Introduction to Accounting and Finance for Civil Engineers
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Module No. #04
Lecture No. #18
Risk Analysis (Part-2)

Good morning, Namaskar, and welcome to the course, once again. In the last lecture, if you remember, we discussed, Risk Analysis, introductory part. We introduced you to four terms, primarily, the additive probability, the joint probability, the expected values, and measures of variation. Under, measures of variation, we studied variance, we studied standard deviation, and we studied, coefficient of variation. In this lecture, we will continue our discussion on, Risk Analysis.

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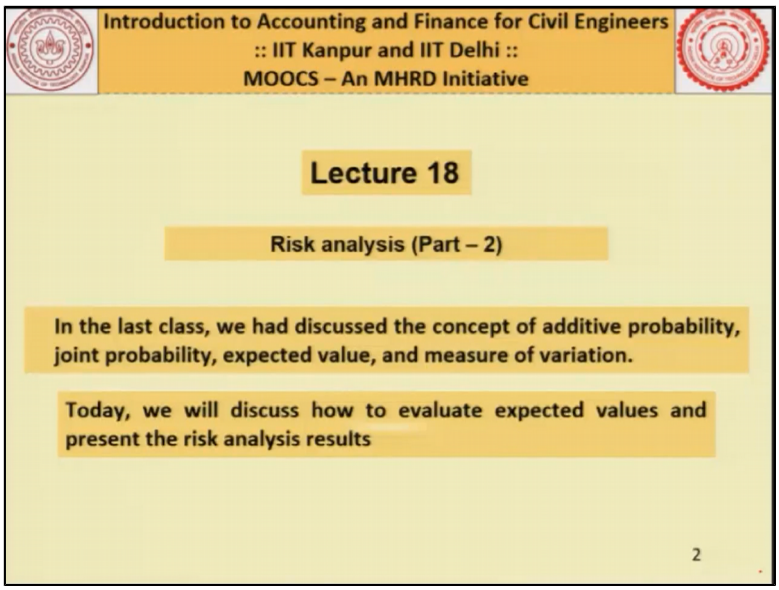

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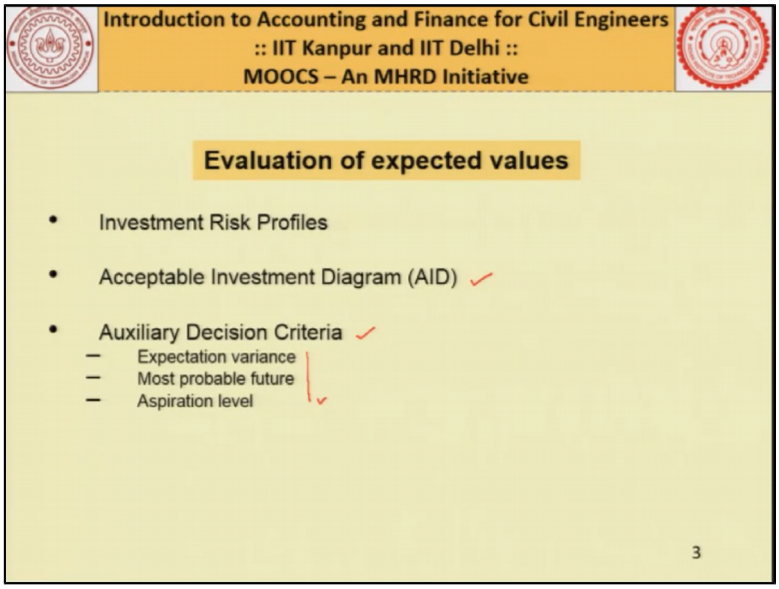

We will discuss, Investment Risk Profiles, Acceptable Investment Diagram, Auxiliary Decision Criteria. Under this, we will see, Expectation Variance, Most Probable Future, and Aspiration Level. As before, all these concepts, we are going to learn, with the help of one small example.

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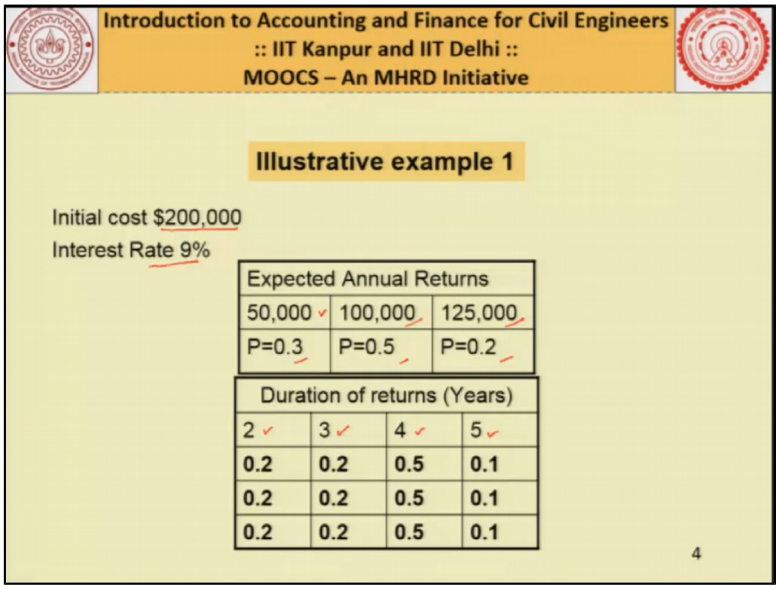

So, let us straight away, move to the example. In this problem, we are given that, the initial cost is 200,000 Dollars, the interest rate is 9%. The investment of this magnitude, is generating a return of, 50,000. Expected return is 50,000, with the probability of 0.3. 100,000, with the probability of 0.5. 125,000, with the probability of 0.2.

The corresponding duration of returns are, 2 years, 3 years, 4 years, or it could be 5 years. So, you can see here, there are changes in expected annual returns, and there are changes in the

duration of returns, itself. Now, this problem, I can represent it, in the form of a Tree Diagram also. And, it would be, something like this, as shown in this particular slide.

