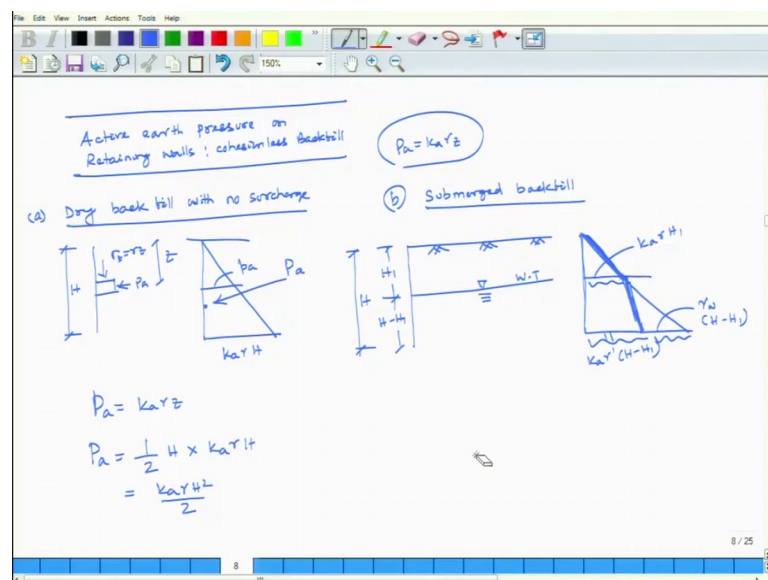


**Foundation Design**  
**Prof. Nihar Ranjan Patra**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 17A**  
**Earth Pressure Theories – Part 3**

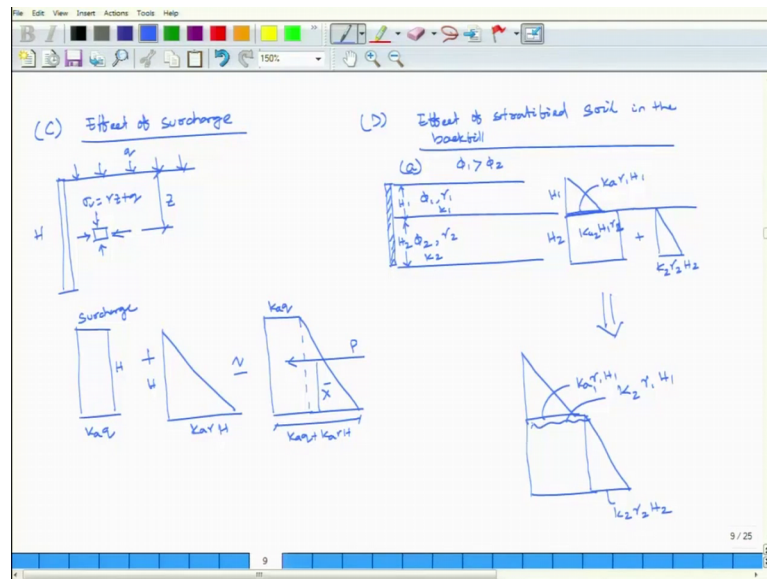
Last class I have started active earth pressure on retaining wall for particularly cohesion less soils. So, previously we have also derived for Rankine's active state and passive state. Now there are 2 cases I have discussed one is your dry backfill with no surcharge it is a very straightforward below is your  $k \gamma h$ , total height is your  $H$  and  $P_a$  is your  $k \gamma z$ .

(Refer Slide Time: 00:18)



So,  $P_a$  is half  $H$  into  $k \gamma H$ . So, this we have  $k \gamma H^2$  by 2  $z$  is any depth. Now submerged backfill I have discussed if there is a soil layer of  $H$  and below a depth of  $H_1$  there is water table. So, from here to here it will be  $k \gamma (H - H_1)$ .

(Refer Slide Time: 01:33)



Below water table there is a regular there is a normal earth pressure  $K_a \gamma'$  because this is your submerged unit weight then there is also water pressure,  $\gamma_w$  into  $H - H_1$  now come to next part C effect of surcharge. So, this is my wall and this will be; in this wall this is your height  $H$  and this will be your soil element. So, there is a surcharge of  $q$ . So, here it is your  $\sigma_v$  is equal to  $\gamma z + q$  and this distance from here to here is your  $z$ .

