Structural Dynamics for Civil Engineers – SDOF Systems

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Lecture – 13
Response to Pulse Excitations

In this lecture we will discuss the Response of Single Degree of Freedom System to Pulse Excitations. A pulse excitation is a force time varying force which is acting for a short duration.

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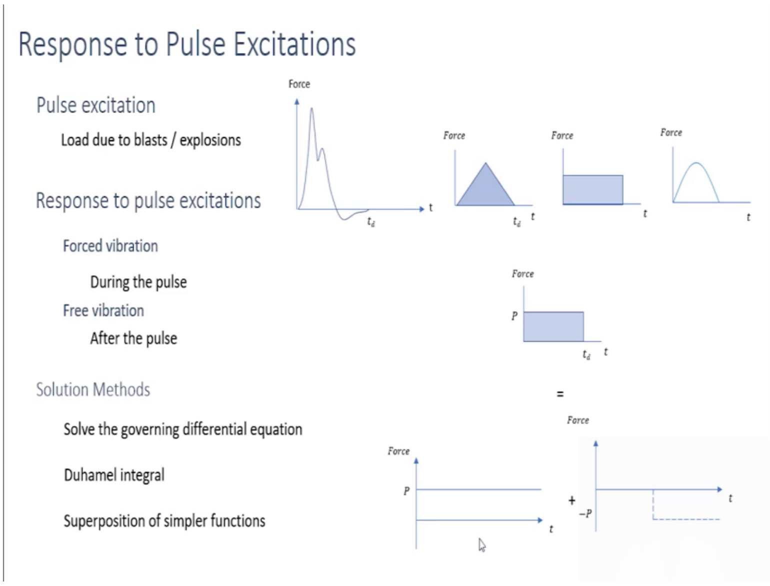

And, these type of excitations happened during blast or explosions. So, this is how a forcing function will look for pulse excitation. So, the force is a time varying force, but it will act only for a short duration. This is another example of a pulse a triangular shape pulse, this is a rectangular pulse. So, in this case the force will be constant, but again it acts for a short duration. So, this is another example and this is a half sine pulse.

Now, let us see how we evaluate the response of a system due to this type of pulse excitations, because of this pulse excitation the system will have a forced vibration and a free vibration. So, when this force is acting the system will be under force vibration. And, once the force stops that is after this time td the system will be under free vibrations. And, that free vibration will depend on the velocity and displacement of the system at the end of this pulse that is when t is equal to td.

So, far we have learned different methods to find the response of a single degree of freedom system under various type of forcing functions. We have learnt to solve the differential equation that is the equation of motion of the system and find the response, we also learnt Duhamel Integral.

So, we can treat any type of force as a series of impulse functions and then we can use Duhamel Integrals to find the response and for these pulse excitations we can also use superposition of simpler functions; that means, we can express a pulse as a superposition of simpler other forces. For example, this rectangular pulse can be treated as a sum of these two forcing functions.

So, this is a step force starting at time is equal to 0 and which has a magnitude p and, this is another step force which is starting after some time and it has a magnitude minus p. So, if you add the response due to these two step forces we would get the response due to this pulse. And, in the previous lectures we have learnt how to find the response due to step functions.

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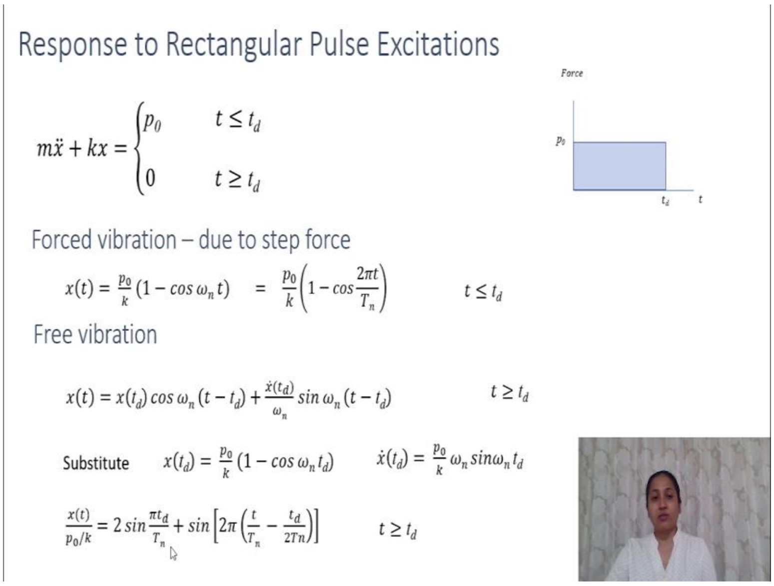

Now, let us find the response of a single degree of freedom system to rectangular pulse excitations.

$$m\ddot{x} + kx = \begin{cases} p_0 & t \le t_d \\ 0 & t \ge t_d \end{cases}$$

This is the equation of motion of an undamped single degree of freedom system and the pulse excitation is acting on it; a rectangular pulse is acting on it so, p naught is the amplitude of the pulse and it is existing for a duration td. So, we can express the force as this, the force is equal to p naught if t is less than td and the force is 0 if t is greater than td.

So, we can solve this equation and find the response. So, when t is less than td so, during this pulse the system will be under forced vibration. So, we can find the response as this, in the previous lectures we have derived the response due to a step force. So, the same expression is valid when t is less than td, because at that time that is during the pulse the step forces acting on the system. So, the response is same as for a step force with magnitude p naught and it is p naught by k is equal to 1 minus cos omega n t.

$$x(t) = \frac{p_0}{k}(1 - \cos \omega n\, t) = \frac{p_0}{k}\left(1 - \cos\frac{2\pi t}{Tn}\right) \qquad t \le t_d$$

So, we are considering undamped system there is no damping in the system so, the response is in terms of omega n. And, we can rewrite this in terms of natural period and we would get this expression and this response is valid when t is less than td. So, after this pulse that is when t is more than td, then the system will experience free vibration. So, because of this step force this is vibrating and at td it will have some displacement and some velocity.

So, if that displacement and velocity values are non-zero then that will cause a free vibration to the system, we have derived the equation for free vibration earlier. So, we can use that to find the free vibration response. So, the free vibration of a single degree of freedom system an undamped system is expressed like this and x td and x dot td are the displacement and velocity as time is equal to td, that is at the initiation of the free vibration.

$$x(t) = x(t_d)\cos \omega n\,(t - t_d) + \frac{\dot{x}(t_d)}{\omega n}\sin \omega n\,(t - t_d) \qquad t \ge t_d$$

So, we can find the values of displacement and velocity at td by using this expression. We can substitute the value of td here and get x td and we can differentiate it once and substitute td and we will get x dot td.

$$x(t_d) = \frac{p_0}{k}(1 - cos\,\omega n\, t_d) \qquad \dot{x}(t_d) = \frac{p_0}{k}\omega n\, sin\omega n\, t_d$$

So, we can substitute these two in this and get the expression for the free vibration. Make the substitution and simplify it we would get this expression, that is x t divided by the static response, p naught by k is the static response is equal to 2 sin pi td by Tn plus sin 2 pi t by Tn minus td by 2 Tn. So, this expression is valid when time is greater that td.

$$\frac{x(t)}{p_0/k} = 2\,sin\frac{\pi t_d}{Tn} + sin\left[2\pi\left(\frac{t}{Tn} - \frac{t_d}{2Tn}\right)\right] \qquad t \geq t_d$$

So, if we look at this expression we can understand that, this response is a function of t by Tn that is the ratio of the time to the natural period. And, this quantity depends upon td by Tn that is again a time ratio that is the duration of the pulse to the natural period. So, the response will depend on the ratio of this duration and the natural period. So, now, let us see let us calculate the response for different values of td by Tn.

