Structural Dynamics for Civil Engineers - SDOF Systems
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Lecture - 01
Introduction to Structure Dynamics

Welcome all of you to this course on Structural Dynamics. So, what is structural dynamics? It is the study of behavioral structures under time varying or dynamic load.

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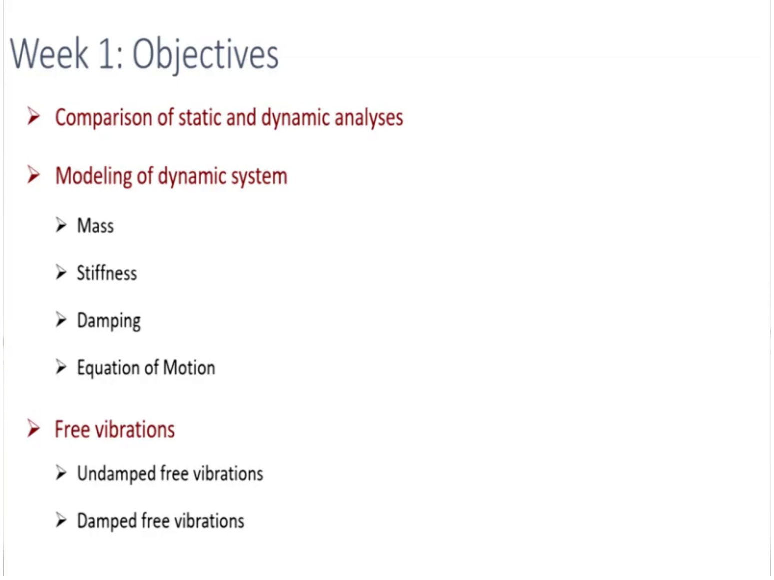

These are the objectives for the first week. First, we will discuss the differences between static and dynamic analysis, then we will learn how to model a dynamic system, we will discuss various elements in a dynamic system, mass, stiffness and damping. After that we will formulate the equation of motion. Then we will discuss free vibrations. In this, we will talk about undamped and damped free vibrations.

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So, now let us look at some examples of dynamic loading on structures. First, we have wind loading, this is very predominant in tall structures. Then we have earthquake loading, this is very critical and very damaging. After structures experience wave loading which is also time varying in nature. Bridges vibrate under traffic loads especially when high speeds trains are moving over it; it vibrates.

And when rotating machinery is placed on a structure it imparts a time varying load to the structure. So, in this case, this foundation will vibrate because of the rotation machinery. These are only a few examples and the list goes on. So, in this course, we will learn some of the basic theories which are needed to understand the behaviour of the structures and these loads.

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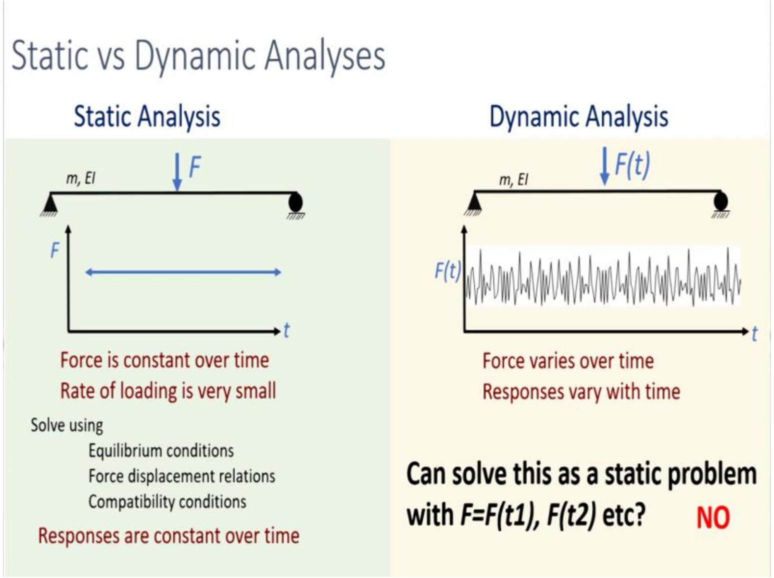

Now, let us see how dynamic analysis is different from static analysis. All of you have learned static analysis. This is an example of a static analysis problem. Here we have simply supported beam with mass m and flexural rigidity EI. We have a force F at the mid span of the beam. In static analysis, it is assumed that the force is constant over time. So, this assumption is valid only when the rate of loading is very small. In such cases the variation of forces of a time will be very insignificant and we can treated as a constant force.

