Introduction to Accounting and Finance for Civil Engineers
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Lecture-21
Simulation-(Part-2)

Good morning namaskar and welcome to the course once again. In the last lecture if you remember we started our discussion on simulation, we tried to understand the application of simulation in 2 situations, the first one in the forecasting of demand and in the second one we tried to evolve a maintenance strategy. In this class also we are going to see few more applications of this particular simulation, will try to first apply this in context of inventory analysis.

Subsequently we will apply this in queuing problems and lastly will try to apply this simulation in the context of finding the net present worth. So we straightaway move to our problem.

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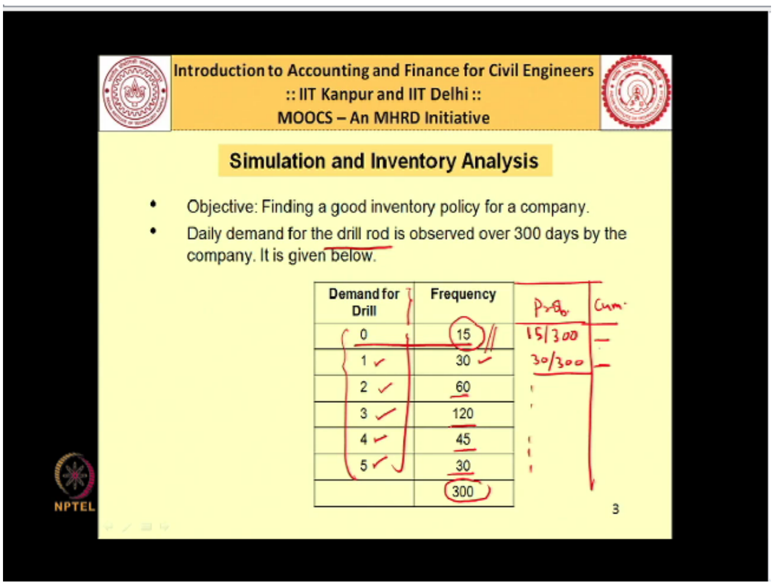

Let us say we want to find a good inventory policy for a company, not for this what the company has done is, it has collected the record for past 300 days and it has found that the demands are varying anywhere from let us say 0 to 5 units, let us say the company is into dealing in drill rods, you know drill rods are used for drilling and it is widely used in the construction industry.

So what it has done is the company has compiled all its past data for last 300 days and it has found that out of 300 days on 15 occasions there was no demand for this particular drill machine or drill rod for that matter. On 30 occasions there was demand for 1 unit, on 60 occasions demand for 2 units, on 120 occasions there was demand for 3 units and on 45 occasions the demand watch for 4 units and finally on 30 occasions it observed that the demand is for 5 drill rods.

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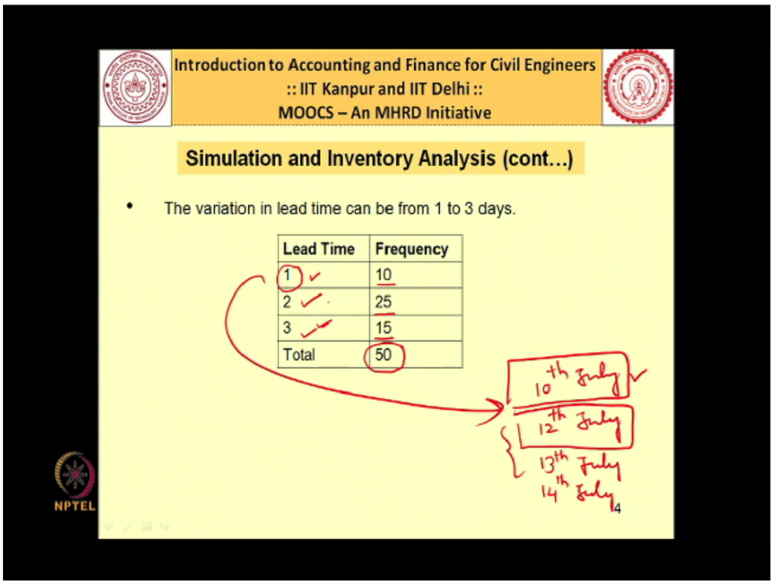

Now the company also collected the data on the lead time. So what is this lead time. So lead time is basically the time which you take when you are placing the order and when you are receiving the material. So suppose the lead time is 1 day what does this mean, lead time of 1 day means let us say today is let us 10th of July you are placing order on this day. So you are going to get the material on 12th July morning.

So this is what the lead time of 1 day in our problem means, you place the order on 10th July it is going to come to you on the 12th July morning, suppose the lead time is 2 days, so you will receive the material on 13th July, if you are placing the order on 10th July and finally if you lead time is 3 days you are going to receive your order on 14th of July. This is what we have to understand in the context of a problem as far as this lead time is concerned.

So what the company has done is it has compile the data again this time for 50 days and it has found that on 10 occasions the lead time was 1 day, on 25 occasions the lead time was 2 days, remember we are trying to do Monte Carlo simulation and their we assume that all these phenomenons are offering on a random basis. So we are generating random numbers and depending on the random number interval we are forecasting the demand.

We are forecasting the lead time and we are also forecasting the let us say demand for drill machine or drill rod in this particular problem. So the same concept we will use. So what we do as a first step is to all these frequency values into a property value, so how do I do this for getting the probability of 0 drill demand we are finding that frequency here is 15, so the probability is going to be 15/300.

So this probability is going to be 15/300 likewise the property for 1 unit of drill rod is going to be 30/300. So likewise you can go on finding the probability then in the next step what you do this is probability column we have to add and then the next column if you remember we are adding cumulative probability, so we are going to write this and then when you write the next one you are going to add this value.

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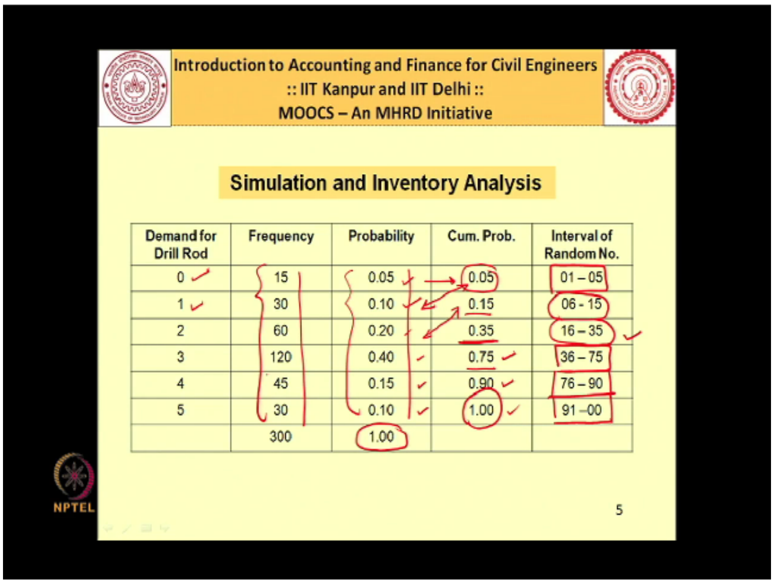

Demand for Drill Rod	Frequency	Probability	Cum. Prob.	Interval of Random No.
0	15	0.05	0.05	01 – 05
1	30	0.10	0.15	06 - 15
2	60	0.20	0.35	16 – 35
3	120	0.40	0.75	36 – 75
4	45	0.15	0.90	76 – 90
5	30	0.10	1.00	91 –00
	300	1.00		

So what I have done, I have done these calculations for you in this particular slide. So you can easily find I have converted these frequency values here from this column into this column. So 15/300 I am getting 0.05 30/300 I am getting 0.1, likewise 0.2, 0.4, 0.15 and 0.1. So when you take the sum of all these values you find the total sum is coming to be 1, finding the next column is not difficult.

