**Introduction to Accounting and Finance for Civil Engineers**

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**Module No. #02**
**Lecture No. #10**
**Sensitivity Analysis (Part – 2)**

Good morning, and welcome to this lecture on, Introduction to Accounting and Finance for Civil Engineers. In the last lecture, if you remember, we discuss some aspects of Sensitivity Analysis. In this lecture also, we will discuss, some more aspects of Sensitivity Analysis. If you remember, Sensitivity Analysis aims to find, the impact of various changes in the variables, on a particular decision

We learn that, we can carry out Sensitivity Analysis for, one variable at a time, two variables at a time, and more than two variables at a time. This can be done for, single Alternative. This can be done for, more than one Alternative, as well. In the last class, if you remember, we perform the Sensitivity Analysis, for single variable. And, that was only limited to, single Alternative.

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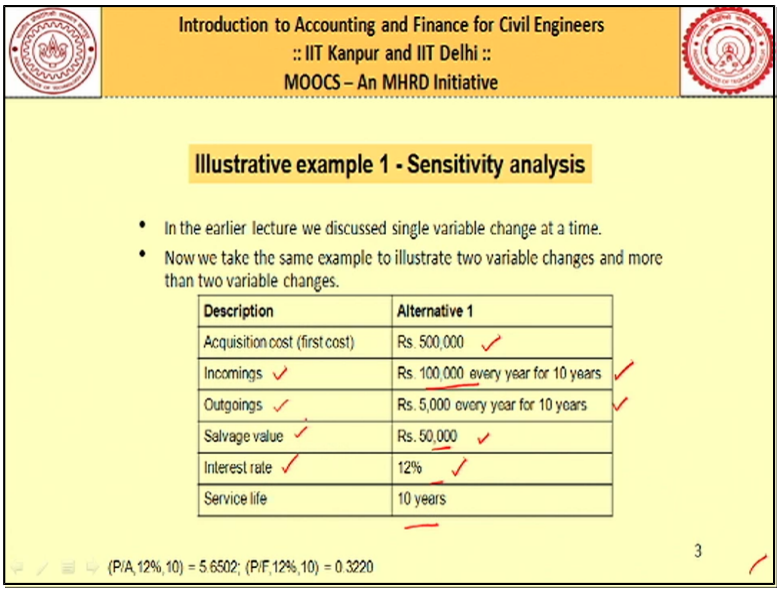


In this class, we are going to carry forward, with the same example. And, we will perform the Sensitivity Analysis for, two variables at a time. And subsequently, we will see it for, more than two variables at a time. So, this was the example, we did it in the last class, if you remember. We

were given the first cost as, Rupees 500,000. You can see it here, the incomes or the incomings were, 100,000 every year, for 10 years. Outgoings were, 5,000 Rupees every year, for 10 years

Salvage value was, Rupees 50,000. Interest rate considered for the analysis was, 12%. And, service life was, 10 years. In the last class, we found that, the variables such as, incomings and interest rate, were highly sensitive. We also learned that, the variables such as outgoings and salvage values, were not that sensitive. We are defining sensitive variables as those variables, wherein even a small change, will bring the reversal of decision.

So, earlier, if we are getting Net Present Worth of positive, by changing these sensitive variables, you will quickly find that, they are turning out into a negative Net Present Worth. We did the calculation, based on absolute value, as well as, based on some percentage value also. The results were shown on a spider web diagram. And, there we can clearly see, depending on the slope of the lines, we can see, which variable is sensitive, and which variable is not.

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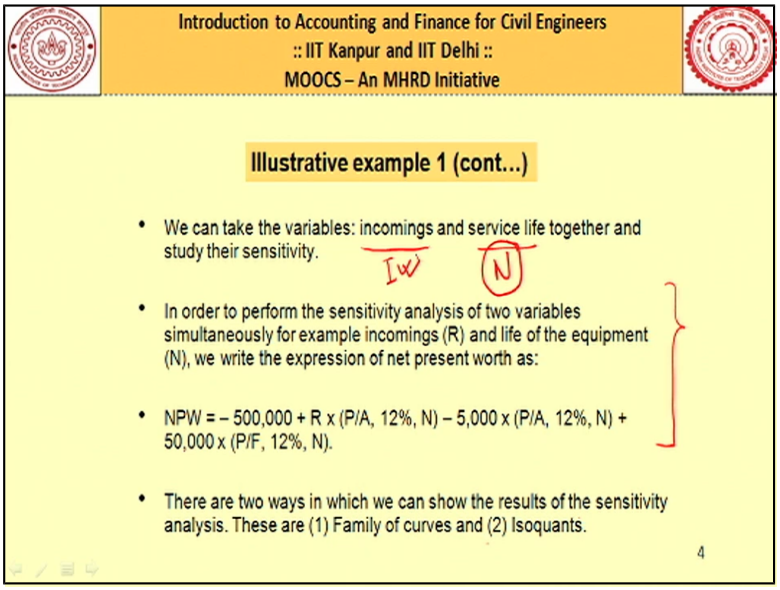


Now, we move forward for the same example, and try to take two variables at a time. Let those two variables be, incomings and service life. So, we are studying the changes in these variables together. Now, there are two approaches in this. Let us denote incomings by I, and let us denote service life with N. So, in one case what we do is, we keep the N value at a particular value, and then we keep on changing I. So, what we will get is, we will have a large number of points, for a

constant value of, let us say, N is equal to 10.