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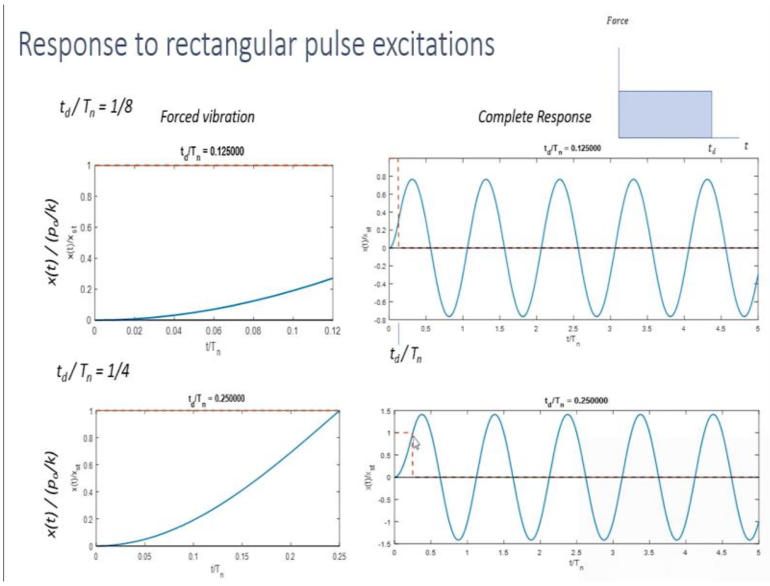

For td by Tn is equal to 1 by 8, if we evaluate these two expressions and if we plot this we would get the response, the complete response would be this. And, during force vibrations, that is when t is less than td this is the response. This is in fact, the normalized response so, we are plotting x t divided by p naught by k, this p naught by k is the static deformation. So, these plots are for normalized displacement or deformation. So, this red dotted line indicates the static response.

So, in this case when td by Tn is equal to 1 by 8 the duration of the pulse is very short and during the pulse that is when forced vibration is happening, the amplitude of the response is very low compared to the static response. So, this dynamic response is very low, but during the free vibration phase this is having a higher amplitude.

So, depending upon the displacement and velocity at td time is equal to td, the system will undergo a free vibration. And, the free vibration amplitude is higher than the forced vibration amplitude, as you can see from this figure; this is the free vibration amplitude and it is higher than the force vibration amplitude.

So, the maximum amplitude of this response happens after the pulse. So, let us check this for td by Tn is equal to 1 by 4. So, in this case the duration is more than the previous case so, as the duration of the pulse is increasing the response is also increasing during the forced vibration. So, now, the duration is more so, the displacement also grows. Again in this case the maximum displacement, the maximum response happens after the pulse ends.

So, the maximum response is happening during the free vibration phase. So, this is the force vibration, this one force vibration response and again the red dotted line shows the static response. So, during the forced vibration, the dynamic response is less than the static response, but the response increases after the force ends that is during the free vibration phase the displacement is high.

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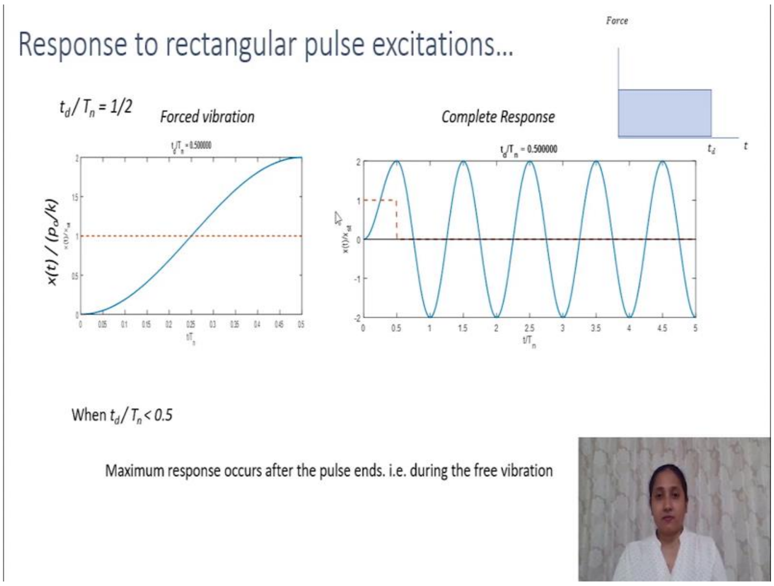

Now, let us see it for another value of td by Tn. So, td by Tn is half. So, now, the duration of this pulse has increased earlier the td by Tn was 1 by 4 now it is double of that. So, as the duration is increasing the response is also increasing. So, after t by Tn is equal to 0.25 this response is higher than the static response.

So, as the duration of this force is increasing the response is also increasing. At the end of the pulse the normalized deformation becomes equal to 2; that means the dynamic response is twice that of the static response and we had derived this result when we derived the response force for a step force.

So, when a step force is acting the maximum amplitude of the single degree of freedom system will be twice that of the static response. So, here when the duration of the pulse is half of that of the natural period we are getting the similar response. The amplitude of this free vibration response is also equal to 2; that means, even during the free vibration the maximum displacement will be twice that of the static displacement, that is p naught by k.

So, based on the results which we have seen so far we have seen that when td by Tn that is the duration of the pulse divided by the natural period of the system is less than 0.5 the maximum response occurs after the pulse ends, that is the maximum response occurs during the free vibration. So, this is the result for td by Tn is equal to half. So, in this case the maximum response occurs at the last moment, that is when the pulse ends and that is when the free vibration starts. So, when td by Tn is less than half then the maximum response is happening during the free vibration phase that is after the pulse after the force stops. Now, let us find the response for some other values of td by Tn.

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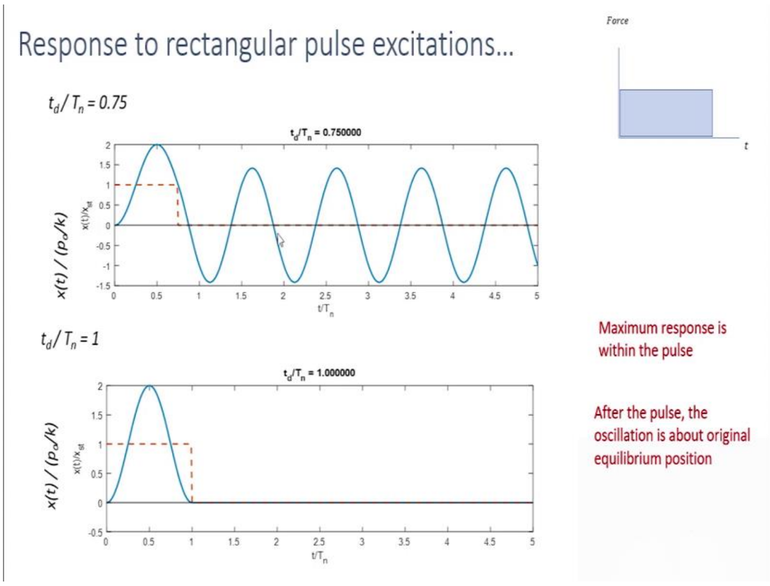

So, when td by Tn is equal to 0.75, we have the complete response as this. So, this is the duration of the pulse that is when the static response is not 0. So, td by Tn is 0.75. So, here as you can see the maximum displacement is happening during the pulse; so, the maximum response is happening during the forced vibration. So, during the force vibration the response grows from 0 to the maximum value of 2 and after that it is oscillating back, but the force stops there. So, based on the displacement and velocity at that position it is continuing the free vibration. So, the free vibration response the maximum response is less than that of the force vibration maximum.

Now, let us see the response for td by Tn is equal to 1. So, when td by Tn is equal to 1 the response is like this so; that means, during the forced vibration phase, the response increases from 0 to the maximum value that is 2, that is the normalized response. So, this is similar to the previous case the response grows to 2, then it reduces it is oscillating back. So, the system is oscillating during the forced vibration phase about the static displacement position.

So, this is oscillating about p naught by k that is x by p naught by k is 1. So, this is starting to oscillate, but when t by Tn is equal to 1 that is when the pulse stops the displacement and the velocity both are equal to 0. So, that is when the pulse stops, so, the initial conditions for the free vibration phase is 0. So, because of that there would not be any free vibration. So, this just oscillates once and stops there. So, the response will be there only during the pulse that is during the force vibration phase, there would not be any free vibration for this particular case that is when td by Tn is equal to 1.