You can find the cumulative probability, so this remains 0.5 itself for generating this what I have done I have added 0.5 with this value. So I am getting 0.15, likewise I am getting this as 0.35, how I am getting this by adding this 0.15 with this 0.20. So this now I am getting it likewise you will get 0.75 as the next value, 0.9 as the next value and finally 1 as the last value here.

Now if you remember in the last class we generated the random number in this fashion. So cumulative probability of 0.05 I am giving a random number interval of 01 to 05, what does this mean, any number that I generate in between this 01 and 05 I will assume that the demand for the drill rod on that particular day is going to be 0, likewise if my number appears between 06 to 15 I will assume that demand for drill rod to be 1 unit, 16 to 35 how I am getting this.

This is the 35 here, you can see 0.35, so for that I am writing 16 to 35, this is 0.75, so the next number starts from 36 and it goes up to 75, likewise for 0.90 I am writing it between 76 to 90 and likewise for 1 I am writing anywhere from 91 to 00. So this is one of the ways in which you generate the random number interval. So what is happening is depending on the frequency or depending on the probability of occurrence I am assuming that my demands are going to be happening in the future as well.

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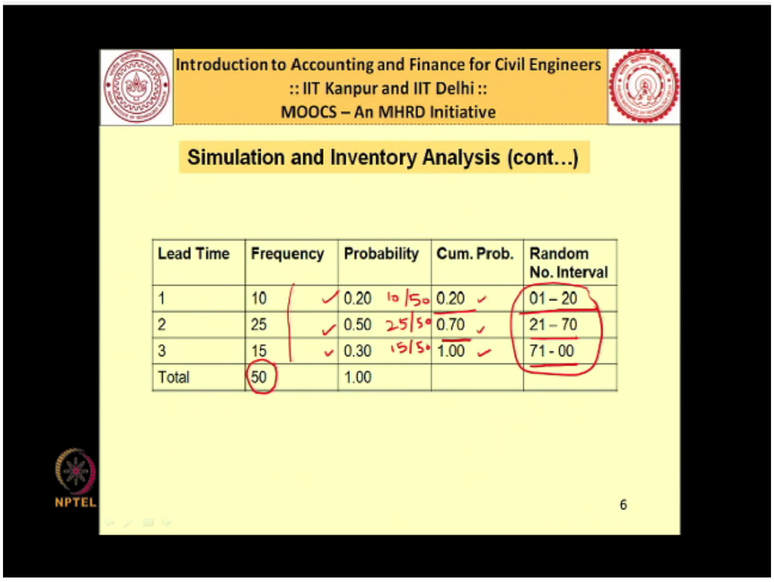

Now in the same manner I am going to generate the random number interval for the lead time also. So lead time you remember the frequencies where given 10, 25 and 15 and the total frequency was 50. So I convert this into a property value by doing this calculation, this is coming from 10/50, this is coming from 25/50, and finally this is coming from 15/50. So you can get this 0.2, 0.5 and 0.3.

So you can generate the cumulative probability values also, it will be 0.20, 0.70 and finally 1, now in the same manner I am also generating a random number interval here. So 0.20 cumulative probability I am writing the interval as 01 to 20 for this one I am writing it 21 to 70 and finally I am writing this as 71 to 00. Now remember we are generating only 2 digit random number.

If the number of digits in a random number is 3 accordingly you have to revise your random number interval, but this is quite easy or 2 digit random number and in fact I had shown you the random number table also.

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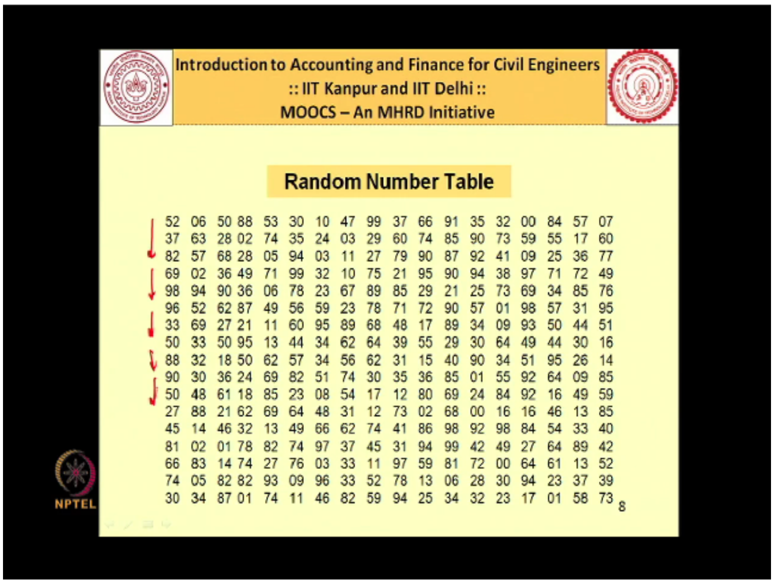

52	06	50	88	53	30	10	47	99	37	66	91	35	32	00	84	57	07
37	63	28	02	74	35	24	03	29	60	74	85	90	73	59	55	17	60
82	57	68	28	05	94	03	11	27	79	90	87	92	41	09	25	36	77
69	02	36	49	71	99	32	10	75	21	95	90	94	38	97	71	72	49
98	94	90	36	06	78	23	67	89	85	29	21	25	73	69	34	85	76
96	52	62	87	49	56	59	23	78	71	72	90	57	01	98	57	31	95
33	69	27	21	11	60	95	89	68	48	17	89	34	09	93	50	44	51
50	33	50	95	13	44	34	62	64	39	55	29	30	64	49	44	30	16
88	32	18	50	62	57	34	56	62	31	15	40	90	34	51	95	26	14
90	30	36	24	69	82	51	74	30	35	36	85	01	55	92	64	09	85
50	48	61	18	85	23	08	54	17	12	80	69	24	84	92	16	49	59
27	88	21	62	69	64	48	31	12	73	02	68	00	16	16	46	13	85
45	14	46	32	13	49	66	62	74	41	86	98	92	98	84	54	33	40
81	02	01	78	82	74	97	37	45	31	94	99	42	49	27	64	89	42
66	83	14	74	27	76	03	33	11	97	59	81	72	00	64	61	13	52
74	05	82	82	93	09	96	33	52	78	13	06	28	30	94	23	37	39
30	34	87	01	74	11	46	82	59	94	25	34	32	23	17	01	58	73

If you see this is how the random number tables where shown in the last lecture also. So you can see either you can generate your own random number with the application of some calculator or you can use computers and depending on how many digits you want to prefer you can accordingly generate 2 digit random numbers or 3 digit random numbers. In this table you can see random numbers.

If you go column wise it is 52, 37, 82, 69, 98, 96, 33, 50, 88, 90 and so on. So you can find here is that there is no pattern as such which you can decipher out of these random numbers.

So any time any number can appear. So this is the beauty of these random number. So looking at this random number now we have to start the simulation process.

