Principles of Construction Management
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Lecture – 20
Crashing of Networks

[FL] and welcome once again to this series of lectures on Principles of Construction Management. In the last couple of lectures, we will talking about planning, scheduling, resource management and today, continuing from that discussion we will be talking of what is called crashing of a network.

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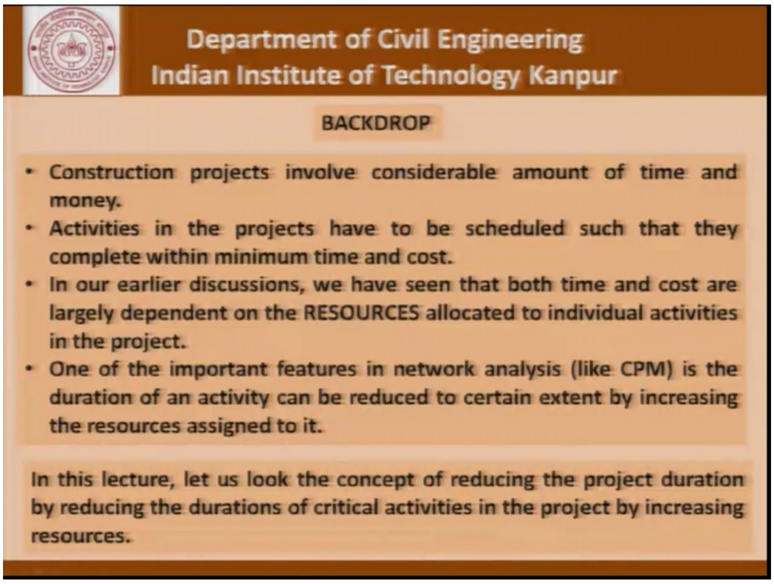

Now, what is crashing? Construction projects involve considerable amounts of expenditure in terms of time and money; activities in the projects have to be scheduled such that they are completed with the minimum time and cost. In our earlier discussions, we have seen that both time and cost are largely dependent on the resources allocated to the individual activities in the project. One of the important features in network analysis applied to construction management like CPM is that the duration of an activity can be reduced to a certain extent by increasing the resources assigned to it.

Now, in this lecture, let us look at the concept of reducing the project duration by reducing the durations of critical activities in the project, by increasing the resources allocated to them. What we had said last time, and I would like to retread that today, there are 2 kinds of activities let us say critical and noncritical. By doing re allocation of resources, leveling resources, we try to rationalize the resource deployment over the period of time and that is done with non critical activities, because the duration of the project is governed by critical activities. So, now, the discussion today is, is there a possibility of reducing the overall duration of the project, now, in order to reduce the duration of the project, obviously, it is not important to concentrate on the non critical activities, obviously, we must concentrate on the critical activities.

Meaning thereby, that if we want to reduce the duration of critical activities, obviously, there will be a requirement or a commitment of additional resources, which will cause additional expenses. What has to be seen is whether this additional expense of resources can be justified in terms of the benefits that may accrued to the contractor by finishing the project early, that is something which is relevant because as far as the contractor is concerned the project cost has a direct cost and indirect cost and so on. So, we have to see the overall perspective of balancing the additional resources, of balancing the expenditure in the additional resources and the benefits accruing there from. So, that basically is the essence of what we will talk about today and that is what is called crashing of network.

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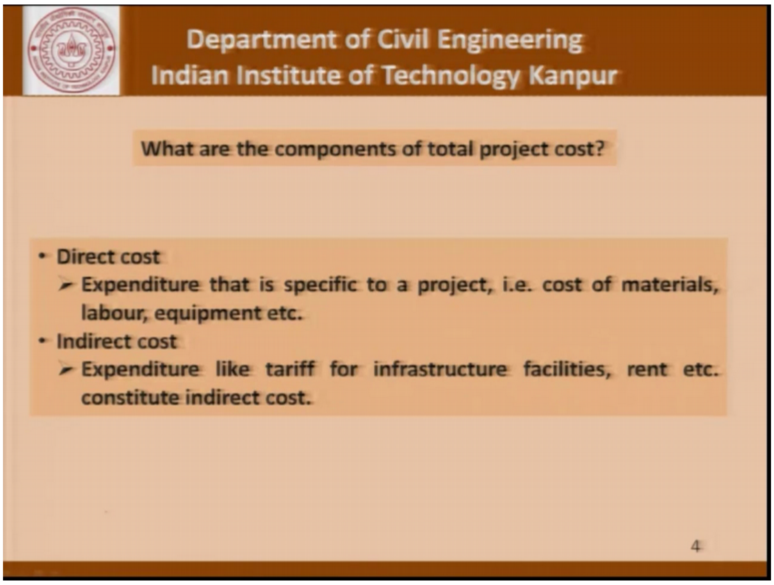

Now, what are the components of total project cost one component is, obviously, direct cost which comprises of expenditure, that is specific to a project, that is cost of materials, labour and equipment. Indirect cost includes expenditure like tariff for infrastructure facilities, rent and so on.

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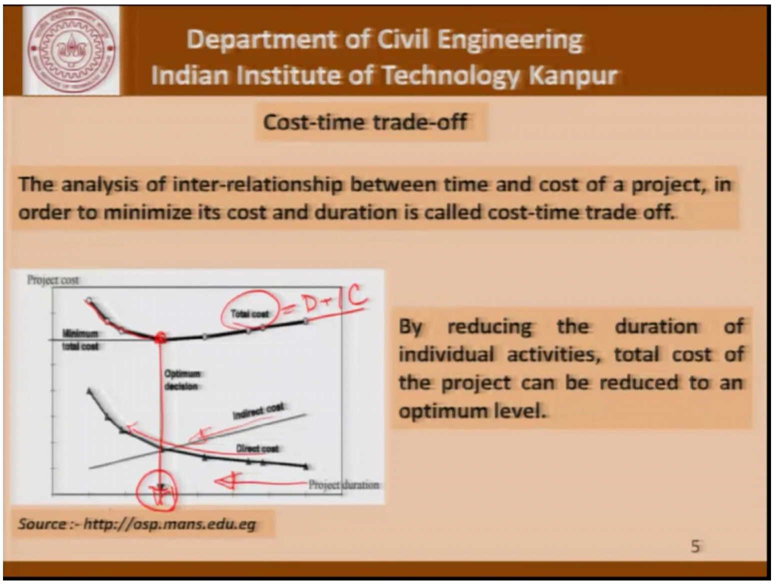

Now, cost time trade off is what we talk about really, the analysis of the inter relationship between the time and cost of a project in order to minimize its cost and duration is called the cost time trade off. This picture here is a schematic representation of what goes on, if we try to reduce the project duration about direct cost increases, but our indirect cost decreases. So, the total cost, which is essentially the sum of the direct cost and the indirect cost that is what has to be seen. So, it will reduce up to a certain point in time and then start increasing again. So, we should be targeting at getting to this duration of the project, in other words, by reducing the duration of individual activities the total cost of the project can be reduced to an optimum level.