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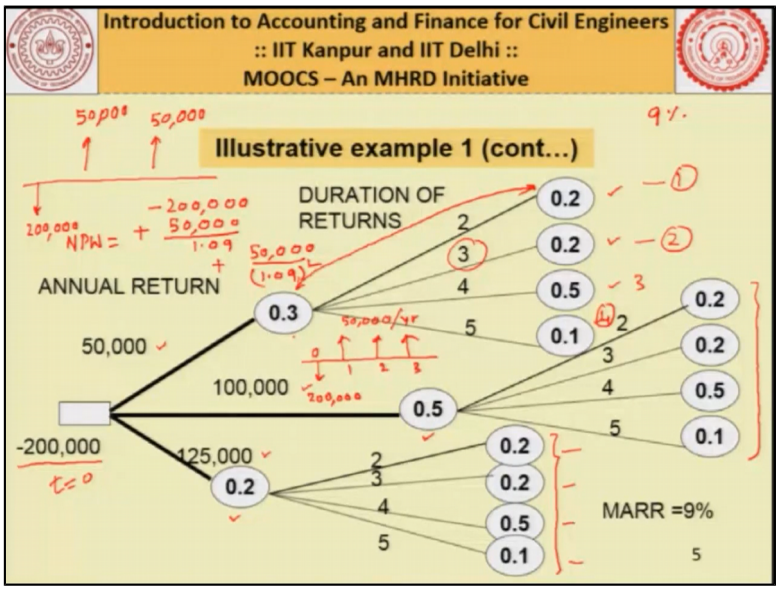

So, you can see, you are investing 200,000 Dollars here, at time t is equal to 0. This is likely to give me a return of, either 50,000, or 100,000, or 125,000. The associated probabilities are, 0.3, 0.5, and 0.2. Now, the duration of returns could be, 2 years with the probability value of 0.2, 3 years, again with the probability of 0.2, 4 years with the probability of 0.5, and 5 years with the probability of 0.1.

Likewise, for annual return of 100,000, we are given, the duration of returns, and their corresponding probabilities. And, at the end, you can see, for an annual return of 125,000, the four return periods, and their associated probabilities, are also mentioned, in this particular slide. Now, the question is, how do we present the analysis results. Now, as I told you, we are right now discussing about, Investment Risk Profile. So, for that, the first step is to, calculate the joint probability, along each of these paths.

If you look at this slide, you will find that, altogether, there are 12 paths. Path number 1 here, 2 here, 3, 4. Likewise, here 5, 6, 7, 8. And, here 9, 10, 11 and 12. So, in total, there are 12 paths. For each of the paths, we can generate the joint probability. For example, for Path-1, it is going to be the multiplication of, 0.3 and 0.2. It is going to be, 0.06. For Path-2, it is again going to be, 0.06. For Path-3, it is going to be, 0.15. And, for Path-4, it is going to be, 0.03. So, this information, I have arranged it, in the form of a table.

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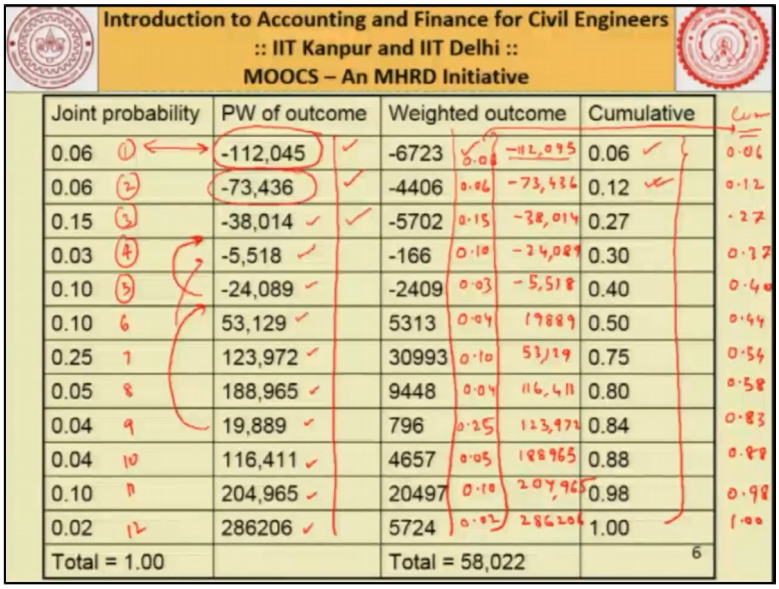

And, your joint probabilities, for each of these paths, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 paths, are calculated like this. Now, for each of these paths also, we have to calculate, the Net Present Worth. So, I will just illustrate you, for one case, let us say, for Path-1. So, for Path-1, if you draw the Cash-flow Diagram separately, it would be like this. You are investing 200,000 Dollars here, at time t is equal to 0. And, you are getting a return of 50,000, every year, for the next 2 years.

So, this is the Cash-flow Diagram, corresponding to Path-1. Now, you know, the rate of return is 9%, for this particular example. So, you can calculate, the Net Present Worth, NPW is, - 200,000 + 50,000, divided by 1.09, + again 50,000, divided by 1.09 raised to power 2. So, if you do this, you will find, you are getting a value of, - 112,045. Now, in the same manner, you can calculate the present worth, corresponding to Path-2.

Now, if you see the Path-2, you will find, the return is now, for 3 years. So, my Cash-flow Diagram would be, something like this. You have, 200,000 here. And then, every year 50,000, for next 3 years. 50,000 per year, for next 3 years. So, you can see, this is 200,000 here. So, this is the Cash-flow Diagram, corresponding to path number 2. So, if you calculate the Net Present Worth, at 9% interest rate, you will find, you are getting a value of, - 73,436.

Likewise, for Path-3, it is - 38,000 something. For Path-4, it is - 5,000 something. For Path-5, it is - 24,000 something. For Path-6 onwards, you find that, Net Present Worth is coming to be, in positive. Now, so far, nothing new. We have already done, similar exercises, in our previous lectures. Even, the generation of this column, is nothing new. What I have done is, I

have multiplied, these two columns. So, joint probability, if you multiply with present worth, you are getting the weighted present worth, like this.