Now, how do you find it out considering these if I consider only surcharged load only, we have to apply method of superpositions, this is I have pressure because of your surcharge, this is your  $K_a q$  and this side is your  $H$  plus if I take out the surcharge and soil, apply only surcharge then applying with your soil backfill, or backfill cohesion less soil it comes out to be  $K_a \gamma H$  and this side is your  $H$ . Now if I draw it how it looks? It looks like this is your  $K_a q$  and this part is your  $K_a q$  plus  $K_a \gamma H$  and this is your earth pressure  $P$  at a distance  $\bar{x}$  this is my  $c_g$ . So, basically in this case of effect of surcharge method of superposition has been applied.

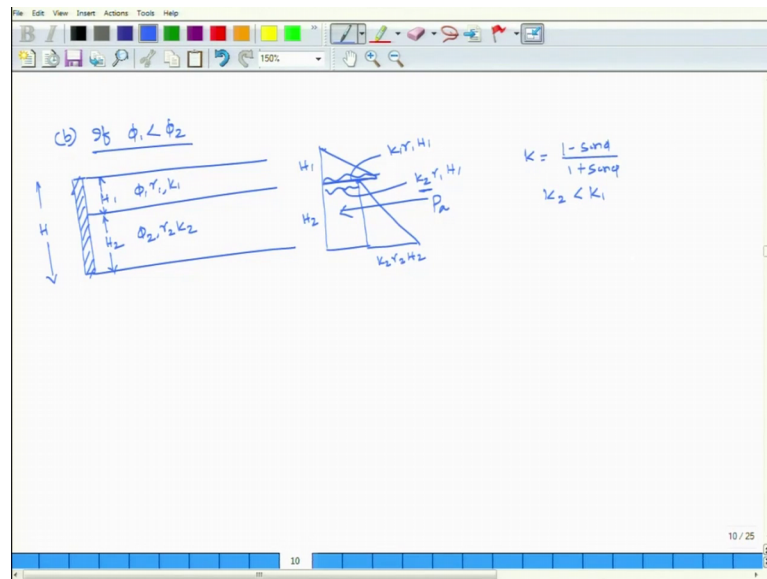
So, surcharge load has been taken independently, then backfill soil taking into this backfill soil what is a earth pressure because of surcharge what is your earth pressure then add it this is what we are getting, then second part is your point D that is your effect of stratified soil in the backfill. These are all homogenous soil same soil properties are there, then effect of stratified soil in the backfill. Let us consider a wall this is my retaining wall with this wall let it be this is my  $H_1$  this is  $\phi_1 \gamma_1 k_1$  this is  $\phi_2 \gamma_2 k_2$  this is  $s_2$ .

Now, how is your pressure distribution diagram? Earth pressure distribution diagram. So, in this case  $\phi_1$  is greater than  $\phi_2$  topsoil soil angle of internal friction  $\phi_1$  is greater than you have  $\phi_2$ . If  $\phi_1$  is greater than  $\phi_2$  then draw it cut of soil from here to here this is my  $K_a \gamma_1 H_1$ . Now for bottom soil means at the junction what will happen at this point just before touching this is your  $K_a \gamma_1 H_1$  at this junction if the  $\phi_2$  starts, as  $\phi_1$  is greater than  $\phi_2$  then  $K$  value will be for  $\phi_2$ , then it is coming out to be here then this will be your  $K_a \gamma_2 H_1$ , then for this value for this soil independently this is your  $K_a \gamma_2 H_2$ .

Now, if I draw it if I rewrite it completely, then how it looks like? It looks like this way then this way then this way. So, this will be your  $K_a \gamma_1 H_1$  and the this to this will be your  $K_a \gamma_2 H_1$ , this is your  $K_a \gamma_2 H_1$  and this is your instead of  $K_a \gamma_2 H_1$  I can write it  $K_a \gamma_2 H_2$  this is your  $K_a \gamma_2 H_2$ . So, effect of stratified soil in backfill. So, particularly in a layer soil let us say case one if  $\phi_1$  is greater than  $\phi_2$ , this is your wall a retaining wall in this retaining wall there is a height or depth  $H_1$  where this a  $\phi_1$   $\gamma_1$  and  $K_a$  and there is a depth  $H_2$  for  $\phi_2$   $\gamma_2$  and  $K_a$ .