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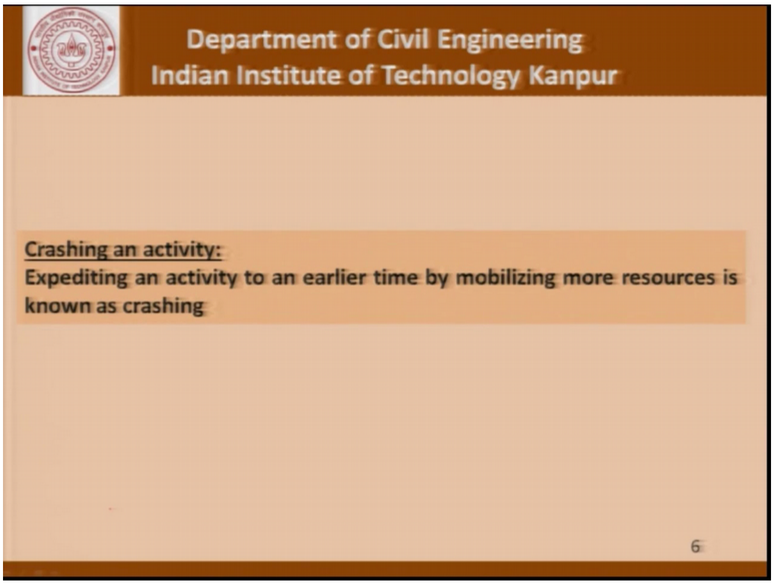

Now, let us technically define what is crashing. Expediting an activity to an earlier time by mobilizing more resources committing more resources to it is known as crashing.

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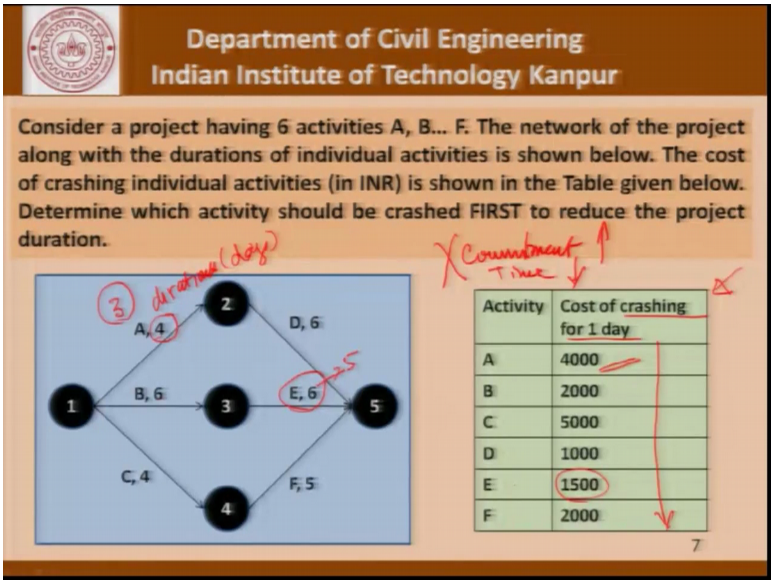

Now, let us look at an example, to explain this concept consider a project having 6 activities A, B, C, D, E, F and the network of the project along with the durations of the individual activities is shown here, the cost of crashing individual activities, it could be in an units, it is given in Indian rupees, here it shown in the table given below. Now, this is the table determine which activity should be crashed first in order to reduce the project duration.

So, suppose, we have this project has which 6 activities and these of the durations let us say that is given in days and this table gives us the cost of crashing for 1 day. So, if we want to carry out activity a in 3 days, it is possible even though it is 4 days here, it is possible to do it in 3 days provided we are able to mobilize additional resources of 4000 rupees. Similarly, activity E instead of 6 can be done in 5 days provided we are willing to spend another 1500 rupees. We must remember, that at the end of it, it is not possible for any activity to be infinitely crashed, that is we cannot say that if we are willing to increase the commitment of resources, indefinitely, that time can be reduced indefinitely. This cannot be done there may be minimum times for any of these activities beyond which no matter how we crush it, it will not work. So, there is a minimum time which is required, there is a reasonable time which we are willing to gave and beyond that reasonable time yes, it is possible to reduce it to that minimum time by putting in additional resources. So, this is the back drop of carrying out this exercise.

So, once we have this information listed with us, which is the cost of crashing for one day for the different activities, obviously, it make sense to begin with the activity which requires the minimum amount of cost. So, we should do that activity first which involves minimum additional commitment of resources.

The second part of it the discussion is that is that activity critical, will really making that commitment help as far as reducing the project duration is concerned that will happen only when the activities critical. So, the first thing to do perhaps is to find out which are the critical activities and then try to see what are the resources required if you want to crash those activities.

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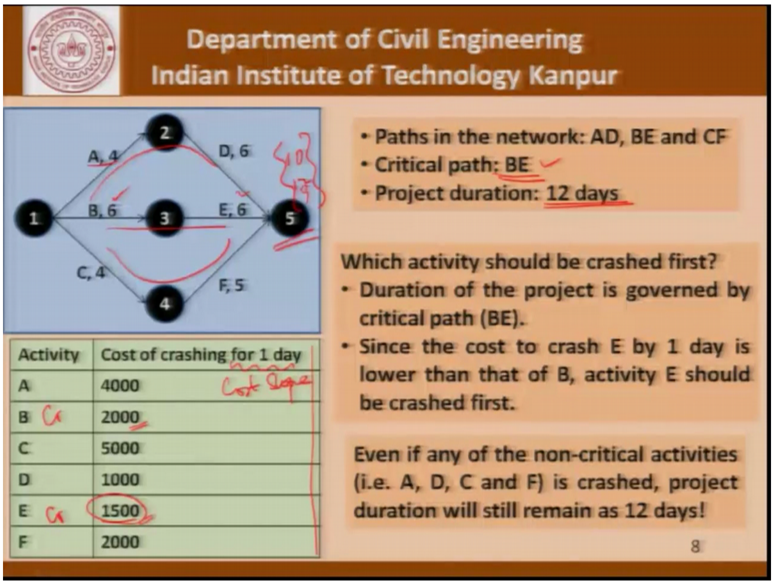

So, let us move forward and we do the network analysis, we try to find out that the paths in the network A, D, B, E and C, F. These are the 3 paths, in this network comprising of activities A, D on one side, B, E on the other and C, F is the 3rd and the critical path is B E, this takes 6 and 6, 12 days; this takes 10 days and this is 9 and we are looking at the maximum of this and therefore, the critical path is B E which is 12 days long. So, this project with the given information can be completed in 12 days at the certain cost. Now, let us try to see how we will use this information on crashing.