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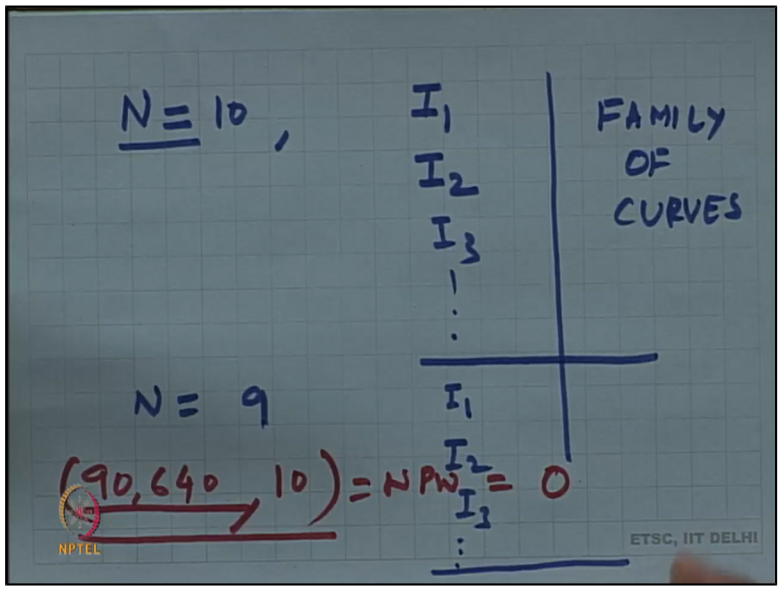


For this, we will go on changing I. So, I1, I2, I3, so on. So, we get, set of points here. Now, what I do? I make changes in N, NI. N is equal to 9, I keep. And, then again, I go on changing I as I1, I2, I3, and so on. So, by doing this, you will find that, I am getting a Family of Curves. We are saying this is as, Family of Curves. And, it would look something like this.

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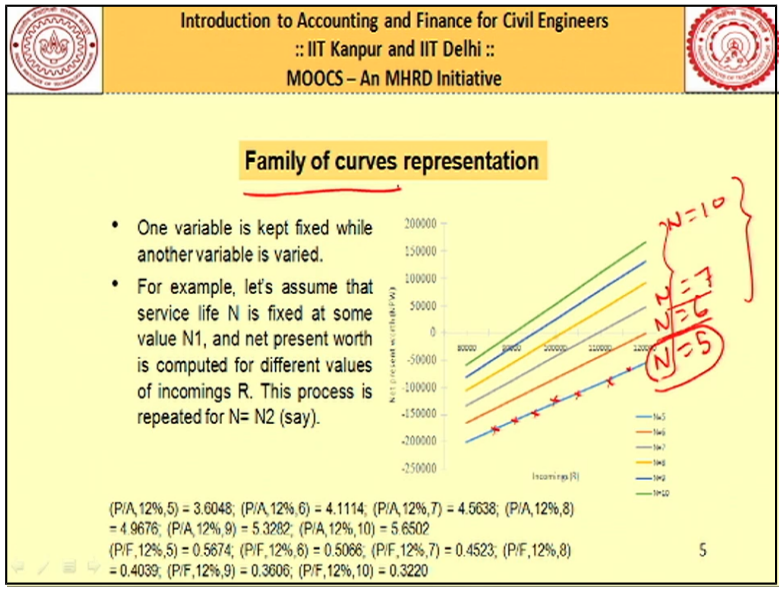

So, you can see on your screen, these are your Family of Curves. How we are deriving this. We are keeping, N as constant. So, this could be, let us say, N is equal to 5. Then, we have N is equal to 6, N is equal to 7, and so on. Now, for N is equal to 5, we are constantly changing the value of I. So, I1, I2, I3, I4, and we are getting, lots of points like this. Now, I am joining these points, and I am getting curve.

In the same way, I am doing it for, another value of N. It could be, N is equal to 6. It could be, N is equal to 7. It could be, N is equal to 10. So, this representation to show the sensitivity of two variables at a time, we are calling them as Family of Curves. So, this is Family of Curves representation. There is another way also in which, I can show the result of Sensitivity Analysis of two variables at a time. And that, we are calling it as, Isoquants. For drawing Isoquants, what I do is, I try to find, all those coordinates at which, my Net Present Worth is 0.

So, if you remember from the previous lecture, we had obtained one such value, where NPW was 0. So, for N is equal to 10, we had obtained an incoming value, which was something like 90,640. So, at this value of incoming, and at this value of service life, if you remember, we had obtained a Net Present Worth of 0. Remember, for 90,640, and service life of 10 years, we adopted a Net Present Worth of 0.

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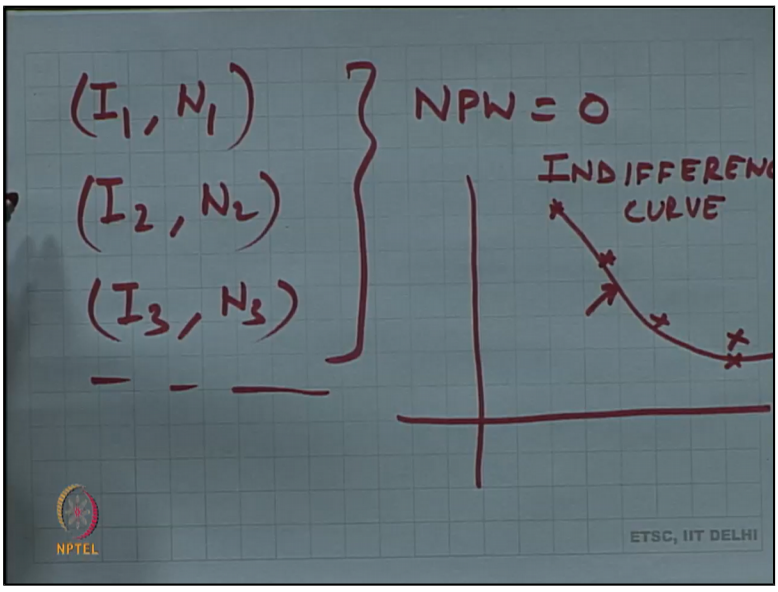


