Introduction to Accounting and Finance for Civil Engineers

Professor. Sudhir Misra
Department of Civil Engineering, IIT Kanpur
Professor. Kumar Neeraj Jha
Department of Civil Engineering, IIT Delhi

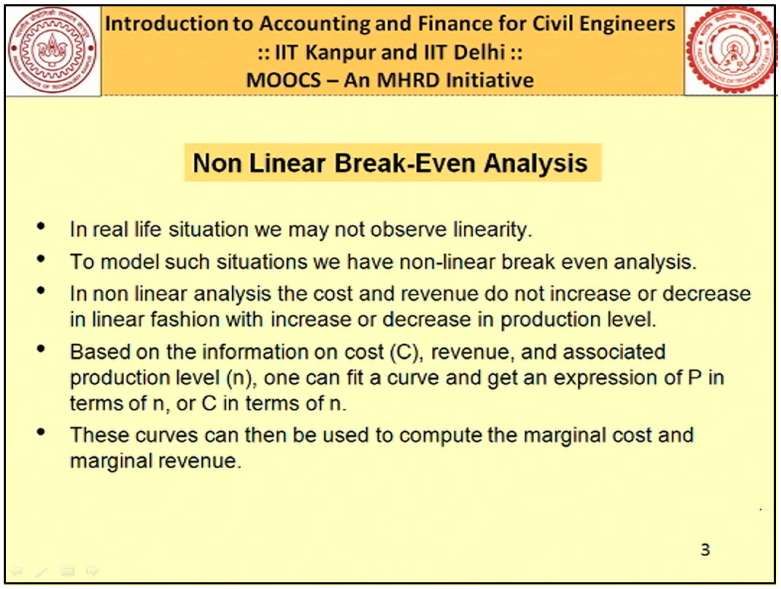
Module No. #03
Lecture No. #12
Break-Even Analysis (Part – 2)

Good morning, Namaskar, and Welcome to the course once again. In the last lecture, we discussed, Break-Even Analysis for Linear problems. When I say, Linear, it was essentially meant to be like this. Whatever be the quantity, that we are producing, or we are selling, it was independent of the prices. That means, if you are selling the quantity X, at P price, we will sell that quantity 2X also, for the same price, P only.

We also made, some other assumptions, for example, we made assumption that, the company is into only 1 product. Later, of course, we release that assumption. We said that, company is into multiple product also. And, we could see, how multiple Break-Even Analysis, can be done. One of the other assumption was, whatever quantity is being produced, is being sold.

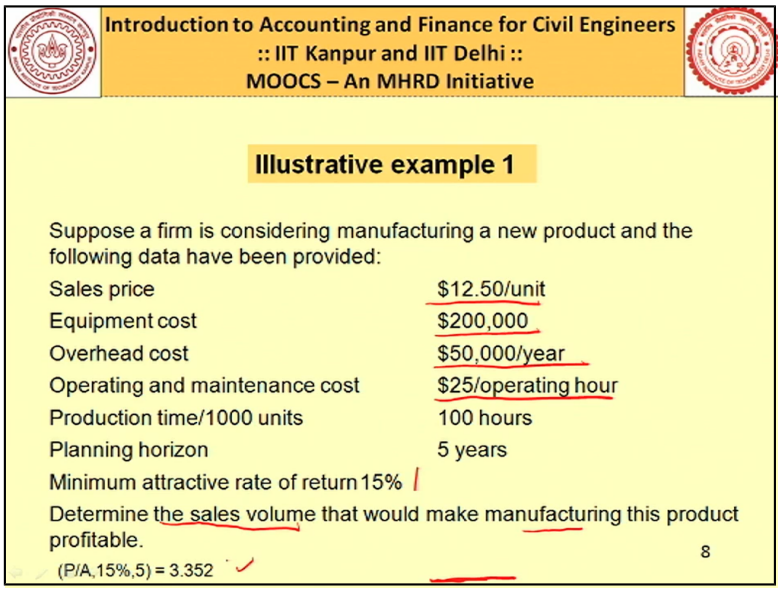
Now, in this lecture, we are going to cover, Non-Linear Break-Even Analysis. Now, as the name suggests, here, the relation between, price and the quantity, the cost and the quantity, is not Linear. Rather, they are following, some Non-Linear relationship. So, how to carry out, the Break-Even Analysis, under such situation. That is what, we are going to discuss, in this particular lecture.

(Refer Slide Time: 01:40)



Now, we will straightaway move with one small example, so that it becomes clear to you, how to perform the Break-Even Analysis.

(Refer Slide Time: 01:47)

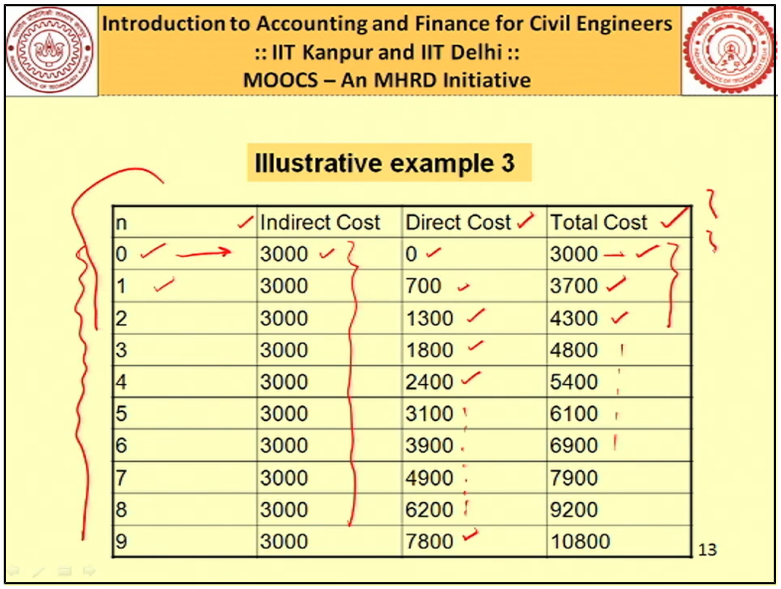


So, let us assume that, a firm is considering, manufacturing a new product. And, these data are available to us. The sales price is, 12.5 Dollar per unit. The equipment cost is, 200,000 Dollars. Overhead cost is, 50,000 Dollars per year. Operating and maintenance cost is, 25 Dollars per operating hour. Production time is, of course 100 hours, for 1,000 units. And, planning horizon is, 5 years. The rate of return is 15%. We are supposed to determine the sales volume, that would make manufacturing this product profitable.

