation like this and from this equation, you have to solve the values for X and Y. Now if you solve it, then it comes to 1 and 0, that is the eigenvector for lambda 3 and if you remember this is exactly what we have considered.

So you see that mathematically we can derive, we do not have to draw the curves or plots and so on. Of course we can do it for better understanding, but otherwise just doing some simple matrix algebra, you can calculate this. By the way, just to let you know that from this equation to solve it for X and Y, you can use reduced row echelon form RREF to find the solution of X and Y. And you can find the process of RREF from any matrix algebra standard book or text.

So similar way we found that it was 1, 0 for lambda 3 the eigenvalues, eigenvectors and then for lambda 2 eigenvector 2 we can similarly replace the lambda, lambda value in this equation as 2 here and here by replacing this lambda and this lambda. And then we see sort of an equation of similar way that we have derived here and again, if you solve for X and Y using RREF method, you can find the values for X and Y as minus 1, 1. And this is exactly what we found or what we have seen before, which was the eigenvectors for eigenvalue 2. So this is how mathematically you can figure out what are your eigenvalues and eigenvectors.

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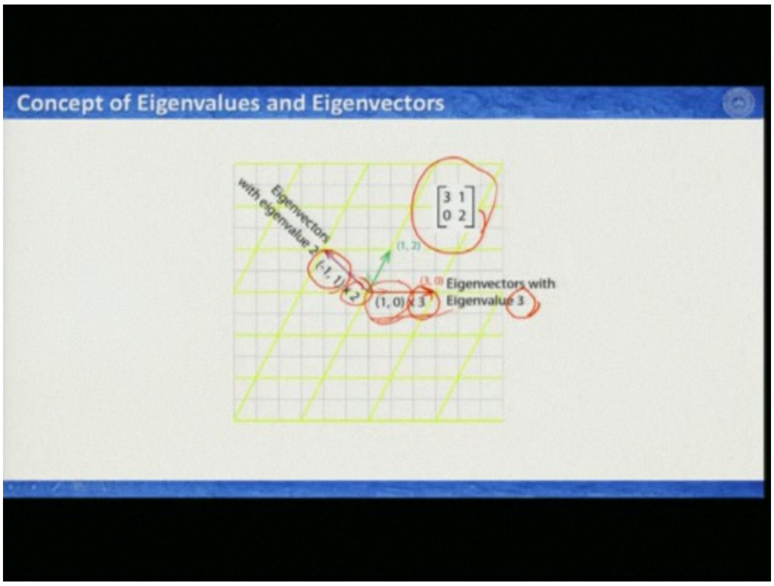

If we go back to our previous slide, we can see that these are your matrix. So 1 minus 1, 1, when you have this and it is when the eigenvalue is 3 you get 1, 0. Now, we can apply this concept now of this eigenvector and eigenvalue to calculate the principal directions and principal stresses and we learn this in the next segment of this lecture.

Okay, so in the previous part we learned how to calculate or what is the meaning of eigenvectors, eigenvalues and how to calculate them. And now we will apply the same technique for our calculation of principal stresses and the principal directions and I remind you that we need to find 6 values, three for principal stresses and three for principal directions.

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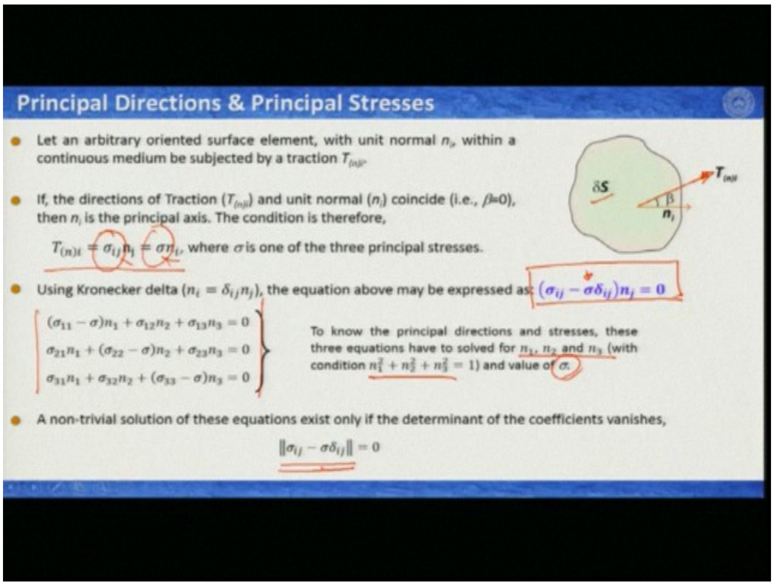

So let us start with this consideration in this slide. So we can consider very similar way the same arbitrary element which is Del S as surface area. And it has an unit normal which is N i. That means N1, N2, N3 in three different directions and this is the traction you are working and with the traction it is making an angle which is beta.

Now clearly if this beta angle is 0, then you are N i is essentially your, along the N i directions, you do not have any shear stress. So therefore it automatically gives you the direction of principal stresses and at the same time if you can calculate the magnitude then these are your magnitude of the principal stresses. But if that does not happen, then you have to figure out, particularly the orientations where your shear stresses are 0.

That means, where Sigma is one of the principal stresses and it must follow this convention that Sigma i j must be replaced by Sigma and these two vectors would switch their positions that means N j to and N i. Now to do that, you can actually figure out an equation, I am not going into the details of how to get this equation to this. Where you get this particular term Sigma involved here and if you expand this equation in 3 directions, so you would get things like this. And based on this, you actually have to calculate the 3 directions which are your principal directions and you have a condition where that we have learned also with the force that N1 square plus N2 square and N3 square, they should be the sum of this should be 1 and then of course you have to figure out the value of Sigma.

Now very similar way, if you have this equation then you can only get a nontrivial solution, where your vector is not 0. That means it is not aligned so that means of beta is 0. Then you can get a nontrivial solution of these equations, these three questions only if the determinant of the coefficient vanishes. So this is the part that you have to solve the determinant and you have to figure out what your eigenvalue.