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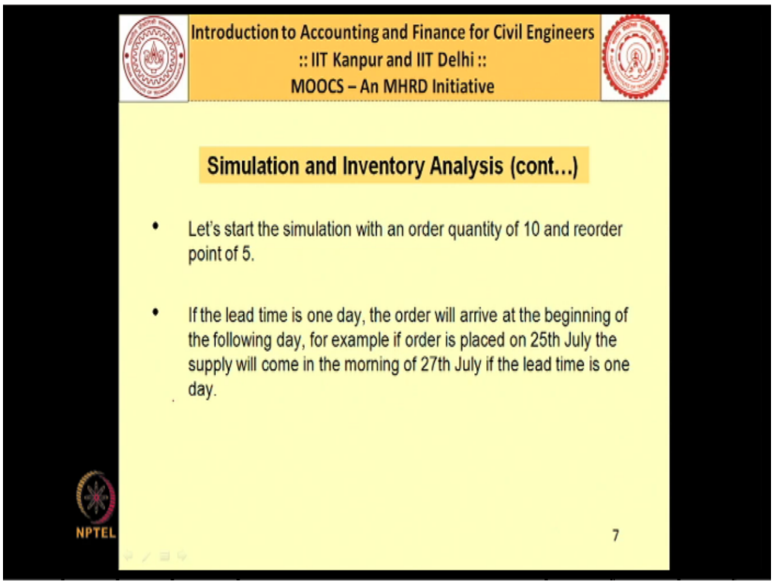

So let us start the simulation with an order quantity of 10 and reorder point of 5, so when you say reorder point it means the moment your stock goes below 5 you will place the order and the minimum order quantity that you are placing is 10 right. So you can say this is an economic order quantity, what happens in inventory analysis you will find that are there is a particular number in which you should be ordering to make maximum money out of it to optimize the this particular ordering process.

And the one of the policies that the company has assume for the time being is to ordering it whenever the stock possession below 5 right. Now as I told you when I say lead time of 1 day the order will arrive at the beginning of the following day. For example if the order is placed on let us say 25th of July, the supply will come in the morning of 27th July, if the lead time is 1 day.

If the lead time is 2 days it will come to you on 28th of July, if the lead time is 3 days it will come to you on 29th of July. So now what I do I generate simulation table and I start entering values depending on my requirement.

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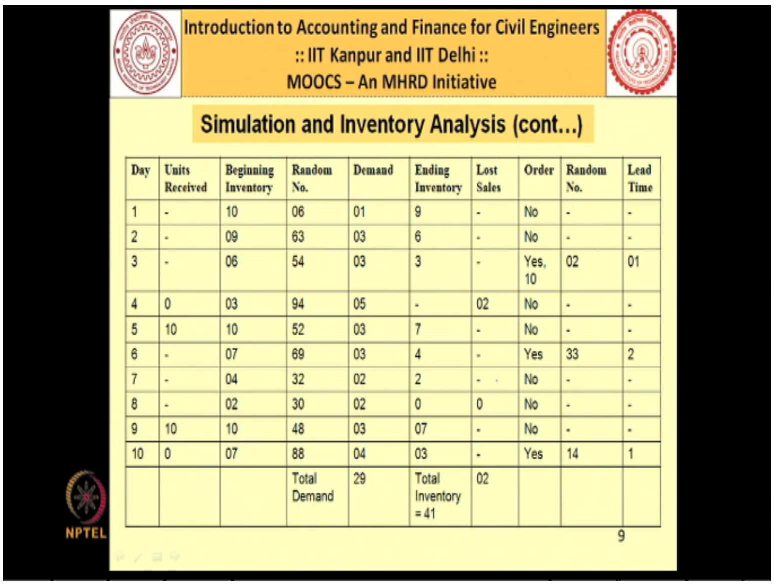

Day	Units Received	Beginning Inventory	Random No.	Demand	Ending Inventory	Lost Sales	Order	Random No.	Lead Time
1	-	10	06	01	9	-	No	-	-
2	-	09	63	03	6	-	No	-	-
3	-	06	54	03	3	-	Yes, 10	02	01
4	0	03	94	05	-	02	No	-	-
5	10	10	52	03	7	-	No	-	-
6	-	07	69	03	4	-	Yes	33	2
7	-	04	32	02	2	-	No	-	-
8	-	02	30	02	0	0	No	-	-
9	10	10	48	03	07	-	No	-	-
10	0	07	88	04	03	-	Yes	14	1
			Total Demand	29	Total Inventory = 41	02			

So let us try to generate this particular table, so let us say I am trying to do this analysis for 10 days, this is because I am doing it manually, if I am to use these computers or some softwares you will find that the number of days that you carry out the simulation could be running in 1000 or in 5000. So let us say there is no order pending, so I have not received any unit here. So my beginning inventory as I told you it is given to be 10.

So I am staring the simulation assuming that I have 10 numbers in stock which I have received today itself, now I am generating a random number and random number that I got here is now let us assume it to be 06. So when you get a random number 06 your demand for that particular day is likely to be 1 from where I am getting this for this you have to go back to the particular table here.

So you can see between 01 to 05 any number appears demand for that day is 0, between 06 to 15 the number appears the demand will be assumed to be 1. So right now the number is 06 so I will assume that the demand for drill rod on that particular day is 1, so you can see here I am writing this has 1. So what will be my ending inventory for that day you started the day with 10 units and you spent I mean you already consumed 1 unit.

So the ending inventory is going to be 9, now there will be no loss sale because whatever was the demand you were able to fulfil. So lost sale will be nil, you did not lose any sale for that day. Now the next question that we ask is, do we have to order, no, why because my ending inventory is still 9. So it has not reached my reorder point, so I am not ordering. So I will not generate the random number for order and lead time also will not be calculated.

Now when it comes to the day 2 my beginning inventory is the same as the ending inventory of yesterday so which is 9, now I generate the random number again, random number is now 63. So I go back to the table you can see here corresponding to 63 my demand is 3, So you can see any number generated between 36 to 75 my demand for that day is 3 units. So I will write this as 3 here.

So you can see the demand for this day is likely to be 3 days, so my ending inventory is going to be 9-3 which is 6 units, lost sales no why because whatever was the demand I was able to fulfil it. So no need of losses, so I am not writing anything here, do I need to order no because my in the inventory is still 6, so no need to order it yet, so no need to generate the random number here, no need to calculate the lead time here.

Now when it comes to the third day you can see my beginning inventory will be the ending inventory of yesterday, so 6 was here, so this 6 will be here, now my next random number that I generated is coming to be 54. So I go back to the table again and see where does 54 lie, so 54 you see again it is coming in this interval 36 to 75. So my demand for that day again is going to be 3.

So I write this 3 here, so what you see now my ending inventory becomes 6-3, so this ending inventory is 3 here, was there any lost sale, no because your demand was 3 you are able to meet the demand, so that is it. Do you need to order, yes why because now it has dropdown below your reorder point, your order point assumed was 5 unit, now it is at 3 units, so we need to place the order.

So we are placing the order and what will be the order quantity it is going to be 10 as we assume in the problem in the beginning. Now I generate the random number, so random number that I am getting is 02, so I go back to this particular table which is here 02 will lie in this interval. So we will assume that the lead time for this order is going to be 1 day, so I write lead time here as 1 day.

So that means this order is being placed on day 3, so when will I get the order if the lead time is 01. So I will be getting this order on day 5 morning. So we will see where it exactly it will come. Now when it comes to day 4 units receive 0, because order will be received tomorrow only. Now the beginning inventory is 03 from where I am getting this, this is nothing but this value in the inventory of yesterday is beginning inventory of today.

I generated the random number it is coming to be 94 and corresponding to 94 if you go back to the table you can see between 91 to 00 I am getting the demand as 5 unit, so I will write this as 5 years. So you can very clearly see the demand here is 5. So ending inventory is going to be 0 because you had beginning inventory 03 and you had a demand of 5. So 3 you would fulfil, but 2 was your lost sales.

Because they was demand for 5 units and in your stock they were only 3 units. So you lost the sale as far as 2 unit drill rod is concerned. So you need to order today?, no why because 1 order is still pending. So you cannot place the order till you get the material, random number no need to generate here, lead time no need to calculate here. Now when it comes to day 5 you can see here I am receiving 10 units.