So, based on these two responses we can see that the maximum response is happening within the pulse; that means, the maximum response is happening during the forced vibrations. So, we have seen earlier that when td by Tn was less than half, the maximum displacement the maximum response was happening during the free vibration phase, but now the maximum response is within the pulse. And, we can see that during the pulse the oscillation is about this static deformation and after the pulse that is during the free vibration, the oscillation is about the original equilibrium position that is when the displacement is 0.

Now, let us see the responses for other values of td by Tn.

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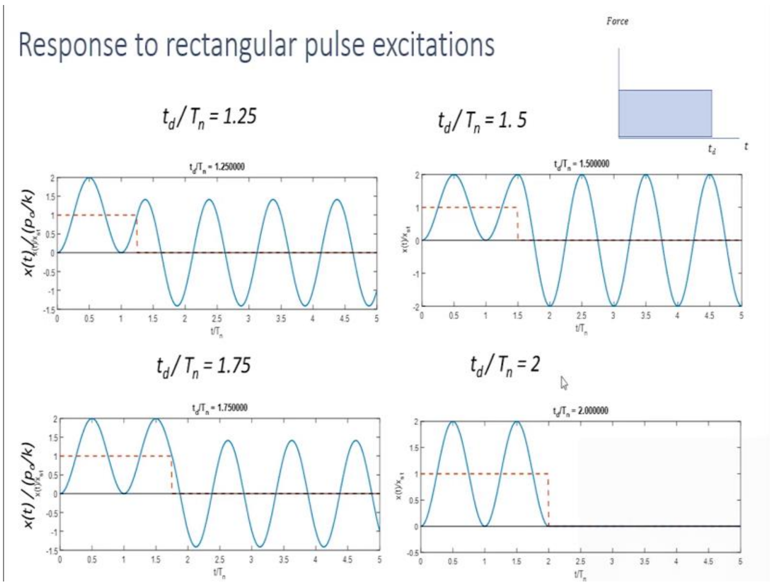

So, when td by Tn is equal to 1.25 the response is like this. During the pulse the system is oscillating about the static deformation and after that it oscillates about the original equilibrium position that is the 0 response. And, again the maximum displacement is during the force vibration that is during the pulse. Similarly, this is the response for td by Tn is equal to 1.5. So, the pattern is same as the previous.

So, here also during the force vibration phase, the system oscillates about the static deformation. And, then it starts oscillating about the original equilibrium position, but here the amplitude is the same during the free vibration and the forced vibration phase so, that is because of the initial conditions here. At the beginning of the free vibration phase the displacement is maximum 2 and the velocity is 0. So, for the free vibration also the maximum amplitude will be same as the initial displacement that is 2 here.

So, now, let us see the response for td by Tn is equal to 1.75. The similar pattern continues so, during the pulse the system oscillates about the static equilibrium position and after that it goes back to the original equilibrium position and the maximum response is happening during the force vibration. So, this is when td by Tn is equal to 2. Here during the force vibration the system oscillates, but at the end of the force vibration the displacement and the velocity both are equal to 0. So, because of that there would not be any free vibration in this case. So, this is similar to the case when td by Tn is equal to 1.

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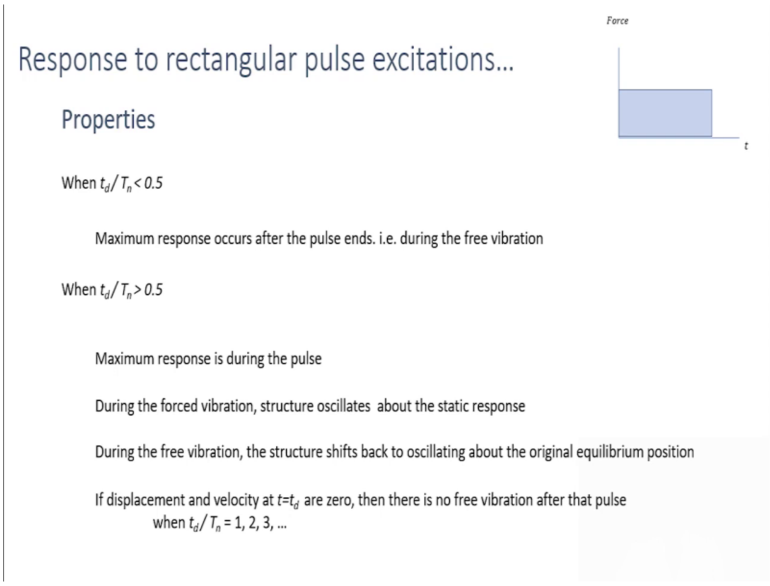

So, based on the results we have seen so far we can conclude the properties of the response of a system to rectangular pulse excitations. So, we have seen that when this td by Tn is less than half, the maximum response occurs after the pulse ends that is the maximum response is during the free vibration. And, when td by Tn is greater than 0.5 the maximum responses during the pulse, that is during the forced vibration.

So, during the forced vibration the structure is oscillating about the static response and, during the free vibration the structure shifts back to oscillating about the original equilibrium position. And, if the displacement and velocity at t is equal to td that is when the force ends, if that displacement and velocity are 0 then there is no free vibration after the pulse. So, this criteria is satisfied when td by Tn is equal to 1 2 3 etcetera so, for these values there would not be any free vibration.

So, at the end of the forced vibration the displacement and velocity are 0; that means, the mass comes to rest at the end of the force vibration and there would not be any free vibration phase.

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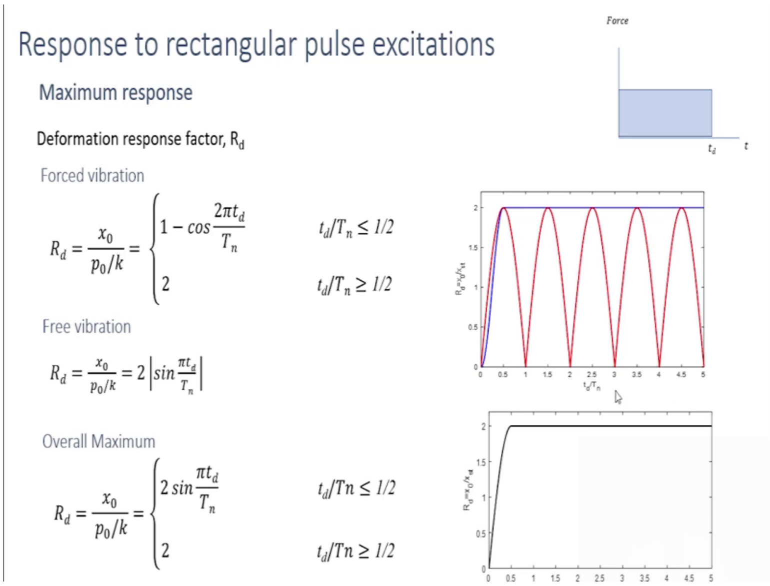

Now, let us find the maximum response during the rectangular pulse excitation. So, we can calculate the deformation response factor Rd. So, we have calculated this earlier also so, this Rd is equal to the dynamic amplitude divided by the static deformation. So, during the forced vibration phase we can calculate the Rd by maximizing the expression for the response, for the forced vibration response. So, we just have to find the maximum of the response.

So, if you do that we will get the expression for Rd that would be equal to the amplitude of the dynamic response divided by the static response. So, when td by Tn is less than 0.5 the expression for Rd is this that is 1 minus cos 2 pi td by Tn. And, when the td by Tn ratio is more than half, then the maximum amplitude is 2. The maximum amplitude will be 2 times the static deformation, but the maximum value of Rd is 2.

$$Rd = \frac{x_0}{p_0/k} = \begin{cases} 1 - cos\frac{2\pi td}{Tn} & t_d/Tn \leq 1/2 \\ 2 & t_d/Tn \geq 1/2 \end{cases}$$

And, during the free vibration phase the Rd value will be similar to the free vibration phase.So, here during the free vibration phase we can calculate the value of Rd as this so, this is x naught by p naught by k will be equal to 2 modulus of sin phi td by Tn.

$$Rd = \frac{x_0}{p_0/k} = 2\left|sin\frac{\pi td}{Tn}\right|$$

So, if we are plotting these two expressions, this blue one is for the forced vibration response. So, initially when td by Tn is less than half we have this expression 1 minus cos 2 pi td by Tn. So, that is this portion. So, after td by Tn is equal to half that is if td by Tn is more than half, then the maximum amplitude will be two irrespective of the value of the td by Tn.