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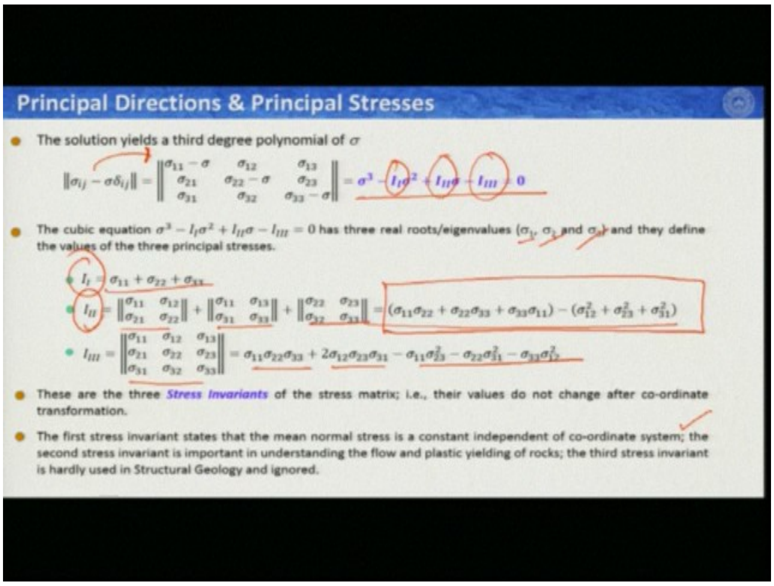

Now if you do that, then essentially it would take the shape. So this equation would if you expand it in matrix form, it would come to this part and if you solve it, I recommend that you try to solve it, if you know how to get the determinant of a matrix, a three-dimensional matrix, recursive matrix, then you will arrive to third-degree or third-degree polynomial or a cubic equation like this. Now if you have that equation with you, where I1, I2 and I3 are three constants, you learn what these are soon.

Essentially this would yield three values, 3 real roots of this equation and you can say root 1 is Sigma 1, root 2 as Sigma 2 and root 3 is Sigma 3 and they are the values of the principal stresses and these are also eigenvalues of the stress matrix that we are dealing with. So this is the way you get your principal stresses. Now this I1, I2 and I3 that you also get along with these principal stresses, they have very important contribution in the understanding of mechanics and also in structural geology.

You can express them in terms of stress components like this. That there are many forms you can write these three terms, you can express these three terms. But this I1, I2 and I3 these are known as stress invariants. These are not eigenvalues but they also do not vary, they also characteristically represent the stress matrix.

And most important that their values also do not change with the coordinate transformation. That means you are not be deforming your material, but if you are transforming your coordinates, these values would remain constant, they would not change. So therefore these are some sort of characteristics but these are not eigenvalues. Now the first one I1, I1 is stress invariant. That is a mean normal stress and is a constant independent of the coordinate system. We learn soon what is mean by normal stress. The second stress invariant which is commonly expressed this way or in the matrix from this way is generally applied when you try to understand the flow or plastic yielding of rocks.

So that means, you would learn soon later, that what the (())(36:18) criteria and other things there you use, it is failure criteria, so the yielding of the materials this I2 is used very frequently. The third stress invariant is not used in structural geology or hardly used and is mostly ignored. At least I did not see any application of this 3rd stress invariant. So this is how we derive the principal stresses.

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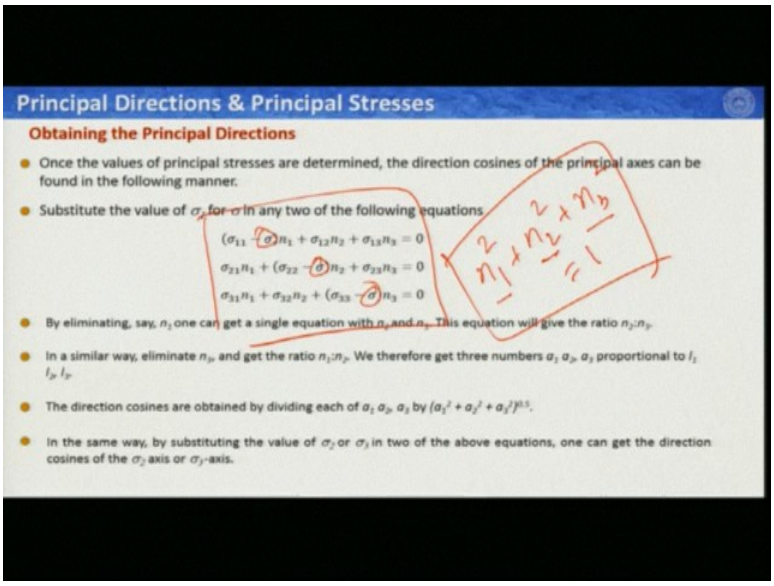

And for the principal directions I have given some instructions, how to derive that? But otherwise it is very easy because you now have solved your equations. So you have actually some number of unknowns and you have also this relation N1 square plus N2 square plus N3 square equal to 1. So using this equation and these three equations, where you Sigma are known and for each Sigma 1, each Sigma 2 and each Sigma 3 or each Sigma, that means Sigma 1, Sigma 2, Sigma 3, you can get the values of N1, N2 and N3. The instructions are given here and also you have these stress invariants.

So I am not going into the detail of this, you can read it and it is better if you get any strain matrix, any stress matrix, stress tensor. And you try to deform it or you try to rotate it, and then figure out what are the invariants or what are the principal directions and principal stresses, you can do these exercises by yourself. And again, as well as I tell that if you have any confusion, if you cannot solve it, you are more than welcome to come back to us. So you can contact the TAs and also you can write to me.