This recede is against the order placed on third day you remember we place the order here, so I am receiving the material on 5th morning 10 units. So my beginning inventory is going to be 10 because I had exhausted all my stocks. Now the random number generated is 52, so the demand corresponding to that is 3. So my ending inventory is going to be 10-3 7, lost sales will be 0 here because whatever demand was there I was able to meet no lost sales.

I will not place the order today again, no need to generate the random number, no need to generate the lead time, so this is how I continue to carry out my simulation if I am doing it manually I will not go beyond this 10 or 15 days cycle and then I can calculate the total values. So you can see here the total inventory that I am carrying is 41, how I am getting this I am summing it up this particular column. So 9+6+3+0+7+4+2+0+7+3, so I am waiting total inventory that at anytime I have in my stock in this 10 days is 41 units.

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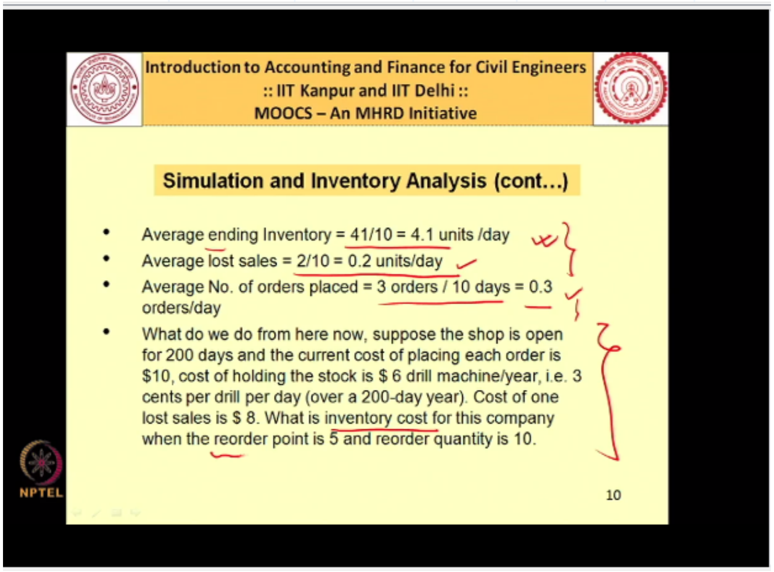

Total demand that I simulated is coming to be 29 units which is nothing but the sum of this particular column. So what exactly we are getting out of this analysis, we are finding that average ending inventory is 41/10, 41 I have already showed you that divided by 10 is 4.1 units per day, average lost sales if you see in the 10 days analysis period you could see that only on one occasion we lost sales.

When you say lost sales means demand was there but there was no material available in your stock. So you lost that particular sale, so you are getting this as 2 here. So if you calculate the average you will get 2/10. So you lost sales on an average 0.2 units per day, average number of orders that we placed let us go back and see, so you place the order on day 3, you place the order again on day 6 and we place the order again on day 10.

So out of 10 days we placed order 3 times, so this what we are getting average number of orders placed in 3 orders/10 days which is 0.3 orders per day, so what do we do from here onwards, we will calculate the cost what we will do we will calculate what is my inventory carrying cost, likewise I will also calculate what is my lost sales cost and for these values of 4.1 unit and 0.2 units per day and finding the number of orders that we are placing.

All these cause if I add it up I will find I have a particular cost to maintain this policy, what is the policy that I am adopting here, reordering point of 5 and economic order quantity of 10. Now what I will do is I will change these values. So let us say instead of reorder point of 5 and reorder quantity of 10 I may change is values to let us say 3 and reorder quantity may be 5, I will try to find out what is the cost associated with this.

So this way I will try to do large number of simulations corresponding to different situations and that way I will come to know which policy I should be following it up for economising my inventory carrying cost. Now as you remember this particular process does not guarantee optimal solution but yes it very closely imitates the real life situation and based on your past experience based on your past that are you are able to closely resemble your day today business cycle.

So depending on the analysis of various scenarios we will be in a position to find the best inventory policy for my problem. Now I will take it up 1 more example that is also very commonly used in the context of any engineering applications and that is the example of a queuing problem, what happens you take queuing business what happens let us say in this case I am assuming that there is a port wherein number of barges are coming every night.

These barges carrying goods which are to be taken to different cities from this particular port. Now there are workers employed to unload this barges, you know the more the time a barge spends in this particular ports the more money they have to lose because these barges when they are standing there when they are there or they have to pay money to the port authorities. So they just cannot tolerate wastage of time due to late unloading.

So the port authorities what they have done, they have collected the data on the number of barges that are arriving in any particular night on that particular port and they have also collected the data how many of those barges are able to be unloaded. So these data they have captured it and they have given us you can have a look at this data.

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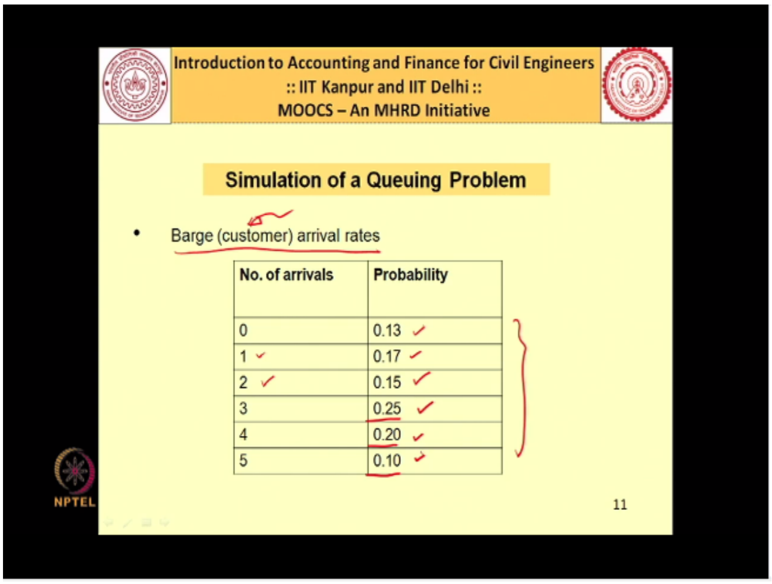

So let us say the barge arrival rates, this bare could be any customer, for example you are going in for a reservation counter, so you are the customer if ships are coming on a particular port, they are the customers, so the number of arrivals and their properties have been captured in this particular table, so 0 arrival the property is 0.13, 1 arrival that means 1 customer is arriving on that particular night the probability is 0.17, 2 customers arriving on the particular night the probability is 0.15 corresponding to 3 0.25+1*4 it is 0.2.

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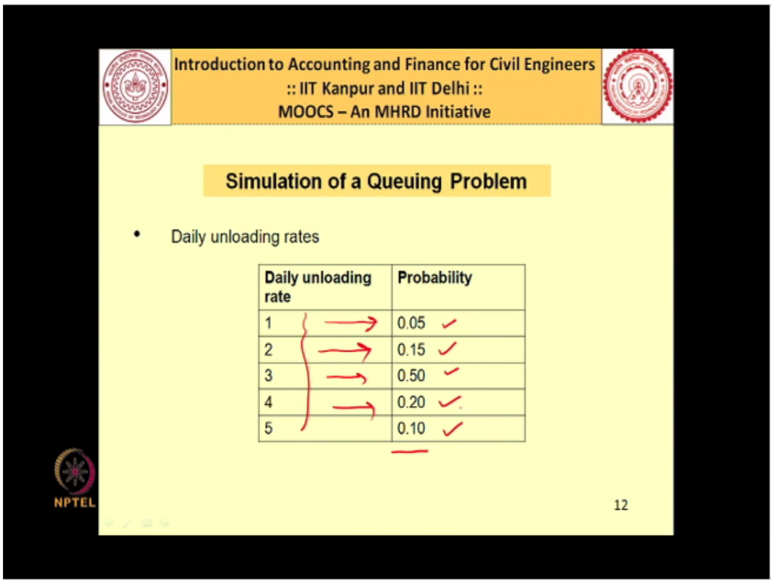

And corresponding to 5 it is 0.1, likewise the unloading rates also vary, why they vary because these barges carry different types of material, so sometimes the unloading could be quiet fast, sometimes the unloading could be delayed. So these probabilities have also been captured, so the probability that 1 barge will be unloaded on that particular night probability is 0.05, 2 getting unloaded probability is 0.1.