Likewise, I can calculate all those points, I1-N1, I2-N2, I3-N3, in such a way that, for all of these

points, I am getting a Net Present Worth of 0. Now, if I connect all these points, having coordinates I1-N1, I2-N2, I3-N3, I4-N4, you may get a curve like this, something like this. So, this we are calling it as, Indifference Curve. That means, as long as you are on the curve, one such example, I have drawn it here, on the computer screen.

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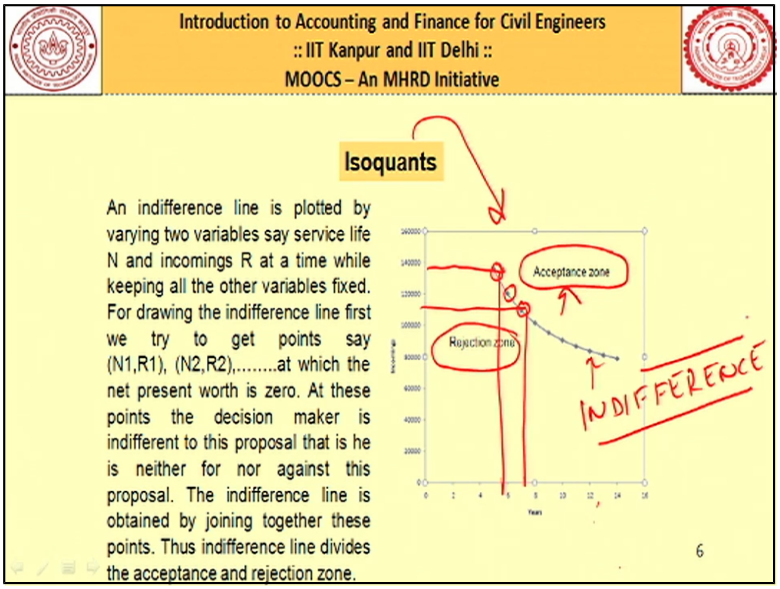


You can see, this particular curve, which is also known as, the whole representation is known as, Isoquants. This Isoquants is drawn, by joining all those points. You can see, all these points here, here, here, at which, the Net Present Worth is 0. So, for this coordinate, the Net Present Worth is 0. Likewise, for this coordinate, the Net Present Worth is 0. So, we are joining all those points, at which, my Net Present Worth is 0.

Now, anything above this, we are calling this as acceptance zone. And, anything below this line, we are calling them as rejection zone. As long as, you are on this particular curve, you are indifferent towards, either acceptance of that particular Alternative, or rejection. You are not sure, whether to accept it, or reject it. Because, for you, both of them are same. Because, at that point of time, Net Present Worth is 0. So, I am not at all concerned, whether you tell me to accept this Alternative, or whether you tell me to reject this Alternative.

So, this is what is meant by, Isoquants. And, the curve, if you see, the line obtained using these points, we are also some time calling them as, Indifference Curve. Because, at these points, we

are not at all interested in, either A or B. So, we are indifference, we are showing indifference, and that is why, they are also known as Indifference Curve. So, this is as far as, the representation of two variable changes at a time is concerned, in the Sensitivity Analysis.

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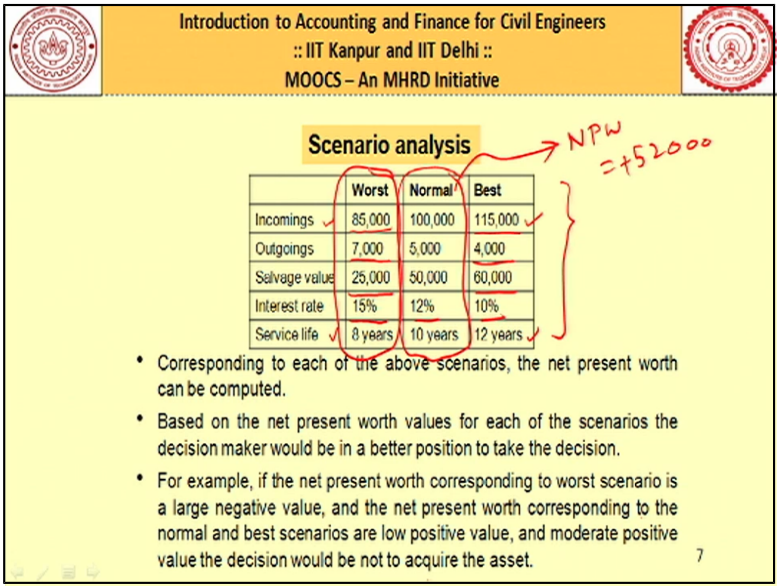


Now, we will see, what happens, when more than two variables are changed at a time. So, for example, we would like to change the interest rate, we would like to change the service life, we would like to change the incomings, we would like to change the outgoings. So far, either we have been changing, one variable at a time, or we are being changing, two variables at a time. Now, you want to see, how to analyse situation in which, all variables are changing at a time.

