Structural Dynamics for Civil Engineers - SDOF Systems

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Lecture – 07
Forced Vibrations Part 2

Now, we will see the harmonic vibrations of damped systems.

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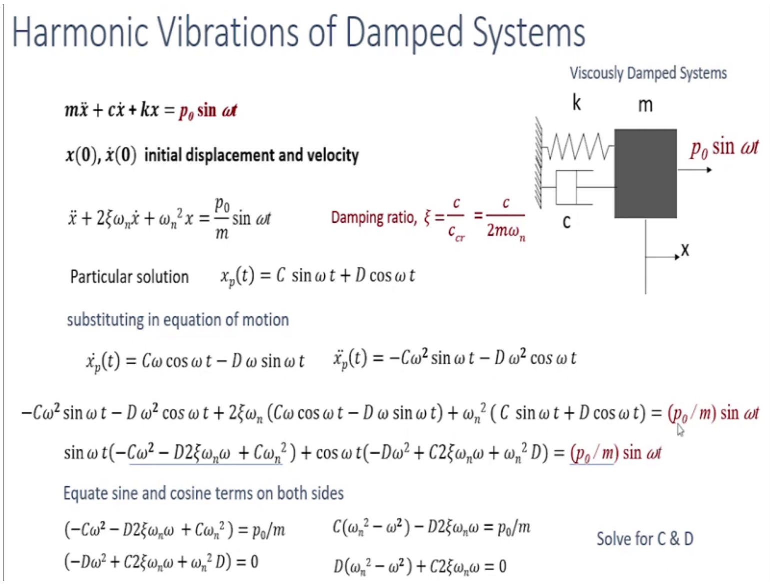

So, in free vibrations, we have already seen damped free vibrations and damping is the property by which the free vibration of a system diminishes with time and damping is because of many energy dissipation mechanisms present in the system.

So, now, let us look at a viscously damped system. So, in viscously damped system the damping is represented by a viscous damper and the damping force will be equal to the damping coefficient multiplied by the velocity of this mass.

So, the equation of motion of this type of systems, we have written for free vibrations and in forced vibration when a harmonic force of p naught sin omega t is acting on the system, the equation of motion becomes this.

$$m\ddot{x} + c\dot{x} + kx = p_0 \sin \omega t$$

It is mx double dot plus cx dot plus kx is equal to the harmonic force p naught sin omega t and the system will have some initial displacement and velocity and we assume that we know this initial conditions. We can rewrite this equation of motion like this and we can define this zeta as a damping ratio which is the damping coefficient divided by the critical damping coefficient and we have also seen in free vibrations that the critical damping coefficient is equal to 2 m omega n or 2 root k m.

$$\ddot{x} + 2\xi\omega_n\dot{x} + {\omega_n}^2 x = \frac{p_0}{m}\sin\omega t \qquad \xi = \frac{c}{c_{cr}} = \frac{c}{2m\omega_n}$$

So, now we have this equation of motion and we have to find the solution of it. So, here also the solution will consist of 2 parts; the complementary solution and the particular solution. The complementary solution will be equal to the homogeneous solution that is the solution to free vibrations. So, now, we will find the particular solution for that we will assume that the particular solution is of the form C sin omega t plus D cos omega t. So, now, we have to find out these constants C and D.

$$x_p(t) = C\sin\omega t + D\cos\omega t$$

So, for that we have to substitute this equation in the equation of motion in this equation of motion and then we have to find out these constants. Let us substitute this equation and its derivatives in this equation of motion. So, we have to find the velocity function by differentiating this once. So, if you differentiate this once, you get C omega cos omega t minus D omega sin omega t that is the velocity of this particular solution. Similarly we can find out the acceleration by differentiating this velocity once more. So, the acceleration would be minus C omega square sin omega t minus D omega square cos omega t.

$$\dot{x}_p(t) = C\omega\cos\omega t - D\omega\sin\omega t \qquad \ddot{x}_p(t) = -C\omega^2\sin\omega t - D\omega^2\cos\omega t$$

Now we can substitute the particular solution expressions for displacement velocity and acceleration in this equation of motion. So, if you do that substitution we will get this expression. So, first we have x double dot. So, the first 2 terms will represent this acceleration and 2 zeta omega n multiplied by the expression for velocity then omega n square multiplied by the displacement terms.

So, we have this expression which is equal to p naught by m sin omega t which is the force. So, now, we can rearrange this equation. So, we can collect all the sin omega t terms. So, sin omega t multiplied by minus C omega square, then this term 2 zeta omega n D omega then the next sin omega t term is this C omega n square. Then we have the cos omega t terms. So, minus D omega square plus this term C 2 zeta omega n omega then we have this term which is equal to omega n square D. So, we just rearrange these terms in sin and cosine terms. So, now, we can equate the sin and cosine terms on both sides. So, these are the sin terms on left hand side and the right hand side we have one sin term sin omega t term.

So, we can equate these terms to this one we can also equate the cos omega t terms to 0 because in the right hand side, there are no cos omega t terms. So, if you do that we will get two expressions one is this equal to p naught by m the second is this expression equal to 0. So, we can rearrange this again and we will get an expression like this from this equation and another expression from this equation. So, you have two equations and 2 unknowns C and D. So, we can solve this algebraic equation and find out the expressions for our constants C and D.

So, after solving these two, equations we will get the expressions for C and D.