3 getting unloaded probability is 0.5, 4 getting unloaded probability is 0.2 and finally 5 getting unloaded probability is 0.1. So we are trying to find out what is the best combination of number of workers, so that we do not get complaints from these barge owners that the unloading has been delayed and they have to be unnecessarily paying the excess charges.

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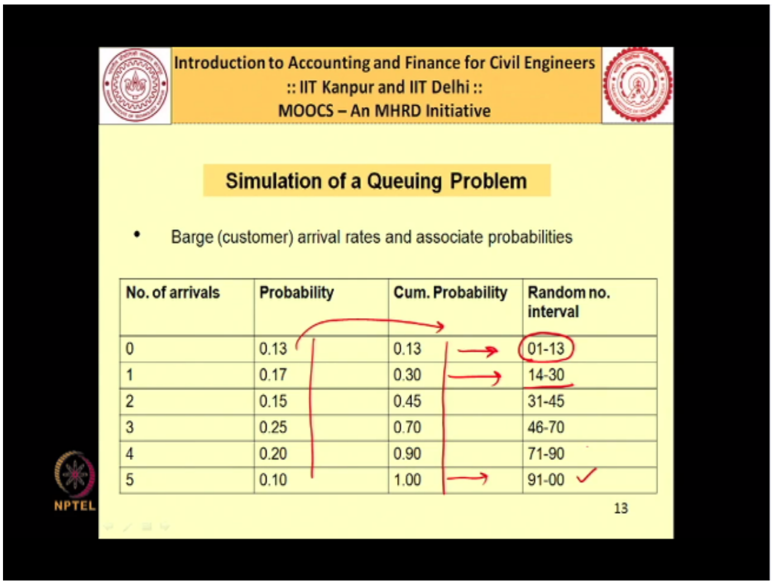

So what I did as a first step the customer arrival rates and associated probabilities have been captured in this particular slide So from these probabilities value I could calculate the cumulative probability values and now you know how to generate the random number. For example corresponding to 0.13 I am writing a random number interval as 01 to 13 for 0.3 I am getting a random number interval of 14 to 30.

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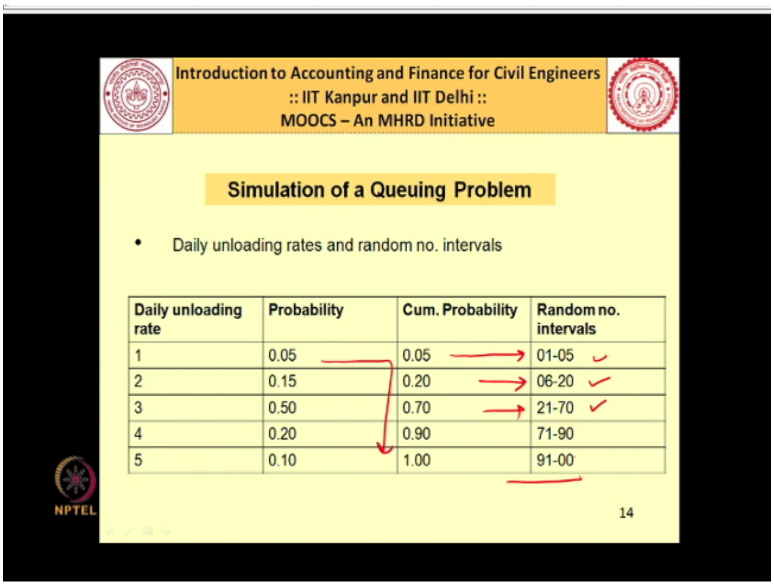

And likewise corresponding to 1 here 91 to 00. In the same manner for unloading rates also I am generating random number interval, so from probability I am getting the cumulative probability like this and this is how I am generating the random number interval for 0.05 I am writing 01 to 05 for 0.20 I am writing 6 to 20 for 0.7 I am writing 21 to 70 and so on. Now I write the particular table of simulation in a particular style.

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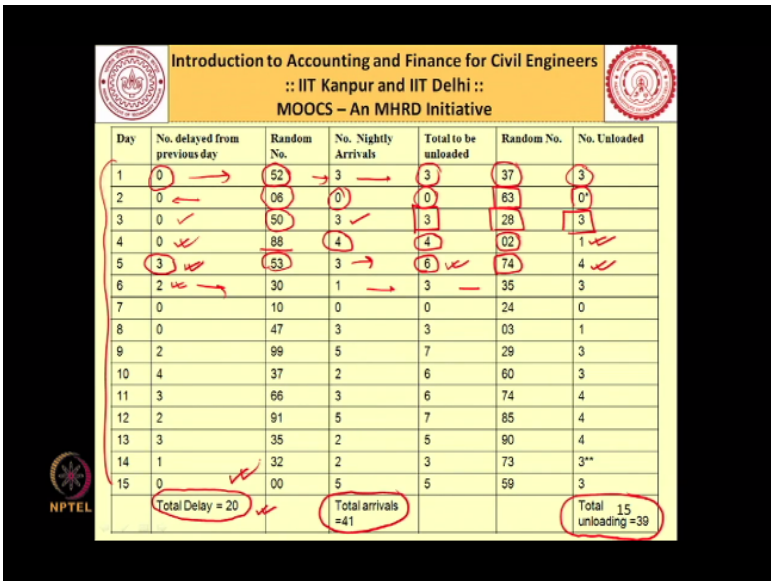

Day	No. delayed from previous day	Random No.	No. Nightly Arrivals	Total to be unloaded	Random No.	No. Unloaded
1	0	52	3	3	37	3
2	0	06	0	0	63	0*
3	0	50	3	3	28	3
4	0	88	4	4	02	1
5	3	53	3	6	74	4
6	2	30	1	3	35	3
7	0	10	0	0	24	0
8	0	47	3	3	03	1
9	2	99	5	7	29	3
10	4	37	2	6	60	3
11	3	66	3	6	74	4
12	2	91	5	7	85	4
13	3	35	2	5	90	4
14	1	32	2	3	73	3**
15	0	00	5	5	59	3
	Total Delay = 20		Total arrivals =41			Total 15 unloading =39

So that it becomes very easy for me to analyse, so what I am doing let us do this analysis for 15 days and let us see how it goes out, so what I am writing in the second column is number delete from previous day. So let us say out of n number of barges that arrived on that particular night on that particular port, let us say there is no delay from the previous day. So whatever came on that particular night everything was unloaded right.

So number delayed from previous day is 0, now I generate a random number it is coming to be 52. So corresponding to 52 I will see how many barges are expected to arrive on that particular night. So 52 is in this range 46 to 70. So I will assume that there would be 3 barges arriving on that particular day. So that number of arrivals is 3 you can see here, so how many has to be unloaded since there was no from the previous day.

And today there were 3 nightly arrivals, so total to be unloaded is 3+0, so 3. Now I generate the random number for unloading so I am getting a random number of 37 so I will go back to this table, so 37 lies here, so you can see I will be able to unload all 3 on that particular night. So the number unloaded is 3 here. So again you can calculate 3 came and 3 were unloaded so no delays, so 3-3 is 0 here.