So,  $\phi_1$  is greater than  $\phi_2$ , then what will happen? For this is a straight forward now for this layer this will be  $K_a \gamma_1 H_1$  and this is my  $H_1$  and this will be my  $H_2$ , but at the junction there are 2  $\phi$  values one is your  $\phi_1$  because of  $\phi_1$  this because of  $\phi_2$  this will be a  $K_a \gamma_2$  because of  $\phi_2$ , then  $H_1$  and  $\gamma_2$  this is what it is coming and entirely because of this  $K$ .

(Refer Slide Time: 08:52)



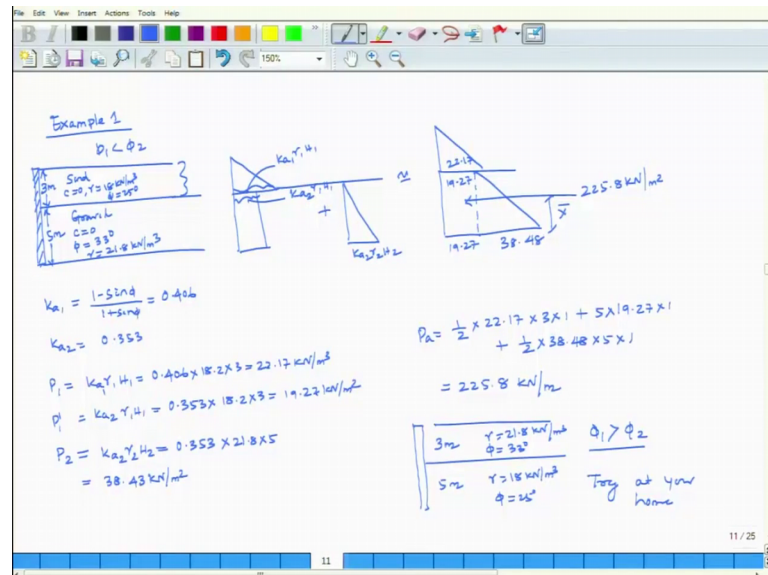
Two gamma 2 s 2 this is how it looks like your earth pressure distribution diagram, now come to b part. If phi 1 is less than phi 2 if phi 1 is less than phi 2 pi one is less than phi 2 draw a wall this is my retaining wall, in this retaining wall total height is H and this will be H 1 and this will be your H 2 and in this case it is your phi 1 gamma 1 k 1 phi 2 gamma 2 k 2.

In this case phi 1 is less than your phi 2, if I draw it how it looks like or first soil this is my case this will be your k 1 gamma 1 H 1, then for at junction this looks like this. So, from here to here it is your k 2 gamma 1 H 1, then k 2 gamma 1 H 1 then for this soil this soil particularly this soil this is your k 2 gamma 2 H 2. I did a mistake here it will be your here it will be your gamma 1 here it will be your gamma 1 this soil, but here it is a phi because of phi k 2 gamma 1 H 1. So, because of at interface this phi changes, but this gamma 1 will be same earlier gamma and H 1 is your this height. So, in this case phi 1 is less than phi 2 once phi 1 is less than phi 2 then what will happen? K 2 value will be less than k 1 what is the k? K is equal to 1 minus sin phi by 1 plus sin phi then k 2 is less than k 1 k 2 is less than k 1 .

Considering first layer of the soil particular these are the layering of the soil. So, this is my earth pressure distribution diagram. So, this will be H 1 and this will be H 2 and from here to here, here to here this will be a k 1 gamma 1 H 1 at the interface phi value has been changed, but H 1 is same gamma is same then here only because phi value has been changed it has been changed to k 2 gamma 1 H 1, and this is your H 2 and particularly

for second layer of the soil second layer of the soil it should be a  $k_2 \gamma_2$  and  $H_2$  and find it out the resultant and from where your  $P_a$  will be acting.