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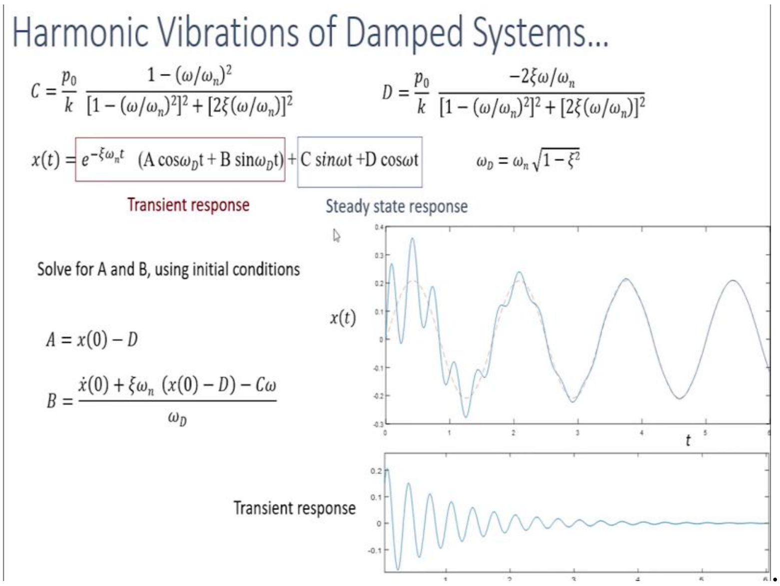

And that will be equal to C is equal to p naught by k 1 minus frequency ratio square divided by 1 minus frequency ratio square the whole square plus 2 zeta frequency ratio square and D is equal to this p naught by k, the numerator will be minus 2 zeta frequency ratio, the denominator will be same as C. Now we know the values of C and D so, we can write the expressions for the particular solution as C sin omega t plus D cos omega t. And we know that the total solution is the sum of complimentary solution and the particular solution.

$$C = \frac{p0}{k} \quad \frac{1-(\omega/\omega n)2}{[1-(\omega/\omega n)2]2+[2\xi(\omega/\omega n)]2} \qquad D = \frac{p0}{k} \quad \frac{-2\xi\omega/\omega n}{[1-(\omega/\omega n)2]2+[2\xi(\omega/\omega n)]2}$$

So, we have seen in harmonic vibrations of undamped systems that the complementary solution would be equal to the free vibration response. So, here we have the total solution as the complementary solution and the particular solution. So, this is if you remember the free vibration response of the damped system which is equal to e to the power minus zeta omega nt multiplied by A cos omega Dt plus B sin omega Dt. So, this omega D is equal to the natural frequency of the damped system which is equal to omega n which the natural frequency of the undamped system multiplied by 1minus zeta square zeta is the damping ratio.

$$x(t) = e^{-\xi\omega nt} \quad (\text{A } \cos\omega_D \text{t + B } \sin\omega_D \text{t}) + \text{C } sin\omega \text{t +D } \cos\omega\text{t} \qquad \omega_D = \omega_n \sqrt{1-\xi 2}$$

So, now, we have the total solution. So, if you look at the solution, these two terms they decay with time. So, this exponential function is in terms of t. So, as time increases this these 2 terms decrease with time. So, this part of the solution is known as transient response because this decays with time and this part is known as steady state response. So, this will not decay with time, this will be the predominant response because this part decays with time and this will be remaining as long as the force remains. So, this is the predominant part of the response. So, as we have did earlier, we can calculate the constants A and B using the initial conditions. So, we have the initial displacement and initial velocity.

So, you can substitute the initial velocity in this expression and we can find out the velocity expression by differentiating these ones and substitute the initial velocity value in that. And then we will have two equations; if you solve those two equations, we will get the value of A and B. So, if you do all that we will get A is equal to x naught the initial displacement minus this expression D and B will be equal to this. I am not showing this derivation in this video, but I would like you to try this out and calculate these constants.

$$A = x(0) - D$$

$$B = \frac{\dot{x}(0) + \xi\omega_n\ (x(0) - D) - C\omega}{\omega_D}$$

So, now let us see how the solution looks like. So, if you plot this total solution for some values of initial conditions, we will get the solution like this the blue line represents the total solution, the red dotted curve represents the steady state response. So, and if we plot only the transient response, it will look like this. So, as expected this transient response will decay in time and become zero after some time. And if you look at the total solution initially, the total solution will vary like this; it is the sum of the steady state response and this transient response. So, after some times as the transient response decays, the total solution will become equal to the steady state response and that is why this part of the solution is known as steady state response because that part of the solution remains as long as the forces available.

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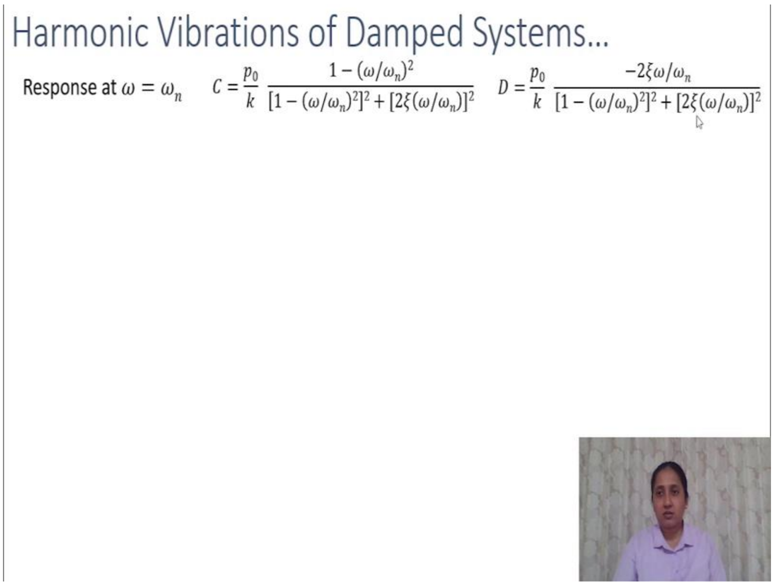

Now, let us look at the vibrations response at omega is equal to omega n. In undamped systems, we have seen that when the forcing frequency is equal to the natural frequency; the response becomes unbounded. The amplitude of the steady state response increased in each cycle. Now let us see how the damped system behaves when the forcing frequency is equal to the natural frequency.

$$C = \frac{p0}{k}\ \frac{1-(\omega/\omega n)2}{[1-(\omega/\omega n)2]2+[2\xi(\omega/\omega n)]2} \qquad D = \frac{p0}{k}\ \frac{-2\xi\omega/\omega n}{[1-(\omega/\omega n)2]2+[2\xi(\omega/\omega n)]2}$$

So, we have the expressions for the constants C and D. When omega is equal to omega n, this ratio will become 1 and C will be equal to 0. This numerator will become 0. So, here in the expression for D, this term will become minus 2 zeta and this term will vanish and this term in the denominator will become 2 zeta the whole square. So, together this will become p naught by k minus 2 minus 1 divided by 2 zeta.