The first question to be asked is which activity should be crashed from our analysis given here? The duration of the project is governed by the critical path B E and since the cost to crash E by 1 day is lower than that of B activity E should be crashed first. So, we are looking at E is critical and B is critical to crash B requires 2000 to crash E it requires 1500.

And therefore, it make sense to crash E first and then, see how things change only to reiterate even if any of the non critical activities A, B, C and F is crashed the project duration will still remain 12 days and it makes no sense to do that.

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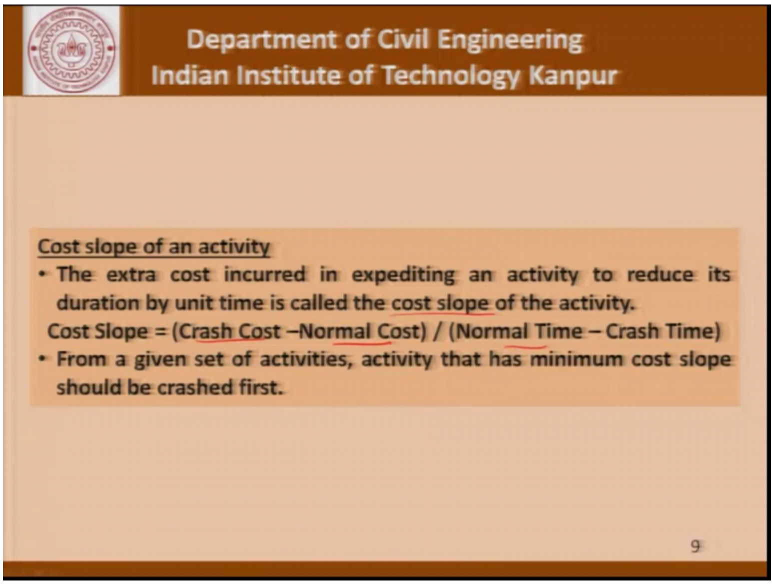

Now, let us try to formally define what is called the cost slope of an activity the extra cost in expediting an activity to reduce its duration by unit time is called the cost slope of that activity that is given by crash cost minus normal cost divided by normal time minus crash time that is the cost slope. In fact, in this table when the cost of crashing for one day was given what was essentially given is the cost slope of all these activities because what we mean by this is that this is the cost that is incurred for crashing that activity by a single day and; obviously, from the given set of activities the activity that has the minimum cost slope should be trashed first.

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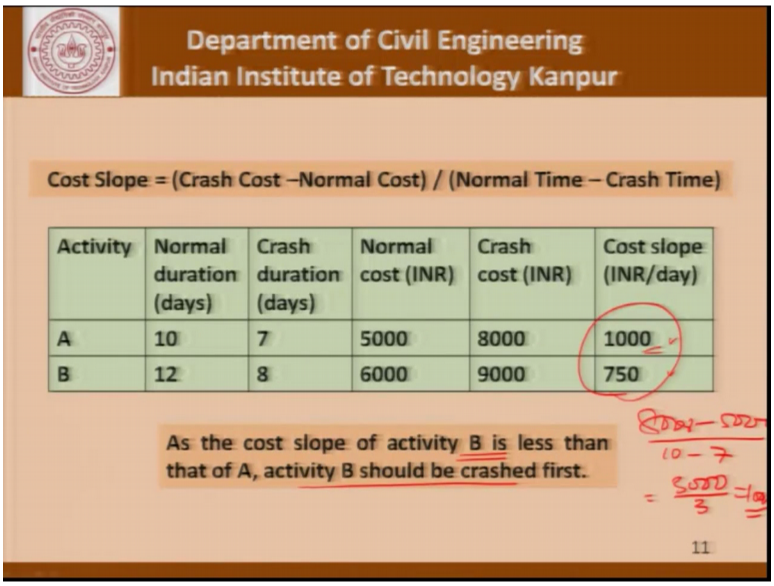

Let us consider two activities A and B the normal durations, the crash durations, normal cost and crash cost of A and B are given in the following table and if you were asked the simple question as to which of these should be crashed first, the analysis would basically be that A has a normal duration of 10 and a crash duration of 7, B has 12 and 8. So, these are essentially the minimum times that is required for the activities A and B. Beyond this, it cannot be crashed. Now, the crash cost which is the cost at the crash duration, this is not the cost slope. In the previous example, what was given was crashing per day, here, we are giving the crash cost, that is the cost at the crash duration and the normal cost is given here.

So, if you want to calculate the cost slope of these activities we have to carry out the small calculation crash cost minus normal cost divided by normal time and crash time difference and we get 1000 here and 750 here. This 1000 here is nothing, but 8000 minus 5000 divided by 10 minus 7 which is equal to 3000 divided by 3 and that I s equal to a 1000. Similarly, the cost slope for activity B is 750 given this data, activity B cost slope is lower and therefore, if it is required activity B needs to be crash first.

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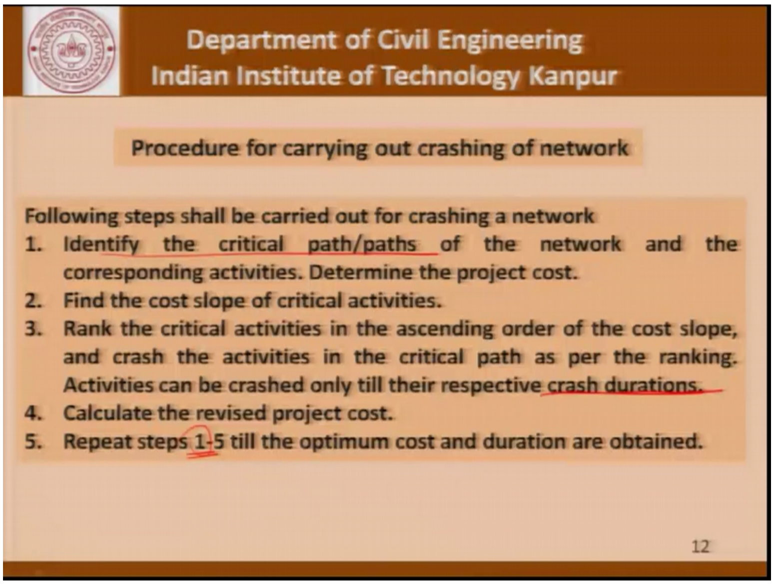

So, having done the simple example, we can lay down the protocol or the procedure to be followed for crashing a network. The first step is identify the critical path or paths of the network and the corresponding activities, determine the project cost, find the cost slope of critical activities, rank the critical activities in ascending order of the cost slope and crash the activities in the critical path as per the ranking; activities can be crashed only till their respective crash durations. Obviously, we cannot try to crash them beyond that crash duration.