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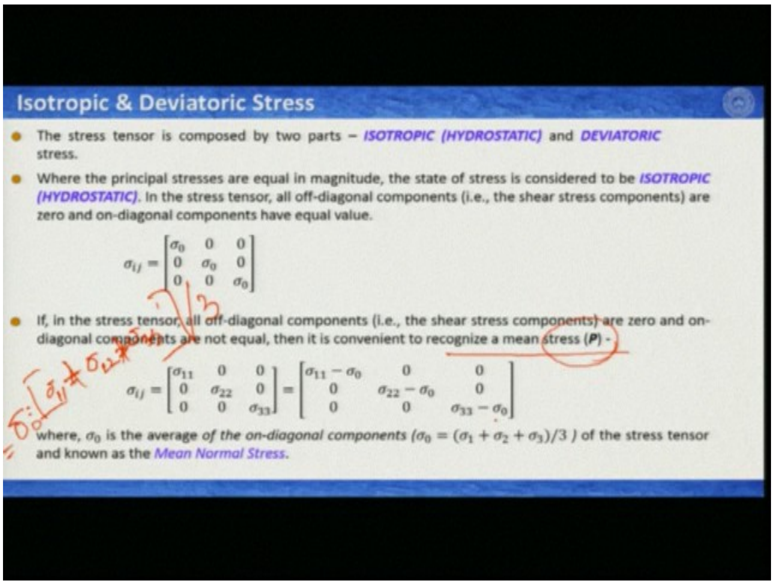

So let us go to another very important topic of this stress subject that we always consider, particularly in structural geology which is isotropic and deviatoric stress. And you can actually decompose your stress tensor, the general stress tensor to the isotropic or sometimes we call it hydrostatic and deviatoric stress matrices or stress tensors.

Now what is isotropic state of stress? As it defines it is written here, where the principal stresses are equal in magnitude, the state of stress is considered to be isotropic or hydrostatic. That means the magnitude of Sigma 1, Sigma 2 and Sigma 3 are equal, but of course they are mutually perpendicular to each other.

In the stress tensor, so now if I have Sigma 11, Sigma 22, Sigma 33 all are equal, then I can replace them with a single value. In this case what I have done is Sigma 0. Okay, so all other components are 0, you have only three principal stresses, have similar magnitude. Now if the stress tensor all of diagonal components, that means the shear stress components are 0 and on diagonal components are not equal, that means you do not have any shear stress acting but your Sigma 1, Sigma 2 and Sigma 3 are not equal to each other.

Then you can actually get something which is called mean stress. A mean stress is actually the sum of the principal axes of the stresses divided by 3 or is the average or of the on diagonal components gives you the mean normal stress. So if I have non-equal values, here Sigma 11 is not equal to Sigma 22, not equal to Sigma 33, in that case I can sum them, and then divide them by 3 and therefore I get something called Sigma 0 and this I can separate out and then can write the stress matrix also in this way.

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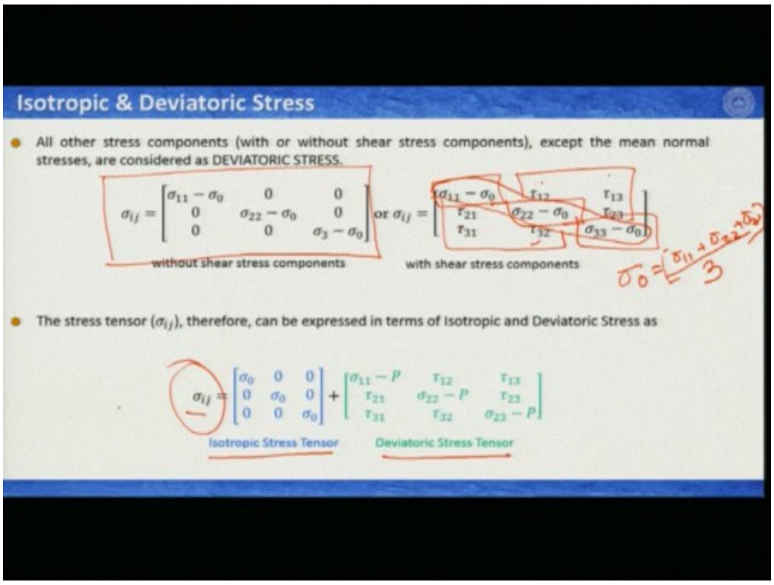

Then all other components which are not isotropic in the stress matrix or stress tensor matrix with or without shear stress components except the mean normal stresses are considered as deviatoric stresses. So you can write it this way. Okay, so this is written without shear stress component and you can also add your share stress components, sorry, not this one. So your on diagonal components and off diagonal components are working here. And these are deviatoric stresses where you have taken the mean stress Sigma 0 which was actually Sigma 11 plus Sigma 22 plus Sigma 33 divided by 3.

So this is your deviatoric stress component. So in general, if I consider the overall stress matrix Sigma i j I can decompose it in two stress tensors. The first one I came keep only the diagonal components, keeping all the diagonal values equal that would give you the isotropic stress tensor. And the deviatoric stress tensor is whatever is remaining and if you sum them, you will get your total stress tensor.

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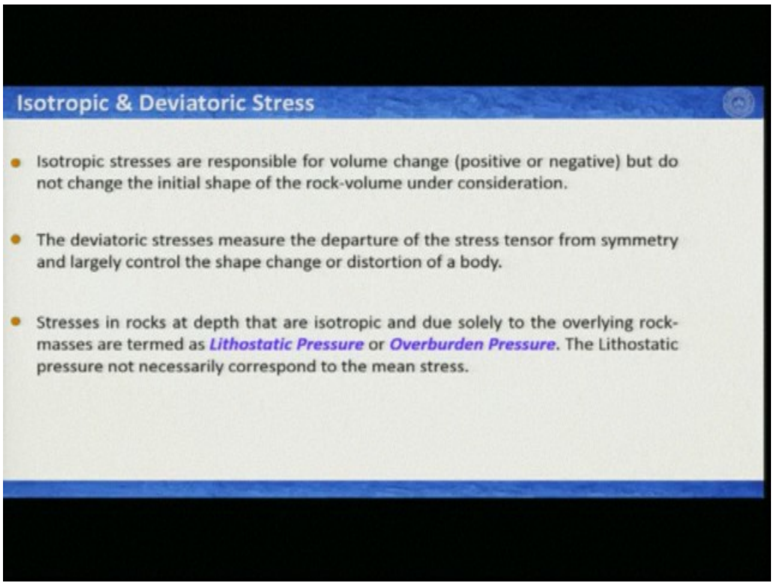

So now we will discuss what are the meanings of isotropic and deviatoric stresses? Now isotropic stresses as we can understand that all the values are equal. Their magnitudes are equal. So therefore, isotropic stresses are mostly responsible for the volume change. It could be positive or negative, depending on the direction it is working. But it does not change the initial shape of the rock volume under consideration.