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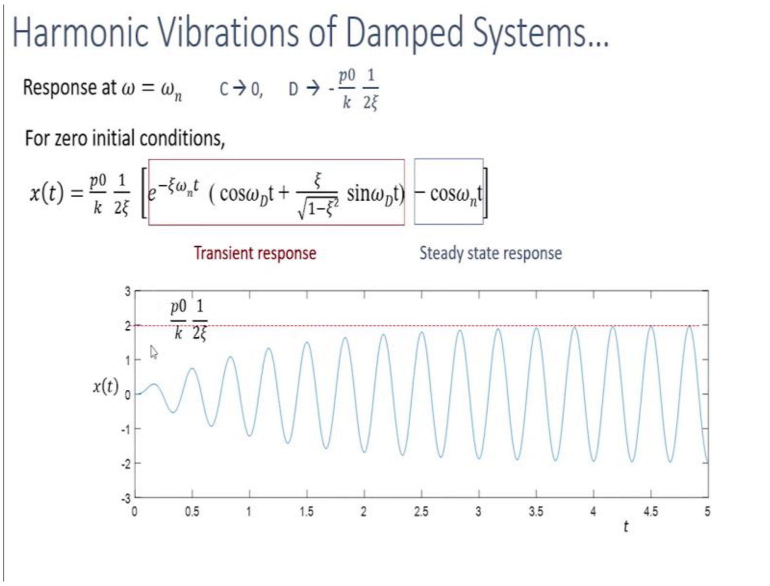

So, C becomes 0 and D becomes this value. The complementary solution can be found out using the initial conditions for zero initial conditions the total response of this damped system will be this and as you can see these terms decay with time.

$$x(t) = \frac{p_0}{k}\frac{1}{2\xi}\left[e^{-\xi\omega_n t}\ \{\cos\omega_D\, t + \frac{\xi}{\sqrt{1-\xi 2}}\sin\omega_D\, t\} - \cos\omega_n t\right]$$

So, they represent the transient response and the steady state response is this which is a function of omega n as the forcing function is equal to the natural frequency. Now let us see the solution graphically. So, the solution looks like this. The amplitude of this steady state response is equal to p naught by k multiplied by one by 2 zeta unlike in the case of undamped systems, this amplitude is not a function of t. So, this is a constant and this does not increase with time.

So; that means, in the steady state response, we have a bounded response. The amplitude of the steady state response is equal to this value and this does not change with time this stays constant with time and as you can see here in the beginning when the time is small we have the effect of this transient response. So, after some time the transient response will decay and will become equal to 0 after some time. So, the total response converges to the steady state response and continues as long as the force remains and the amplitude of the steady state response is also constant. So, the amplitude does not increase with time.

So, now you would be able to appreciate the effect of damping in systems. In undamped system, we have seen that at omega is equal to omega n the response increases after each cycle. So, as the time goes by the response keeps on increasing, but damping makes that response bounded and it makes it bounded to this value. No matter how many how much time passes, the maximum amplitude of the response will be this much p naught by k multiplied by 1 by 2 zeta. So, if the damping is high, the amplitude will be smaller and the damping is small, the amplitude will be larger and we know that the p naught by k is equal to the static response.

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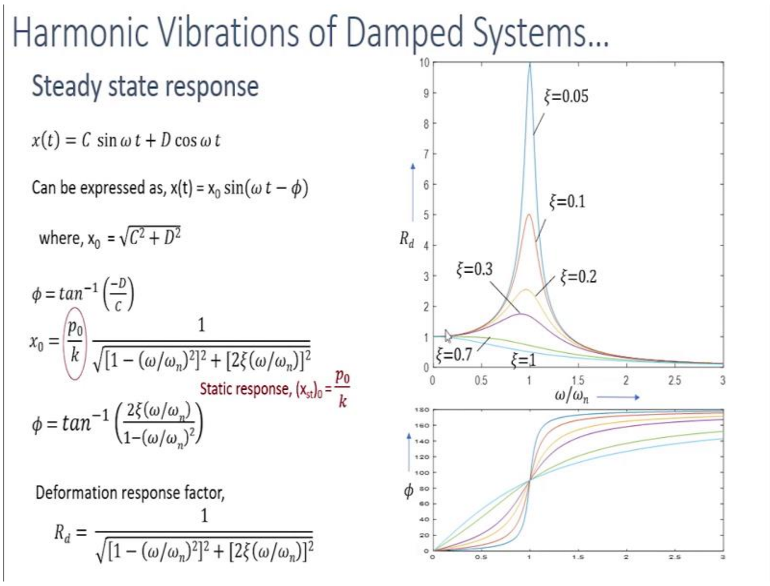

Now, let us explore the steady state response in detail as the transient response decays with time the steady state response is the predominant form of vibration. So, as we have seen already, this is the expression for the steady state response and this can be alternatively expressed as x naught multiplied by sin omega t minus phi. So, this sin and cos terms can be written as a sin term with some phase angle and here the x naught is equal to the square root of C square plus D square and the phase angle this tan inverse mins D by C. So, we can calculate we know the expressions for C by D using that we can calculate the amplitude. It will be equal to p naught by k multiplied by 1 by square root of 1 minus frequency ratio square, the whole square plus 2 zeta frequency ratio the whole square.

$$x_0 = \frac{p_0}{k} \frac{1}{\sqrt{[1-(\omega/\omega_n)2]2+[2\xi(\omega/\omega_n)]2}}$$

$$\phi = tan^{-1}\left(\frac{2\xi(\omega/\omega n)}{1-(\omega/\omega n)2}\right)$$

So, we have seen that this p naught by k is equal to the static response and this is a multiplication factor by which the amplitude of this response amplifies or decreases. And the phi can be found out as tan inverse 2 zeta frequency ratio divided by 1 minus frequency ratio square. We can denote this factor as the deformation response factor which is Rd and this is equal to this factor.

$$R_d = \frac{1}{\sqrt{[1-(\omega/\omega_n)2]2+[2\xi(\omega/\omega_n)]2}}$$

Now, let us see how this deformation response factor varies with the damping ratio, we have seen similar result in undamped systems also. So, in damped systems depending, upon the damping ratio the value of Rd changes.