You are given this, factor also, P given A, 15%, 5, is equal to, 3.352. Now, this is the kind of problems, we will be encountering in, Non-Linear Break-Even Analysis. What you have

finding is that, depending on the production, your prices are changing. Your sales price is changing, your cost is changing, unlike the Linear analysis, where these were supposed to be constant. Now, in Non-Linear Break-Even Analysis, we come across, two types of problems.

(Refer Slide Time: 03:16)



In one problem, we are given the data corresponding to, let us say, the data could be something like this, wherein, we are given the Indirect Cost, we are given the Direct Cost, and we are given the total cost, for different level of production. So, for different value of N, that is production level, we are given, what is the Indirect Cost. For example, for zero production, that means, no production at all, we are incurring an Indirect Cost of 3,000. The Direct Cost is 0. Total cost is 3,000.

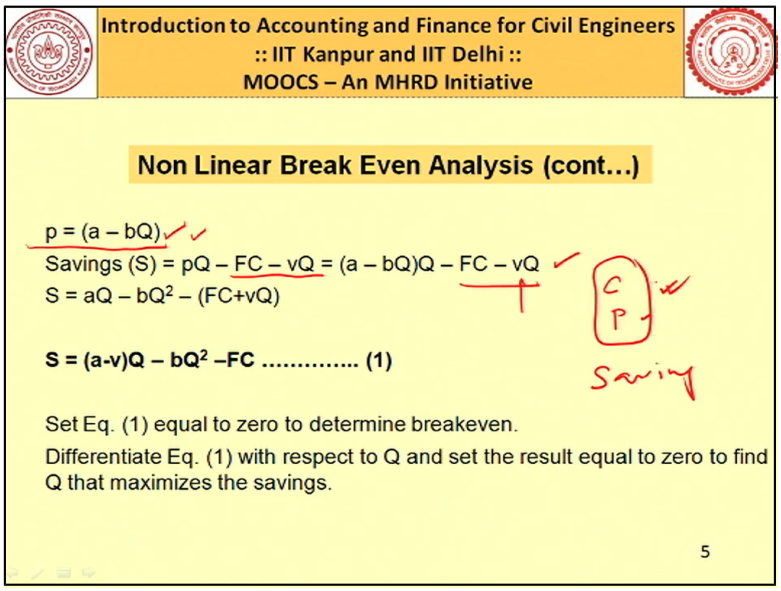
If the company is producing 1 unit, Indirect Costs remain same. In fact, irrespective of your production, your Indirect Cost is constant. On the other hand, Direct Cost is changing, with respect to, the level of production. So, for example, 1-unit production, the Direct Cost is 700. 2 units, it is coming out to be 1,300. 3 units, it is coming out to be 1,800. 4 units, it is 2,400 and so on, you find that, if the company is producing 9 units, the Direct Cost is 7,800.

So, total cost, as you know, is the sum of Indirect Cost and Direct Cost. So here, for no production, it becomes 3,000. For 1 unit, it becomes 3,000 + 700. So, 3,700. For two units, 3,000 + 1,300, it is 4,300, and so on. So, this is one set of problems in which, we are given the hard data corresponding to, different production level.

So, we are given the Indirect Cost, we are given the Direct Cost, and we are given the total

cost also. Now, looking at these data, we can derive a relationship between, cost and the level of production. Likewise, you can also have data, pertaining to purchase price, with respect to production level. And, we can try to find certain equations, we can try to fit in certain equations, which can explain the relationship, given by these data. So, this could be, one type of problem.

(Refer Slide Time: 05:13)



On the other hand, the other type of problem could be, in which, we are directly given the equation. For example, it could be that, okay, my purchase price, or sales price rather, is given in terms of A - B into Q. Q is the Quantity, that we are producing, A and B are constant. So, these values will be, known to us, in the problem. And, let us say, the relationship is given. Likewise, they will tell you, okay, the total cost is, fixed cost + variable cost.

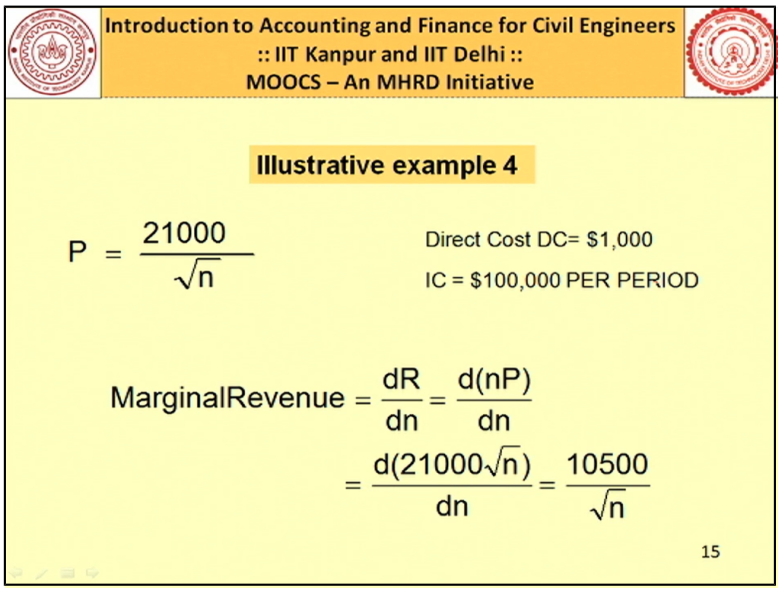
So, FC + VQ. So, let us say, V is Unit Variable Cost, and Q is the Quantity. So, Total Variable Cost becomes VQ. So, the idea is, in some cases, we will be directly given, these kind of equations, to establish the relationship between, let us say, Sales Price versus Quantity, Cost versus Quantity, and so on. Now, knowing these cost and the price, we can always find, what is the profit, or what is the saving, that we are getting.

So, let us say, if S represents Saving. So, the difference of the Revenue and Cost, is going to give me the Saving. Now, what I do is, at the break-even point, as you know, the cost and the price, both are going to be the same. That means, it is that particular production level at which, we are just able to recover our cost. We have already seen, how to find out, this break-

even point, in case of Linear Break-Even Analysis.