Now I generate the random number again it is coming to be 06. so 06 if you look at this particular table it is in this interval 01 to 13 and we are resuming that corresponding to this the number of arrivals for that particular night is 0, that means no barges would be arriving on that particular night. So 0 here so total to be unloaded of course 0. Now you generate the random number 63, number unloaded 0 because there is no arrival of barge on that particular night right.

So the number delayed from previous day is 0, again now you generate the random number 50 so estimated arrival is 3 total to be unloaded is 3 again right, so random number I generate I am getting 28, so 28 is in this interval, so assumed unloading rate on that particular night is 3. So 3 unloaded so again there will be 0 unloading to be carried out, random number is 88 for the next day, number of nightly arrivals is 4.

So total to be unloaded 4, I am getting a random numbers 0 to 2 here, so the number that will be unloaded on that particular night is going to be 1 only. So what is happening 4 barges arrive on that particular night and only one got unloaded. So the remaining 1 have to be unloaded, the next day so 4- 1 3 is pending for the previous night number. You generate another number here random number 53 corresponding to that 3 nightly arrivals.

So 3+3 6 to be unloaded, random number again to be generated 74, so you can unload 4 on that particular night. So this is here so 6 to be totally unloaded out of that only 4 got unloaded. So 2 are remaining. So this way you continued doing this exercise for let us say 15 to 20 times and you can find these values, how many total arrivals for this 15 days. So you find total of 41 barges arrived in this 15 day period.

And in that 15 day period 39 barges were unloaded, the total delayed that you observed from last 15 nights was 20. So that on 20 occasions whatever barges arrived on the night they were not unloaded on the same night. So that is why you can see the delay is 20 here.

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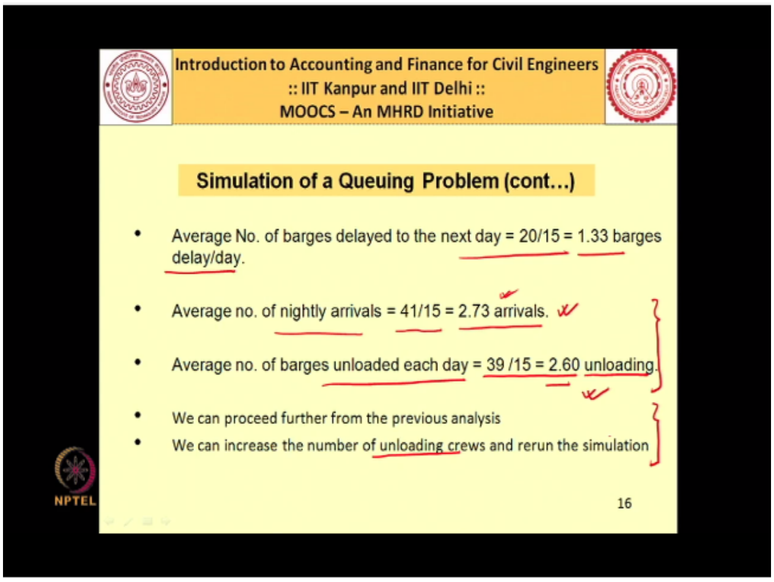

So we can do this calculations average number of barges delayed to the next day 20/15, 1.33 barge, delay/day, average number of nightly arrivals 41/15 which is 2.73, average number of barges unloaded each day 39/15 2.60. Now you see average is 2.73 and unloading average is 2.60. So in 15 trials although they may not be able to take the decisions based on these of 15 days trial itself.

But this is just giving an idea that the situation is not too bad, so average number is 2.73 as far as arrival is concern and as far as unloading is concern it is very close to 2.60. Now you can carry on this analysis for large number of days and if you are not happy with the rate of arrivals and the rate of loading you can try to increase the crew and rerun the simulation and see whether this is trying to help you out.

So this is how you try to apply simulation in train problems, you can find plenty of examples of such type of problems in the case of civil engineering applications. Now I will show you 1 last application of simulation in the context of computation of net present worth. For this if you remember towards the end of our discussion on net present worth we took 1 example in which we said that at time t=0.

I am supposed to invest some money and in return I am waiting cash flows at the end of year 1, at the end of year 2 and at the end of year 3. If you remember in the very beginning of a lectures we said that all the cash flows are certain to occur, later we said no these values may change and subsequently towards the end we try to assign probabilities also associated with each of these cash flows.

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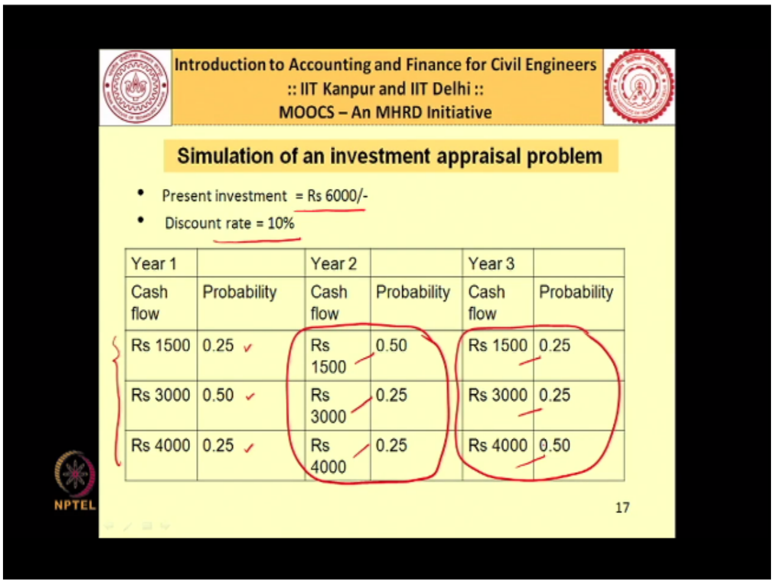

So we go back to those problems and let us see this problem once again, if you remember in one of the lectures we already exposed used to this particular problem in which you are investing 6000 rupees at time t=0 discount rate is 10% and the cash flows for year 1, year 2 and year 3 are known and their probability values have also be computed. So at the end of year 1 you are likely to get 1500 or 3000 or 4000.

And the associated probabilities are 0.25, 0.5, and 0.25, on the other hand at the end of year 2 you are likely to get these 3 values and the associated problems are also shown here. So 1500 the likely probability is 0.5, 3000 the likely probability is 0.25 and 4000 the likely probability is 0.25 again. For year 3 if you look at the cash flow it is same here but that properties are changing corresponding to 1500 the properties 0.25 corresponding to 3000 the probabilities 0 25 again.

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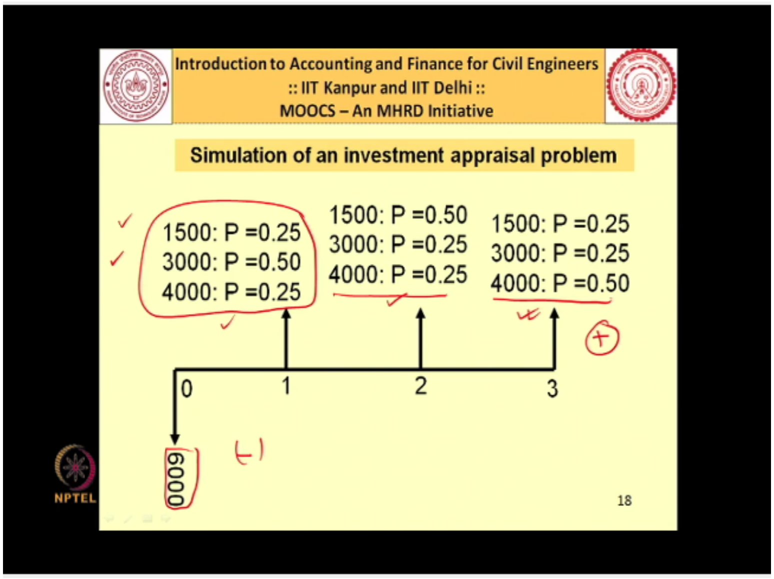

And corresponding to 4000 the probability is 0.5, I have converted this into a pictorial representation which is something like this. So this is the investment that you need to put at time t=0 and the cash flow for year 1 and their associated problems are here for year 2 it is here, for year 3 it is here. So these are all your plus value and these are your minus value. Now so far nothing new, now what we would like to do here is.