Calculate the revised project cost, repeat steps 1 to 5 till the optimum cost and duration have been obtained. Remember, that what we are ask to do is repeat steps 1 to 5, what is 1, 1 is to find out the critical path having done, the critical path first or having identified the critical path first, but having change the durations of those activities by crashing and so on. It is likely that some other activities of some other paths would become critical and therefore, as we crash activities in a stepwise manner it is important that at each step, we check if other activities have become critical or not because if they have, then the cost slope there also will become important this something which you have to keep in mind as we do an actual example and that is what we will do.

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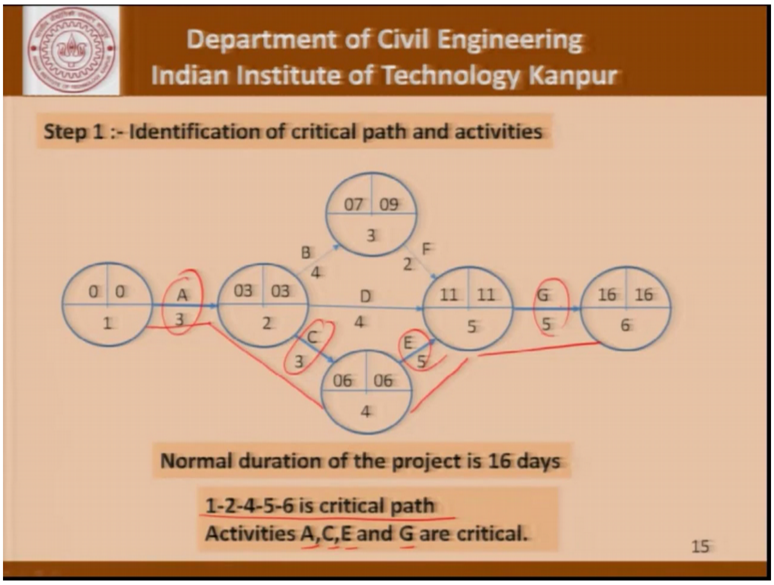

We will consider a project having 7 activities as shown below for that dependences, normal durations and crash durations of activities in days and the event times are shown in the network. So, this number here is Tn that is the normal time, this is the crash time which is Tc.

So, with this information about the crash time and the normal times given for all the activities, we will proceed to find out what activities to crash, what not to crash and so on and we are also given the information, that the indirect cost as far as the project is concerned is 6000 rupees per day.

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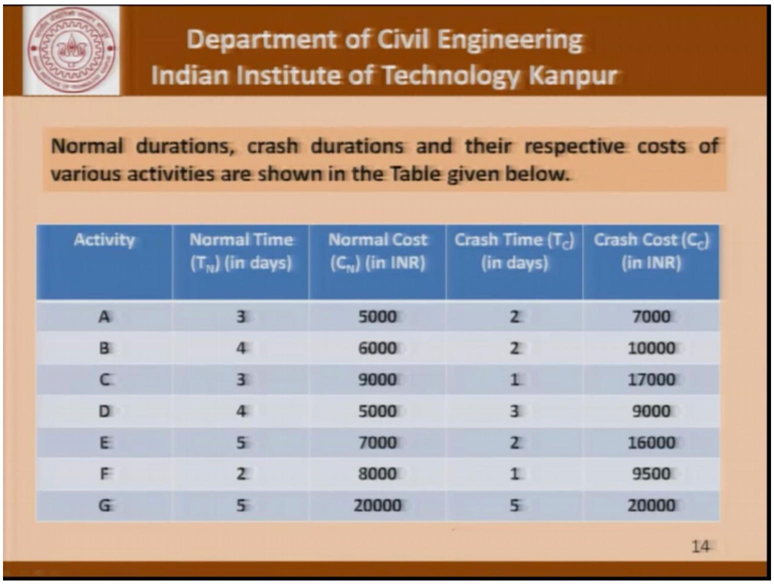

Activity	Normal Time (T_N) (in days)	Normal Cost (C_N) (in INR)	Crash Time (T_C) (in days)	Crash Cost (C_C) (in INR)
A	3	5000	2	7000
B	4	6000	2	10000
C	3	9000	1	17000
D	4	5000	3	9000
E	5	7000	2	16000
F	2	8000	1	9500
G	5	20000	5	20000

So, this table here in addition to the normal times and the crash times it gives you the information about the normal cost and the crash cost for all the activities. So, if we look at A for example, the normal times for 3 and 2, as I explained in the previous slide and the crash cost is 7000 and 5000. So, that analysis we can do for all the activities and what we proceed to do is the first step, which is identification of the critical path and activities.

Now, this we are already familiar with and I am not going to spend time on this, we find that the normal duration of the project is 16 days and 1, 2, 4, 5, 6 is the critical path, that is our critical path runs like this 1, 2, 4, 5 and 6. So, the activities A, C, E, N, G are critical. So, there is activity A, C, E and G these are the critical activities, it s not a very difficult network; I am sure, you can find out that there are other paths, but they are not critical. So, now, as far as the project cost is concerned, at this point in time assuming that the activities will take place at their normal durations, will have the total cost which is the sum of all the activities that is activities A plus B plus C and so on write up to G. The sum of the normal cost will give us the direct cost of the project and that in this case turns out to be 60000 INR.