So, when the damping is small, the value of Rd at damping ratio is equal to 1 is very high and when the damping is increasing the response factor at damping ratio 1 becomes smaller and smaller. As we have seen in the undamped systems when the damping ratio is close to 0 that is when the forcing frequency is close to 0, we can see that Rd is equal to 1; that means, the amplitude of the steady state response will be equal to the static response p naught by k.

When the frequency ratio increases this response factor will also increase and that increase will depend upon the value of damping. So, for light damped systems this amplification will be larger at frequency ratio is equal to 1 and when the frequency ratio increases beyond one, this factor becomes smaller and smaller and when the frequency ratio is much higher than 1, this becomes 0 irrespective of the damping present in the system. So, the damping is controlling this factor when the frequency ratio is near 1. Let us understand the steady state deformation response in little bit more detail.

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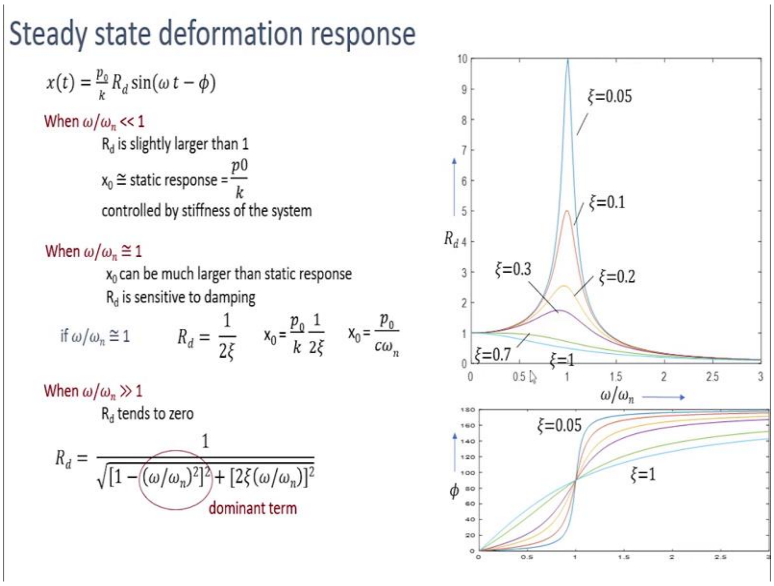

So, the steady state response can be written as the static response multiplied by the deformation response factor multiplied by sin omega t minus phi. So, Rd varies like this and phi varies like this. So, what happens when the frequency ratio omega by omega n is much less than 1?

So, we have seen that when the forcing frequency is equal to 0 or the frequency ratio is close to 0, this factor Rd is equal to one and the static response will be equal to the dynamic steady state response. So, when the frequency ratio is smaller than one when it is in this range, then also this Rd value is very close to 1 which means the steady state response is approximately equal to the static response. So, in this region that is when the frequency ratio is smaller than 1 much smaller than 1, the response is equal to p naught by k.

So, as you can see from this expression this response is controlled by the stiffness of the system if the stiffness is high, the response is smaller the displacement will be smaller and if the stiffness is very low this response will be higher. So, this is similar to the static case. Now when the frequency ratio is near one it is equal to or approximately equal to 1 as you we have seen earlier this amplitude of the response can be much larger than the static response this value of Rd can be much larger than 1. The value depends upon damping for very high value of damping like damping is equal to 0.7 say it can happen that the Rd value is less than one when the frequency ratio is one, but for all real structures the damping will be much smaller. So, the Rd value will be more than 1.

So, we can say that when the frequency ratio is close to 1, the steady state response is controlled by the damping in the system. So, when the frequency ratio is approximately equal to one the value of Rd would be 1 by 2 zeta and the steady state amplitude becomes p naught by k multiplied by 1 by 2 zeta. So, if we substitute the damping ratio we would get x naught is equal to p naught by C omega n. As you can see in these two expressions the amplitude will be controlled by the damping as the damping increases, the value of x naught decreases. So, for small values of damping, the x naught that is the steady state response will be much higher than the static response. So, we can see the same behavior in this curve also.

So, now what happens when the frequency ratio is much larger than 1? When the frequency ratio is larger than 1 as you can see in this curves so, the value of Rd becomes equal to 0, it tends to 0 as the frequency ratio increases.

$$R_d = \frac{1}{\sqrt{[1-(\omega/\omega_n)2]2+[2\xi(\omega/\omega_n)]2}}$$

So, this is the expression for Rd and in this if you observe the denominator, the terms under the rule. So, 1 minus frequency ratio square the whole square. So, this term is frequency ratio to the power 4 and this term has the frequency ratio to the power 2. So, compared to this term the value of this term is negligible. So, you can say this is the dominant term in this expression for Rd, when the frequency ratio is much higher than 1.

So, we can write the expression for the steady state amplitude as p naught by k, the static response multiplied by this term that is square root of frequency ratio to the power 4; that means, omega n square divided by omega square. So, we can substitute the value of omega n as root of k by n. So, this becomes p naught by m by omega square. So, as you can see here, this expression for the steady state amplitude is controlled by the mass of the system.

So, when the frequency ratio is higher than 1, Rd tends to 0 and the value of Rd will be controlled by the mass of the system. If the mass is high the response, the steady state response will be smaller the mass is very low this response will be larger. However, as the frequency ratio goes higher than 1, the response goes to 0. It approaches 0 and this is how the phase angle varies with damping.

We have noticed that when the damping is 0 that is for undamped system, the phase angle was 0 when the damping ratio was less than 1 and it was equal to 180 when the damping ratio was higher than 1. So, when there is some damping present in the system the nature of this phase angle also changes. When the frequency ratio is close to 0, the phase angle is also close to 0 and when the frequency ratio is much higher than 1, the phase angle approaches 180 degree and the nature of the changes in phase angle will depend upon the value of damping. So, as the damping increases, this curve becomes flatter and it changes its values gradually.

So, if we want to design a structure to resist a harmonic force, how will we design that structure, what should be its natural frequency? As we can see from these curves we know that when the natural frequency of the system is close to its forcing frequency, the amplitude of the steady state response is much higher than the static response. So, we can either design the system with the natural frequency such a way that the frequency ratio is much less than 1. In that case our response will be same as the steady state response or the another way is we can design it in such a way that the frequency ratio is much larger than one.