So, so far nothing new. Now, what we are going to do additionally, in this particular problem, in order to draw the Investment Risk Profile, is like this. We are going to draw, the last table, or rather, last column. And, in this column, we are going to write, the cumulative probability. So, for Path-1, it is 0.06. For Path-2, it is 0.06. So, when I add them up, it is 0.12. Now, one more thing that we have to do here is, we have to have, one more step, additional here. That is, to increase, that means, to organise this particular column, in ascending order.

So, the least present worth, I will be writing it, in the beginning. Then, it will be followed by, the next one, then the next one. So, this column, will come here, rather this row, will go there. So, immediately after - 38,000, you will have - 24,089, and then you will write - 5,518. So, this is how, you have to make changes. And then, the next value will come, like this, 19,000 value will come, immediately after - 5,000.

So, I will just write it here. Let us say, we are adding, one more column here. So, this is - 112,045, this is - 73,436, this is - 38,014. Then, I write the next one, - 24,089. And then, my next one comes, - 5,518. So, you can see, what I am doing. I am arranging them, in ascending order. Then, the next higher one is, 19,889. Then, the next one is, 53,129.

Then, the next one is, 116,411. Then, the next higher is, 123,972. Then, we have 188,965. Then, we have 204,965. And, finally, we have 286,206. Now, I also write the exact joint probability, alongside, these weighted outcomes, or rather, these present worth. So here, it is no change, 0.06. Here also, there is no change, 0.06. Here, there is no change, 0.15. Then, I write, 0.10 here.

Then, the next one is corresponding to - 5,000, so it is 0.03. Corresponding to 19,889, it is 0.04. Then, corresponding to 53,000, it is 0.10. Then, you have corresponding to 116,000, this is 0.04. Then, for 123,000, this is 0.25. For 188, this is 0.05. And, for 204,965, you have 0.10.

And, then finally for this, it is 0.02. Now, what I do is, I take the cumulative value for this column. So, 0.06, and I write it here. Cumulative probability, I write it here. So, this is 0.06. Then, it becomes 0.12. Then, it becomes 0.27. Then, it becomes 0.37. This becomes, 0.40.

And then, this becomes 0.44, 0.54, 0.58. Then, you have 3 here, 0.83, 0.88, 0.98. And, finally 1.00.

So, what additionally, you have to do, in addition to the columns, which are shown in black, is to generate another set of column, in which, you are arranging the present worth, in ascending order. So, the least one, I am writing it, in the beginning, and then subsequently, I am writing the other higher values. Now, corresponding to each of these present worth, I am also writing the joint probability. And, for joint probability values, I am taking the cumulative value. So, the last column is, cumulative value.

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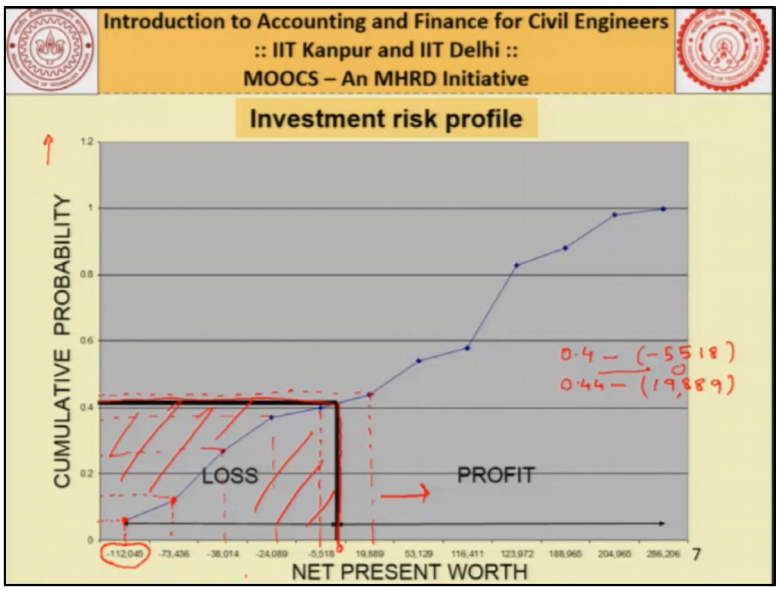

Now, what I do henceforth is, to draw a graph, between the Net Present Worth, and the cumulative probability. So, on this axis, on Y axis, we are showing the cumulative probability. And, on this axis, I am showing the Net Present Worth. So, if you remember, the first value is - 112,045, and the probability is 0.06. So, this is the point, 112,045 in negative, and the probability is very low, 0.06. So, this is the point, I have obtained.

Likewise, for the other point, this is the cumulative value, and this is the Net Present Worth, - 73,436. Likewise, for this point, the cumulative value is here. For this point, the cumulative value is here. Now, if you go back, we are getting 0.4 probability, corresponding to - 5,518, and 0.44 probability, corresponding to 19,889. So, you can see here, 19,889, we are getting this value as 0.44. So, this is 0.44. And, this one, a negative value, - 5,518, I am getting it as 0.4.

So, you can see that, zero will be somewhere between, these two values. So, Net Present Worth of zero, will be somewhere between, 0.4 and 0.44. You can, either exactly plot it on the graph, or you can do the interpolation. So, I will write it here, corresponding to 0.4 probability, we are getting a Net Present Worth of - 5,518. And, corresponding to 0.44 cumulative probability, we are getting a value of 19,889. So, you know, how to interpolate between these two, to find, where will zero lie.

Now, this is how, you draw the Investment Risk Profile. So, what is happening here. Once you draw this line, thicker line, if you see here, thicker line at this point, this is representing, what is the probability of making losses. We are measuring losses, for those values, which are coming below zero. So, you can see, all this area, we are calling them as, a loss area. And, area this side, we are calling them as, profit area.

So, by drawing this particular diagram, you can find that, it is very easy for even a lay man, to appreciate, what is the potential of making losses, in this particular proposal. So, what we were given? We were told that; 2-lakh is your initial investment. And then, we were given, different values of return, and for different periods of time. We were given, the associated probabilities, for each of these paths also. What we did? We first calculated, the joint probability.