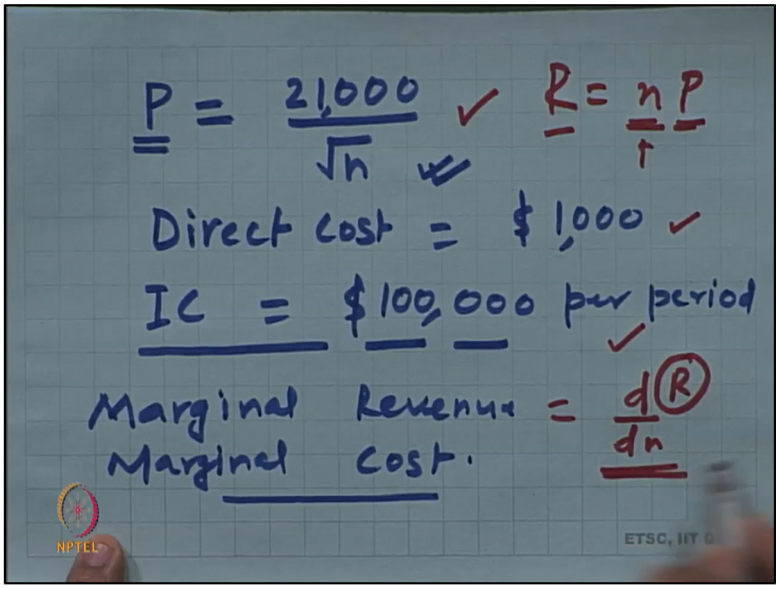
Now, with the help of one small example, we will see, how to find out, the Break-even point. As the relationship is Non-Linear, sometimes you might find that, break-even point could be more than 1. So, we may have to find out, what is the level of production, that we should go in for, so that, it results in maximum profit. So, we are adding, one more dimension. So, if you know the principle of calculus, you should be in a position to find out, that production level at which, you are able to maximise your profit.

(Refer Slide Time: 07:39)



Now, we will try to explain this concept, with the help of one small example, wherein I give you the relationship of various variables, in the form of an equation. And then, from there onwards, you can see, how to carry out the Break-Even Analysis.

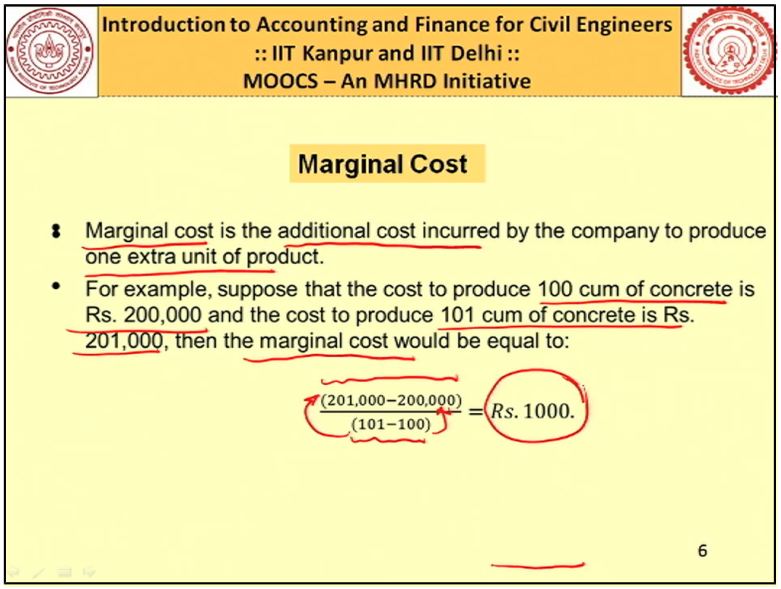
(Refer Slide Time: 07:51)



So, let us assume, we are given for a particular product, the sales price is given to be, 21,000, divided by Under Root N. So, that means, the relationship between, the quantity sold, and the price that you are charging, is captured by this equation. We are also given, the Direct Cost as, let us say, 1,000 Dollars.

Let us say, the Indirect Cost is given to be 100,000, for a particular period. So, we say, it is per period. So, these are the input. Now, I define two terms, we call them as Marginal Revenue, and there is another term called, Marginal Cost. So, these two definitions also, need to be understood. And then, subsequently, we will see, how to carry out the Break-Even Analysis.

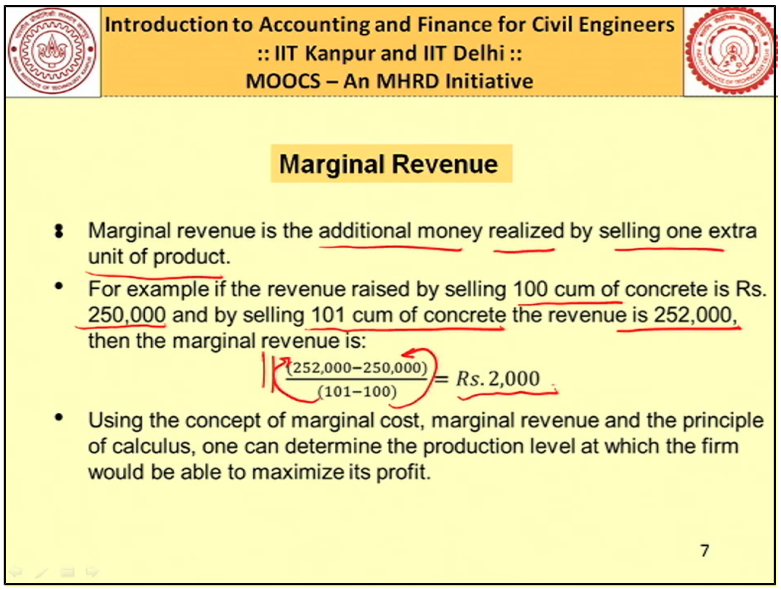
(Refer Slide Time: 08:59)



So, when you say Marginal Cost, it is basically the additional cost, that we are incurring, to produce one extra unit of product. This is how, you define the Marginal Cost. So, what is the additional expense, that you are incurring, in order to produce, one extra unit. Now, to understand this concept of Marginal Cost, just look at this example. Let us say, the cost to produce, 100 cubic metres of concrete is, Rupees 200,000. And, the cost to produce, 101 cubic metres of concrete is, let us say, Rupees 201,000.