So, this plot case Rd versus td by Tn; so, we can find the value of Rd for any value of td by Tn using this. And, the overall maximum amplitude, the overall maximum response will be the envelope of these two curves that is the maximum of these two red and blue values. So, we know these two curves so, we can find out the envelop curves. So, the overall maximum Rd will be 2 times sin pi td by Tn, when td by Tn ratio is less than half and, the overall maximum Rd will be 2 if td by Tn is greater than half.

$$Rd = \frac{x_0}{p_0/k} = \begin{cases} 2\, sin\frac{\pi td}{Tn} & t_d/Tn \leq 1/2 \\ 2 & t_d/Tn \geq 1/2 \end{cases}$$

So, if we draw the envelope of these two plots we would get this. So, we can find out the value of Rd for any td by Tn value using this type of curves. So, this is the deformation response factor and this curve is also known as response factor, because this gives the maximum response for all the values of td by Tn.

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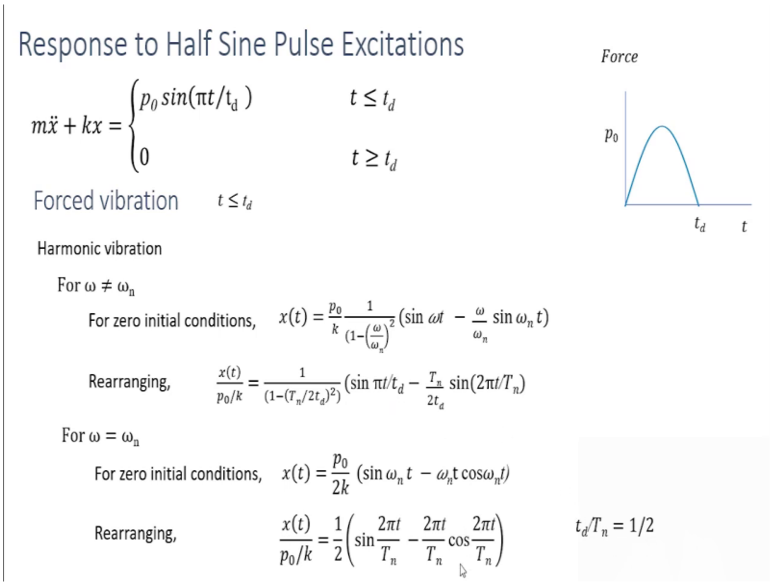

So, now let us see the response to another pulse excitation that is half sine pulse. So, the shape of the pulse is half sine wave and the amplitude is p naught and the duration of the pulse is td. So, we can write the equation of motion as this. So, when t is less than td the force will be p naught sin pi t by td. So, this is a harmonic force during this time between 0 and td. So, we can find the force vibration response for t less than or equal to td, that is during this time the system is under forced vibration.

$$m\ddot{x} + kx = \begin{cases} p_0 \sin(\pi t/t_d) & t \le t_d \\ 0 & t \ge t_d \end{cases}$$

So, since it is a harmonic force the system is under harmonic vibration and we know the response of harmonic vibration. So, when omega that is the forcing frequency, if omega is not equal to omega n, then we know the expression for the displacement response for harmonic vibrations. So, if the initial conditions that is the initial displacement and initial velocity are 0, then the expression for the displacement responses this, we had derived it during the harmonic vibration discussion. So, this is valid for omega naught equal to omega n.

$$x(t) = \frac{p_0}{k} \frac{1}{(1-(\frac{\omega}{\omega n})^2} (\sin \omega t - \frac{\omega}{\omega n} \sin \omega n\, t)$$

So, we can rearrange this in terms of natural period. So, we can convert this natural frequency to natural period and rewrite this expression and we can write that this x t that is dynamic response divided by the static response is equal to this expression. Here td is the duration of the pulse and Tn is the natural period of the system. So, when omega is equal to omega n that is the forcing frequency is equal to the natural frequency, this expression is not valid because this is indeterminate if omega is equal to omega n.

$$\frac{x(t)}{p_0/k} = \frac{1}{(1-(Tn/2td)^2)}\left(\sin \pi t/td - \frac{Tn}{2td}\ \sin(2\pi t/Tn)\right)$$

So, during harmonic vibration discussion we had derived the expression for the response for omega is equal to omega n. So, for 0 initial conditions the response is this.

$$x(t) = \frac{p_0}{2k}\ (\sin \omega n t\ - \omega_n \text{t} \cos\omega n t)$$

So, we can rearrange this expression to this form and this is valid when td by Tn is equal to half, when td by Tn is not equal to half this equation is valid, but when td by Tn is equal to half we can use this expression.

$$\frac{x(t)}{p_0/k} = \frac{1}{2}\left(\sin\frac{2\pi t}{Tn}\ - \frac{2\pi t}{Tn}\cos\frac{2\pi t}{Tn}\right) \qquad td/Tn = 1/2$$

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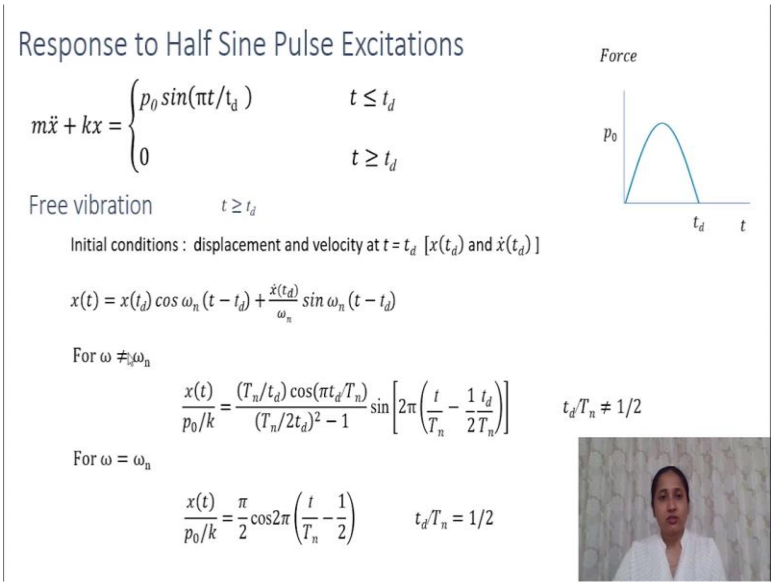

Now, let us find the free vibration response. So, the free vibration response, depend upon the displacement and velocity at t is equal to td that is when the force stops. So, we can find out x td and x dot td and we can find out the expression for the response when omega not equal to omega n, that is if we substitute the value of x td and x dot td. In this expression and simplify it we would get this expression.

$$\frac{x(t)}{p_0/k} = \frac{(Tn/td)\cos(\pi td/Tn)}{(Tn/2td)^2-1}\sin\left[2\pi\left(\frac{t}{Tn} - \frac{1}{2}\frac{t_d}{Tn}\right)\right] \qquad td/Tn \neq 1/2$$

So, here I am not going to the detailed derivation of this because of time constraints, if you are interested you can carry out the substitution and you would get this expression. So, this expression is valid for td by Tn is not equal to half. So, when td by Tn is equal to half as we did in the previous case we would derive the another expression. So, that would be this. So, this expression is valid when td by Tn is equal to half. So, now, we know the forced vibration response and the free vibration response for omega is equal to omega n and omega not equal to omega n.

$$\frac{x(t)}{p_0/k} = \frac{\pi}{2}\cos 2\pi\left(\frac{t}{Tn} - \frac{1}{2}\right) \qquad td/Tn = 1/2$$