We calculated the Net Present Worth, for each of these paths. Then, in the next step, I arrange the Net Present Worth, in ascending order. And, just adjacent to these Net Present Worth, I also wrote down, the probability values. One more column, I introduced, in the name of cumulative probability. Now, what I did? I draw a diagram between, cumulative probability value, and the Net Present Worth. And, in that particular diagram, we are showing, what is the probability of making losses.

That means, in how many cases, out of say 100, there are chances, that I will make money in negative. That means, I will go below zero. So, this is what, if you look at this particular diagram, this is how, you draw the Investment Risk Profile. So, even a lay man, looking at this chart, will say, okay, this is a slightly riskier proposition, because, there is about 40-44%, 40-42% chance of making losses. So, this is one aspect, that we had to discuss.

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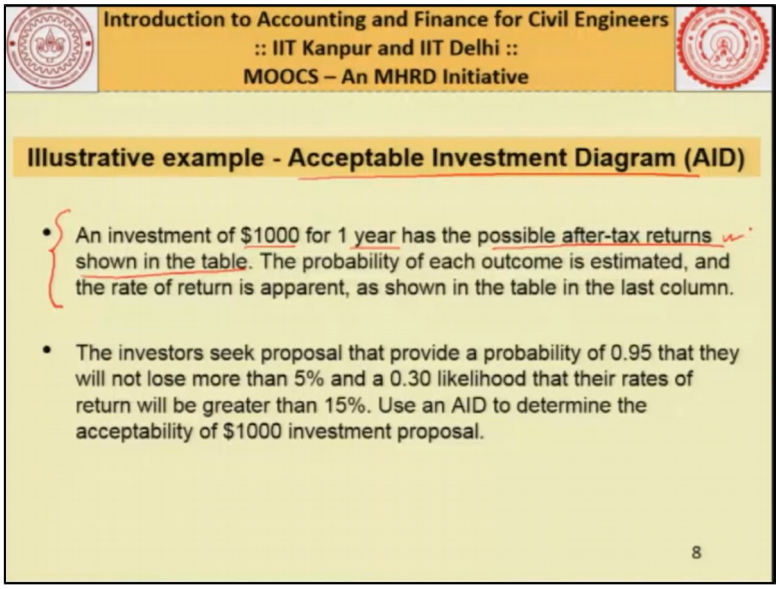

Now, we move into another diagram, what we call it as, Acceptable Investment Diagram. Now, this is one step beyond, what we have drawn, just now. So, what is happening, on the Investment Risk Profile, we are also introducing, one more line, which will capture the aspiration level of the investor. Now, this will be more clear, when we take up, again a different example, but of course a very small example. And, in this example, if you see, it is told to us that, an investment of 1,000 Dollars, for 1 year, has the possible after tax returns, shown in the table. I will show you the table.

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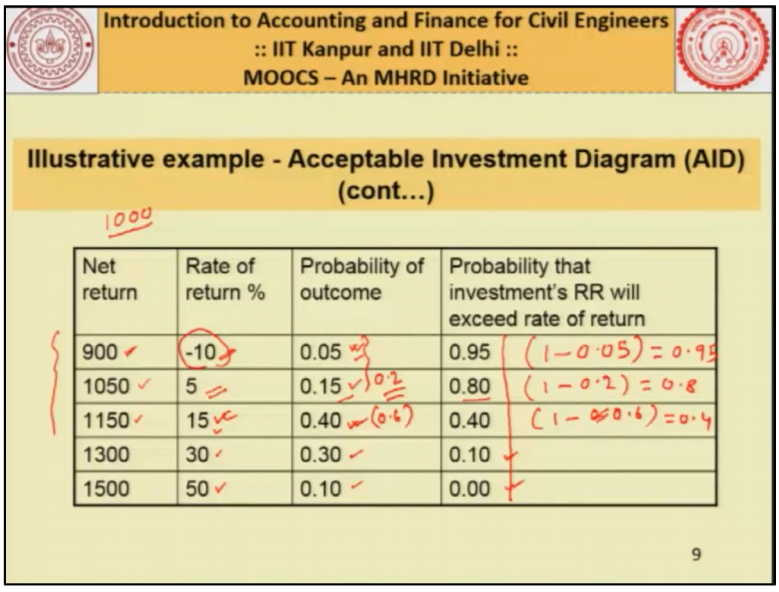

So, we are investing 1,000 Dollars. And, the return corresponding to this, remember, we are doing Post-Tax Analysis. So, it is given to us that, these are after tax returns. So, I will just straight away, show you the table. So, we can see, for an investment of 1,000, there are

chances that, rate of return could be, - 10%. That means, the net return, that you are likely to get is, 900. The probability corresponding to this is, 0.05.

That means, there is a 5% chance that, you will make 10% loss. So, your return on 1,000, is going to be 900. In 15% probability, you find, there is a net return of 1,050. 40% probability associated with, getting a net return of 1,150. 30% probability that, you will get a return of 30%. And, 10% probability that, you will get a return of 50%. So, this is, as far as, the data is concerned.

Now, it is also given to us, remember, I told you that, in order to draw, Acceptable Investment Diagram, we have to proceed forward, from the plot of Investment Risk Profile. So, this data is given to us, for drawing the Investment Risk Profile. Now, some more data is required, to draw this particular diagram called, Acceptable Investment Diagram. And, for drawing that, the information given is, the investors seek proposal that, provide a probability of 0.95, that they will not lose more than 5%.

So, investor is wanting that, they should not lose by more than 5%. And, for this, they want a guarantee probability of 0.95. Other aspiration for the investor is, a 0.30 likelihood, that the rates of return, will be greater than 15%. So, two things, the investor is aspiring for. The first one is, a probability of 0.95, that they will not lose more than 5%. And, the probability of 0.3 that, the rates of return will be greater than 15%.