The deviatoric stress on the other hand, it measures the departure of the stress tensor from the symmetry and therefore that means, it does not consider the symmetric part of it. Okay, so therefore the deviatoric stress is responsible for the strain or distortion of the body and therefore it changes the shape of the body.

Now sometimes we use few different terms like Lithostatic pressures or Overburden pressures. So stresses in rocks at depth that are isotropic and due solely to the overlying rock masses. That means if I am at 600 kilometres then I have a huge 600 kilometre pile of rocks above me or I have of the grain you are looking at. Of course I cannot go to 600 kilometres. So this pressure due to this overlying rock mass is known as lithostatic pressure or overburdened pressure. And it is important to understand these lithostatic pressures are not necessarily correspond to the mean stress. Okay.

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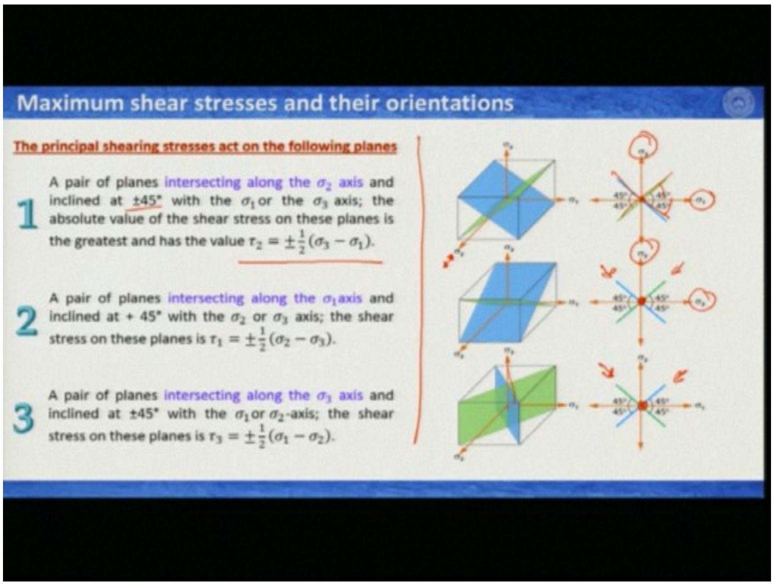

Presently will look at the share stress part and we will do it a very different way. You can get the derivations or the detailed maths involved in it from any book. I again recommend Professor Ghosh's book, you can see how we have arrived to this type of considerations. But the basics, I am going to explain you how it is done.

Now, clearly, I have three orthogonally oriented principal axes of stresses and if I consider about the shear stresses then there are three possibilities and these 3 possibilities are shown in point 1, 2 and point 3. Now in the first possibility you can get a pair of planes which are intersecting along the Sigma 2 axis. So I have these two planes, one is blue, one is green, there are intersecting along the Sigma 2 axis. Okay.

And they are inclined to the Sigma 2 axis with a value of plus and minus 45 degrees. So if I make a cross-section then I see this is my Sigma 2, which is projecting away from the board and then you have Sigma 1 and Sigma 3. So it would intersect like this and each intersection would indicate you, would have the value of 45 degrees if the rock you are considering is perfectly isotropic.

So the shear stress values you would have along these two possibilities, one is plus of this half of Sigma 3 minus Sigma 1 and another is minus of 0.5 Sigma 3 minus Sigma 1. A similar case would happen if you have intersection happening along the Sigma 1 axis. So the two planes are intersecting along the Sigma 1 axis and you can get a very similar way. So in this case, the Sigma 1 is projecting upward of this board and you have Sigma 3 and Sigma 2 left on this plane you are considering. And you would get two different planes.

Again for the third consideration what is left? That you can have the intersection along the Sigma 3 axis and therefore here in this case, Sigma 3 is this direction. So it is perpendicular to board and you have two different planes that you generate. Now from this discussion it may sound little abstract that why you would have these two planes intersecting along a particular stress, principal stresses and its orientations and then why it has to be 45 degrees. These are theoretically calculated but do we see this in nature? The answer is yes.

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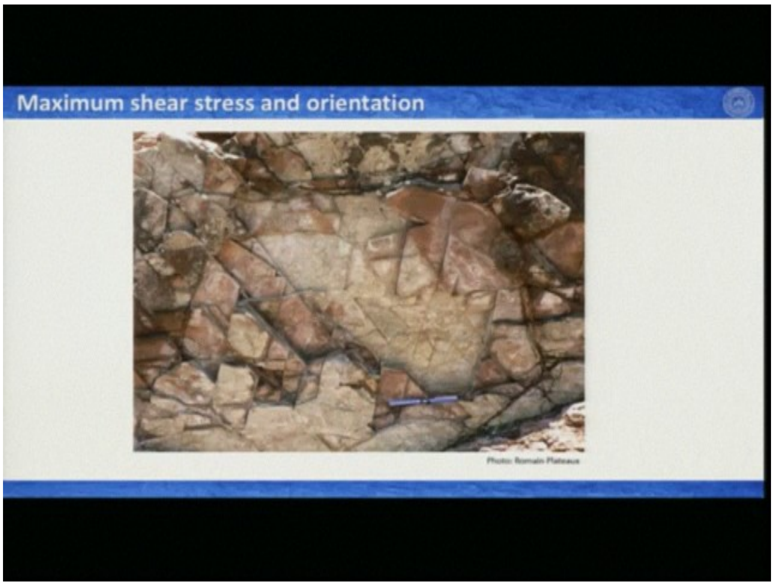

So here I have a few examples for you. What you see here? That this is a piece of sandstone, we took the photograph from the field, he took this image and what do we see here? This rock is apparently homogeneous but is characterised by a set of fractures, one set is like this and then there is a second set of fracture which goes like this, is it not?