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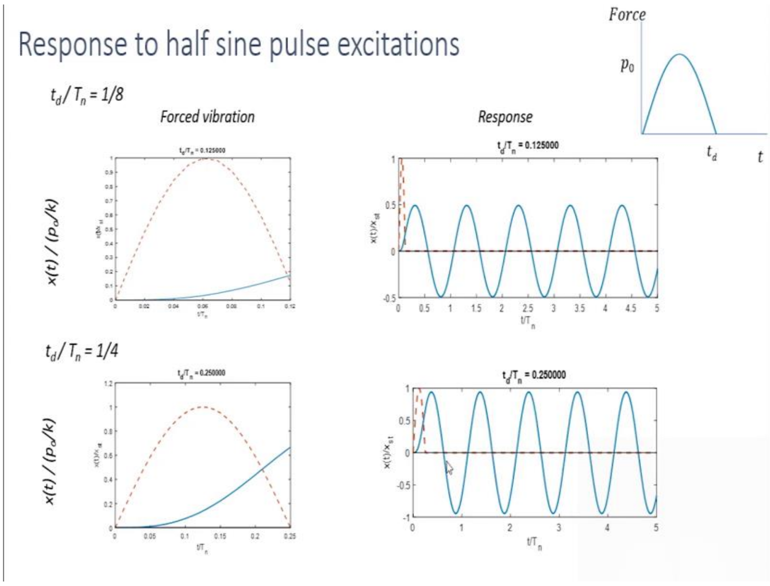

So, we can plot the response curves, when td by Tn is equal to 1 by 8 the response of the system is like this. So, this is the normalized response that is the dynamic response x t divided by the static response amplitude, that is p naught by k. So, this red dotted line indicates the static response. So, that would be the force this harmonic force divided by k. So, this will have the shape of this harmonic force, but the amplitude will be p naught by k. So, in this normalized curve that amplitude will be 1 so, in this blue line is the dynamic response.

So, as in the case of rectangular pulse here also when td by Tn is 1 by 8 the dynamic response is much smaller compare to the static response. And, the dynamic response attains its maximum amplitude after this force ends that is during the free vibration phase. So, the maximum amplitude is during the free vibration phase.

Now, let us see it for 1 by 4 td by Tn is equal to 1 by 4; the similar pattern continues as the duration of the force is increasing, the dynamic response will also increase and for some duration the dynamic response is more than the static response. So, here also the maximum response is during the free vibration that is after the pulse stops.

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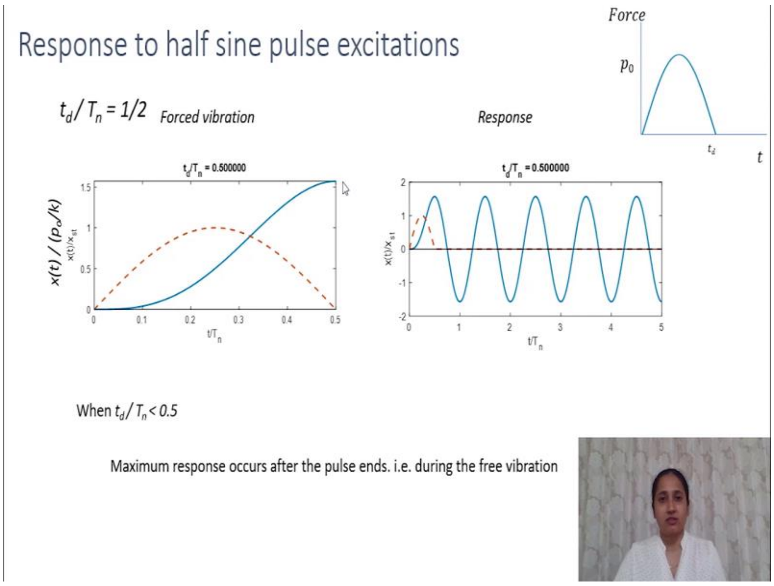

Now, let us see the response for td by Tn is equal to half. So, as in the case of the rectangular pulse here also as the duration of the pulse increases the dynamic response also increases and it reaches a maximum when the pulse is ending so,as the pulse ends this dynamic response reaches a maximum value. So, after that the free vibration is starting. So, here the velocity is 0, but the displacement is maximum. So, the free vibration starts with the same amplitude as this. So, free vibration continues with the same amplitude.

So, here also we can observe of that when td by Tn is less than 0.5 maximum response occurs after the pulse ends, that is the maximum response occurs during the free vibration. And, when td by Tn is equal to 0.5, then the maximum response occurs at the end of the parts or at the beginning of the free vibration. And, during the free vibration also the amplitude is same as this maximum amplitude.

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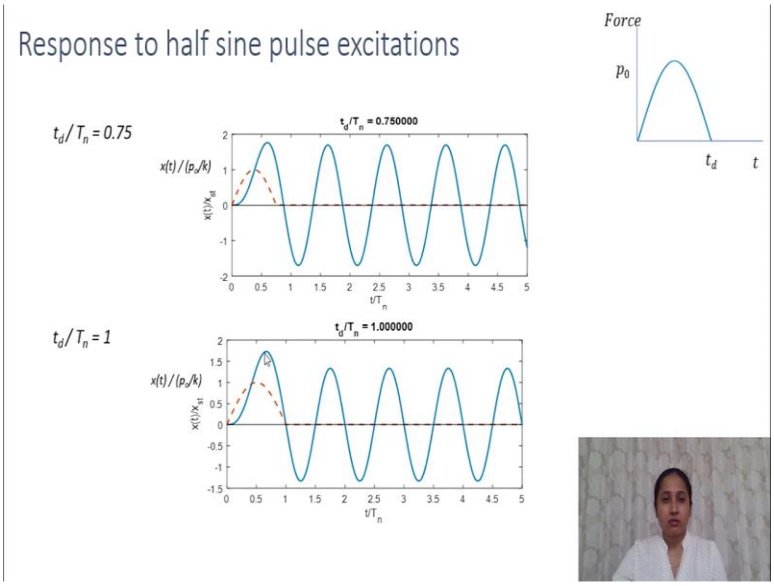

Now, let us see the response for another value of td by Tn that is 0.75. So, here the maximum response is happening during the pulse itself. So, this is the static response the maximum value of the dynamic response occurs here, that is during the force, it is within the pulse. So, now, let us see this for td by Tn is equal to 1. So, here also the same pattern continues; so, during the force vibration the maximum response occurs and after the pulse ends the free vibration continues with some reduced amplitude. Now, the maximum response is within the pulse.

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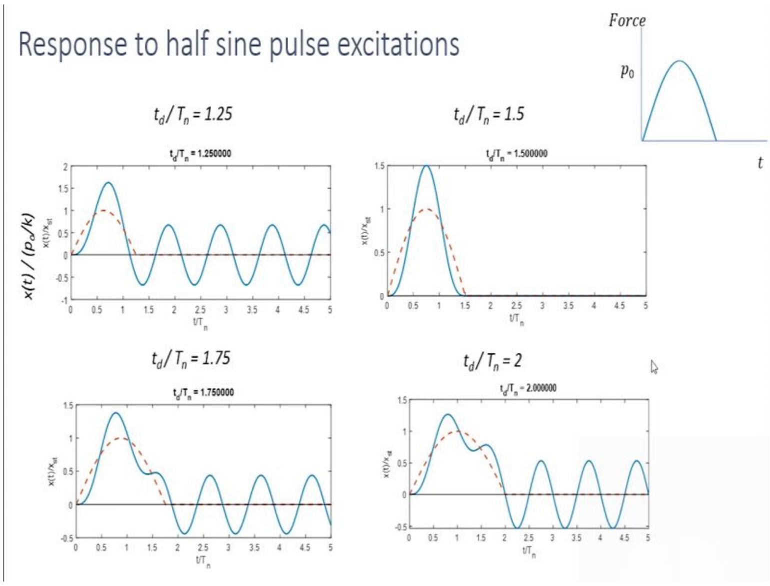

So, let us see the response for td by Tn is equal to 1.25. So, here also the same pattern continues the maximum response occurs during the pulse and after the pulse ends the free vibration starts with the reduced amplitude so, when td by Tn is equal to 1.5.