(Refer Slide Time: 12:18)



Now, let us start with one example, example one what is given in example 1? Example one is given here it is 3 meter, here it is 5 meter then it is a sand  $c$  is equal to 0  $\gamma$  is equal to 18 kilo newton per meter cube,  $\phi$  is equal to 25 degree and this is your gravel in this case  $c$  is equal to 0,  $\phi$  is equal to 33 degree  $\gamma$  is equal to 21.8 kilo newton per meter cube. So, find it out the value of  $P_a$  find the value of  $P_a$  then let us start with this  $\phi_1$  and  $\phi_2$  considered  $K_{a1}$  or I can find it out what is the value of  $K_{a1}$  is equal to  $\frac{1 - \sin \phi_1}{1 + \sin \phi_1}$  which is equal to 0.406 then  $K_{a2}$  which is equal to 0.353.

Now, find it out the considering soil one what is the value of  $P_1$ ?  $P_1$  is equal to  $K_{a1} \gamma_1 H_1$  which is equal to 0.406 into 18.2 into 3 which is equal to 22.17 kilo newton per meter cube and second one  $P_a$  prime at the junction at the interface this is my interface where  $\phi$  value changes,  $P_a$  prime is equal to  $K_{a2} \gamma_1 H_1$  which is equal to 0.353 into 18.2 into 3, which is equal to 19.27 kilo newton per meter square.

In this case  $\phi_1$  is less than  $\phi_2$ . So, then come to  $P_2$  value for second layer at the junction I have calculated this is for first layer soil for second layer soil, this will be  $K_{a2} \gamma_2 H_2$  which comes out to be 0.353 into 21.8 into 5 which is equal to 38.43 kilo

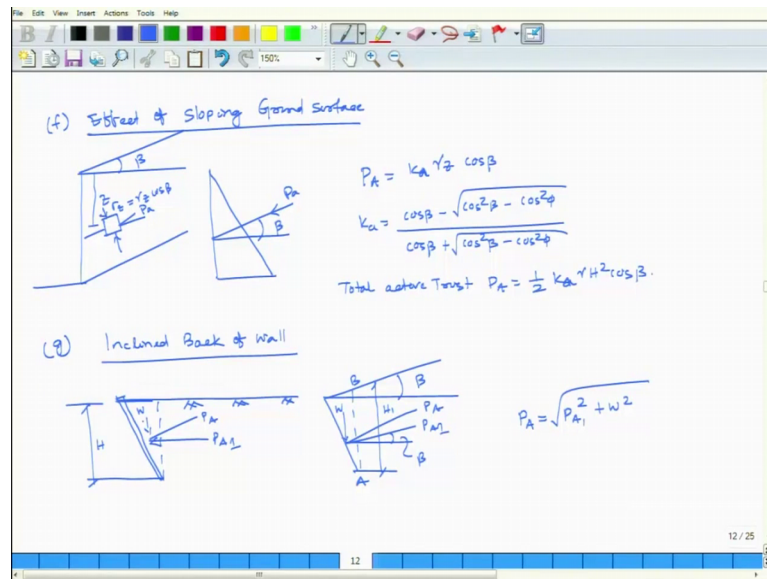
newton per meter square. Now if I plot it how it looks?  $\phi_1$  is less than  $\phi_2$  definitely this pressure diagram is more than here it is coming this this is your  $k a_1 \gamma_1 H_1$  and this part this is this this part is your  $k a_2 \gamma_1 H_1$ , then plus at this stage for second solid comes out to be  $k a_2 \gamma_2 H_2$ .

Then if I re plot it how it looks like? This part is your 22.17 this is your 19.27 and this is your 38.48 this is your 19.27. Now you can calculate what is the value of your  $P_a$  the value of your  $P_a$  will be half into 22.17 find it out the area into 3 into 1 plus 5 into 19.27 into 1 plus half into 38.48 into 5 into 1, which is equal to 225.8 kilo newton per meter kilo newton per meter. So, this acts here 225.8 kilo newton per meter and you can find it out also  $c_g$ , all total where is your  $c_g$  is are proving.