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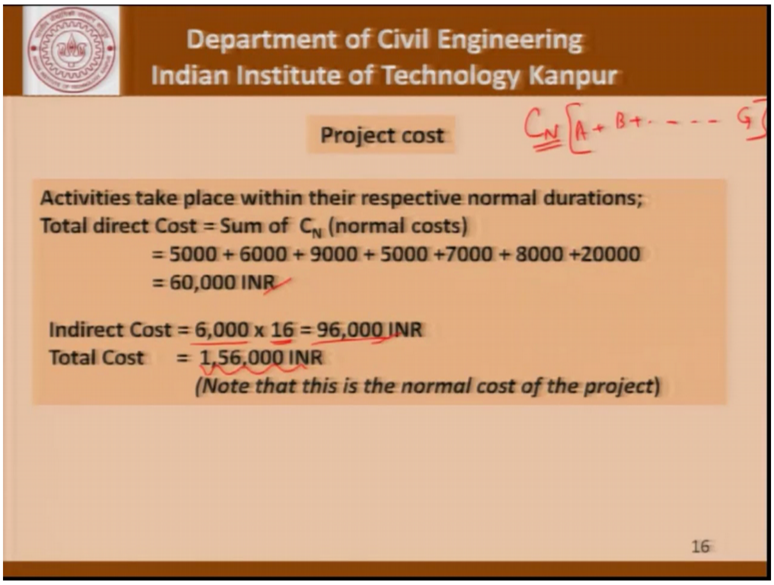

As far as the indirect cost is concerned, we have the information that it is 6000 rupees per day and 16 being our duration of the project, the total indirect cost is 96000 and we have the total cost as the sum of this plus this which is 156000 INR. This of course, is this normal cost of the project.

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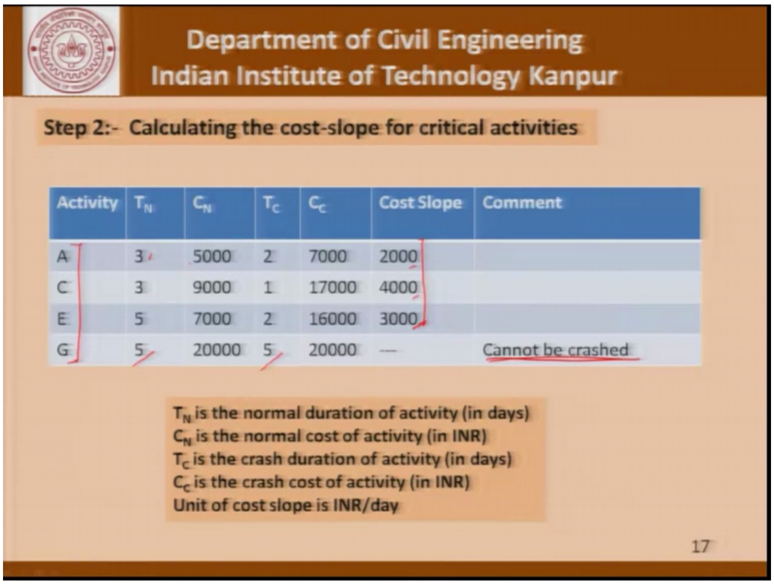

Now, let us try to move forward and try to do the crashing business. We know that activities A C E N G are the critical activities; from the information given Tn and the Tc as for as the activity G is concerned is both the same, which means that this activity simply cannot be crashed, it will take that amount of time. As far as other activities are concerned, A C and E, they can be crashed by different amounts; E can be crashed by 1 day, C can be crashed by 2 days and E can be crashed by 3 days, provided we are willing to put in a certain amount of additional resources and that is where we have to find out the cost slope. The cost slope for each of these activities is 2000, 4000 and 3000 calculated by dividing the difference in cost by the difference in time. So, once that is done.

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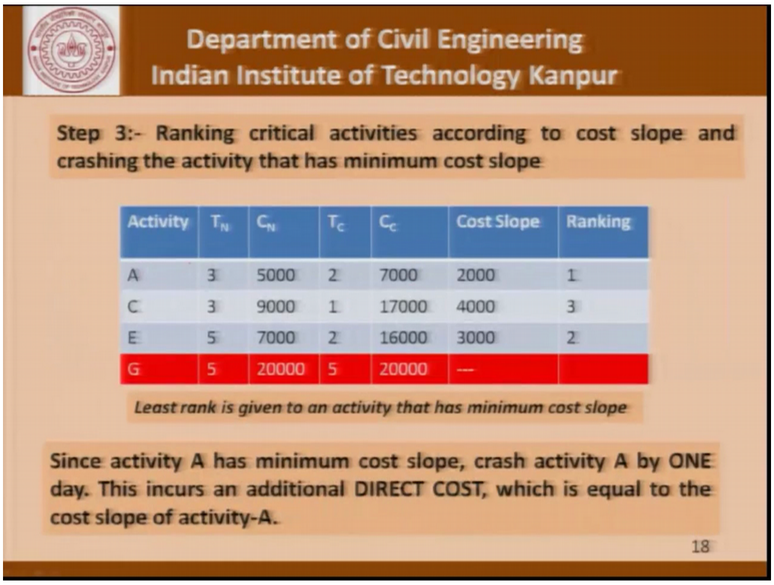

We know that if we rank these activities A C and E, A has the minimum slope; then, it comes to E followed by C. So, since a has the minimum cost slope let us try to crash activity A by 1 day and this requires an additional cost in terms of direct cost which is equal to the cost slope of activity A, but it also results in a saving of 1 day which is equal to the indirect cost of the project itself.

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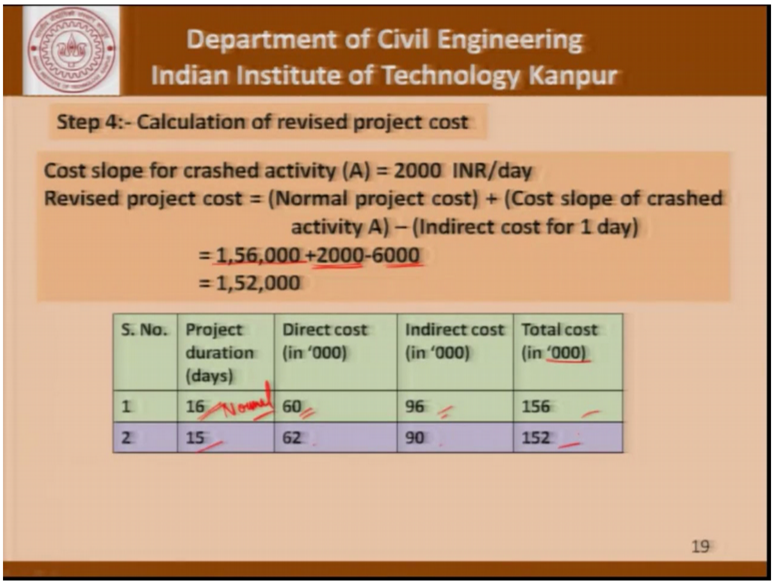

Which means, that as far as the revised cost calculation is concerned, the revised project cost becomes 156000 plus 2000 which is the cost slope of activity A minus 6000 which is the indirect cost for 1 day and that is 152000 which is lower than 156. So, basically, we have a table like this where, we say that if you are completing the project in 16 days which was the normal duration, the direct cost was 60, indirect cost was 96 and the total was 156 in thousands. Now we have crashed the project by changing the duration of activity A, 215 in the direct cost and indirect cost the sum of this is 152 which is 4000 lower than the previous total cost. So, this obviously, is a better solution, but now can we find an even better solution is what is the next step.