We will like to run the simulation for this particular problem, now depending on the probabilities associated with these cash flows we can generate cumulative probability table and thereby we can generate a random number interval for each of these cash flow. So what I will do is first I will generate the cumulative probability table, let us say for year 1 likewise I will do it for year 2, likewise I will do it for year 3.

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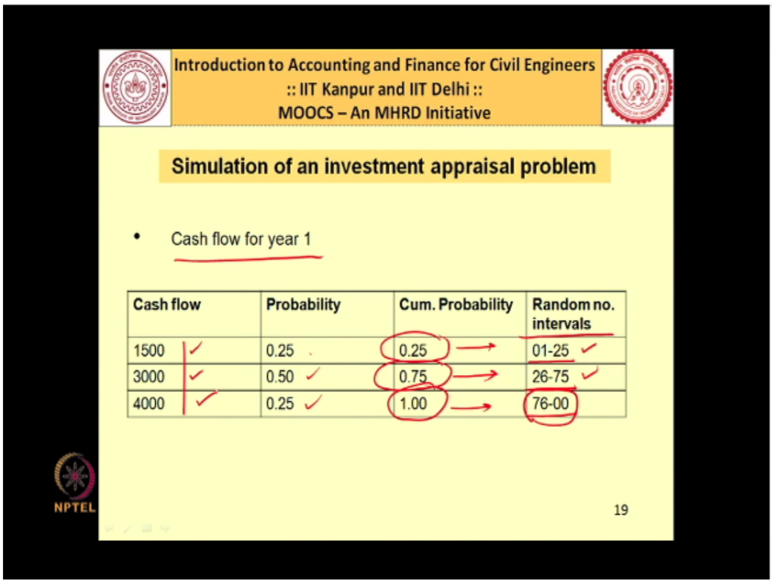

Let us do that, so cash flow for year 1 1500 probability 0.25, 3000 probability 0.5, 4000 probability 0.25. So cumulative probability is 0.25, 0.75 here and 1.0.Now you can very easily understand how to generate the random number intervals for 0.25 I will write 01 to 25, for 0.75 I will write 26 to 75 and for 1 I will write 76 to 00. So anytime I generate the random number and I find that it lies in this interval I will assume that the cash flow for that there is going to be 1500.

If the random number falls in this range I will assume the cash flow for that year to be 3000 and finally if my random number falls in this range I will assume the cash flow to be 4000.

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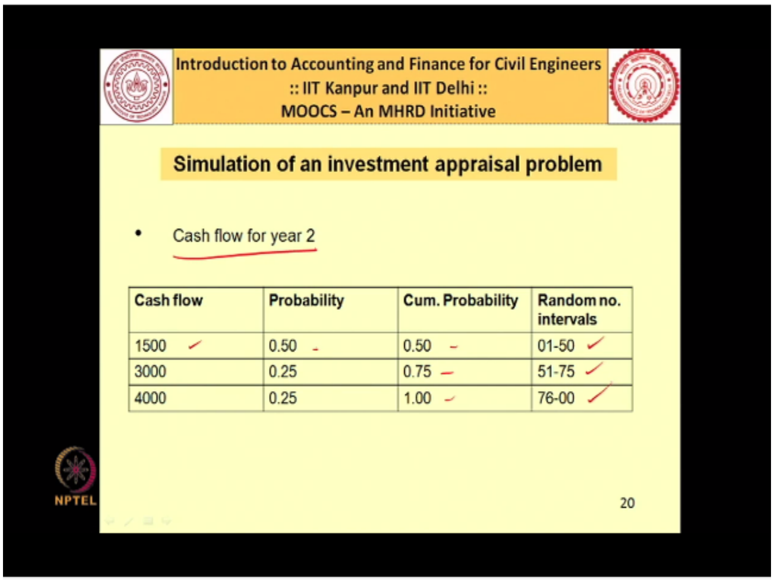

In the same manner I will do this analysis for year 2 cash flow 1500 probability 0.5, so cumulative probability 0.5, 0.75 here, 1.0 random number interval 01 to 50, 51 to 75 and 76 to 00.

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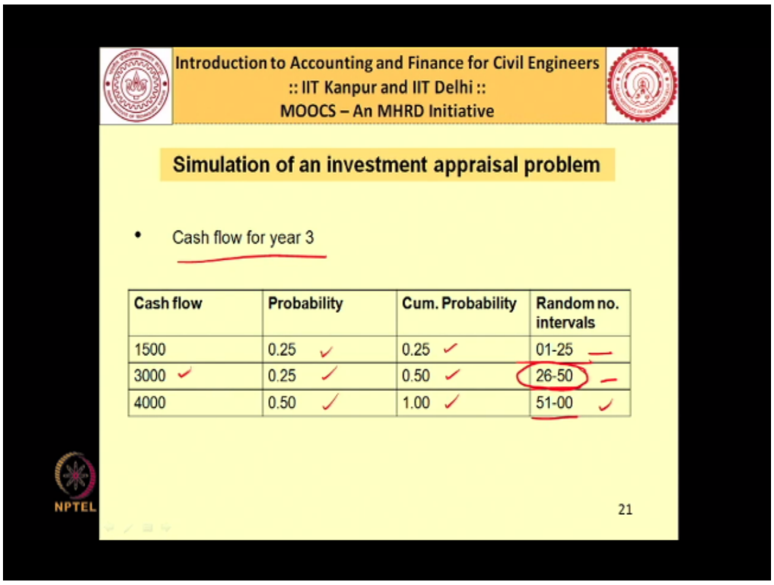

Cash flow	Probability	Cum. Probability	Random no. intervals
1500	0.25	0.25	01-25
3000	0.25	0.50	26-50
4000	0.50	1.00	51-00

And finally for year 3 cash flow the same calculations can be done here also, probability values are changing for each of these cash flow, for the 1500 cash flow probability is 0.25, 3000 0.25 and 4000 0.5, cumulative probability here is 0.25, 0.5 and 1.0. So that means I can write the random number interval as 1 to 25, 26 to 50 and 51 to 00. So that means for this year if my random number falls in let us say this range 26 to 50 I will assume the cash flow to be 3000.

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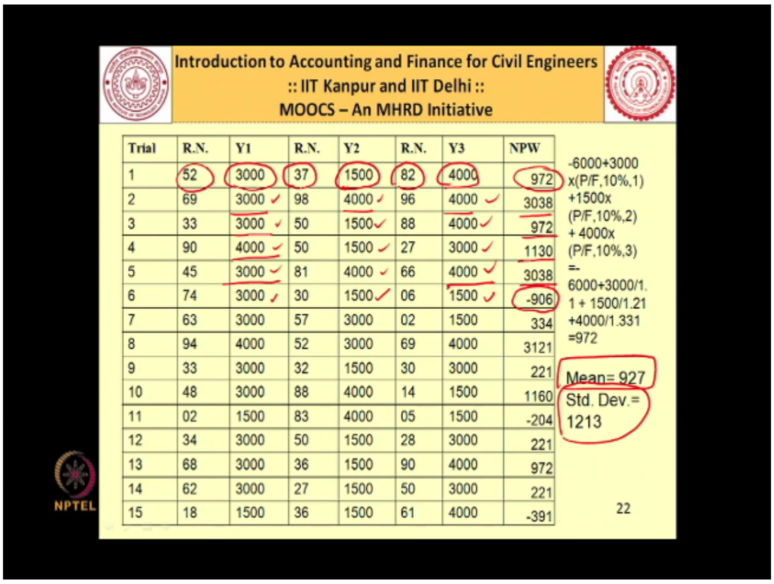