This is how is your example one of the layering soil one example I have make it and this is for your assignment I am putting it you can try at your home, same problem I am just making it here it is 3 meter, here it is 5 meter on changing it  $\gamma$  is equal to 21.8 kilo newton per meter cube,  $\phi$  is equal to 33 degree here  $\gamma$  is equal to 18 kilo newton per meter cube and  $\phi$  is equal to 25 degree; that means, in this case  $\phi_1$  is less than  $\phi_2$ , in these case  $\phi_1$  is greater than  $\phi_2$  you try at your home. These are the 2 took it means same cases I interchange this value of the  $\phi$ . So, that one case it is  $\phi_1$  is less than  $\phi_2$  and other cases it is  $\phi_1$  is greater than  $\phi_2$ , and you can find it out what is the total active earth pressure  $P_a$  and where its acts at a distance  $x$  from the base of this wall.

Now, come to the next part, now next part is your effect of sloping ground I have finished up to here that is your effect of stratified soil in the backfill, now it has been extended now come to the effect of sloping ground surface.

(Refer Slide Time: 19:37)



So, effect of sloping ground surface this is my wall generally what happened we provide the soil at a sloping at an angle beta. So, what will happen? If this is my  $z$  what will happen? If I take it in this way then it will be your  $\sigma_z$  which is equal to  $\gamma z \cos \beta$  and this will be your  $P_a$  instead of doing this inclined if I take it. So, it will be my  $P_a$  at an angle beta you take it in this way parallel to that then it will be very easy then you rotate at an angle beta.

So, in that case what will happen  $P_a$ ?  $P_a$  will be  $K_a \gamma z \cos \beta$  and  $K_a$  is equal to  $\frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$ . So, this derivation I can derive how it has come this derivation it can be very easily with this this can be derived with your  $K$  value. So, total active thrust  $P_a$  is equal to  $\frac{1}{2} K_a \gamma H^2 \cos \beta$ , then second case is your e f g this case is your inclined back of wall; here inclined surcharge or ground surface is sloping in this case wall is not vertical rather it is inclined.

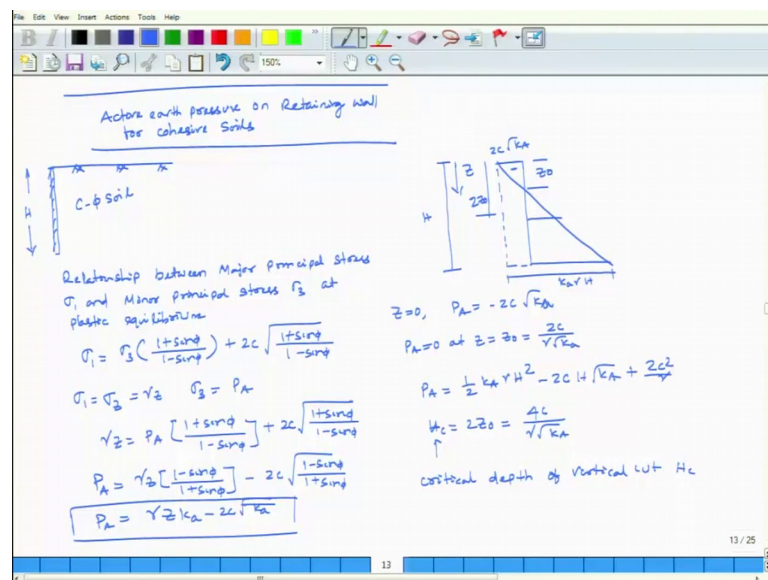
Let us consider this is my wall and this is your  $H$  and this is my ground surface I put it a dotted line. So, what will happen? Here there is a weight  $w$  then there will be a  $P_{a1}$ , then this will be my  $P_a$ . So, if I draw it let me put it other way, this way this way then also I can put it as an inclined both the cases it is a planner as well as there is a sloppy, I can put it is sloping also this will be my beta. So, this comes out to be here then here it will be  $w$ , and here it will be a  $P_{a1}$  or beta and here it is  $P_a$  then this is my  $A$ , this is  $B$  if I take it from here to here say this distance is my  $H$ .

So,  $P_a$  is in this case  $P_a$  is equal to  $P_a$  1 whole square plus  $w$  whole square, square root  $w$  is your weight of this soil in the inclined phase if I take a vertical this is particularly weight of this soil in the inclined phase. Then next part up to this we have covered the cohesion less soils, there are seven cases we have consider. First one is your for cohesion less soil first one is your dry backfill with no surcharges second one is your submerged backfill then backfill with a surcharge.