So, in that case, the response will be close to 0; the response will be even much lesser than the steady state response and in designing a system, there will also be some other constraints which will limit the value of the natural frequency which we can adopt. So, the general rule is that keep the natural frequency away from the forcing frequency as much as we can. So, either make it lesser than the forcing frequency or make it higher than the forcing frequency.

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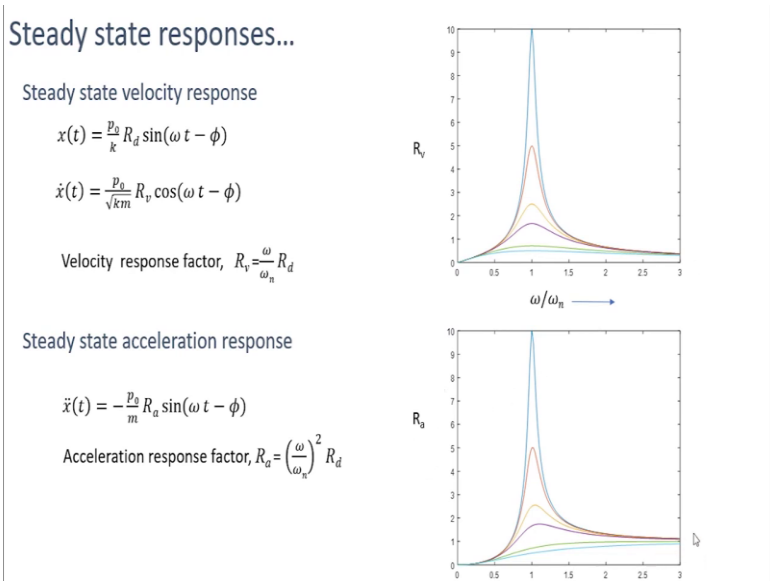

Now, let us see the velocity and acceleration response at steady state. So, this is the displacement response at steady state. This is equal to p naught by k Rd sin omega t minus phi. So, if you differentiate this once, you will get the velocity response. So, the velocity response can be written as p naught by root k m Rv which is the velocity distress response factor and multiplied by the cos function in omega t minus phi. And this velocity response factor Rv is frequency ratio times Rd.

So, if you differentiate this, you would get this expression with Rv is equal to omega by omega l multiplied by Rd. So, if you plot this we get curves similar to Rd, it will look like this and the values will change with the damping ratio. So, this highest curve is for the lowest damping ratio. When the frequency ratio is close to 0 the value of Rv is also close to 0 and when the frequency ratio is near 1, the value of Rv increases; it is much higher than 1 and it is value depends upon the damping.

And when the frequency ratio is much higher than 1, the value of Rv converges to 0 again. Now let us see the steady state acceleration response. So, if we differentiate this velocity response once, we would get the acceleration response. So, the acceleration response is equal to minus p naught by m Ra that is the acceleration response factor multiplied by sin omega t minus phi and the acceleration response factor Ra is equal to omega by omega n that is the damped frequency ratio square times Rd.

So, if we plot this Ra we would get curves similar to Rv and Rd and here also when the frequency ratio is near 1, the value of Ra is much higher than 1 and again it will depend upon the value of the damping. When the frequency ratio is smaller than 1, the Ra will have values very close to 0 and when the frequency ratio is much higher than 1, the value of Ra will converge to 1. So, the behavior of Ra and Rv away from 1 is different from Rd. So, in the case of Rd, when frequency ratio was close to 0; Rd was equal to one, but Rv and Ra becomes equals to 0 when the frequency ratio is close to 0. And when the frequency ratio is much higher Rv is equal to 0 similar to Rd, but Ra converges to 1.

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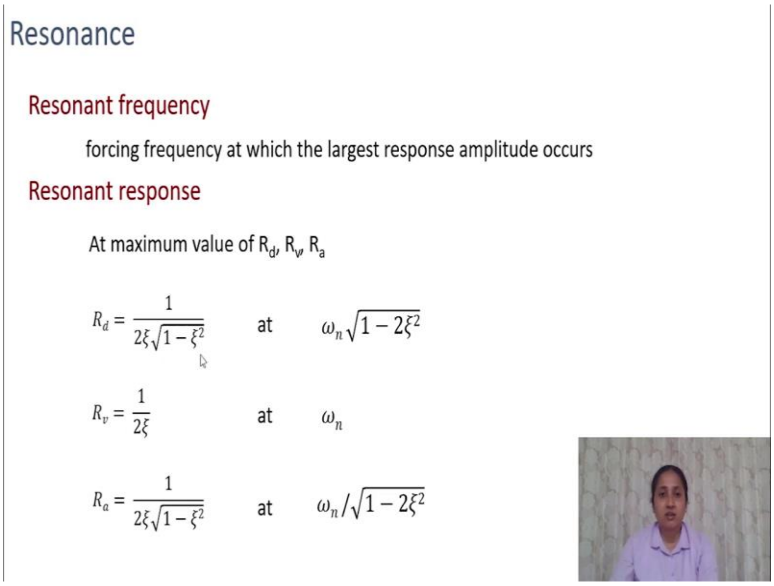

Now, let us understand resonance in damped systems. So, in undamped systems we have seen that at resonance the response becomes unbounded, the response increases in each cycle and that happened when the forcing frequency is equal to the natural frequency. So, in damped systems the response will never be unbounded. So, because of damping it will be bounded. So, we can say that the resonance occurs when the response is maximum. The response is maximum when the forcing frequency is near natural frequency, it need not be equal to natural frequency.

So, the resonance frequency can be defined as the forcing frequency at which the largest response amplitude occurs. The forcing frequency will be different for the displacement, velocity and acceleration responses and the resonant response will be equal to the maximum response for displacement velocity and acceleration and that will be when the value of Rd, Rv and Ra are maximum.

So, we can calculate the value of resonant frequency and resonance amplitude by maximizing the expressions for Rd, Rv and Ra. So, how will you find the maximum value of a function? We can differentiate it with respect to the variable here, in the case we can differentiate the expressions for Rd, Rv and Ra with respect to the forcing frequency and we can equate it to 0 and that would correspond to the maxima or minima and we can find out the second derivative of Rd, Rv and Ra. And when the second derivative is negative that indicates a maxima. So, we can differentiate the expressions for this and calculate the maximum value of Rd, Rv and Ra and they are equal to these.