So, for this, what we are doing is, we are considering, the worst value, the best value, and the normal value, for each of the variables. Normal values are already known to us. If you look at this particular slide, you can see here, the normal values are same as, what we have been considering so far. So, the normal value of incomings is 100,000, outgoings it is 5,000, salvage value it is 50,000, interest rate it is 12%, and service life is 10 years. Now, as I told you, for each of these Alternatives, I am considering their worst, and the best also.

So, for example, incomings, I am estimating that, the worst that can happen is 85,000, and the best that can happen is 115,000. Likewise, for outgoing, the worst scenario could be, to have an expenditure of 7,000, and the best scenario is to have an expenditure of 4,000. Likewise, salvage

value, the worst situation could be, to get a salvage value of 25,000, the best could be 60,000. Interest rate, the worst could be 15%, normal as you know, it is 12%, and the best is 10%.

Likewise, service life can be reduced to 8 years, that could be the worst possibility. And, the best could be that, service live gets extended to 12 years. So, what I have done is, for each of these variables, I have considered the worst, the normal, and the best. Now, in this case, for the normal case, you already know that, the Net Present Worth is coming to be, something like 52,000, in positive. Now, what I do. I calculate the NPV, for all these worst values also. How do we calculate for this? So, we can calculate it, in a normal manner.

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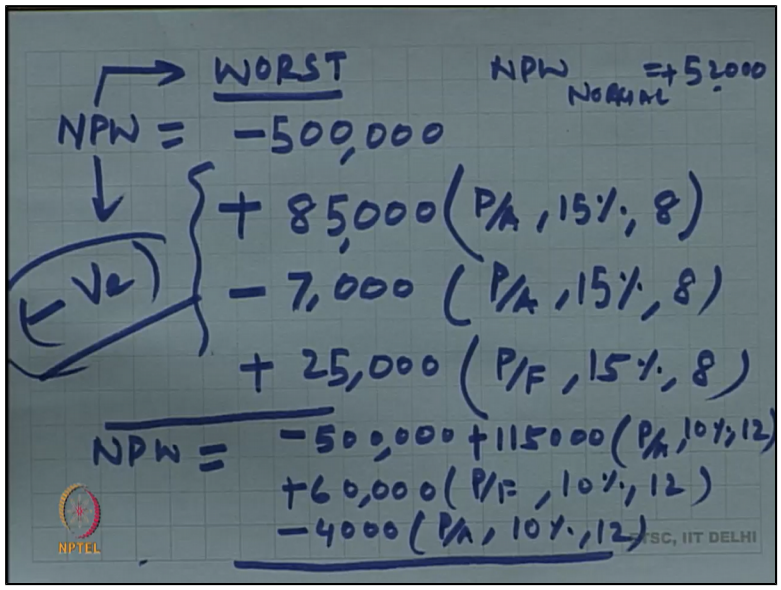


NPW for the worst case could be, - 500,000 + 85,000, which is the worst income, P given A, now the worst interest rate could be 15%, and life could be 8 years. So, this is the value. Likewise, the outgoing is written in negative. So, it is 7,000, P given A, 15%, 8. And finally, for salvage value, it is + 25,000, this is the worst value of salvage value, P given F, 15%, 8. So, this is the NPW, corresponding to the worst scenario. For the normal scenario, already we have calculated, in the previous lecture.

Likewise, we can calculate the NPW, for the best scenario also, which could be, - 500,000, + 115,000, multiplied by P given A, the best interest rate could be 10%, and the number of years could be 12. Likewise, the salvage value is 60,000. So, I write, + 60,000, P given F, 10%, 12.

And, then finally, for outgoing, this is - 4,000, P given A, 10%, 12. So, you can calculate these values. Now, the manner in which, I had to take the decision, is like this.

So, you remember, the NPW for normal situation, was coming to be, 52,000 something, + 52,000. Now, worst case, if you find that, this is a very large negative number. Suppose, you find that, this is a very large negative number, for worst scenario. And, of course, for best scenario, you are getting some positive value, which is better than this, 52,000. But, the key point here that we have to observe is, if we are getting a very high negative value, for the worst situation, then I may not even like to go for this Alternative.

Because, there are chances that, I might lose, if I go for this investment. So, what is happening? NPW normal is, I am getting positive, about 52,000. NPW best scenario, I am getting still higher value. But, if I find that, the NPW for the worst scenario, is very large negative number, in that case, I may not like to go in for this Alternative. However, if I find that, the NPW for worst scenario is also slightly negative, or very close to 0, in that case, I may still prefer to go with, this Alternative.

So, this is how, using a multiple variable changes at a time, we carry out the Sensitivity Analysis. This particular condition, we are also referring to it, by the name of Scenario Analysis. So, Scenario Analysis is nothing but the Sensitivity Analysis, involving more than two variable changes at a time. So, whether you call it Sensitivity Analysis, or whether you call it Scenario Analysis, both are same. Now, what I do is, we move to next set of option like, conducting Scenario Analysis, for more than one Alternative.

So far, we have been observing, Scenario Analysis, for single Alternative. Now, we move to next level, wherein, we try to carry out the Sensitivity Analysis, for more than one Alternative. Here also, as before, we will discuss, the variation in one variable at a time, to start with, we will discuss two variable changes at a time, and then, more than two variables at a time. Now, if you remember, we had been given one Alternative.

Now, to illustrate this case, we are adding one more Alternative. So, earlier one, we are calling it

as A1. Now, the second Alternative, we are calling it as A2. Now, for A2 if you see, the acquisition cost is 400,000. That is, the first cost is 400,000. The income is 80,000 every year, for 10 years. Outgoings if you see, it is 10,000 per year, for 10 years. Salvage value is 40,000. Interest rate is 12%. And then, you have service life of 10 years.