You are familiar with the solution of static analysis problems. This can be solved using equilibrium conditions, force displacement relations and compatibility conditions. We can find the responses of the beam to support conditions, deflections, stresses and strains. Since the loading is constant the responses will also be constant. Now, let us see what happens in dynamic analysis. This is an example of a dynamic problem. The force varies with time the function F is a dependent on time. So, this is an example of a time varying force, the value of force changes at each instant.

So, what will happen to the responses when the force varies with time, the responses will also vary with time. Now, how will we solved these types of problems? These courses about solving these types of problems. So, we already know how to solve the problem when we have a constant force. So, here the value of force changes at each instant. So, can we treat this as multiple static analysis problems that means, can we take each forces at each instant and solve static analysis problem, will that give us the correct dynamic response? We cannot do that and we will see why.

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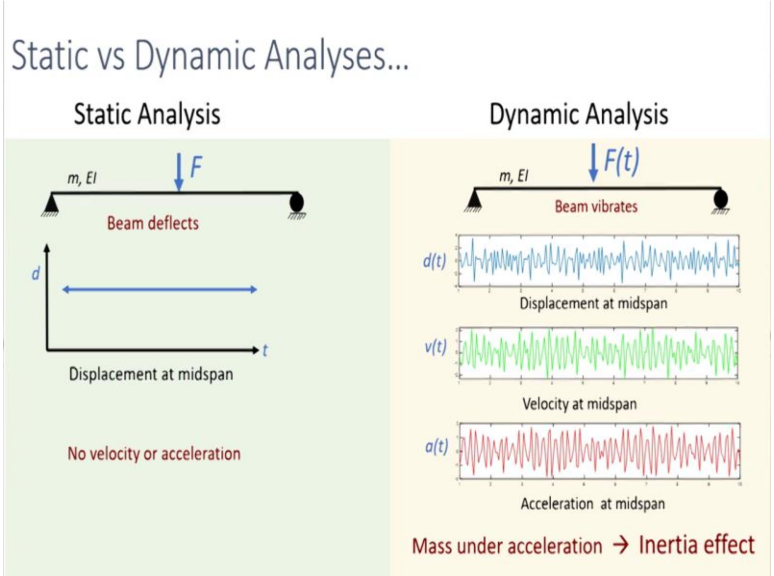

Let us go back to our static analysis problem. So, in the static analysis, we had a constant force and the beam deflects under that force. The deflection will also be constant in time. So, this is the displacement at the mid span, it is constant over time. Since the displacement is constant the beam will not experience any velocity or acceleration. Now, let us see the dynamic analysis scenario. Here the as we discussed earlier the force is a function of time and the beam will vibrate under that time varying force.

So, this is an example of the vibration response the displacement response at mid span of the beam. So, the displacement will look something like this. The value of it changes at each instant. So, what will happen when we have a time varying displacement? When the displacement varies with time we have velocity, that means, the beam will experience velocity and the velocity at the midspan is like this. We will get this velocity, if you differentiate is displacement data. So, as we can see the velocity will also change in time, that means, we have acceleration.

So, this is how the acceleration at this point will look like. And we know that are beam has some mass m. So, what happens when a mass under acceleration? So, when a mass is accelerating, that will lead to some additional effects called inertia effect. And in dynamic analysis, in addition to the applied load, we have to consider the effect of this inertia also. We will look at this inertia effect in detail using Newton's laws of motion.

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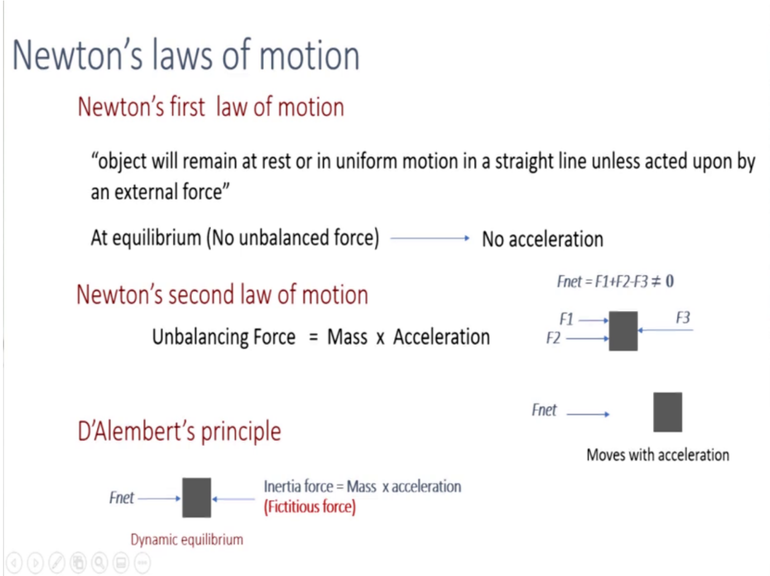