So, we say that, the Marginal Cost at this level is, 201,000 - 200,000. So, this is the extra, that we are incurring, for producing what, from 100 to 101. So, that means, if right now, you are producing 100 units, your cost is 200,000. And, if you are producing 101 cubic metres, your cost is coming to be, 201,000. So, what you find, the Marginal Cost here is, 1,000 Rupees. So, Marginal Cost is that extra cost, that you incur, in order to produce, one more unit.

(Refer Slide Time: 10:13)



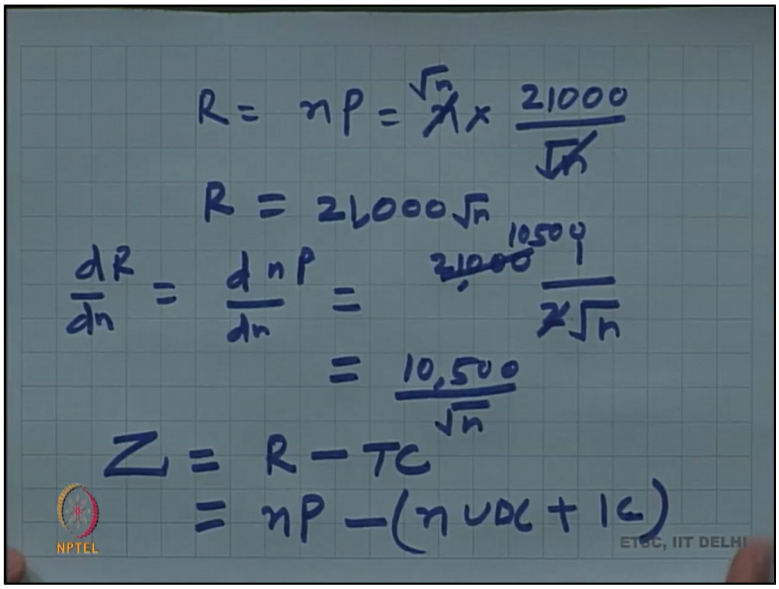
In the same manner, you also define, Marginal Revenue. So, marginal value, in the same manner you can define it as, the additional money, which you realise, by selling one extra unit of product. So, suppose right now, you are selling 100 cubic metres of concrete, for a price of, let us say, 250,000. And, by selling 101 cubic metre of concrete, you are realising 250,000. So, what is the marginal value. Your Marginal Revenue becomes, 252,000 - 250,000, divided by 101 – 100.

So, when you were selling 100, you were getting 250,000. When you were selling 101, you were getting 252,000. So, what is your Marginal Revenue. It is 2,000 Rupees. So basically, this is how, you define Marginal Revenue, and Marginal Cost. Now, we go back to the

problem, which we are trying to solve. And, in this problem, if you remember, I told you that, P is given by an expression of, 21,000 divided by Under Root N. And, the Direct Cost is given to be, 1,000 Dollars.

The Indirect Cost is, 100,000 Dollars per period. Right. Now, Marginal Revenue is defined as, D by DN of this revenue R, where R is what, R is N times P. So, N is the number of units, we are producing. And, P is the sales price. So, revenue that you will get is, N times P. Suppose, N is 10 units, and P sales price is 1,000 Rupees, so 10,000 Rupees would be your revenue. So, Marginal Revenue is, DR upon DN. So, first we try to find out, what is going to be the Marginal Revenue, for this problem.

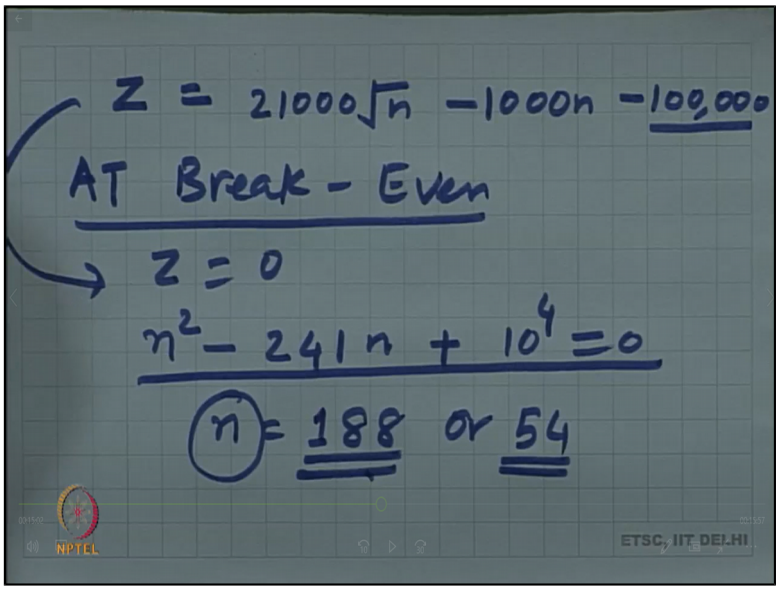
(Refer Slide Time: 12:02)



So, you are already given P, and you know R. So, R becomes, N times P. And, N multiplied by P is, 21,000, divided by Under Root N. So, this gets cancelled. And, you are getting a value of, 21,000 Root N. So, this is your R. Now, we have to differentiate it, with respect to N. So, I say, okay, DR upon DN, which is D of NP upon DM. This is nothing but, 21,000. And, for Root N, if you remember, from your class in calculus, it is 2 by Root N, so this gets cancelled.

So, this is 10,500. So, this is coming out to be, 10,500 upon, Under Root N. So, this is the Marginal Revenue, that we are getting. Now, my profit is represented by Z. Z is nothing but, my revenue - total cost, TC. Revenue is nothing but, N times P. And, C is nothing but, N times UDC + Indirect Cost, which is IC. So, all these terms, you are already familiar with, from the previous lecture.