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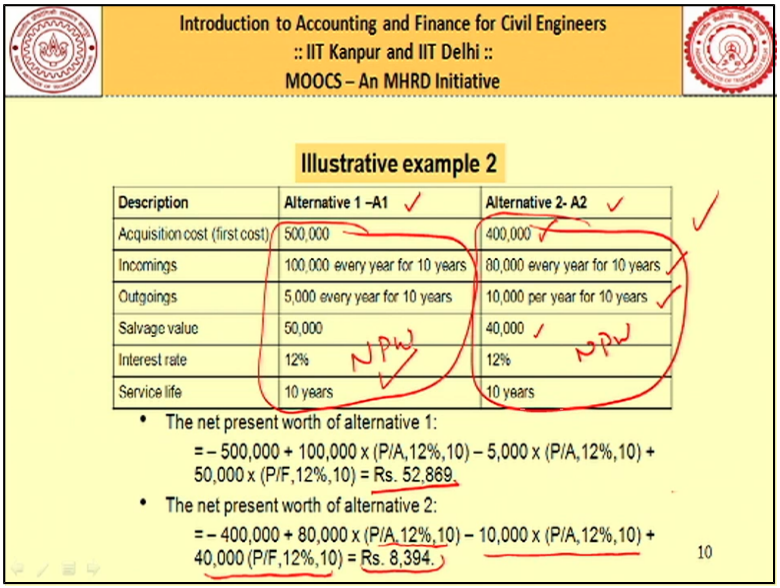


So, this is the Alternative, that I have added just now, to illustrate you, the Sensitivity Analysis for this case. Now, what I do, if we are not considering Sensitivity Analysis, assuming that, all these values are constant, and they are not changing, we can find, which Alternative to choose. So, what I do? I calculate the NPW, for this Alternative. I calculate the NPW, for this Alternative also. And then, I simply choose, which one is having the maximum NPW, and then, I accordingly take the decision.

So, for this Alternative A1, we have already obtained, the Net Present Worth. It is 52,869. Now, for Alternative A2, I can carry out the analysis, in the same manner, and I can calculate the Net Present Worth. If you see here, I am getting the Net Present Worth like this. It is - 400,000 + 80,000, in to P given A, 12%, 10 years, - 10,000 in to P given A, 12%, 10, + 40,000, which is the salvage value, P given F, 12%, 10. So, you find that, you are getting 8,394.

So, for this example, you find that, A2 is having lesser Net Present Worth, compared to the Alternative-1. So naturally, the decision would be to go for, A1. Now, we would like to see, if I

change different variables, one at a time, of course, to start with. Suppose, I change my incomings of Alternative A1, will I still be able to prefer A1, or will it result in selecting A2. Likewise, I will see, if I find that, interest rate is changing for A1, will I still go for A1, or will I move to A2. Likewise, I go on selecting each of these variables, and make changes there, and then try to see, what happens on my decision.

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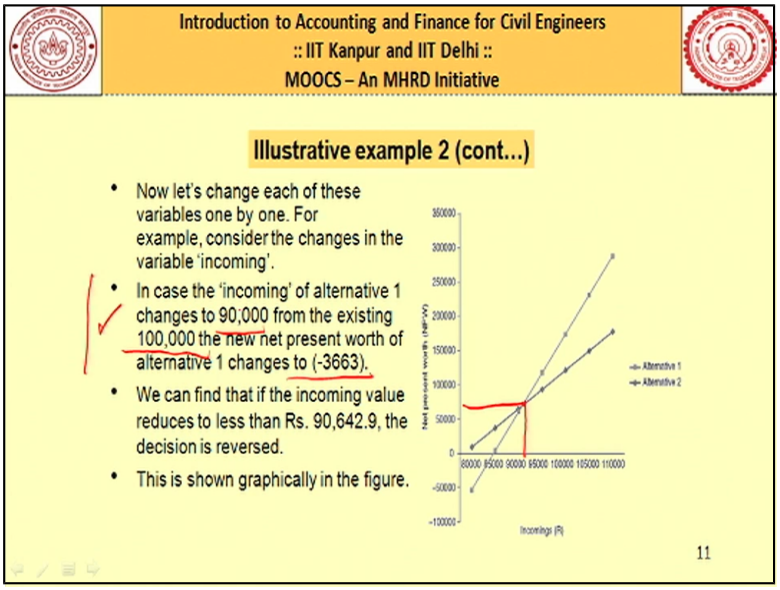


So first, I am taking one variable at a time. And here, you can see, I have done the analysis for incomings. So, you find that, if the incoming changes from, 100,000 to 90,000, from our past knowledge, from the previous lecture, you find that, the Net Present Worth is coming to be, -3,663. So, straight away we can say that, if the incoming in Option-1, comes down from 100,000 to 90,000, we will have to switch to Alternative A2. So, this is shown graphically.

So, this is the point at which, the decision reversal takes place. So, sometimes, this is also known as, Breakeven Point. So, you can see, this is the point at which, we are changing the decision. So, below this, one Alternative is preferred, above this, some other Alternative is preferred. This was, as far as, the variable incoming was concerned. The same analysis, I can carry out for other variables also. And, I can develop a table, like this.