So, now, let us review Newton's law of motion. The first law of motion says that the object will remain at rest or in uniform motion in a straight line unless acted upon by an external force what is this mean. So, if a body is at equilibrium, that means, there is no unbalance force acting on the body; in that condition there won't be any acceleration. So, this is equivalent to a static analysis problem. Let us look at the second law of motion. It says to accelerate a body, they should be an unbalanced force acting on the body and the acceleration will depend upon the unbalanced force applied. And unbalanced force needed to accelerate the body will also depend upon its mass.

So, the unbalanced force needed is equal to mass times acceleration. So, let us understand this in detail. So, we have a body and set of forces are acting on it. And the sum of those forces can be notated as F net, net force. So, if the sum is 0, then that means, the body is at equilibrium and that is the static analysis case. So, what if the sum was not equal to 0, that means, some force F net is acting on the body. So, this is the body and the forces acting on it.

$$Fnet = F1+F2-F3 \neq 0$$

What happens, so when an unbalanced force acts on a body, the body will move with some acceleration. This is what is happening in dynamic analysis. So, we have already seen that the beam accelerates, when a dynamic force is applied. So, we have to consider this effect in the dynamic analysis. So, how will be consider the effect due to accelerating mass in the dynamic analysis? To do this, we will make use of D Alembert's principle this principle says that if we have an unbalanced force on the body, we can consider a fictitious force which is equal and opposite to the unbalancing force and treat this scenario as a equilibrium and he calls it dynamic equilibrium.

And that fictitious force is called inertia force and it is equal to mass times acceleration. So, if there is only F net (*Fnet*) acting on this body, that means, it is an balancing force it moves. Now, if there is another forces acting opposite and equal to it, the body is an equilibrium. So, now, we can solve this similar to a static problem the only difference is that in addition to the applied force, we have to consider this inertia force also. So, this is an additional force which we need to consider in dynamic analysis and this fictitious force is called inertia force.

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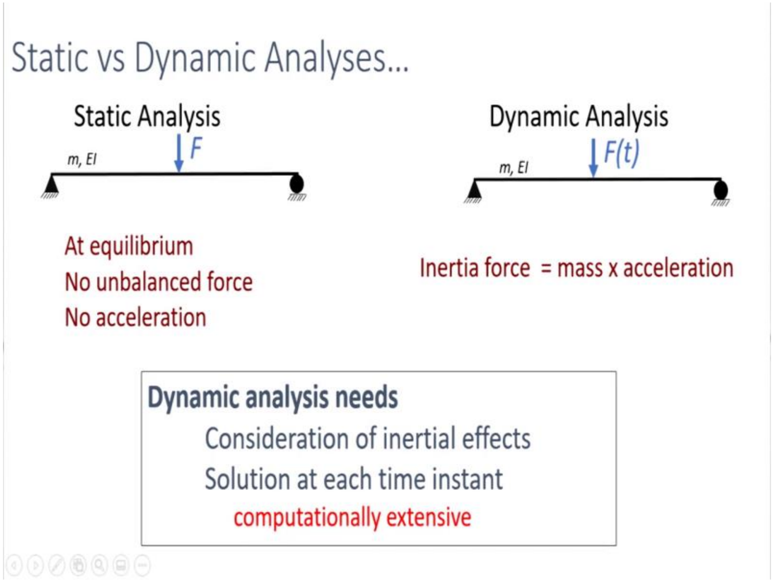

Now, let us compare both the analysis. So, when the load is constant that means static analysis case, the structure is at equilibrium, there is no unbalanced force. And there is no acceleration. The none of the beam parts are accelerating. In the case of dynamic analysis, there will be acceleration, the beam will experience and acceleration. And it will lead to an additional force called inertia force, mass multiplied by acceleration and we need to consider this force in addition to that applied force to get the correct dynamic response of the structure.

So, to conclude the discussion in dynamic analysis, we need to consider the inertial effect that is the effect due to an accelerating mass. And we need to solve the problem at each instant as the force is varying with time. So, this makes dynamic analysis computationally extensive. So, we can say that dynamic analysis is an extension of static analysis. So, it is a generalized analysis and static analysis can be treated as a special case with no inertia force or no time varying force.