(Refer Slide Time: 13:21)

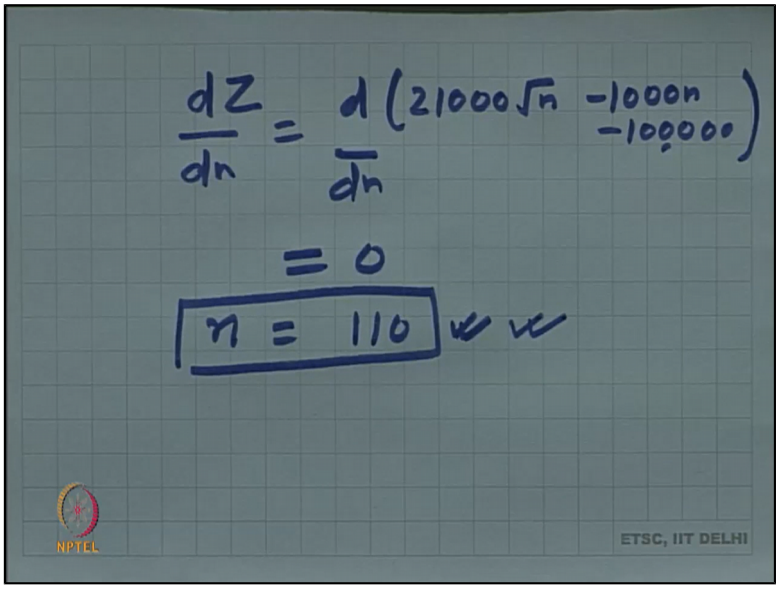


Now, what I do is, I just try to simplify this, and you will find that, you are getting an expression like this. Z is 21,000, Under Root N, - 1,000 N, - 100,000. This 100,000 is the Indirect Cost, which is given for that particular period. Now, at break-even you know, Z become 0. Because, you are not able to make profit, at break-even point. So, at break-even point, Z is equal to 0. So, you equate this expression, to zero.

And, when you do this, you will find, you are getting a Quadratic Equation. And, this is how, it looks. N square, - 241 N, + 10 raised to the power 4. This is equal to 0. So, this is the Quadratic Equation, we are getting. When you simplify this, and when you solve this, you will get a value of N as, 188 or 54. We have got these values, after rounding it off. You can crosscheck these calculations. So, as I told you, just before this, solving a problem, that there could be, multiple values of break-even point.

So, these are the break-even points. That means, whether you produce 188, or you are producing 54, you are able to recover your cost. So, at these two values, you are neither making any profit, nor making any losses. Now, another problem, that we have to find in such problems is, to find out, that particular production level at which, my profit is maximum. So, you know the concept of calculus, you have to differentiate Z, with respect to N, in order to find the maximum value.

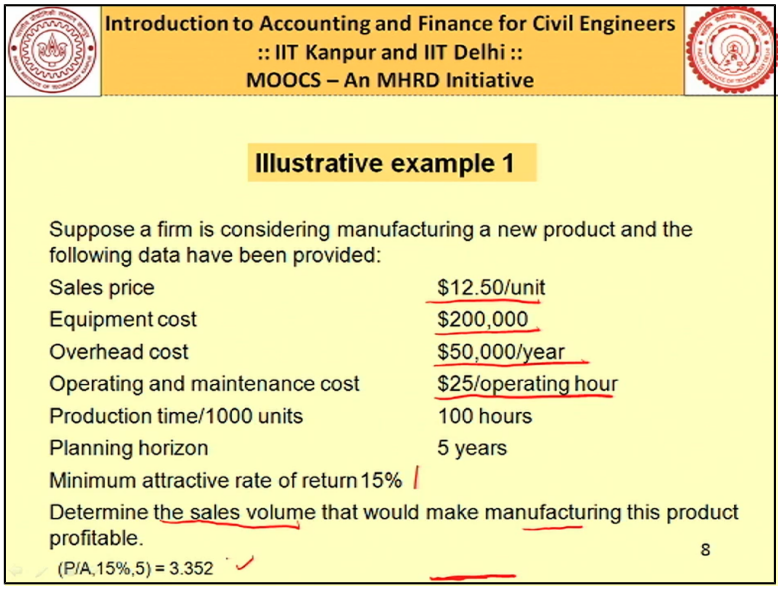
(Refer Slide Time: 15:07)



So, DZ upon DN, we can calculate. Now, Z, you already know, it is 21,000, Under Root N - 1,000 N, - 100,000, D by DN of this expression. Now, I equate it to zero. From the knowledge of calculus, you know, you have to, first find the derivative of this, and equate it to zero. And, when I solve it, I get N is equal to 110. Now, this 110, is that production level at which, my profit is maximum. So, if you remember, we had two values of break-even point, that was 188 or 54.

Now, in between these two value, I find, there is one value of production, 110 at which, my profit is maximum. So, this is how, you have to calculate, your break-even point. And, you also have to find out, that particular production level at which, the profit is maximum. Now, we will take one more example, and that will reinforce your understanding of, Break-Even Analysis. And, in order to do this, I will just tell you, what is there in this particular example. I have already explained you.

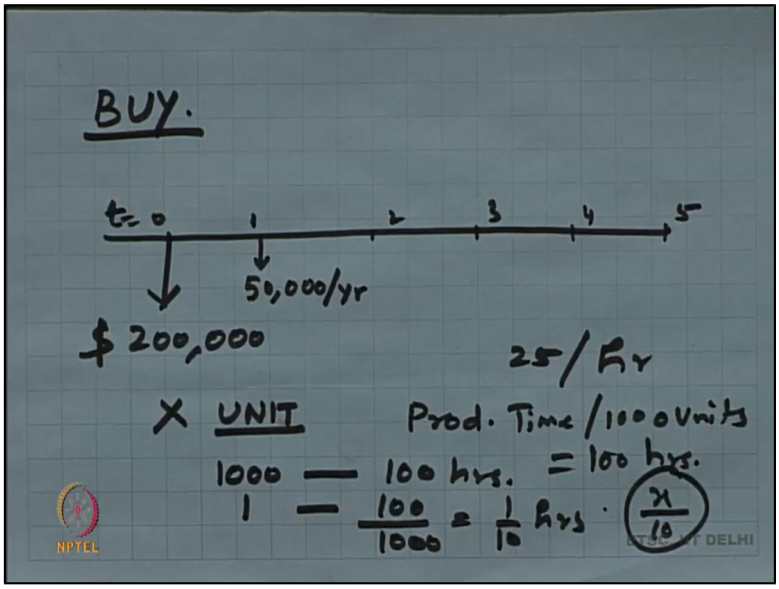
(Refer Slide Time: 16:28)



The data that is available to us, the sales price, if you see, the sales price, it is 12.5 Dollars per unit. Equipment cost is, 200,000. Here basically, we are trying to make a decision between, whether to buy a particular equipment, or whether continue to outsource the product. Right. So, this is what, the statements say. Suppose, the farm is considering, manufacturing a new product. And, if they are trying to go for manufacturing, the equipment is costing, 200,000.