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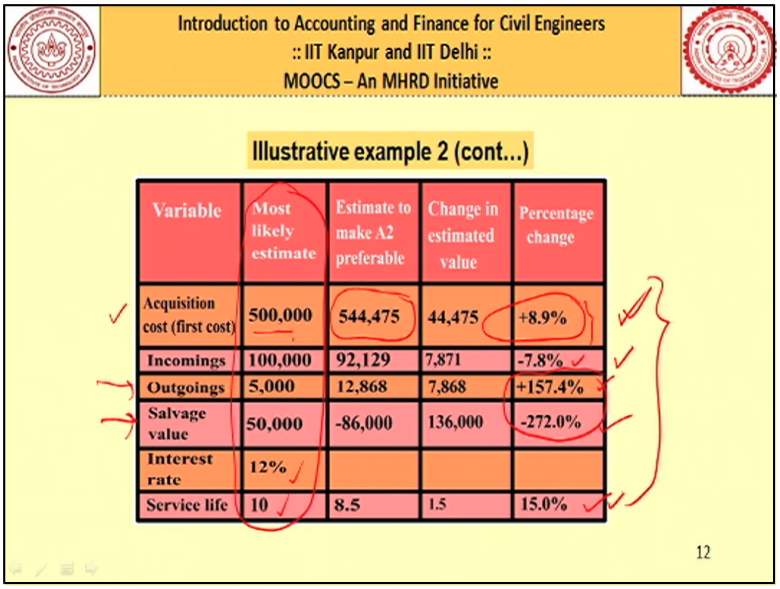


Now, if you look at this table, we are having all these values here, for the original A1 problem, Alternative A1. Acquisition cost, 500,000. Incomings, 100,000. Outgoings, 5,000. Salvage value, 50,000. Interest rate, 12%. Service life, 10 years. Now, I will like to view this problem, like this. By how much amount, this acquisition cost changes, so that I prefer A2, instead of A1.

So, if you carry out this analysis, you will find that, if the first cost of Alternative A1, changes from, 500,000 to 544,000, that means, there is an increase of + 8.39%, I will no longer be preferring A1, rather I will go with A2. So, the moment my initial cost for Alternative A1, changes from 500,000 to 544,000 something, I have to move to Alternative A2. No longer, A1 will be desirable. Likewise, you have already seen for the case of incoming, if the incoming of Alternative A1 drops by 7.8%, we will have to go for A2.

Likewise, if outgoing changes by 157.4% for Option-1, we will have to switch to A2. Salvage value, if from 50,000, it comes down to - 86,000, that means, there is a change of 272%, Alternative A2 will be desirable. Likewise, service life, if it changes by 15% for the Alternative-1, we will prefer A2. So, from this last column of this table, we can very easily find out, which variables are sensitive, and which are not. So, you can see, + 8.9%, - 7.8%.

So, acquisition cost is very sensitive. Incomings, they are very sensitive. Service life, this also is very much sensitive. Remaining, you can see, even for a change like, 150%, 270%, there is no

change in my earlier decisions. So, we call these variables such as, outgoings, and salvage value, these are not so sensitive variables. So, this is how, we carry out the analysis, involving two Alternative, but single variable change. Now, we will see, how to carry out the same analysis, for two variables at a time.

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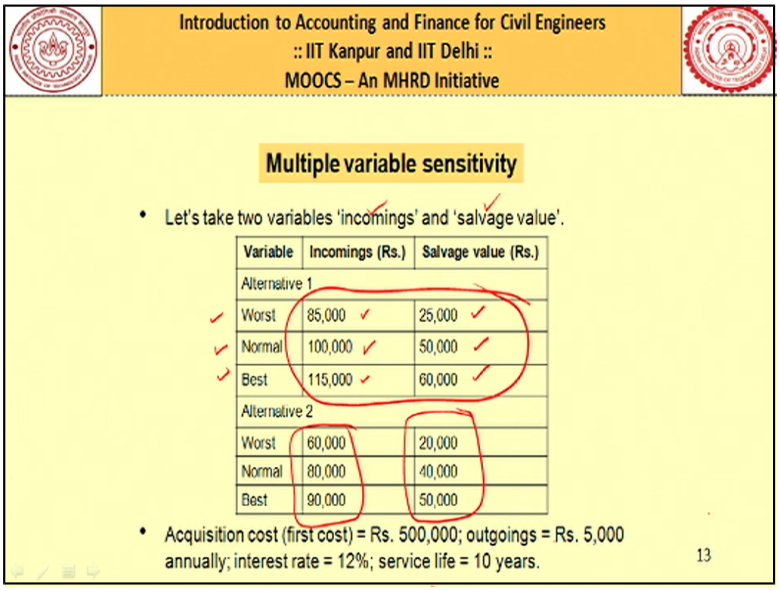


For this, let us take two variables, incomings, and salvage value. So, now we will try to see what happens, when I make changes in these two variables at a time. So, here also, I am taking the worst value of incoming in Alternative-1, the normal is already known, the best is also assumed. So, let us say, the worst case of incoming for Alternative-1 is 85,000, normal is 100,000, best is 115,000.

Likewise, the salvage value, the normal is already 50,000, and the best is 60,000. So, for these two sets of variable values, we will find out the NPW. We will see, how to do it. Likewise, for Alternative-2 also, I have assumed the worst value, and the best value, normal, is already known to you. Likewise, the salvage value also, the worst, the best, and the normal, are known to me.