Now, the question is, use an Acceptable Investment Diagram, to determine the acceptability of 1,000 Dollar investment proposal. So, let us see, how to do this. As I told you, we are given this information, to draw the Investment Risk Profile. Now, in order to draw the Investment Risk Profile, you have to find out the cumulative probability values. So, 0.05 is corresponding to rate of return - 10%. So, the probability that, this investments rate of return will exceed this – 10, will be how much?

It will be, 1 - 0.05, which is 0.95. You understand this. What it says, there is a probability of 5%, that you will make a rate of return - 10%. So, what is the probability that, you will make a return in excess of - 10%, will be given by, 1 - this value, 0.5. So, which is 0.95. Now, for the next case, rate of return is 5%, and probability here is, 0.15. So, 0.15 + 0.05 becomes, 0.2.

So, the probability that, we will make a return in excess of 5% will be given by, 1 - 0.2, which is 0.8.

Now, for 15%, the probability of outcome is, 0.4. So, if you add, this becomes 0.4 + 0.2, which is 0.6. So, this probability, that you will make a return in excess of 15% will be given by, 1 - 0.6, which is 0.4. Likewise, for this, it will be 0.1, and for this, it is 0. Right. Now, using these values of cumulative probability, and net return, I can, or for that matter, rate of return, I can draw a graph like this.

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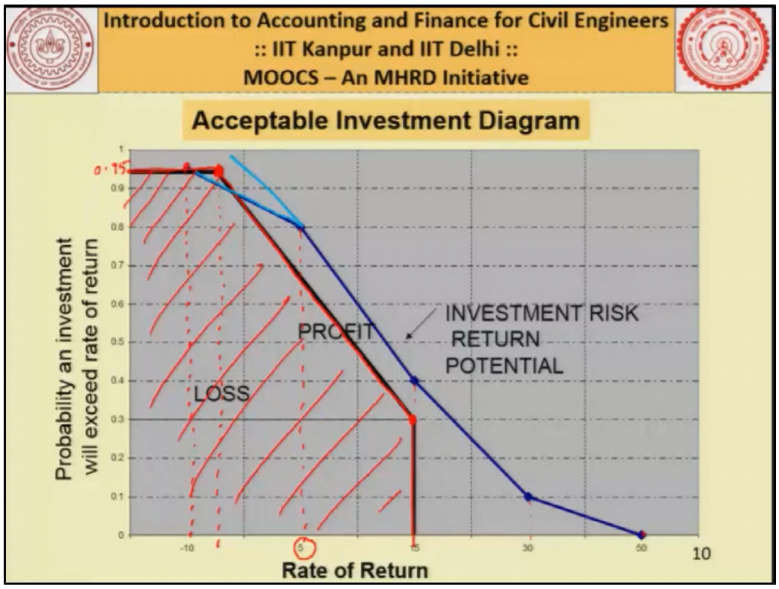

So, you can see, this blue line. So, here it is. This is the first point, that I am drawing. Look at the, blue one. Some corresponding to - 10, what is my probability? Corresponding to - 10, my probability is 0.95. So, you can see here, this point is 0.95. So, for - 10% rate of return rather, my probability is 0.95. Now, look at this point, corresponding to 5% probability, rather 5% rate of return, my probability is 0.8.

Corresponding to 15% rate of return, my probability is roughly 0.4. Corresponding to 30%, my probability is roughly 0.1. And, corresponding to 50% rate of return, my probability is 0. So, I connected all these points, and I am getting a line like this. Now, so far, nothing new. We have already done, this type of exercise, for the previous problem. Now, what is additional is that, investors aspiration. And, investors aspiration is captured by two statements, that is given to us.

If you go back to that statement, it says, the investor wants a guarantee, that he will not lose more than 5%. And, for this, the probability is 0.95. So, I have two points here, two coordinates, corresponding to 0.95. And, if you go back, I have a value 5%. So, this is the line, I can get. Because, this is the point, corresponding to 5% return, here somewhere, he does not want to lose more than 5, and you have a probability here, 0.95.

Another point is, 15% rate of return, probability is 30%. So, these two points, we already have got here. Now, what I do? I join this, with this here. And, this, with this. And then, of course, I connect these two points, like this. So, this area represents, the aspiration level of the investor. Now, in order to see, whether the investor should go in for this option or not, what we have to find is, as long as, this particular blue line, is beyond this area, remember, this is representing the aspiration of investor.

Investor does not want, his return, to fall in this area. So, anything falling in this area, investor is not at all comfortable. Now, in this particular problem, you find that, this line is falling, in this. You can see here. So, what you find here is that, this particular line, is coming here. It is going inside this, which the investor does not want. Had this line, gone out like this, this investment option would have been acceptable. But, just because, this investment plan is not satisfying, the aspiration level of this particular investor, we will say that, this investment is not desirable.

So, what you have seen, in the case of Acceptable Investment Diagram is, I step further beyond drawing, Investment Risk Profile. So, what we have done? In the Investment Risk Profile, we have superimposed, the aspiration levels of the investor. In this example, we were given two aspirations. Corresponding to that, we got two points. And then, from these two points, we could draw a reason. Now, this reason, as long as, my investment is not falling, I am okay.

But, even if there is a slight infringement, in this particular zone, by my investment risk line, I am not comfortable. So, I will not pursue this option. So, this is how, drawing the Acceptable Investment Diagram, you can see, whether a particular investment is desirable or not. Now, in addition to these two situations, we also have to learn, few more thing under, Auxiliary Decision Criteria.