Now it is why is to imagine that these intersection lines because these are planes is one of your principal axes of stresses and directions. And therefore if it considers this, this two are your other principal axis of stresses. So I do not know which one it is, but if I consider this one is which is projecting upward from the board. If it is Sigma 2 then you can consider this one would be Sigma 1 or Sigma 3 and this one would be either Sigma 3 or Sigma 1.

Now there are some considerations as we have seen that this has to be 45 degree, this has to be 45 degree. In this case, it is not and this is because this rock is not perfectly isotropic or they are some other considerations. But generally one angle is acute and another angle is obtuse and there are some relationships for that, we will earn it later. But these two sets of fractures, one like this, one like this, these are known as conjugate sets of shear fractures.

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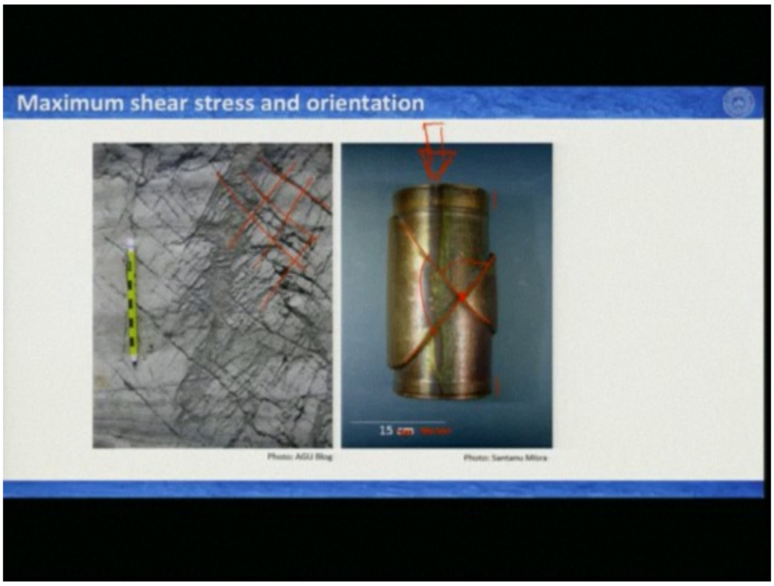

Do we see it somewhere else? Yes, we see different scales. For example here you can see, you are generating two sets of shear fractures, conjugate sets of shear fractures and this is a classic experiment that have done with high-pressure temperature. So this you see a copper jacket, actually rock sample is inside and these two lengths are covered by some alumina disks. So it got compressed from this the diameter of the sample was 15 mm. This must be wrong, there should be 15 mm.

And then what do we see? Because of the compression we generate two sets of fractures. They have some sort of displacements which we are not looking at right now. But we see that this must be intersection of 2 principal axes of stresses and we know this is a compression direction. So if you calculate this must be acute and this is the obtuse angles.

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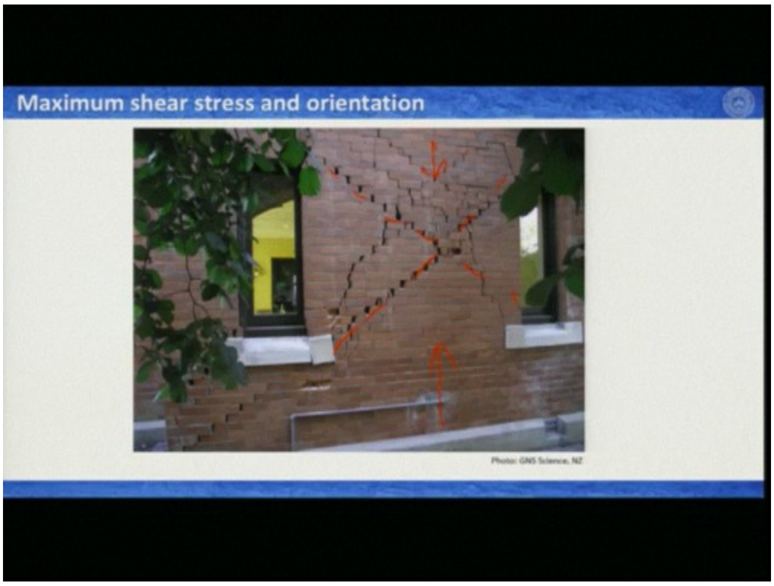

Now we see it also in a very different way, we have learnt it when you have an earthquake. That there are two different motions or you can resolve it in two different motions, one is vertical ground motion and another is horizontal ground motion. Now you can imagine if there is an earthquake, then because of the vertical ground motion, you would have some sort of stress building along the vertical directions and therefore all the walls that we see we get actually some sort of conjugate fractures and where your application of stress must be like this.

So based on these considerations we are convinced that maybe the consideration of stress, so far we learnt are purely based on mathematics and theoretical calculations. But we see strain, we see deformation of rocks, we see there are some sort of similarities or some sort of geometric relationships from one set of deformation to another set of deformation and if we can resolve this through the concept of stress, life become much easier and this is what we have learned because we have seen that it the expresses it perfectly.