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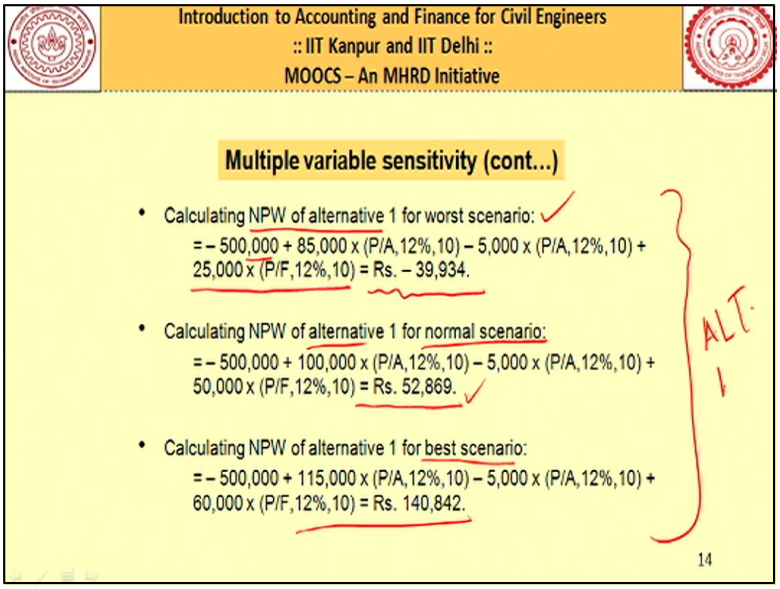


Having done this tabulation, now I can calculate the Net Present Worth of Alternative-1, for the worst scenario. So, this could be, - 500,000 + 85,000, in to P given A, 12%, 10 - 5,000, into P given A, 12%, 10, + this part is for salvage value. So, you find, you are getting a Net Present Worth of - 39,000, for the worst scenario, for Alternative-1.

Likewise, this portion, is already calculated by us. For Alternative-1, normal scenario, it is 52,869. And, when you do the same analysis, for best scenario, we are getting a Net Present Worth of 140,842. So, these are the three values, that we are getting, out of Alternative-1. Now, similar analysis, we are doing it for Alternative-2 also.

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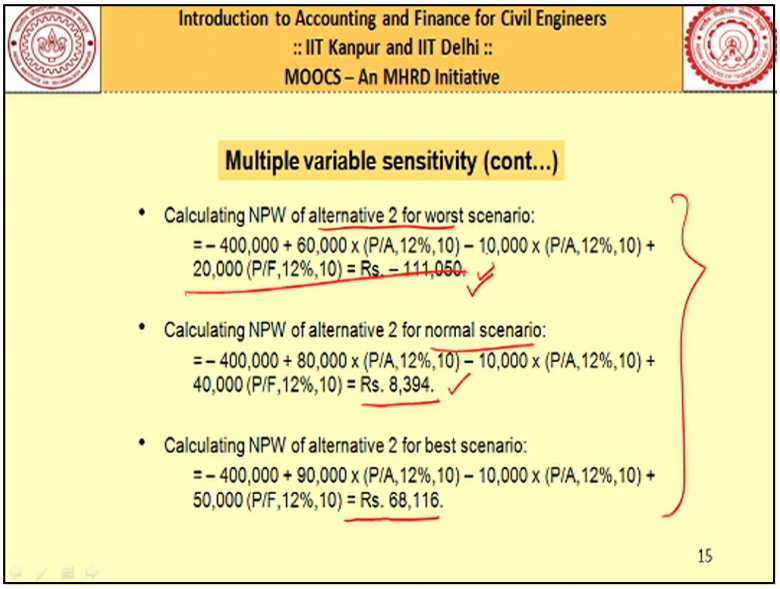


So, for Alternative-2 also, you can calculate the Net Present Worth, for the worst scenario, which is coming to be, very high negative value, - 111,050. The Net Present Worth of Alternative-2 for normal scenario, you can crosscheck, this is coming to be 8,394. And, here you find, for the best scenario, we are getting 68,116.

Now, if you closely compare these two, you find that Alternative-1, even when we are doing this analysis for the worst case, is resulting in a slightly negative value, - 39,934. When compared to Alternative-2, worst scenario, in that case we are getting, - 111,050. So, looking at these two, you can make decisions as to, whether to go for Alternative-1, whether to go for Alternative-2.

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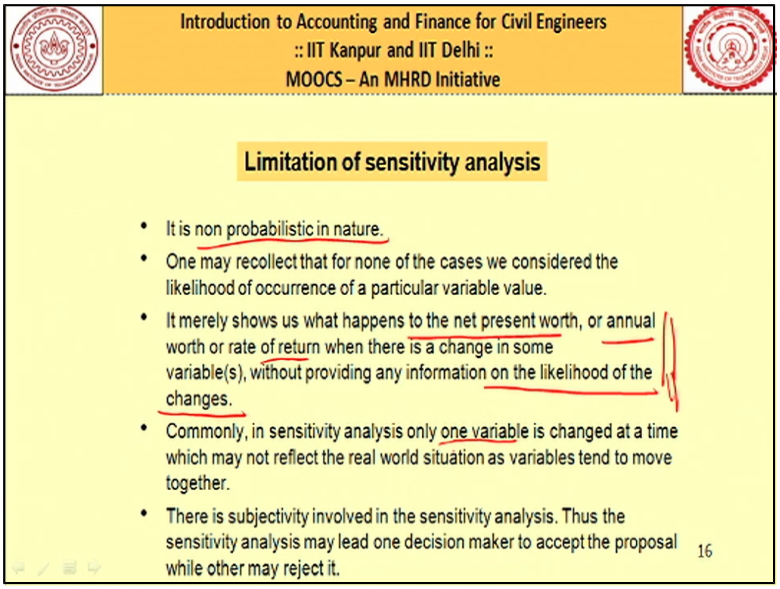


Now, we will quickly go through, the limitation of the whole Sensitivity Analysis. As I already told you, in the previous lecture, it is non-probabilistic in nature. When I say, non-probabilistic, it means, we are not assigning any probability values, corresponding to any of the state of occurrences. So, we are not saying that, okay, worst possibility of income is 85,000, and it has got a probability of 30%. No, we are not assigning any probability.