Overhead cost is, 50,000 Dollars per year. Operating and maintenance cost is, 25 Dollars per operating hour. Production time, per 1,000 units is, 100 hours. And, planning horizon is, 5 years. So, if you try to draw, the cash flow diagram for this, let us say, basically, if let us say, we are going in for, buying of equipment, it would be like this. The equipment is costing, 200,000. So, straightaway, at time t is equal to 0, we have to spend, 200,000 Dollars.

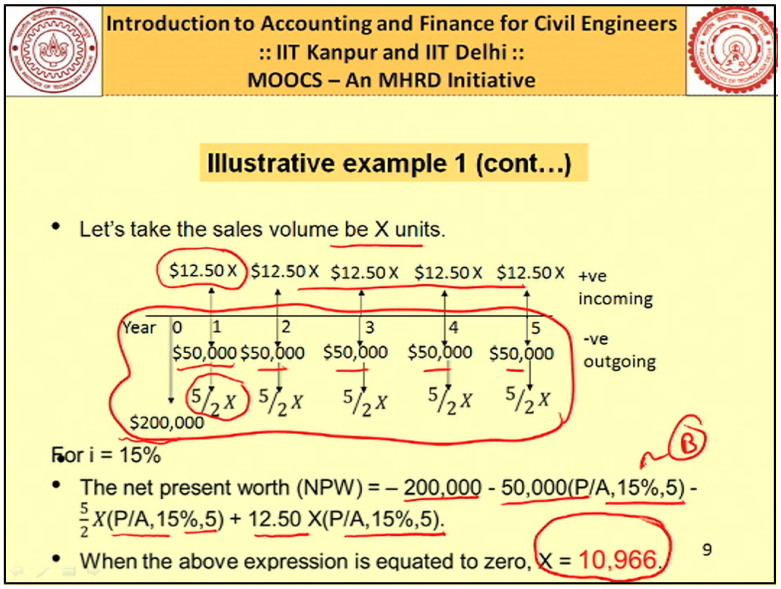
(Refer Slide Time: 17:35)



Now, overhead costs are, 50,000 Dollars again, every year. 50,000 Dollars, per year. This is one, two. Planning horizon is 5 years, as given in this problem. Operating and maintenance cost is, 25 Dollars per operating hour. Now, the question is, determine the sales volume, that would make manufacturing this product, profitable. So, let us assume, I am producing X unit. X unit is being produced. Right.

Now, it is given that, operating and maintenance cost is, 25 Dollars per operating hour. Right. So, it is 25 per hour. Right. And, it is also given that, production time per 1,000 units is, 100 hours. Right. So, that means, 1,000 units are produced in 100 hours. 1,000 in 100 hours. So, 1 unit, I can produce it in, 100 by 1,000 hours. So, it is, 1 by 10 hours. So, X unit, I can produce in, X by 10 hours. X unit, I can produce in, X by 10 hours. Okay.

(Refer Slide Time: 19:04)



Now, I can draw the whole thing, on a piece of paper, in a cash flow diagram, like this. So, if you look at this, the cash flow diagram would be, something like this. If you remember, the sales price is, 12.5 Rupees per unit. So, I am assuming that, sales volume be, X unit. So, this is going to be, 12.5 X every year. Why? I am assuming that, every year, we are selling X unit. And, price for 1 unit is, 12.5, so 12.5 X.

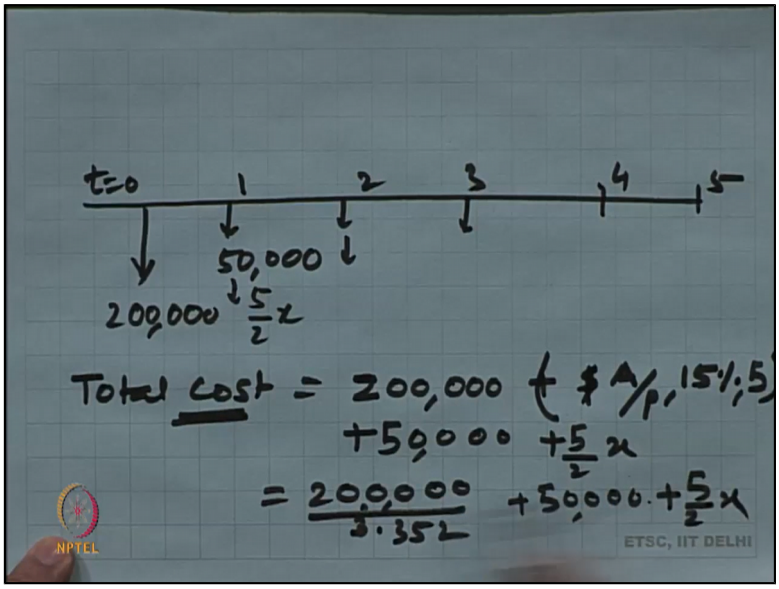
Now, we are also given that, 25 Dollars per operating hour. 25 Dollars per operating hour is the, O&M cost, operating and maintenance cost. Right. And, in 1 hour, we are producing X by 10. So, our cost is going to be, 25 X by 10, which is nothing but, 5 by 2X. So, this is what, we are getting, 5 by 2X, here. And, 50,000 Dollars per year. Year 1, 2, 3, 4, and 5. And, in addition, the variable cost is, 5 by 2X. And, if you are buying the equipment 200,000, you are incurring, at time t is equal to 0.

Now, there are many ways, you can solve this problem. You can, either, try to equate the present worth of these revenues, that you are getting, and equate it with, the cost that we are getting for this whole thing, and then you can find the value of X. This is how, this particular problem has been solved, in this particular slide. So, you can see, the Net Present Worth becomes, - 200,000. This is for buying option.