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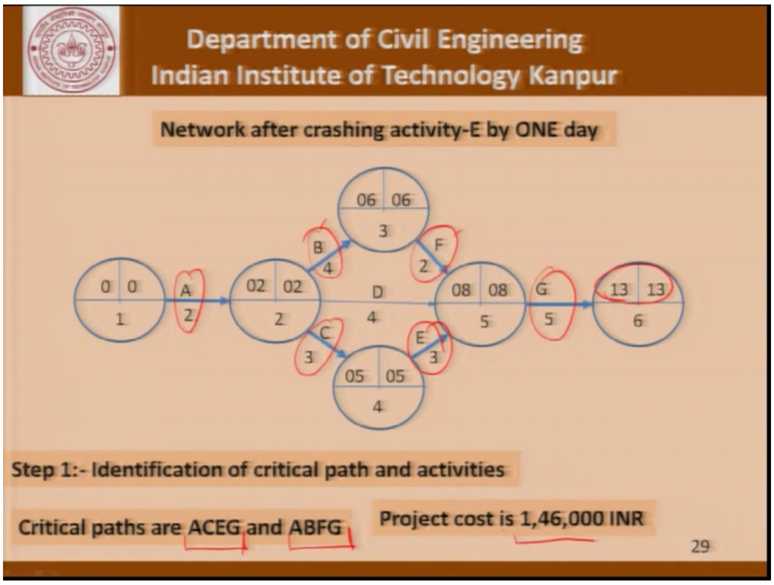

But before that we need to modify our network to check if there are any changes in the critical activities.

So, if we redraw this network and say that instead of 3 this activities been completed in 2 days, we find that there is no change in the critical path in the network, which continues to be having activities A C E and G; even though, the project duration is come down to 15 days and the cost has come down to 152000.

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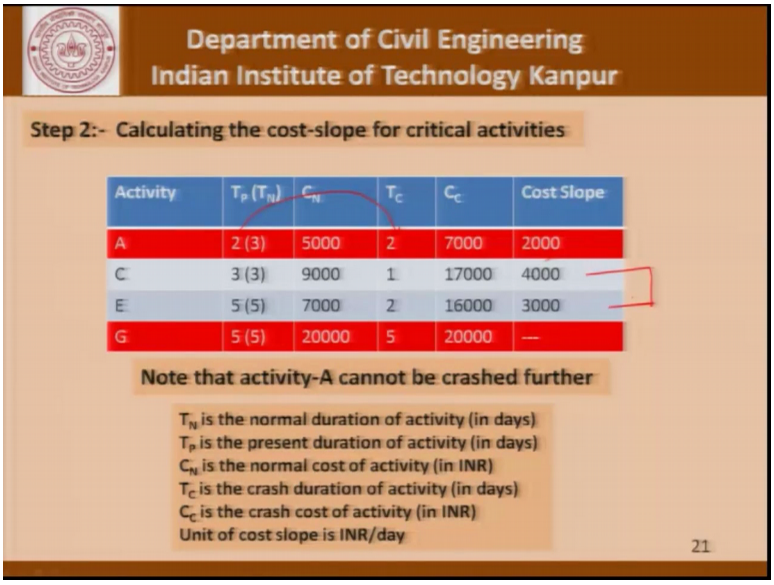

So, let us go to the next step, which we did last time that is the cost slope analysis. We find that even though, A has the minimum slope, but it cannot be crushed any further because it has already reached the crash duration. So, our choice is limited to C and E and between these two.

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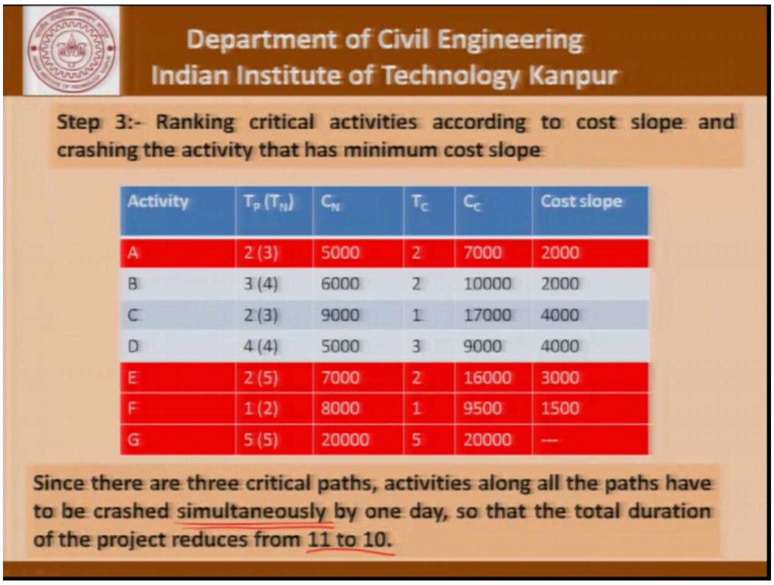

Obviously, we must crash E by 1 day and see what happens to the direct indirect and the total costs.

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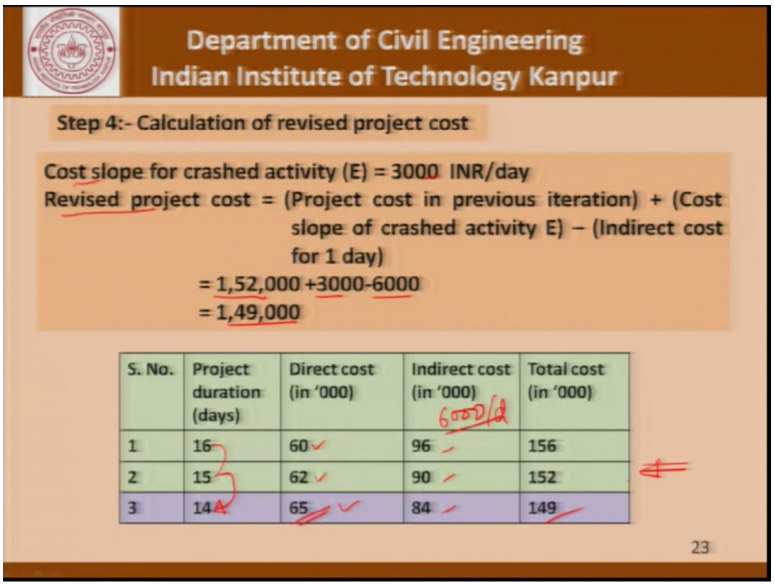

So, that analysis is shown here, given that the cost slope of activity E is 3000; the revised project cost becomes 152 plus 3 minus 6000 and we have a lower number, that is the 149000, which means that we revise the table that for shown last time, in this from now that from 16 to 15 and now to 14; the direct cost has increased from 60 to 62 and now to 65, but the indirect cost has reduced by 6000 for everyday of reduction that were able to achieve and we are now working at 149. The issue is can we do more, let us try to now identify if there are any changes in the network, if you do this we still find that the critical path is A C and G and we are free to go ahead and look at the cost slope of the critical activities.