So, the maximum value of Rd is equal to 1 by 2 zeta square root of 1 minus zeta square and this happens when the forcing frequency is a little less than the natural frequency. So, the resonant frequency for the displacement response is omega n multiplied by 1 minus 2 zeta square and for velocity response the maximum response factor is 1 by 2 zeta and this happens when the forcing frequency is equal to the natural frequency.

Similarly we can calculate the resonance response for acceleration. So, at resonance, the acceleration response factor would be equal to 1 by 2 zeta square root of 1 minus zeta square and this happens when the forcing frequency is equal to omega n divided by square root of one minus 2 zeta square. So, this is little higher than omega n.

So, now we can see that the resonant frequency for displacement velocity and acceleration are slightly different from each other. When the damping ratio is very small, these values under the square root will become equal to 1. So, in that case the resonant frequency would be equal to omega n that is when the damping is very very small and when the damping is very small, Rd, Rv and Ra all of them will be equal to 1 by 2 zeta because this zeta squared term will vanish when zeta is very very small.

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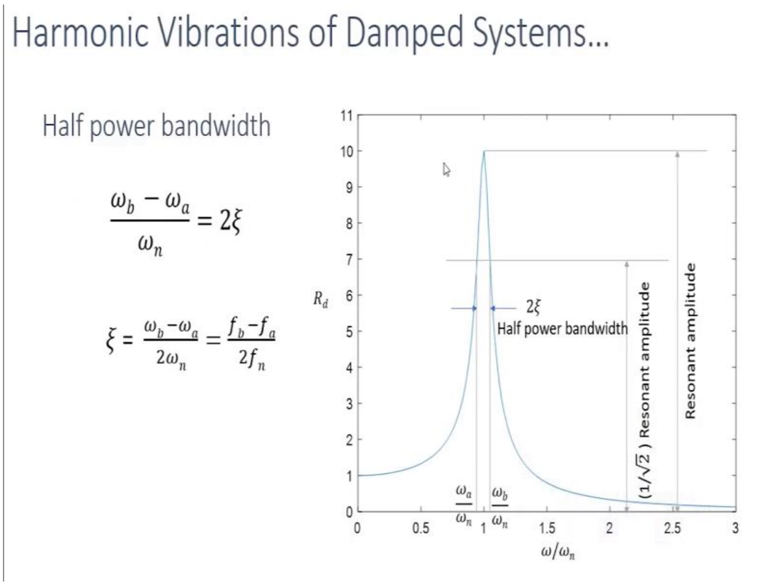

Now, we will understand a property of the frequency response curve called half power bandwidth. So, this is the frequency response curve for the displacement. So, this is the displacement response factor Rd and it is plotted against the frequency ratio.

So, we know that at resonance, this curve has the maximum value. So, this corresponds to the resonant amplitude. Now we can find out two frequencies on either side of this resonant frequency where the amplitude is equal to 1 by root 2 of the resonant amplitude and we can prove that the difference between these frequencies these frequency ratios is equal to 2 zeta. So; that means, omega b minus omega a by omega n is equal to 2 zeta and this is called half power bandwidth because the power in a system is proportional to the square of its displacement. So, here these frequencies correspond to one by root 2 of the displacement amplitude.

So, the power is correspond to half that of the power at resonance. So, this is called half power bandwidth. So, if we can measure this half power bandwidth, we can calculate the value of the damping ratio as omega b minus omega a by 2 omega n. This can be expressed in terms of the cyclic frequencies as well. So, the zeta can be expressed as fb minus fa by 2 fn where f corresponds to the cyclic frequency that is omega divided by 2 pi.

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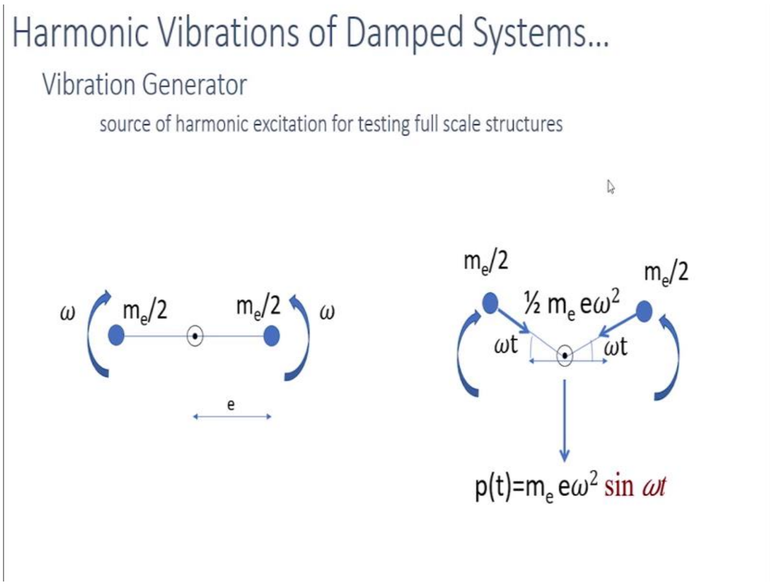

To understand the dynamic properties of structures we often test full scale structures using some harmonic excitation. So, to do that, we need to generate harmonic forces and now we will explore the working of a basic vibration generator.

So, this vibration generator will have 2 masses which are of equal mass and they are placed at some eccentricity a from the center and these two masses rotate in opposite directions with some angular velocity omega. So, this will be rotating in clockwise direction and this will be rotating in anti clockwise direction. At time t, the eccentric masses would have moved by an angle equal to omega t. So, a force will act on each of the eccentric masses towards the center by the amount half me e omega square. The horizontal component of both of these forces will cancel out each other and the vertical forces remains.

So, the vertical component will be equivalent to me e omega square sin omega t that is the sum of the vertical components of both these forces. So, this system will exert a harmonic force which is equal to this much to a structure which is attached to this vibration generator. So, if we compare it with our normal expression for the force. So, the p naught is equal to me e omega square. So, this will exert a force whose amplitude is equal to the eccentric mass times the eccentricity times omega square. So, we can use this and find out the response for a full scale structure. To test a full scale structure we can run the vibration generator at different values of omega and measure the response of that structure.