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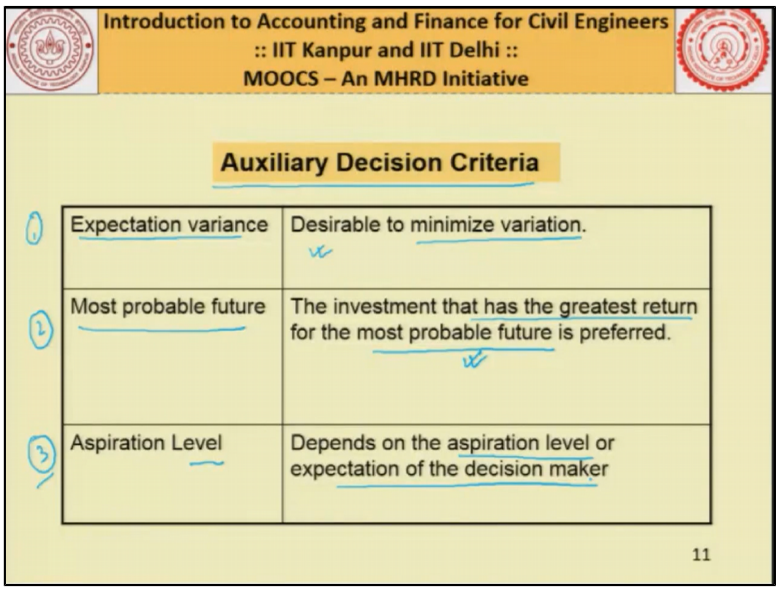

Now, under this, we are going to learn three things. First one, we call this as, expectation variance. Second one, we call this as, most probable future. And, third one is, aspiration level. So, under expectation variance, we try to minimise the variation. So, in this case, it is desirable to minimise the variation. In most probable future, what we do? The investment that has the greatest return, for the most probable future, we will see, what does this most probable future mean, with the help of one small example.

So, under this criteria, the investment that has the greatest return, for the most probable future, is preferred. And then, of course the third one, is again, aspiration level. Now, as the name suggests, it depends on the aspiration level, or expectation of the decision maker. So, all three again, we would like to understand, with the help of one small example.

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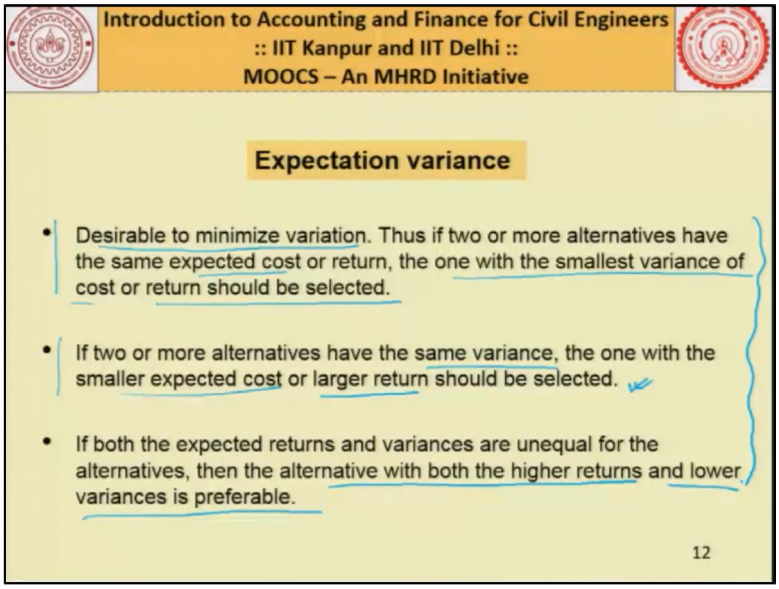

Now, this is what I told you, in the case of expectation variance, it is desired to minimise the variation. So, if two or more alternatives, have the same expected cost or return, obviously the one with the smallest variance of cost or return, should be selected. So, we are trying to minimise, the variance here. On the other hand, if two or more alternatives have the same variance, the one with the smaller expected cost or larger returns, should be selected.

This is quite obvious. On the other hand, if the expected returns and variances are unequal, for the alternatives, then the alternative with both the higher returns, and lower variances, is preferable. Now, all these three statements, will be very clear to you, if we do this small example.

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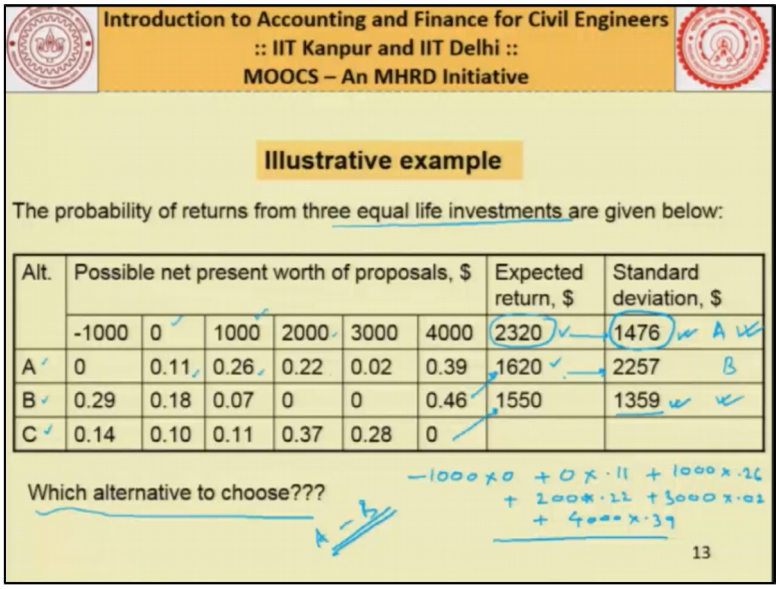

Now, in this example, we are given the probability of returns, from three equal life investments. Here, the life of all the three investments are equal. Now, let us say, what are the three alternatives. Alternative-A, B and C. In Alternative-A, you are investing 1,000 Dollar now, and you are given the Net Present Worth, and the corresponding probabilities. So, for example, you are investing 1,000, the probability that you will get 0, is 0.11.

The probability, that you will get a Net Present Worth of 1,000, is 0.26. Probability, that you will get a Net Present Worth of 2,000, is 0.22. 3,000, 0.02. And, 4,000, 0.39. Right. So, these are the values of return, given for Alternative-A. Likewise, for B, you can see, corresponding to 0 it is 0.18, corresponding to 0.07 it is 1,000, and corresponding to 2,000 it is 0. We illustrate you the concept of Auxiliary Decision Criteria, especially the expectation variance, with the help of following example.