Third one fourth one and fifth one there are stratified soil case one it is your  $\phi_1$  is greater than  $\phi_2$ , then for layer soil  $\phi_1$  is less than  $\phi_2$ , I solved one example considering layers soils then there is a case of effect of slopping ground; ground has been sloped what will be your earth pressures then backfill also inclined backfill also inclined at an angle you can say that this is my  $\alpha$ .

So, these are the all 6 7 cases particularly for cohesion less soils, now let us move to active earth pressure on retaining wall for cohesive soils on retaining wall or cohesive soils.

(Refer Slide Time: 26:11)



Let us consider a wall height is  $H$  total height is  $H$  from here to here, smooth vertical backfill and this is a  $c$   $\phi$  soil the relationship between major principal stress and minor principal stress are plastic equilibrium, relationship between major principal stress  $\sigma_1$  and minor principal stress  $\sigma_3$  at plastic equilibrium  $\sigma_1$  is equal to  $\sigma_3$  one plus  $\sin \phi$  by  $1$  minus  $\sin \phi$  plus  $2c$  root over of  $1$  plus  $\sin \phi$  by  $1$  minus  $\sin \phi$ .



This is standard relationship between major principal stress from the mechanics of solid mechanics engineering mechanics and minor principle stress  $\sigma_3$  at plastic equilibrium this one. So, let us say  $\sigma_1$  is equal to  $\sigma_z$  which is equal to  $\gamma z$  and  $\sigma_3$  I equal to is your active earth pressure. So,  $\gamma z$  is equal to  $P_a$  into  $1 + \sin \phi$  by  $1 - \sin \phi$  plus  $2c$  root over of  $1 + \sin \phi$  by  $1 - \sin \phi$ . It is your  $\gamma$  into  $z$  not subscript then is your  $P_a$  which is equal to  $\gamma z$  into  $1 - \sin \phi$  by  $1 + \sin \phi$  minus  $2c$  root over of  $1 - \sin \phi$  by  $1 + \sin \phi$ , then which comes out to be  $\gamma z k_a$  minus  $2c \times$  root over of  $k_a$  this is your  $P_a$  value.

So, for cohesive soils  $P_a$  is equal to  $\gamma z k_a$ , minus cohesion force  $2c$  root over of  $k_a$ . So, let us try it this is my wall this is wall say this is height  $H$  in this wall and let me put it in this direction this is my  $z$ . So, at  $z$  is equal to  $0$ ,  $P_a$  is equal to  $z$  is equal to  $0$   $P_a$  is equal to minus  $2c$  root over of  $k_a$ . So,  $P_a$  is equal to  $0$  again, at  $z$  is equal to  $z_0$  when it will be  $2c$  by  $\gamma$  root over of  $k_a$ . Now if I put it if I draw it how it looks let us say this is my  $2z_0$  and in this case this will be  $k_a \gamma H$ , and then here it I minus this is your  $2c$  root over of  $k_a$  and this is your  $z_0$ .

Now, if I said  $P_a$  is equal to  $0$ . So, it will be minus  $2c k_a$  minus  $2c k_a^2 z_0$ , now let us put it  $P_a$  is equal to half  $k_a \gamma H^2$  square minus  $2c H k_a$  root over plus  $2c$  square by  $\gamma$ . So, net earth pressure is  $0$  are  $2z_0$ . So, if I say  $s c$  is equal to  $2z_0$ , which is equal to  $4c$  by  $\gamma$  root over of  $k_a$ , this is called your this is called your critical depth of vertical cut  $H_c$ . So, just I have started active earth pressure on retaining wall for cohesive soils. So,  $P_a$  is equal to  $\gamma z k_a$  minus  $2c$  root over of  $k_a$ , then  $z_0$  what is the value of at this surface it is your  $2c k_a$  then a  $2z_0$  this is completely  $0$  then you can find out resultant find it out earth pressure distribution diagram. Let me stop it here next class I will start case by case for slopping ground and other cases and I will solve few more examples.

Thank you.