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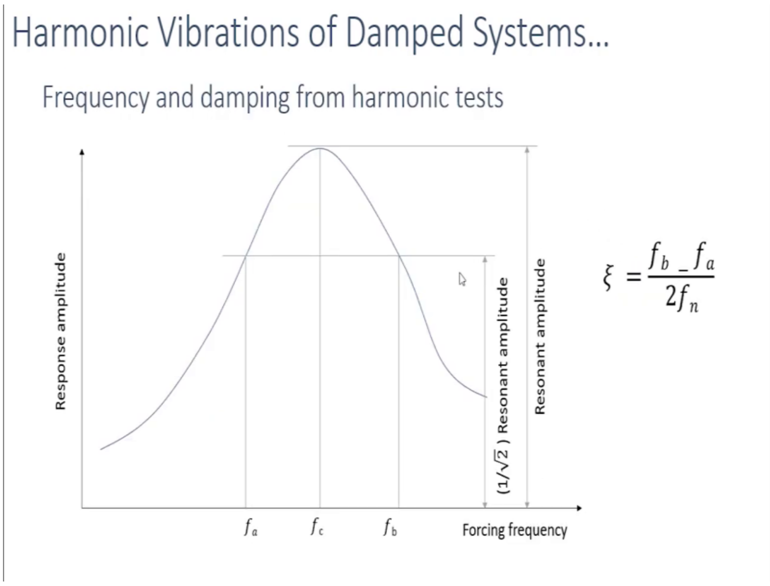

So, from the measured responses we can plot a curve like this. So, the x axis will be the forcing frequency for different values of forcing frequency, we will have the response of the structure. So, from this we can find out the maximum response. As we have seen it earlier when the damping is very less the resonant frequency is equal to the natural frequency. So, we can find the frequency corresponding to this maximum response, we can consider it as the natural frequency and the damping ratio of the structure can be calculated using the property of half power bandwidth.

So, we can find out the resonant amplitude and we can find out 1 by root 2 times the resonant amplitude and we can calculate fb minus fa divided by 2 fn and that would be equal to zeta. So, using harmonic test we can calculate the damping ratio and the natural frequency of the structure.

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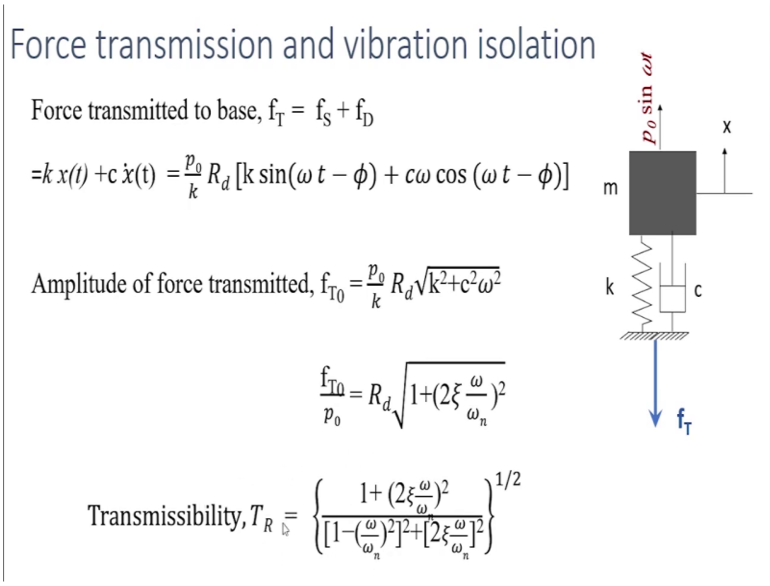

Now, let us understand the force transmitted from a vibrating system to its support and this information will be used in designing the vibration isolation systems. So, we have a single degree of freedom system and it is acted by a harmonic force equal to p naught sin omega t. So, we have to find out the force transmitted to the support and this force will be equal to the force in the spring plus the force in this damper. So, the transmitted force to the base is equal to spring force plus damping force. So, which is equal to k that is the stiffness times the displacement response plus the damping coefficient times the velocity response.

So, that is equal to this value that is p naught by k multiplied by Rd that is the deformation factor; deformation response factor multiplied by k sin omega t minus phi plus c omega cos omega t minus phi. So, the amplitude of this force can be calculated as p naught by k Rd times square root of k square plus c square omega square. So, that would be the amplitude of the force transmitted to the support.

So, we can find out the ratio of the amplitudes of the transmitted force to the amplitude of the applied force and that would be equal to R d times square root of 1 plus 2 zeta omega by omega n the whole square you will get this term from this if you divide this by k you will get this.

So, this ratio is known as the transmissibility. So, the transmissibility is equal to Rd times this term and we know Rd is equal to 1 by square root of 1 minus frequency ratio square the whole square plus 2 zeta frequency ratio the whole square. So, we can write the expression for transmissibility as this.

$$\text{Transmissibility}, TR = \left\{ \frac{1+(2\xi\frac{\omega}{\omega n})^2}{[1-(\frac{\omega}{\omega n})^2]^2+[2\xi\frac{\omega}{\omega n}]^2} \right\}^{1/2}$$

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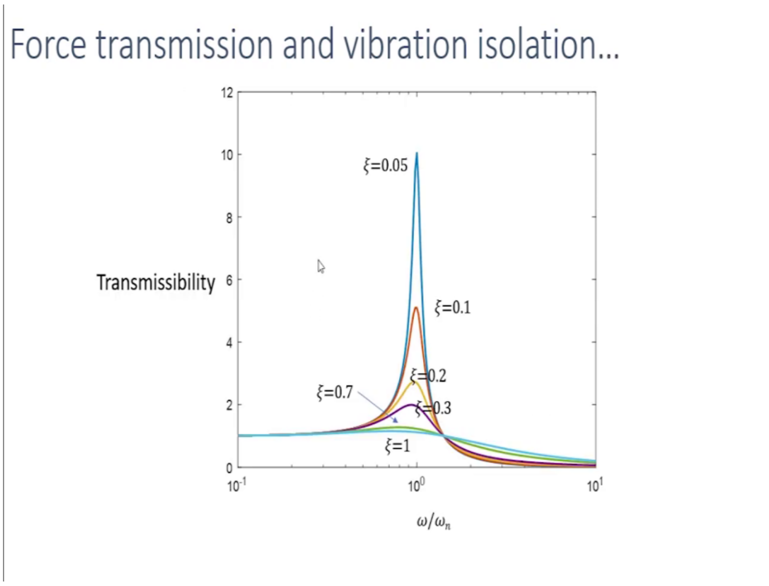

So, if you plot that relation the transmissibility versus frequency ratio, you will get curves like this. So, each curve will correspond to a different value of damping. So, as in the case of deformation response factors the highest peak at frequency ratio is equal to one will be for the lowest damping value, when the frequency ratio is much less than 1 the transmissibility is equal to 1.