In this example, the probability of returns, from three equal life investments are given. In Alternative-A, the possible Net Present Worth of proposal being - 1,000, the probability is 0. 0, the probability is 0.1. 1,000, probability is 0.26. 2,000, probability is 0.22. 3,000, probability is 0.02. And, 4,000, probability is 0.39. Likewise, for B and C, these values are given. Now, the question is, which alternative to choose. So, out of the three alternatives, which one you will choose.

So, the first step here is, to calculate the expected return. So, expected return for Option-A, how do I calculate? It will be, - 1,000 into the probability, + 0 into 0.11, + 1,000 in to 0.26, + 2,000 into 0.22, + 3,000 in to 0.02, + 4,000 into 0.39. So, if you do like this, you will

calculate it to be, 2,320. Now, in earlier session, I had also given you a formula, to calculate the standard deviation. Using that formula, you can calculate, the standard deviation of this option.

This is coming to be, 1,476. Likewise, you can calculate, the expected return for Option-B Here, it is coming to be, 1,620. And, its variance is, or rather standard deviation is, 2,257. For C, the expected value is 1,550, and the variance rather the standard deviation is 1,359. Now, the question is, which one to choose.

So, if you look at, the Option-A and B, can you guess, which one you will prefer. So, here you can see, the expected return is also higher, compared to B, and standard deviation is also lower. So, between A and B, the choice is very simple. You will go for, A. But, what about A and C. Out of A and C, which one will you choose. So, here you find, expected return for A is more, but standard deviation is higher than, the Option-C.

So, here, you will have to make a trade-off, and you will find that, this method fails. You will not be able to choose between, A and C, using this particular method. So, it is quite possible, that sometimes, some methods, fail to deliver your answer. So, does not matter, you will have to switch to, some other method. So, we will see, between A and C, how to choose, the appropriate method. And, for that, we will go for, some other methods.

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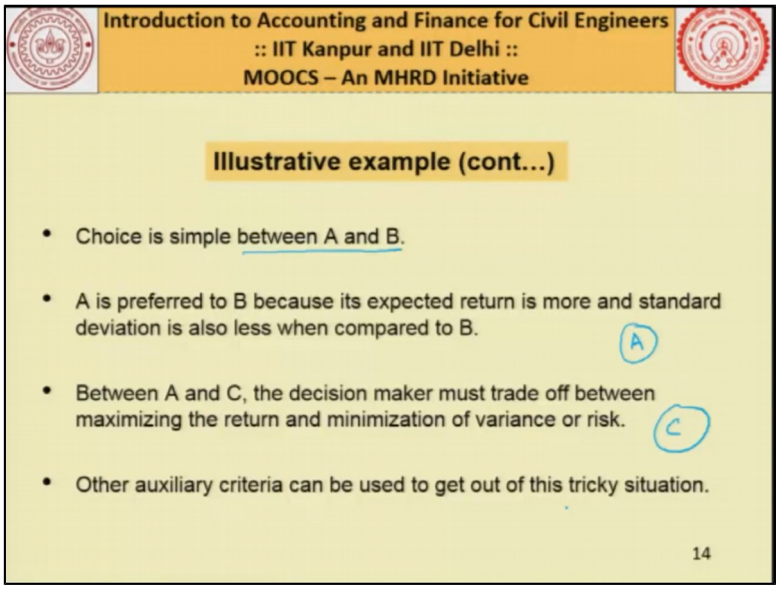

So, here you can see, the choice is simple between A and B. I have chosen A, for obvious reason. Because, there you could see, the expected return is more, and the deviation also is

less. But, the choice between A and C is, difficult to make. And, we will see, some other method.

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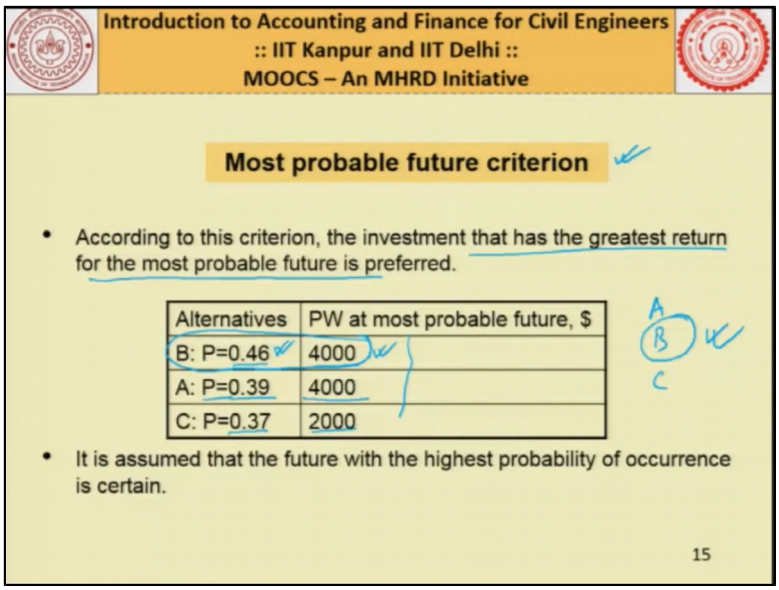

And, one of the methods, which we will be using is, most probable future criterion. Now, in this criterion, the investment that has the greatest return, for the most probable future, is preferred. Now, I have to tell you, what is meant by, most probable future. So, you go back to the table. So here, for Option-A, you find it out, what is the maximum probability, and what is the return, corresponding to that. So, you find that, for A, the maximum probability is 0.39.

So, this is what is, we are calling, maximum probable future. So, you can see here, I am writing for A, it is corresponding to probability 0.39. We are saying that, this is the most probable future. And, the return corresponding to that is, 4,000. On the other hand, for B, if you see, here again, it is 0.46, corresponding to 4,000. And, for C, it is corresponding to 0.37, return of 2,000. So, this is what, I am writing it here.

For C, corresponding to 0.37, it is 2,000. For B, it is 0.46, and present worth here is 4,000. So, the choice is simple, you will go with Option-B. Because, this is giving you the maximum return, and it is at the most probable future. So naturally, whatever is most probable future, you choose the greatest return, corresponding to that. So, out of A, B and C, using this criterion, we will choose B. Now, we will see, some other method.