So, we find that we can still crash activities and 4 can still become 2, now this is the revise network after this revision. We have crashed A from 3 to 2 and now, we have crashed E to 4 days and the project duration has become 14 and the cost is 149 and if we do the network analysis at this point in time, we still find that the critical activities are A C E and G. So, we go back to this table, which gives us the cost slopes and we try to crash E by 1 more day.

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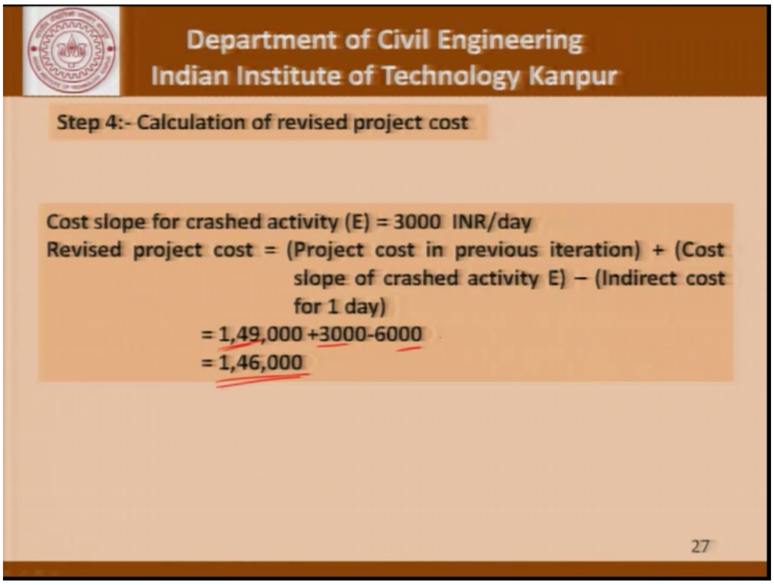

Once we do that we come to a total cost of 146; and 149 was the previous cost at the cost slope of activity E and subtract the indirect cost. So, once we come to 146 we go back to this table and we find that this 146 is still lower than the previous iteration.

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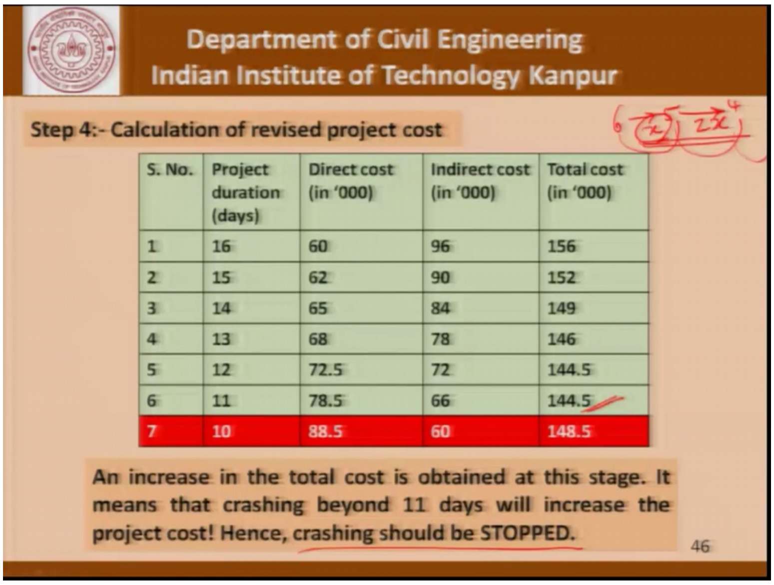

S. No.	Project duration (days)	Direct cost (in '000)	Indirect cost (in '000)	Total cost (in '000)
1	16	60	96	156
2	15	62	90	152
3	14	65	84	149
4	13	68	78	146
5	12	72.5	72	144.5
6	11	78.5	66	144.5
7	10	88.5	60	148.5

So, we perhaps have still some room for doing more crashing. So, we come back to this picture here, now we are begin with the project duration of 13 and the durations of the activities have already been adjusted here. We now, find that the critical paths are A C E G and A B F G. So, A AC E G is not the only critical path, there is another critical path A B F and G. So, activities B and F have also now become critical. So, if we bring it to 13 days, now if we want to reduce the project by 1 more day, we have to ensure that we also account for the cost slopes in activities B and F and then, try to see if we can get a better cost than a 146000 and order to do that, we come to this table where, we have now included activities B and F, earlier we are working only with C and E.

Now, we are working with B and F, of course, A and G are out of the reckoning having reached the crash durations in the first iteration itself. Now, if we look at these cost slopes, we cannot really proceed as simply as we did in the previous case and the reason is the following. There are 2 critical paths and the activities along both paths have to be crashed simultaneously by 1 day and only then the total duration of the project will reduce from 13 to 12.

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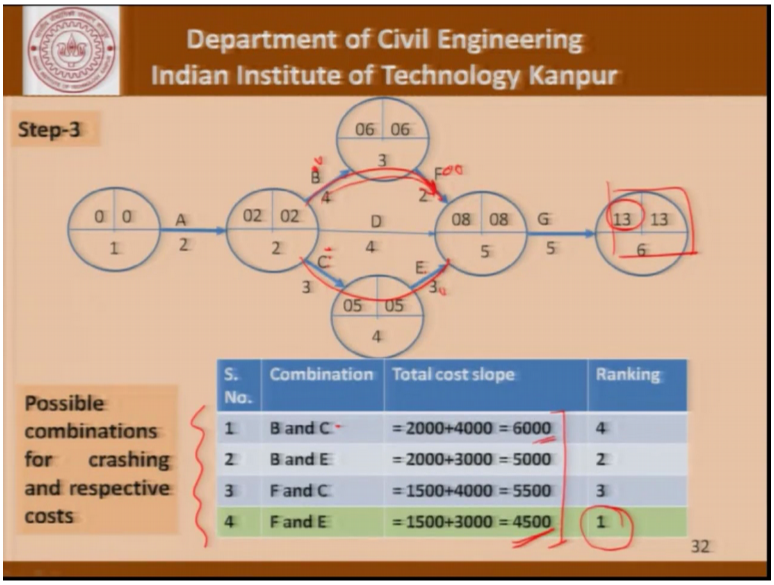

What I am trying to tell you is that, if you look at this project, the way it is shown, there is this path here, which is C and E; there is this path here which is B and F. It makes no sense to just crash an activity whether it is C or E based on its cost slope because then, this path here will still remain critical and we will not get any reduction in the project time.