So, in that sense, it is non-probabilistic approach. Now, what we are doing is, as part of limitation, you can say that, the Sensitivity Analysis merely shows us, what happens to the Net Present Worth, or for that matter, annual worth, or rate of return, when there is change in some variable, without any information on the likelihood of the changes. When we include the likelihood, you will find, you will be able to calculate the risk also.

But, that will be the subject matter of, some other lecture. What happens, in Sensitivity Analysis, commonly we make changes in one variable, thinking that, it would not affect other variable. But, that does not happen, in real life situation. So, the moment you change one variable, there would be some impact, on other variables also. However, this particular thing is neglected, in the Sensitivity Analysis.

Of course, there is subjectivity involved in Sensitivity Analysis, depending on the psychology of the project manager, depending on the background that he had, from his previous experiences,

they may come out with subjective decisions. So, even though, the calculation may show same set of values, you will find that, different project managers may react, differently.

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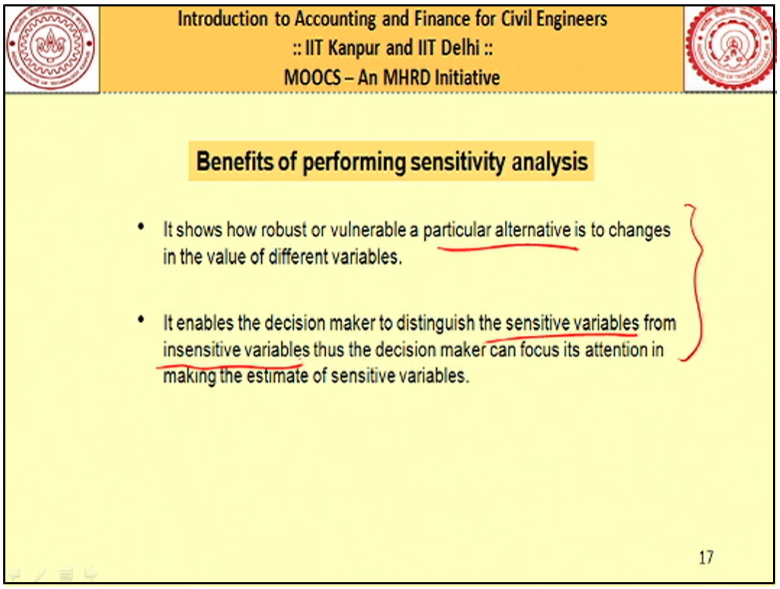


Of course, there are certain benefits of performing Sensitivity Analysis. You can see, how robust, or vulnerable, a particular Alternative is, to the changes, in the value of different variables. So, if I make changes in the variable here and there, what will happen to my decision. This can be very clearly seen. It also enables the decision maker, to distinguish the sensitive variables, from insensitive variables.

We have already seen, depending on the slope of the lines, we can see, which is the highly sensitive variable, which is the non-sensitive variable, and so on. Now, the implication of this is, we will take more care in analysing, the sensitive variables. We will spend more time and money, in analysing those variables. Because, even a slight mistake here and there, might change our decision. So, that is the implication of identifying, the sensitive variables, and insensitive variables.

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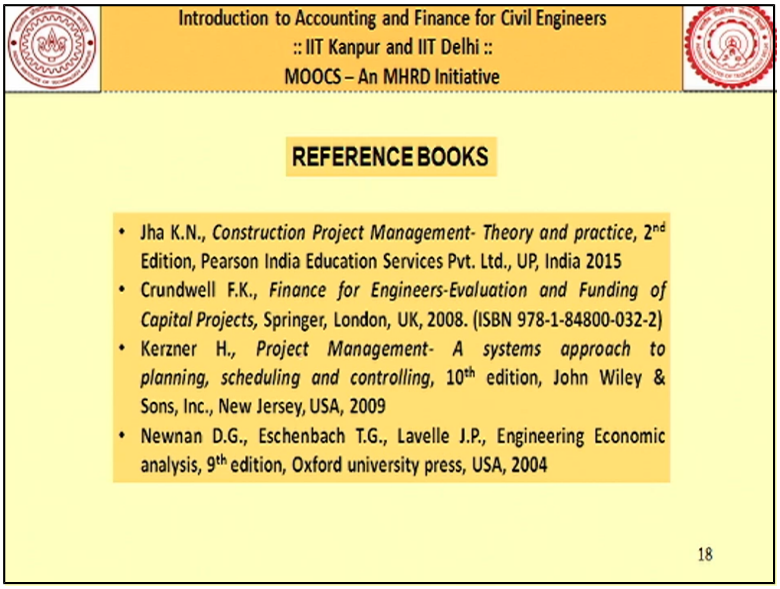


I will show you the reference book, once again, you can just have a close look at this. And, depending on your requirement, you can refer any one of these books, and try to do sums, pertaining to different types of situations. So, in this lecture, we definitely understood, all the aspects of Sensitivity Analysis, involving single Alternative, more than one Alternative, single variable changes at a time, two variable changes at a time, and more than two variable changes at a time. So, we stop at this point, and see you later, in some other lecture. Thank you, very much.