So, here again the maximum amplitude happens during the pulse, but at the end of the pulse the displacement and the velocity are 0. So, because of that there is no free vibration. So, this is similar to td by Tn is equal to 1 result during the rectangular pulse. So, if the duration of the pulse is still more increased that is if td by Tn is equal to 1.75, we get the similar pattern as this td by Tn is equal to 1.25.

Here, again the maximum amplitude happens during the pulse, but the free vibration continuous and the free vibration amplitude is much lesser than the force vibration amplitude. This pattern continues when td by Tn is equal to 2 as well. And, when td by Tn is equal to 2.5 the maximum response happens during the pulse, at the end of the pulse the displacement and velocities are 0.

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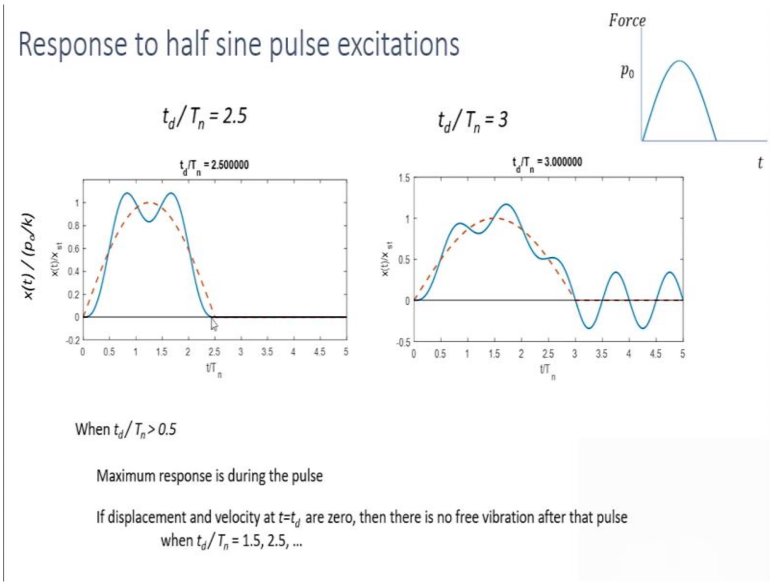

So, there is no free vibrations so, this is similar to td by Tn is equal to 1.5. So, for this there is no free vibration phase, when td by Tn is equal to 3 the response is this. So, here the maximum amplitude is during the forced vibration and the free vibration continues with the reduced amplitude.

So, the maximum amplitude during the forced vibration is reducing as the duration of the pulses increasing. So, when td by Tn was smaller this amplitude was higher. So, as the td by Tn values are increasing during the forced vibration the amplitude is reducing. So; that means, as the duration increases this pulse is getting flatter. And, so; that means, the rate of loading is getting lower as this pulse is getting flatter. So; that means, the dynamic effects are reducing, the dynamic response is becoming closer to the static response. So, as the duration of the pulse increases this difference will come down.

So, we can conclude that when td by Tn is greater than 0.5 the maximum response is during the pulse and that maximum response will reduce as td increases. And, if displacement and velocity at t is equal to td are 0; that means, if the displacement and velocity here is 0 then there is no free vibration after the pulse. So, and this happens when td by Tn is equal to 1.5, 2.5 etcetera.

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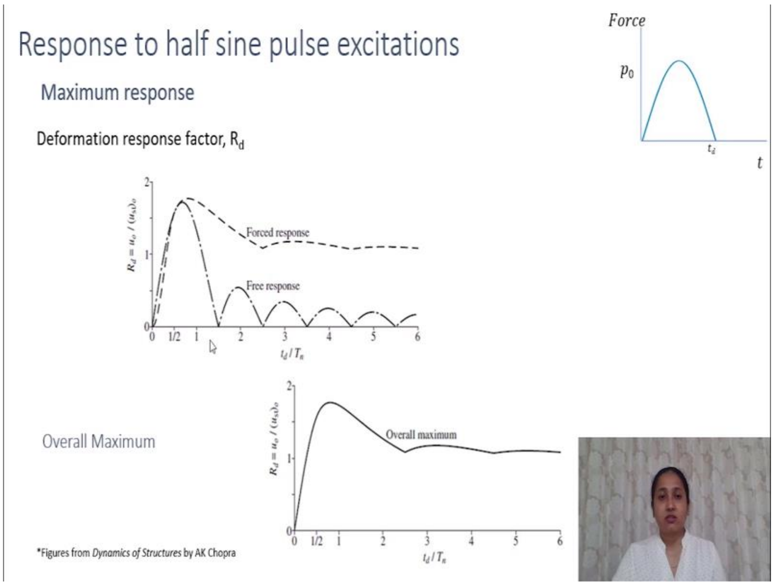

So, now, let us find the maximum response. So, to find the maximum response we have to maximize the response expressions, I am not going to the detail of the derivation. So, this is the deformation response factor. So, during the force vibration the maximum response varies like this. And, during the free vibration the maximum response varies like this for different values of td by Tn. So, we can use this curve to find the maximum response for any value of td by Tn. So, this is also known as a shock spectra or the response spectra for this pulse excitation.

And we can find the overall maximum so, that would be the envelope of both these curves that is the maximum values of both these curves. So, the overall maximum is this. So, for any value of td by Tn we just can find the maximum response. If, you remember this shock spectrum for the rectangular pulse excitation, we would remember that the maximum; the overall maximum response was two for higher values of td by Tn. So, in the case of sine pulse half sine pulse this is much lower this is close to 1.

So, why we are seeing this reduction in the maximum response? So, in the case of a rectangular pulse the load is suddenly applied. So, before the pulse the load amplitude is 0 and when the pulse is starting it is a sudden application of a load p naught. So, the dynamic effects are more there. So, that is why even for higher values of td, we would get a maximum response of 2 this response factor is 2, but in the case of the sine pulse the application of the load is gradual. So, it increases from 0 and gradually it reaches p naught. So, the dynamic effects are lower in this case. So, that is why the maximum amplitude is less than 2.

So, as the duration of the pulse increases as td increases this sine pulse becomes flatter and flatter; that means, rate of loading is lower and lower so, the dynamic effects will also come down. So, if the value of td is very large then the dynamic effects will be very low and this response factor is close to 1; that means, the dynamic response is close to the static response.

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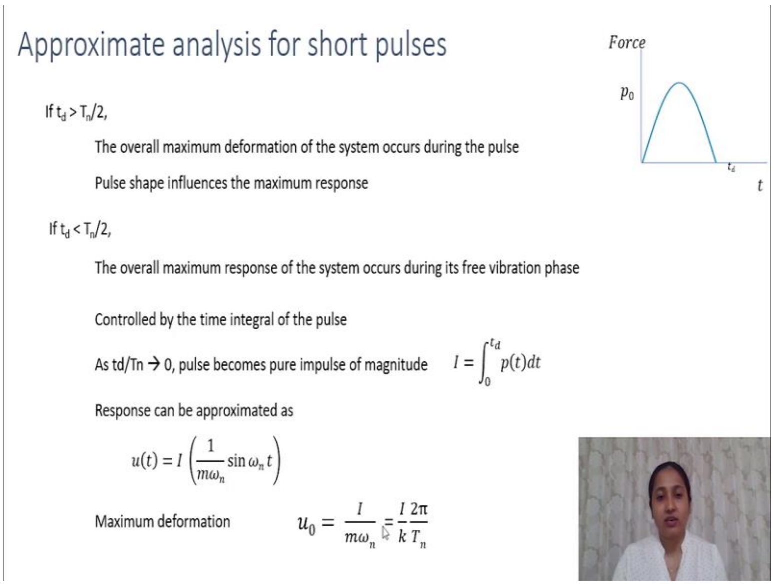

So, we have seen the responses for two different types of pulses. So, based on the results we can find an approximate analysis method for short pulses. So, so far we have seen that if the duration of the pulse is greater than Tn by 2, that is natural period by 2 then the overall maximum deformation of the system occurs during the pulse. And, the pulse shape influences the maximum response. So, in the case of a rectangular pulse the maximum response during the pulse is higher than the response due to this sine pulse, that is because during the rectangular pulse the load is suddenly applied here the load application is gradual.