- 200,000 - 50,000, P given A, 15%, for 5 years, and you will add this 5 by 2X, P given A, 15%, 5. And, you equate it to this, or you bring it here, + 12.5 X, P given A, 15%, 5. You will find that, when you try to solve this, you are getting a value of, X is equal to, 10,966. The same problem, I can solve it also using, the Annual Cost Method. So, the same Cash Flow

Diagram, I will draw once again.

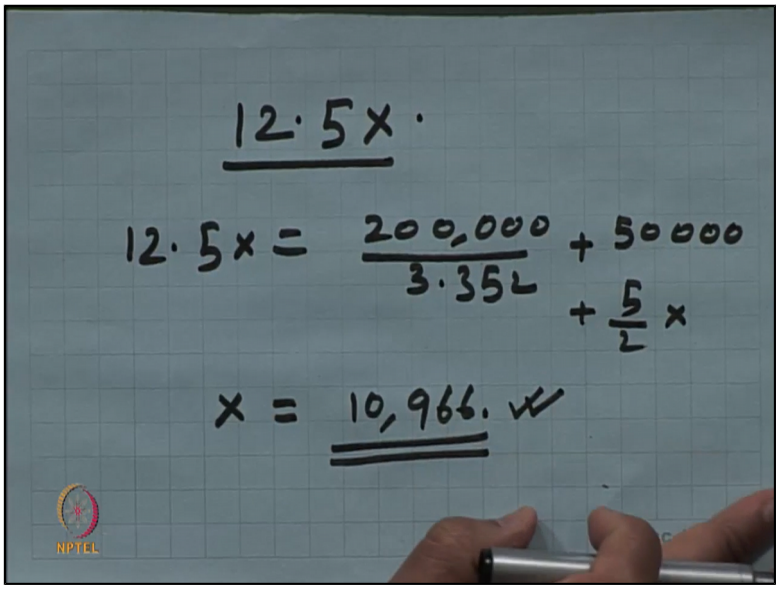
(Refer Slide Time: 21:33)



So here, we are incurring 200,000, at time t is equal to 0. And, 50,000 every year, as overhead cost, + 5 by 2X, I am incurring every year. This is being done for, 5 years. So, 1, 2, 3, 4, and 5. So, my total cost is going to be, 200,000. I want to find its equivalent A.

So, I write A given P, for an interest rate of 15%, for a period 5 years, + 50,000, is in terms of annual cost itself, + 5 by 2X, is also in terms of annual cost itself. So, when I solve this, this expression is, 200,000 divided by 3.352, + 50,000 + 5 by 2X. So, this is the cost, corresponding to, buying option.

(Refer Slide Time: 22:47)

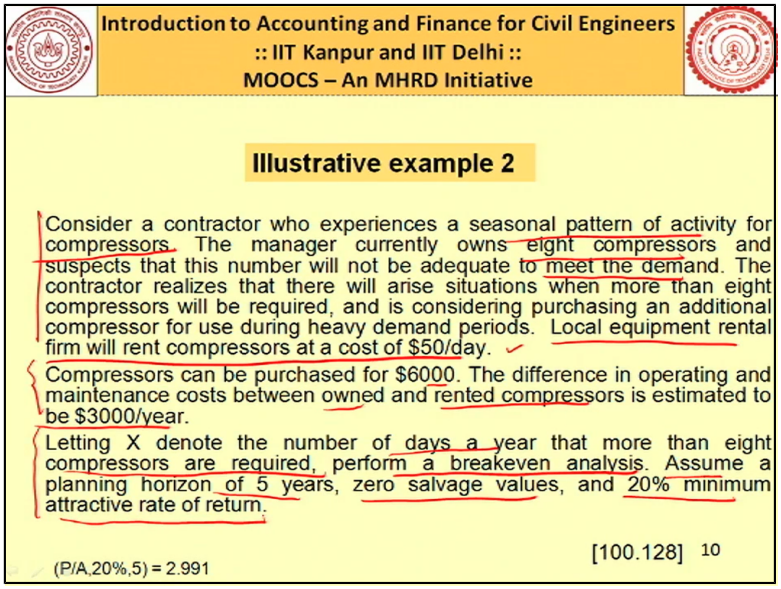


If I am getting it from other place, I am getting it at a rate of 12.5 X. So, what I do? I equate

the two. So, 12.5 X is equal to 200,000, divided by 3.352, + 50,000 + 5 by 2X. So, when you solve this, you will get, X is equal to, 10,966. So, this is how, whether you use Annual Worth Method, or whether you use Present Worth Method, you can find out the solution.

So, you find that, as long as, you are producing X is equal to 10,966, whether you buy, or whether you produce it through your own equipment, both are going to be the same. If the value of X changes from this value, your decision also changes. Now, I take another problem, that will give you more understanding, of this Break-Even Analysis.

(Refer Slide Time: 23:50)



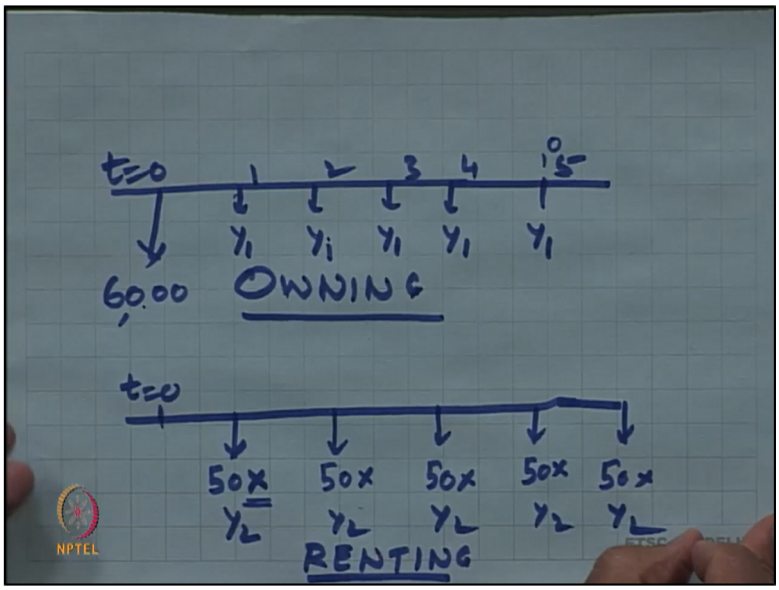
Just look at this, particular problem. It says, there is a contractor, who is experiencing a seasonal pattern of activity, for compressors. Right. So, compressor is a construction equipment, which is very useful at sites. For many purposes, it can be used. Now, the manager is currently owning eight compressors, and suspects that, this number will not be adequate, to meet the demand. So, they already have eight, but they find that, this number will not be adequate, to meet the demand.