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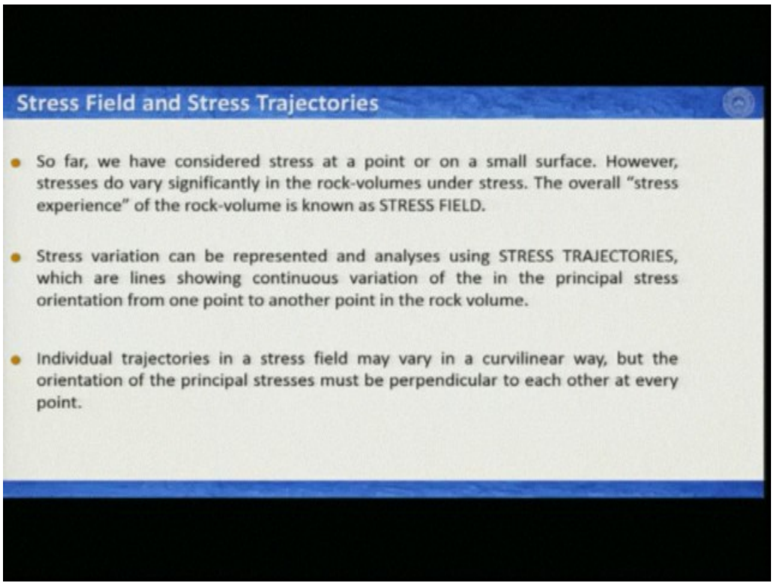

And now there is also one point that we would like to learn at this stage. That we have considers so far stress at a point or stress on a surface. But you consider lithospheric scale or few kilometres or few 100 kilometres. So and we have to visualise this that stress at each and every point is not constant. So they do vary, they do vary significantly in the rock volumes under stress. So the overall stress consideration from each and every point, we generally call it as stress field and this stress field, in this stress field the stresses do vary from one point to another point.

This stress variation can be represented and one can analyse it using stress trajectories, which are the lines showing the continuous variation of the principal stresses, principal stress orientations from one point to another point within the rock volume. Now when you do, we will see the diagram soon. The individual trajectories in a stress field that means in a single point you can draw the 3 different axes which are perpendicularly perpendicular to each other. And then you have to connect it to the next point, next point, next point and so on.

So overall, it may vary in a very curvilinear way, but it is important when you measure and draw the stress trajectories, the orientation of the principal stresses always must be perpendicular to each other and at every point.

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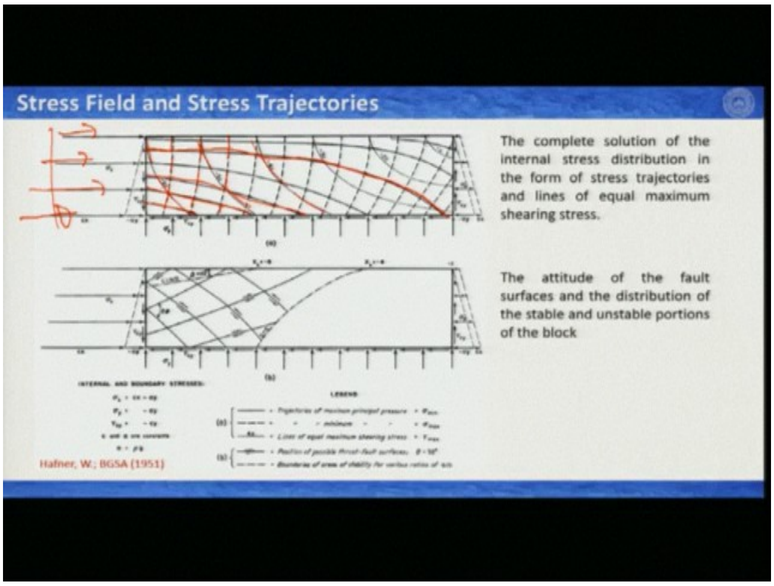

So here is a very classic work, theoretical work by W. Hafner and it is a paper of 1951 where he has calculated the stress trajectories. The different symbols are given here, in the top part, the top diagram that we see these are complete solutions of the internal stress distribution in the form of stress trajectories and the lines of equal maximum shearing stresses.

Accordingly these lines are the, these strong lines are the maximum principal stresses and these dotted lines are minimum principal stresses. The boundary conditions for the theoretical model was that he applied a significant amount of horizontal stress here. And here he applied also horizontal stress but this is much less and what we see? That because we have large horizontal stress, large magnitude horizontal stress in this side, it is slowly declining towards the low magnitude of the horizontal stress and this decline is mostly happening due to gravity.

At the same time, because he has also plotted the shear stresses which are like this and based on this shear stress and these two maximum and minimum principal stresses, it is possible to resolve also that what would be the potential failure planes or where you can generate faults in this theoretical model. So this is one of the very classic diagrams that Hafner gave and people still do follow his model to calculate the stress trajectories in different scales.

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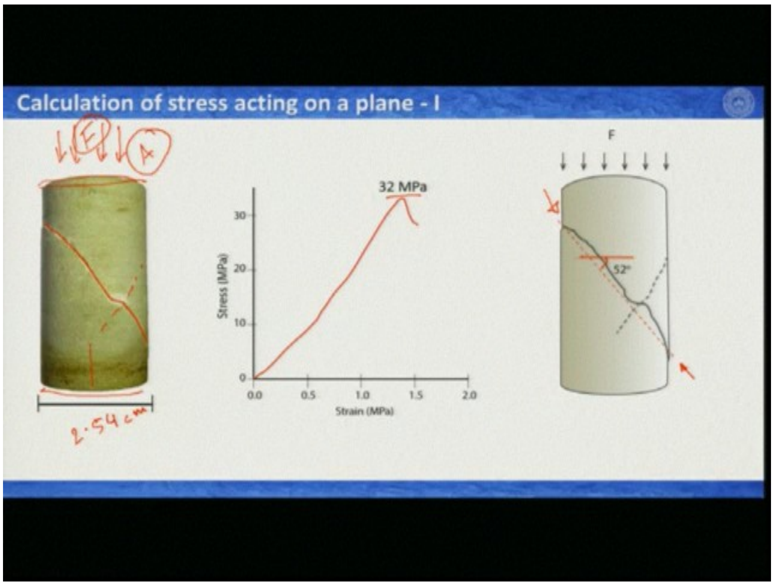

Okay, now we will learn two applications of the theory of stress that you have understood so far or discussed so far. The first is a problem that we generally solve or try to figure out when you do rock deformation experiments in the laboratory and second one is also we apply in the rock deformation experiments and at the same time in the field and in the many other applications. So we will first take the problem that we generally solve in the laboratory for rock deformation experiments.