So, when td is greater than Tn by 2 then the shape of the pulse influences the maximum response. So, when td is less than Tn by 2 then the overall maximum response of the system occurs during the free vibration. So, that is the maximum response happens after the pulse. And, this response is controlled by the time integral of the pulse. So, if the duration of the pulse is less then the response is controlled by the area of the pulse not the shape of the pulse, area under this curve decides the response.

So, if td tends to 0 then this pulse will become equivalent to a pure impulse. So, impulse as we discussed earlier it is force acting force very small time. So, if the steady tends to 0, then this pulse becomes an impulse and the magnitude of that impulse can be calculated as the time integral. So, that would be p t the expression for this force times dt. So, if you integrate it you will get the area under this curve and that is the magnitude of the impulse.

And, if the duration of the pulse is very short if the duration is less than Tn by 2, we can approximate this pulse excitation as an impulse with this magnitude, the magnitude of the impulse will be the area of this force. So, I is the magnitude of the impulse so, the response can be approximated as I times the unit impulse function, we derived the unit impulse function in the previous lectures.

So, the response due to this pulse, this short pulse can be approximated as the unit impulse function times the area of the pulse. And, the maximum deformation can be calculated as I by k 2 pi by Tn or I by m omega n.

$$u0 = \frac{I}{m\omega n} = \frac{I}{k}\frac{2\pi}{Tn}$$

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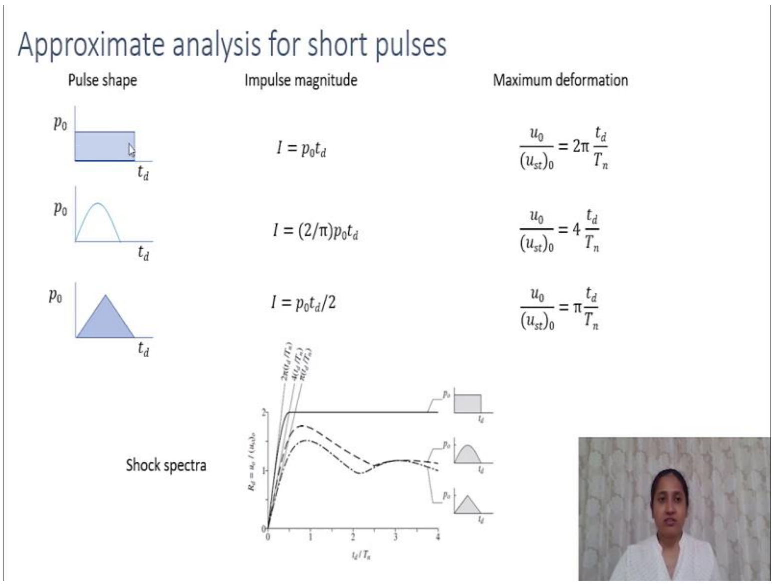

So, for different types of pulse shape of same amplitude, if this td is less than Tn by 2 then we can equate this pulse as a impulse. So, the impulse magnitude will be equal to the area of these curves. So, in the case of a pulse shape it can be expressed in an impulse with the magnitude p naught td. And, in that case the maximum deformation would be this; the normalized deformation that is the dynamic amplitude divided by the static amplitude would be 2 pi td by Tn in the case of a rectangular pulse.

So, in the case of a sine pulse with amplitude p naught the impulse would be equal to 2 by pi p naught td and, the deformation response factor would be this. And for triangular pulse, the area would be equal to the impulse so p naught td by 2 and here the maximum deformation would be this. So, this figure shows the shock spectra for these three different pulses. So, as you can see here if the td by Tn values are less than half, then the shock spectra can be approximated as a straight line and that equation will be approximated as these three expressions.

So, if the duration of the pulse is less than half the natural period, then we can use these approximated expressions to calculate the normalized response. So, the shock spectra can be approximated as this straight line if td by Tn is less than half. So, here we have considered pulse shapes with equal amplitude.

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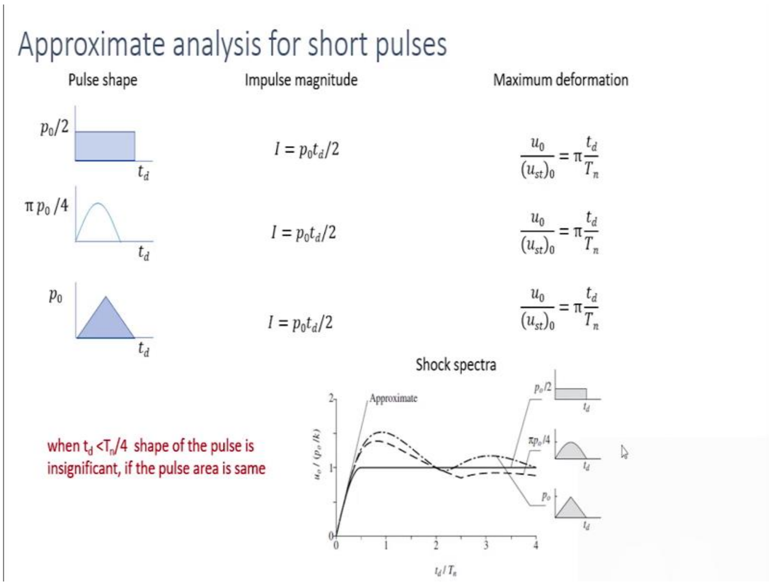

So, now we will consider the same pulse shapes, but with different amplitudes. So, that the impulse magnitude of all these pulses are same. So, here the impulse magnitude of all these pulses are same is equal to p naught td by 2. So, that is the area under this these curves. So, the amplitudes are different. So, the maximum deformation if you calculate that would be same because the area is same so, the deformation is also same the impulse magnitude is same.

So, now, let us see the shock spectra in this case. So, here the amplitudes are different, but the area under these curves are same. So, this is when td by Tn is equal to half and when td by Tn is equal to 1 by 4 that is when here so, until then all the three spectra gives the same value. So, when td is less than Tn by 4, then the shape of the pulse is insignificant if the area of the pulses same.

So, if the area is same irrespective of the shape of the pulse we would get the same deformation. So, we can use these results also to calculate the maximum response of a system under short pulses. So, if the pulse duration is less than Tn by 4, then irrespective of the shape of the function we would get similar responses.

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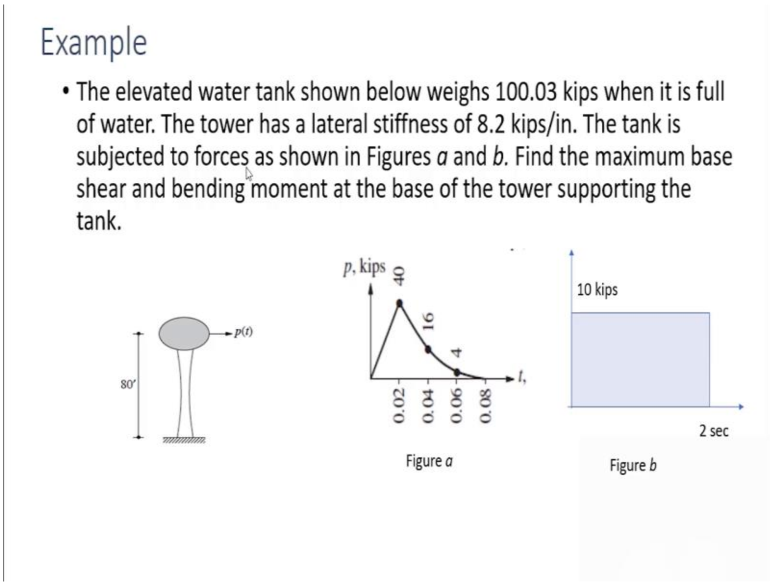

Now, let us solve an example problem we have an elevated water tank. So, this is the same water tank we considered earlier. So, the weight and the stiffness of this system is given the weight is 100.03 kips and the stiffness of the supporting system is 8.2 kips per inch. And, the tank is subjected to forces as shown in figure a and b, find the maximum base shear and bending moment at the base of the tower supporting the tank. So, we have to find the shear and the bending moment at this support. So, now, let us find this out for the first pulse force.