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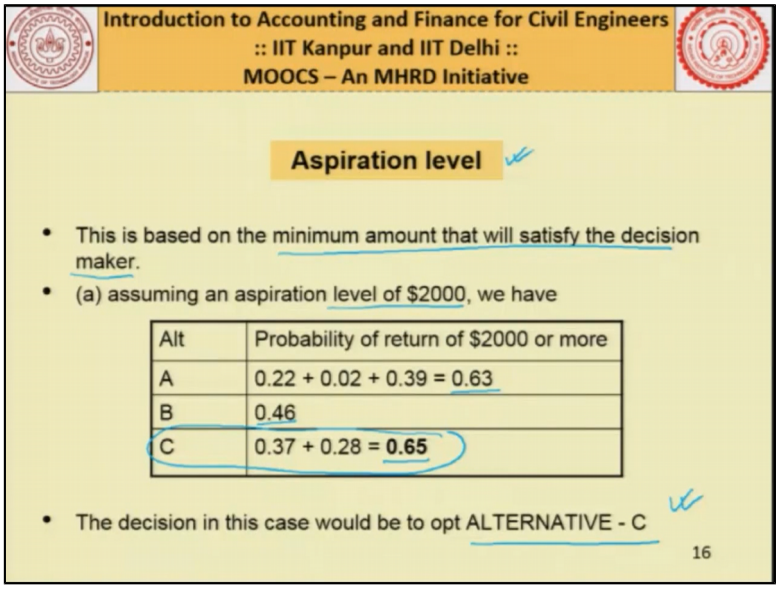

And, in that, we are understanding, aspiration level. Now, this is based on, the minimum amount, that will satisfy the decision maker. So, as it is subjective in nature, it will depend, what aspiration level, has been formulated by the decision maker. Now, aspiration levels could be different. I could be happy, even if I do not make any losses. So, even if slight money I am making, I am ok. Somebody maybe thinking, no, I have to make at least 4,000, or at least 5,000.

So accordingly, he or she, will take the decision. So, let us say, the aspiration level is to earn 2,000 Dollars. So, I go back to the previous table. First, I erase all of them, so that, it becomes more clear. So, you can see, the aspiration is to earn, at least 2,000. So, let us see here, for Option-A. So, 2,000, corresponding probability here is, 0.22. For 3,000, it is 0.02. And, for 4,000, 0.39.

So, all these are, greater than, 2,000 or more. So, the corresponding probability will become, 0.22 + 0.02 + 0.39. So, this is 0.63. So, the probability, that you will make a return of more than 2,000 Dollars is, 0.63, for Option-A. When it comes to Option-B, it is 0 + 0 + 0.46. So, for Option-B, it is 0.46. For Option-C, it is 7 + 8, 15, and this is 6, so, 0.65.

So, this is what, it is written in this particular slide. You can see, 0.65 for C, 0.46 for B, and 0.63 for A. So, which one now you would choose. Obviously, C, because. This is satisfying the investors aspiration. So, I will choose alternative C, using this particular criterion. Now, I change the aspiration level, from 2,000 Dollars, I want to make some money.

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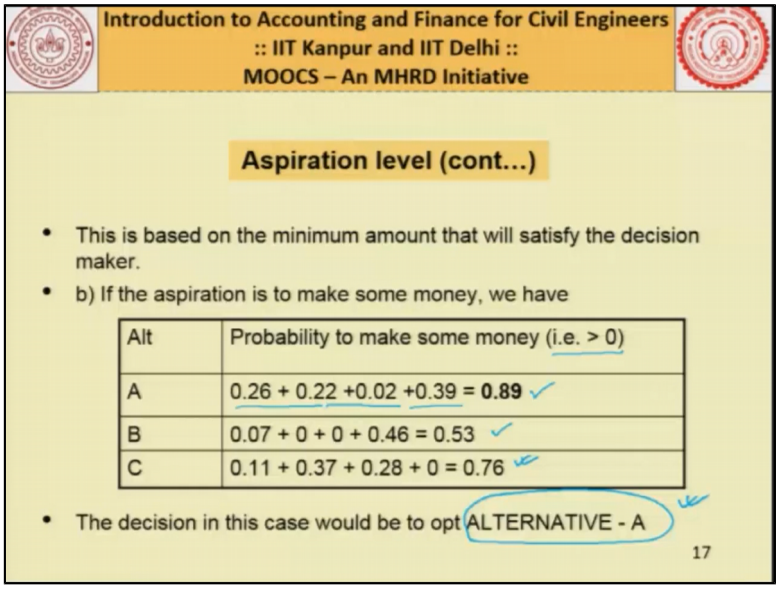

So, that means, I must get more than zero. So, that is what, here I have written, greater than zero. So, for greater than 0, if you go back to the previous slide, it is more than 0, It is, let us say, 0.26 + 0.22 + 0.02. So, all these here, here onwards. So I have just added up here, in this table. So, it is 0.26 + 0.22 + 0.02 + 0.39, so 0.89. For B, same way, it is coming to be 0.53. For C, it is coming to be 0.76.

So, naturally using this criteria, if you want to make some money, you will prefer A. So, what you find, depending on your aspiration level, your decisions are changing. In some cases, it is A. In some case, it is B. In some case, it was C. You may also remember that, sometimes, it may not be possible, to evaluate and answer, using a particular method. In that case, you have to switch to some other method. So, what we have learnt in this particular lecture is, how to draw Investment Risk Profile, how to draw Acceptable Investment Diagram.

And, we have also studied, the three auxiliary investment criteria. The basic ones are the, expectation, variance, most probable future, and aspiration level. As I told you, depending on your aspiration level, your decisions may change. Sometimes, it may not be even possible, to find an answer, using a particular method. That is a signal, to switch to some other method. So, that is it, for this particular lecture. In the next lecture, we will see, some more information on Risk Analysis. But, till then, thank you and goodbye.