Trial	R.N.	Y1	R.N.	Y2	R.N.	Y3	NPW
1	52	3000	37	1500	82	4000	972
2	69	3000	98	4000	96	4000	3038
3	33	3000	50	1500	88	4000	972
4	90	4000	50	1500	27	3000	1130
5	45	3000	81	4000	66	4000	3038
6	74	3000	30	1500	06	1500	-906
7	63	3000	57	3000	02	1500	334
8	94	4000	52	3000	69	4000	3121
9	33	3000	32	1500	30	3000	221
10	48	3000	88	4000	14	1500	1160
11	02	1500	83	4000	05	1500	-204
12	34	3000	50	1500	28	3000	221
13	68	3000	36	1500	90	4000	972
14	62	3000	27	1500	50	3000	221
15	18	1500	36	1500	61	4000	-391

So now using these tables now I am able to find a combined table and let us try to do this analysis for 15 trials, so let us say in trial 1 I am getting the random number as 52, so I go back to the first table and 52 I find it is here. So I will assume that cash flow is going to be 3000 for that year. So I will write 3000 here, year 1 cash flow will be 3000. Now I generate second random number it is 37.

I look at the second table and see where is 37, so 37 lies here between 01 to 50. So I will assume that the cash flow for that year is going to be 1500. So I write this as 1500, now the next random number is 82, I will look at the third table which is here and 82 will be lying in this range. So I will assume my cash flow for third year is 4000. So you see here I am getting 4000 here. Likewise I goon generating random number.

And I go on writing the cash flow for year 1, year 2, year 3. So far here for trial 2 you can see these value to be 3000, 4000, 4000. For trial 3 3000, 1500, 4000. For trial 4 4000, 1500 and 3000. For trial 5 3000, 4000 and 4000. Now for each of these trails what I will do is I will calculate the net present worth. For example for trial 1 if you see the first trial we can draw a cash flow diagram corresponding to first trial.

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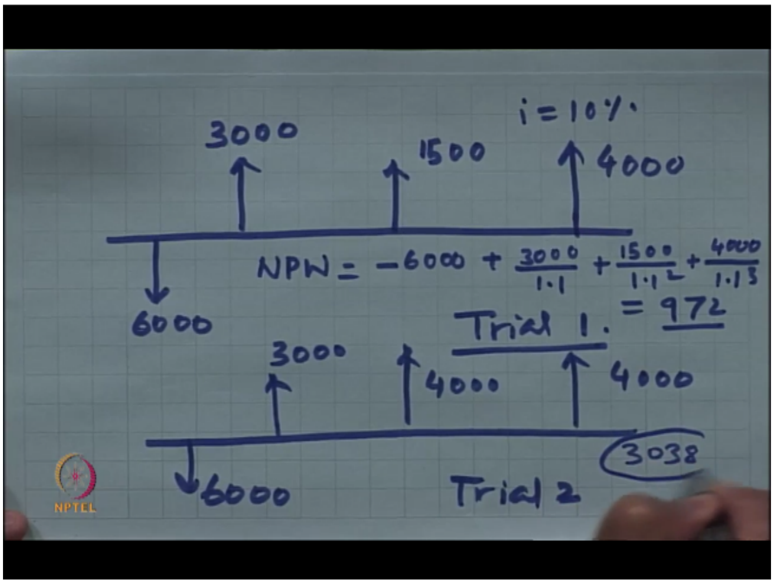

So first trial if you see we are getting a cash flow 6000 here and year 1 just now I found that it is going to be 3000, for year 2 it is going to be 1500 and for year 3 it is going to be 4000. So this is my trial 1, likewise if you remember for trial 2 I got this as 6000. There is no change here, this one I got 3000, this one 4000 and finally for year 3 also I am getting 4000. So likewise for each of these trials you can draw the cash flow diagram.

And you can calculate the net present worth. So for example net present worth for this trial is going to be -6000, let us assume interest rate is 10%, so +3000/1.1+1500/1.1 raise to power 2+4000/1.1 raise to power 3. So you will find this is coming to be about over 972. So likewise you can calculate the NPW for each of these trails. For example for this trial 2 you are getting 3038.

So these calculations I have performed in this computer's slide so for example for first trial you find this is 972. For the second trial it is 30838, for the third it is 972, fourth 1130, 3038. On 1 occasions it is negative also, you can see it is -906, so this is corresponding to a cash flow of 3000 at year 1, 1500 in year 2 and 1500 in year 3. So you go on doing this and you can calculate the mean of all these net present worth.

Of course for 15 trails if you do the mean value is coming to be 927 and standard deviation if you calculate using any one of your scientific calculator you will be getting about 1213. Now you can compare with the expected value of mean of net present worth.

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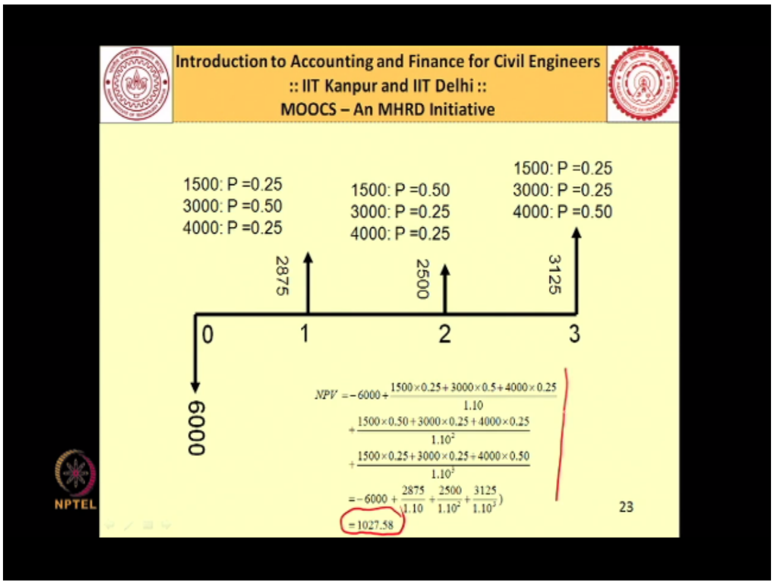

And that you will remember it was coming to be 1027, so the difference is coming here because the number of trials that we are doing is only 50 whereas for finding the expected value you need to carry out this analysis for large number of times. So the reason why I told you this example is to make you understand the implication of these cash flows being random in nature.

In the beginning we assume all of them to be constant, all of them to be certain. So whatever we assume we said it is likely to be occurring with 100% certainty. But in real life situation it does not happen, so equipment any equipment that you buy you might have resumed it to last for 10 years, but it may last only for 9 years or 8 years. So from your past record you can see which equipment was lasting for how many years, so those kind of records you can generate, you can find out the probability corresponding to each of the life.

And then you can also include those variables in your simulation. So the more such that are you collect the better your analysis would be towards the real life situation. So we in this particular lecture learnt 3 problems, 1 the application of simulation in inventory analysis, second one in queuing problems and the third one in this case of net present worth computation. So I hope now you have got fair enough idea of carrying out simulation in our day to day examples.

And I hope you enjoyed this particular topic, next time when we come or we discuss 1 very important topic that of bidding, any civil engineering field you find that bidding is a very very important topic. So we will see what is the probability of a contractor winning a bid at a given profit margin. If he or she increases the profit margin what is the probability that he will win or you will lose the bid. So these are very important questions in the context of civil engineering application. So that is what we are planning to take up in the next lecture, so till then thank you very much and goodbye.