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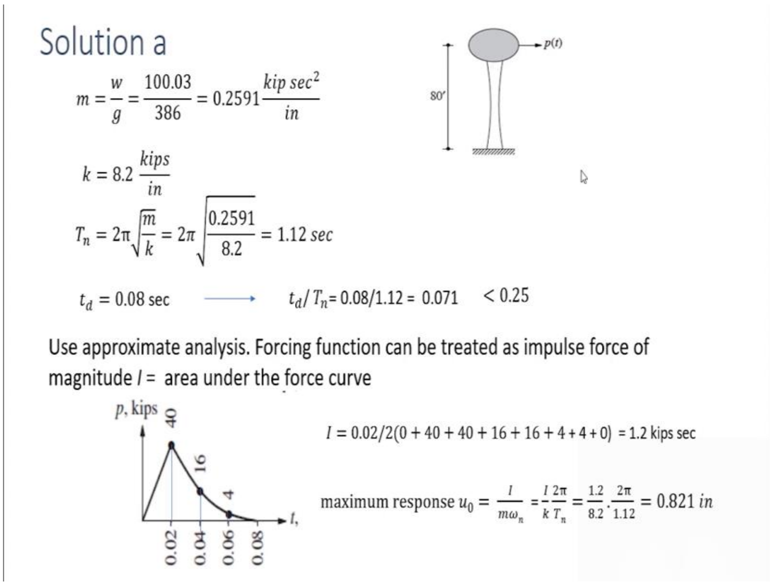

So, we have the weight so, we can calculate the mass. So, weight divided by gravitational acceleration and the stiffness is given 8.2 kips per inch. So, we can find the natural period as 1.12 seconds and the duration of this pulse is given as 0.08 seconds. So, using this we can calculate the ratio td by Tn so, that is equal to 0.071. So, we just learned that if this td by Tn ratio is less than 1 by 4 we can use the approximate analysis to solve this so, this 0.071 is less than 0.25. So, we can use the approximate analysis.

So, the forcing function can be treated as an impulse force of magnitude I and that magnitude is the area under this curve. So, we can find the area under this curve using the trapezoidal method. So, calculate approximate this as trapezoids and we can find the area as sum of the area of this trapezoid. So, for the first one is time step is 0.02, so, 0.02 by 2 this side plus this side, so, that is 0 plus 40.

So, for the next one this is this side plus this side, so, again 40 plus 16. So, for the third one at is 16 plus 4, 16 plus 4 last one it is just 4 plus 0. So, we can find the area as 1.2 kips seconds. So, once we have the magnitude of the impulse we can find the maximum response due to that impulse it is calculated as I by k 2 pi by natural period. So, we can substitute the value I is 1.2, k it is 8.2, then we have 2 pi and the natural period is 1.12. So, we get the maximum response as 0.821 inch. So, now we know the maximum displacement. So, we can find the force. So, we have the stiffness value. So, we can find the equivalent static force.

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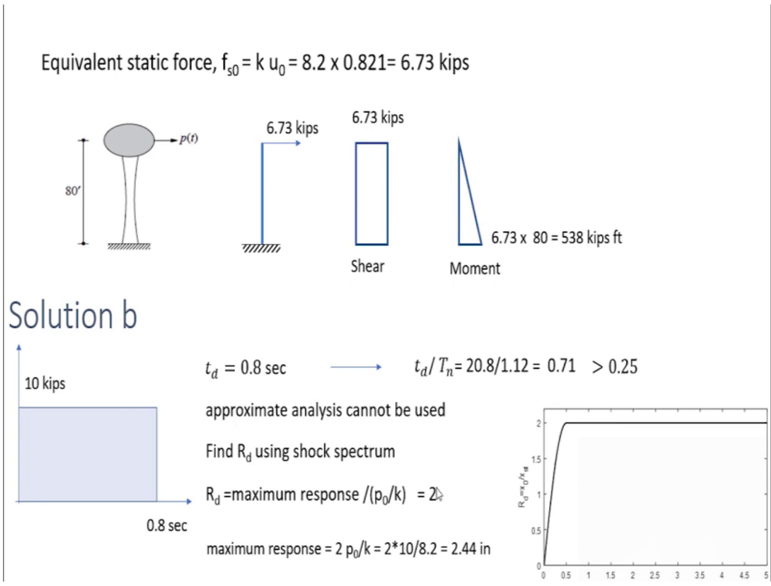

So, the equivalent static force acting on this tank is k times the maximum displacement so, that would be 8.2 times the displacement we just calculated 0.821. So, that would be equal to 6.73 kips. So, this system is equivalent to a vertical cantilever that is this supporting system is like a vertical cantilever and an equivalent force of 6.73 kips is acting here.

So, we can find the shear force diagram of this so, the shear will be constant throughout the section. So, the base shear is also equal to 6.3 kips, you can find the moment this multiplied by the distance would give the moment at this location. So, this is the bending moment diagram. So, the moment at the base is 6.73 times the length that is 80 feet. So, that would be equal to 538 kips feet. Now, for the next pulse force the duration of the pulse is 0.8 second. So, this is a long pulse.

So, this 0.8 duration gives td by Tn ratio is equal to 0.71. So, that is higher than 0.25. So, we cannot use the approximate method to find the maximum response. So, we have to find the deformation response factor using the shock spectrum, because we have calculated the value of Rd for different values of td by Tn we found that expression. So, we can use that shock spectrum and find the maximum response. So, this was the overall maximum response spectrum for a rectangular pulse force. So, we have drawn this before.

So, we will use this now and find the value of Rd for td by Tn is equal to 0.71. So, we can see here that if this td by Tn value is higher than 0.5, then this Rd that is the deformation response factor is equal to 2; that means, the maximum dynamic response is twice that of the static response and the static response is p naught by k. So, the maximum response divided by p naught by k is equal to 2. So, we can find the maximum response we know the p naught that is equal to 10 kips and k is already given as 8.2. So, if we substitute the values we would get the maximum response is 2.44 inches.

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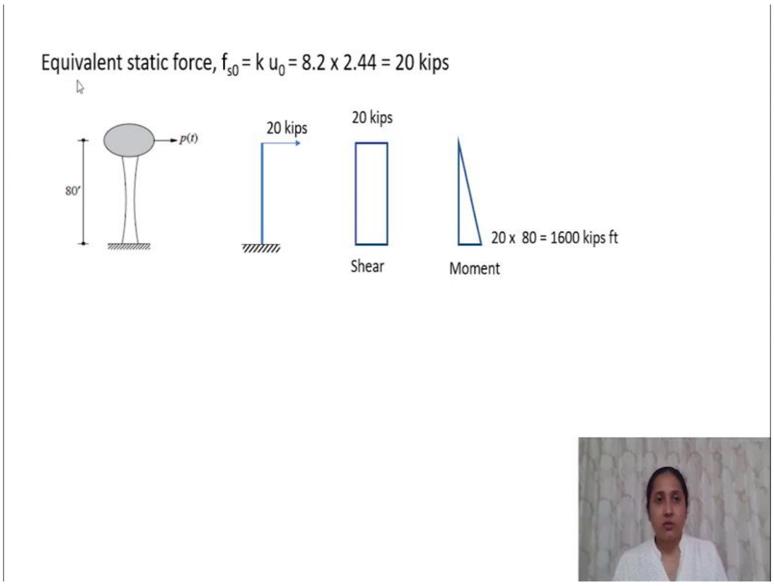

So, now, as similarly as we did here we can find the force and the moment shear etcetera. So, the equivalent static force now is equal to k times u naught that is 8.2 times 2.44 so, that is coming as 20 kips. So, during the second pulse force the equivalent force acting on the system is 20 kips. So, the base shear will also be 20 kips and the base moment will be 20 times 80. So, that is 1600 kips feet.