What we see in this slide? It is a sandstone and you can see that there is a fracture running across the sample. What happened with this sample? So it was a sandstone, we drilled a core and after coring the sample we cut the top and bottom sides, keeping two faces perpendicular to the axis. So that means this surface and this surface, so they were perpendicular to the axis of the sandstone sample, sandstone cylinder to be very specific.

And then a load has been applied from the top, where this bottom was fixed and then it got deformed. And while it deformed, it produced a major fracture and a very minor fracture on this side and we just learnt the theory of shear stresses and you can see here very nicely that two surfaces were formed. One is prominent and another is not that much. But now we know that we have applied the load here, say the load was as well F.

And we know the area of this cylindrical sample. The top part of the cylindrical sample, say for example I know this is 2.54 cm. That means it is 1 inch, so we have the area of this sample surface, we have the force and we can calculate the stress. And this is the stress-strain curve of this deformation, so we see that the sample deformed at 32 MPa strains. So that its strength that is 32 MPa.

This is some sort of routine activities. But if you would like to go further for analysis, then we would like to know that what was the shear stress and what was the normal stress on this fracture surface. So if I summarise this image of this sandstone deformed, sandstone, then it looks like this, you can approximate this plane as this red dotted line here and you can figure out the angle is about 52 degrees with the horizontal plane. Now the challenge is, or the task is to find out what was the normal stress on this surface and what was the shear stress on the surface.

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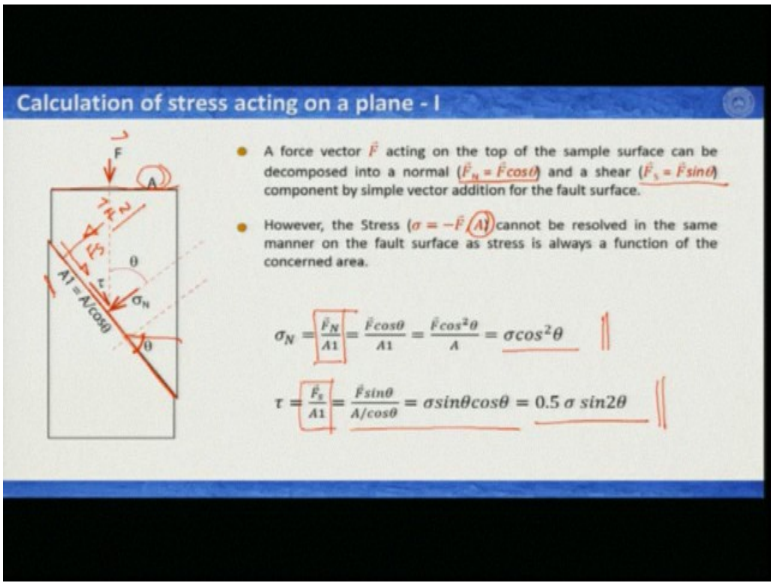

So to do that what we generally do in laboratory? We first have to make some sort of drawings and then figure it out what is working. Sort of working drawing and as we have understood from the force descriptions of this course that if this is the force being applied on this surface, then I can resolve this force on this surface very easily. Say, if I have this force acting on this, then normal to this would be your normal force. That is your FN and then parallel to this surface that would be your FS.

Now this would be a simple vector additions or vector operations. So, FN would be F cos theta and FS would be F sine theta. So this is how we can calculate the forces acting on this plane, one is normal component another is shear component. But for the stress it is not that straightforward, simply because stress is the function of the area. Now clearly the area here is not the same of the area of this fracture plane.

Now if I know this angle, which is theta, so this area if I designate it as A1, then A1 actually is a function of the area A and this angle with cosine. So A divided by cos theta is the A1, the area of the surface. So therefore to calculate now the normal stress clearly we have the FN and we have the A1, so that would be your normal stress working on the surface. And if you resolve it through some algebraic manipulations, you would get it is actually Sigma cos square theta where Sigma is a stress, the peak stress that we got which was 32 MPa.

Similarly for shear stress it would be. The shear stress, shear force component that is FS divided by the area A1 and again, if you do some algebraic manipulations you would arrive, it would be half of Sigma sine to theta. So this is how we calculate the stress acting on a fracture plane after a deformation experiments. And this you can apply in many other fields depending on your problem.

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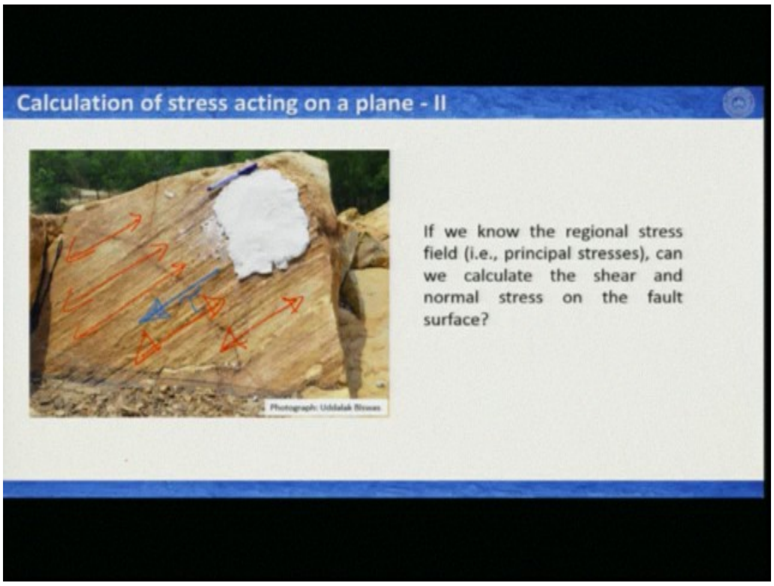

The second problem, we would look is sort of calculating the shear stress and normal stress on an inclined surface. So here I try to give you or give you a demonstration of the problem. Now what we see here in this photograph, this is a sheared surface, fault surface and this, all these striations that you these are known as silicon lines, we learn about it later. And this white thing that you see here, this we have done in the field just to sort of take a cast of this silicon lines for a different purpose. So it is still there.