The contractor realises that, there will arise situations, when more than eight compressors will be required. And, he is considering, purchasing an additional compressor for use, during heavy demand periods. So, they want to go beyond eight, so that they can take care during, heavy demand periods. Now, the local equipment rental firm, will rent these compressors, at a cost of 50 Dollars per day. As I told you, in using Break-Even Analysis concepts, you can have a decision between, Purchasing versus Renting, Buying versus Leasing, and so on.

So, right now, you see here, local equipment rental firm, will rent compressors, at a cost of 50 Dollars per day. On the other hand, if you want to buy it, it can be purchased for 6,000 Dollars. Now, the difference in operating and maintenance costs, between the owned and the rented compressors, is estimated to be, 3,000 Dollars, per year. So, that means, between the owned and rented compressor, if you try to find the difference, for operating and maintenance costs, you find it is, 3,000 Dollars per year.

Now, letting X denote the number of days a year, that more than eight compressors are required. So, suppose X is the number of days for which, more than eight compressors are required, we are supposed to perform a Break-Even Analysis. That means, we are supposed to find out, for how many days in a year, if we are going to use more than eight compressors, buying option would be preferred, or renting options would be preferred. You are given that, planning horizon is 5 years, salvage value zero, and there is a 20% minimum attractive rate of return. So, the interest rate is 20%.

(Refer Slide Time: 26:18)



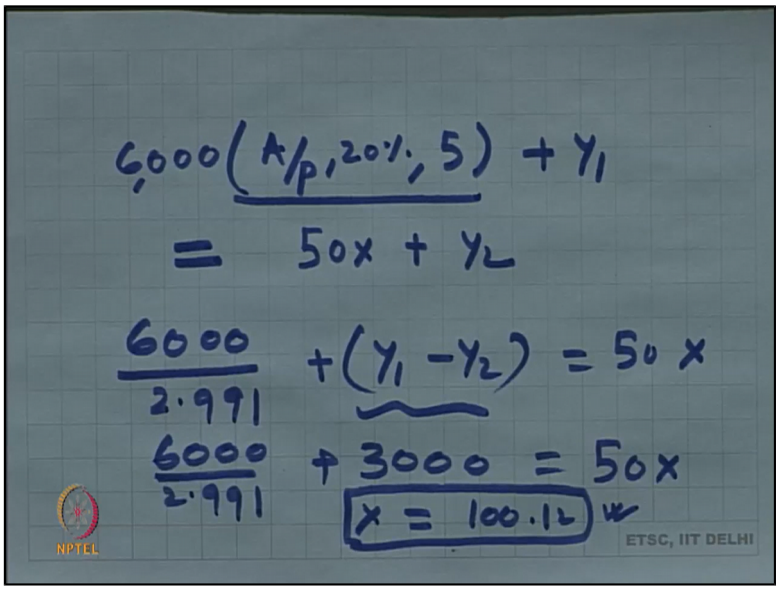
Now, for solving this problem again, we try to draw this cash flow diagram. And, if you see, the cash flow diagram would be, something like this, as far as, this owning option is concerned. Compressor you can own it, for 6,000 Dollars, if you have, this much money, at t is equal to 0. And, let us assume that, a planning horizon is, let us say, 5 years. So, 4 5. And, let us assume Y1 annually, is the operation and maintenance cost, for this particular equipment.

This is the, owning option. Now, there is no salvage value given, so this is zero here. Salvage

value is zero. On the other hand, for renting option, at time t is equal to 0, you do not have to do anything. 50 X is given, and Y2 is given. So, if you remember, on rent, these compressors are available for, 50 Dollars. And, 50 Dollars, per day.

So, if you are using it for X days, it is 50 X. Likewise, 50 X here, 50 X here, 50 X here, 50 X here. And, of course Y2, Y2, every year, as part of O&M cost. And, at t is equal to 0, there is nothing. So, this is renting option. Now, I just compare the cost of the two, and equate them. Because, at break-even, I am indifferent towards, owning, as well as renting. So, what I do? I just equate, the two costs.

(Refer Slide Time: 28:00)



Now, this I could have done it, using Present Worth Method of Analysis, also. But, Annualised Cost Equivalent Method would be, much easier here. So, what I do? I equate it like this, 6,000, A given P, because I know the present value, I want its annual value, for an interest rate of 20%, for 5 years' period, this is for owning, + Y1. This must be equal to, 50 X + Y2.

Now, I bring this Y2, to this side. So, I write 6,000, and A given P, for 20%, 5 years, it is going to be this value, 1 upon 2.991, + Y1 - Y2, is equal to 50 X. And, now in the problem, I am given that, Y1 - Y2, that is the difference in operating and maintenance costs between, owned and rented compressor is, 3,000 Dollars. So, this is 3,000. So, 6,000 divided by 2.991, is equal to, 50 times X.

So, if you solve this X, you will get some value, 100.12. What does this mean. If I use this

compressor, for more than 100 days, it is preferable. As long as, you are using exactly 400 days, then whether you buy, or whether you take it on rent, it does not matter. But, more than 100, buying option would be preferred. If you are using it for less than 100, the renting option would be preferred. So, this is how, we try to carry out the Break-Even Analysis, in case of Non-Linear situations.

So, just to summarise, in Non-Linear Break-Even Analysis, either we are given the data, data for the number of production, the Direct Cost, the Indirect Cost. And, from this data, I can try to find out, some form of equations, by fitting some form of curves. And then, using the principle of calculus, I can find out the break-even point. I can also find out, that particular point at which, the profit is maximum.

In another case, I may directly be given, the relationship between, different variables. Using those variables, and the knowledge of calculus, I can find out my break-even point. I can also find out, at what production level, my profit is going to be, maximum. So, this is how, I carry out my, Non-Linear Break-Even Analysis. So, thank you for listening, and see you in some other class. Thank you, very much.