So; that means, the applied force is transmitted to the support as it is. The vibration is not at all isolated in this case and when the frequency ratio is close to one the transmissibility is higher than one; that means, the transmitted force is more than the applied force. So, instead of isolating the vibration an amplification of vibration is happening when the frequency ratio is higher than root 2 the transmissibility is less than one; that means, the transmitted force is less than the applied force; that means, vibration isolation is happening when frequency ratio is more than root 2 and the relationship between transmissibility and damping also changes when the damping ratio is higher than root two.

So, when the damping ratio is close to one the transmissibility was low if the damping was high. So, the highest peak was corresponding to the lowest value of damping, but after frequency ratio is equal to root 2 this relationship changes. So, beyond root 2 the transmissibility is high for higher value of damping the transmissibility is smallest for the smallest value of damping.

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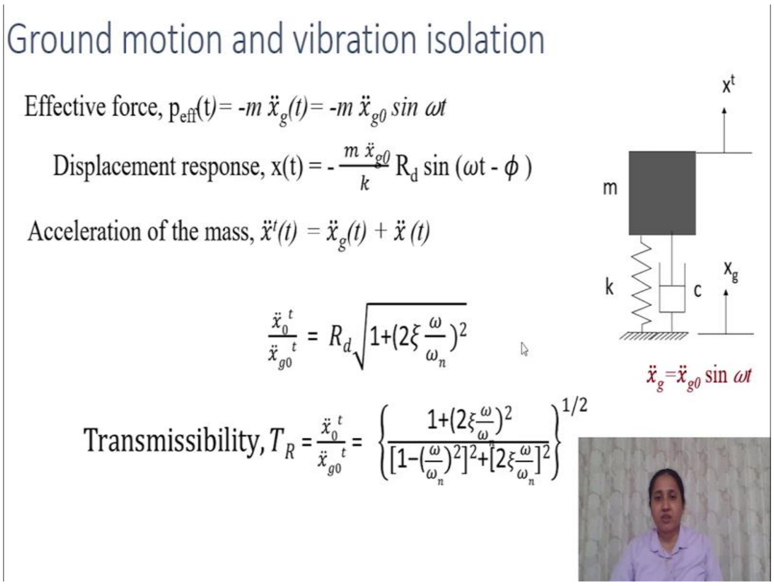

Now, let us see how ground motion is transmitted to a structure. So, we have this single degree of freedom structure with some stiffness and damping and it has a mass m and at the support a ground acceleration equal to x double dot g naught sin omega t is acting. So, this ground vibration will be transmitted to the system and we will find how that transmissibility is. So, the effective force acting on the structure because of this base acceleration would be equal to minus m multiplied by this ground acceleration.

So, this is minus m x double dot g naught sin omega t and the displacement response due to this effective force can be found out like this, it will be minus m acceleration amplitude divided by k Rd times sin omega t minus phi and the acceleration of this mass the total acceleration of this mass will be the sum of the ground acceleration plus the acceleration of this mass that is the acceleration of this mass relative to the support. So, the total acceleration will be the sum of these two.

So, we can find the ratio of the total acceleration of this mass to the ground acceleration amplitude. So, and that can be calculated as from this expression it would be equal to Rd times square root of one plus 2 zeta frequency ratio square, this is similar to the force transmissibility which we have seen earlier.

So, the transmissibility here is the ratio of the amplitude of acceleration of this mass divided by the ground acceleration amplitude and that would be equal to this which is exactly same as the force transmissibility we have seen earlier.

$$\text{Transmissibility}, TR = \left\{ \frac{1 + (2\xi \frac{\omega}{\omega n})^2}{[1 - (\frac{\omega}{\omega n})^2]^2 + [2\xi \frac{\omega}{\omega n}]^2} \right\}^{1/2}$$

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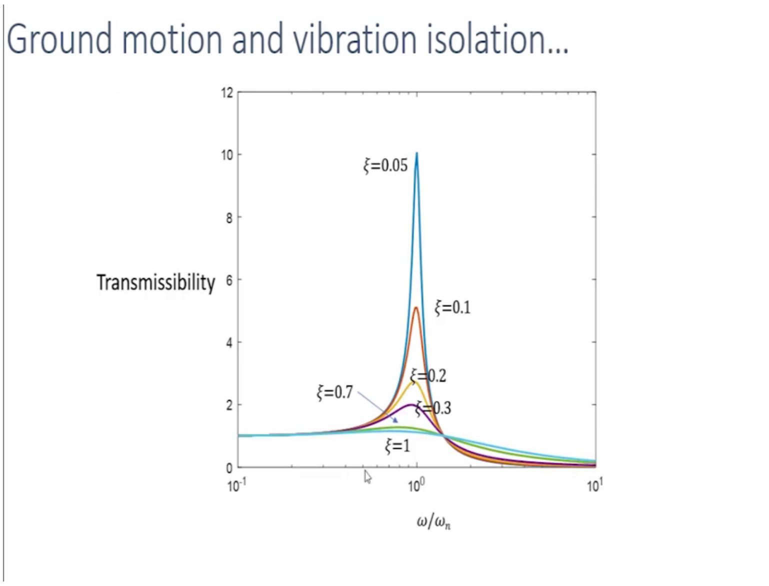

So, here also the nature of transmissibility is similar to the force transmissibility. So, if we have to isolate a structure from ground acceleration, we have to make sure that the frequency ratio is higher than root 2 and when the frequency ratio is higher than root 2 the transmissibility is proportional to the damping; that means, if the damping is less, the transmissibility will also be less. So, if you have to isolate ground acceleration, we need to have a support with very less damping. This concludes our discussion on harmonic vibrations of viscously damped systems in the next lecture we will solve some examples.