But what is important? That because this is a fault plane, what do we see in this photograph? Certainly there was some sort of shear movement along this direction. So shear stress were active there, you see, I am giving double headed arrows because at least from this image, I do not know which way the fault moved. But certainly it moved in this direction. So we would like to know what was the shear stress here.

Imagine that just for this problem imagine the shear stress was in this direction. Okay, downwards, that may not be true for this exposure, but for this particular problem let us assume this it was on the down direction, down deep direction. So this would be your shear stress and then of course something. There must be a normal stress component, which was perpendicular to the surface.

Now if you would like to know that how to get it, the basic thing you need, the primary requirement for this is you have to have the regional or local stress field. That means the principal axes of stresses. Now if we consider this is the 2-D problem, then you need Sigma 1 and Sigma 3.

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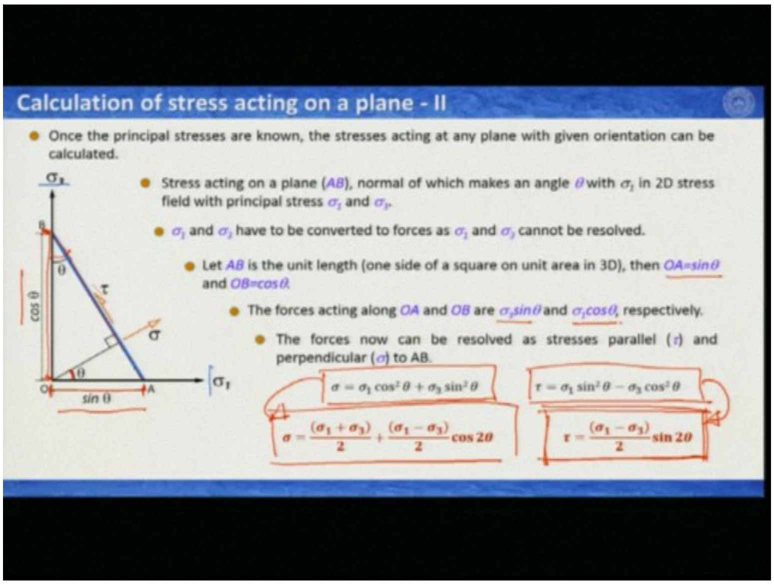

So if I would like to see this problem geometrically, then I can consider it that because Sigma 1 and Sigma 3 these are perpendicular to each other, so this is Sigma 3 and this is Sigma 1 and this is your fault plane, this blue line that you see here AB, right. So that Tau is acting along the fault surface, and this is your normal stress. The consideration is that this normal stress if I project it towards the origin of this coordinate frame Sigma 1, Sigma 3, then it makes an angle of theta.

Now we have to figure out what is the shear stress and what is the normal stress acting on a plane. Now clearly you cannot resolve it directly, so we have to first convert these stress components to the force components. So if we consider AB is the unit length, then OA, that means this would be sine theta and OB would be the cos theta. Remember, this is your theta, so this has to be also theta.

Now the forces acting on OA, that means along this particular side and OB that is this particular side would be Sigma 3 sine theta and Sigma 1 cos theta respectively. Okay, this is simple geometry. So therefore we got the forces along OA and along OB and if we have that, then we can actually calculate these forces, out of these forces we can transfer it to the stress because we have the areas. So therefore we can finally figure out what would be your normal stress component which is Sigma here and what would be your shear stress component which is Tau here and of course you can do some algebraic manipulations and you would arrive in these two equations.

I read these two equations and I will tell you why? Sigma equal to Sigma 1 plus Sigma 3 divided by 2 plus Sigma 1 minus Sigma 3 divided by 2 cos 2 theta and Tau equal to Sigma 1 minus Sigma 3 by 2 sine 2 theta, now these two equations are very, very important not only for structural geologists, but any engineering geologist, someone who does structural stability analysis and so on. Because one has to know that what would be the shear stress of a given plane once we know the forces or stresses in that region.

So with this note I conclude this lecture on stress. I hope we have more or less a very good understanding on the strain in the previous lecture series and now this stress.

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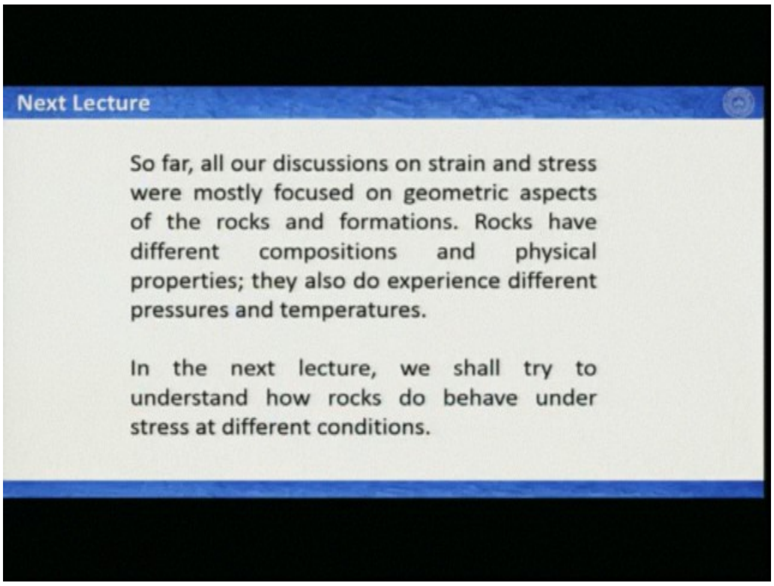

So our discussion so far, all together on strain and stress were mostly focused if we look at on geometric aspects. We did not consider what is a material, what was the material property and so on. But we know that rocks have different compositions and various physical properties. So they also do experience different pressures and temperatures at different regime when they stay in the earth.

At the surface conditions they do not experience that much pressure and temperature, but when you take all these rock materials deep inside the earth, the experience significant amount of pressure and temperature. Accordingly in the next lecture we will actually try to understand how these rock materials at different earth conditions based on the composition, based on their physical properties do behave under stress and this would be a topic of rheology which we will take over in the next lecture. Thank you. Stay tuned.