So, in order to get a reduction in project time, we have to find a combination it could be B and C which is this and this it could be B and E, it could be F and C, it could be F and E. So, any of these 4 combinations, if the cost slope that is the additional cost that is incurred in changing the duration of the activities by 1 day, the sum of that has to be considered and that is what has been done here. So, B and C gives a total cost slope of 6000, B and E gets 5000, 5500 here and 4500 here and this, obviously, then ranks highest that is the cost slope of crashing F and E together is 4500 rupees.

So, if we do that, then we will be able to reduce the project by 1 day. In other words, what we will do is will have this 146000, we will add to it not the cost slope of an activity, but 2 activities that is E and F and subtract the indirect cost for 1 day and we get a number 144500 which is still better than a 146000.

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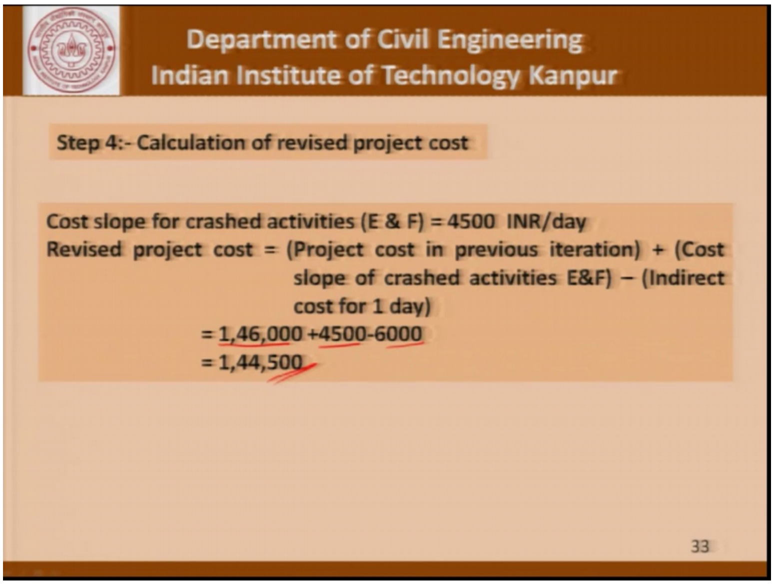

So, if we look at this table now, from 16 days which we started, we have come down to 12 days and we have reduced the total cost from 156 to 144.5. Eventhough, we had to increase our direct cost from 60 to 72 and a half. So, the network after crashing E and F by 1 day looks like this and you would now, like to find out if it is possible to crash it any further; of course, the critical path continue to be having activities A C E G and A B F G, only this activity still continues to be non critical as far as this project is concerned.

Now, if you look at this table, to carry out the cost slope calculations, we find that activities A E F and G have all reached their critical times and therefore, the only possibility that we have for crashing is activities B and C and since, there are 2 critical paths activities along both the paths have to be crashed simultaneously by 1 day and only then we will get a reduction from 12 to 11. Now keeping this in mind, B and C combination has a total cost slope of 6000. Please note that activities A E and F cannot be crashed any further and in that case the revised cost becomes 144.5 plus 6 which is the slope of the crashed activities and minus 6000 which is the indirect cost for 1 day and we find that there is no change in the cost of the project. So, what we have reached now is a Plato here 144.5 to 144.5. And even though, we have reduced the project duration by one day.

In other words, we would still like to crash the project because for the same cost, we are able to complete the project one day earlier. So, this is definitely a better option than the previous one that is trying to complete the project in 12 days at the same cost and we basically say that the project cost is 144.5 thousand with these critical paths; please also take note that once we have reduced the project duration to 11 days, D also has become critical 2 plus 4 plus 5 is 11; 2 plus 3 plus 1 plus 5 is 11; 2 plus 2 plus 2 plus 5 is 11.

So, all the activities basically, as far as this network is concerned are now critical activities and there is no float in these activities, we can complete the project in 11 days at the cost of a 144.5 thousand, but of course, we should try to examine whether or not we would like to crash the project further if the activities can still be carried out in a shorter time, for that we go back to this table. We now find B C and D are the activities which are candidates for crashing and we find that yes there is a scope because you we are still working with 3 2 and 4 as Tp for the activities B C and D which is higher than there crash durations and these are our cost slopes with this information, we try to find out what is the actual cost in terms of crashing the network by 1 day which would involve all the paths.

Now, again the discussion that the activities along all the paths have to be crashed simultaneously and only then, that duration of the project can be reduced from 11 to 10. This takes us to the discussion that what are the possibilities B C and D have all to be crashed by 1 day and only then, we will be able to get the project to 10 days. What is the cost of crashing all these 3 activities? The cost is 10000; now obviously, this 10000 is greater than the saving of 6000 and therefore, we expect the cost be higher. Let us do that and formally close this discussion, 144.5 plus 10000 minus 6 thousand gives you a higher total cost than what you had when you are completing the project in eleven days.

So, this is a project cost in 11 days this the project cost in 10 days. So, if you put it back in a table we find that a 144.5 has now become a 148.5 and therefore, it is a case that crashing may be stopped at this point because we have already reached the optimum. We have now a started increasing the cost of the project even though, the activities are still crashable that is theoretically, if we want we can complete the activities in a shorter duration than what is being planned, but that will involve cost to the extent that the total cost of project will increase. We must also remember that in this model, what we have done is we have assumed that the cost slope is constant from one day to another; it is always a possibility that there is an activity which from 6 days to 5 days cost some x, but if you try to crash it from 5 to 4 it might cost 2 x, this is the kind of thing which we have not done in this discussion.

What we have assumed it whether you are crashing the activity from 6 to 5 or 5 to 4 of possibly 4 to 3, the crash cost is the same or the cost slope is still the same, that is something which we can do as an exercise try to change the cost slope and see how it works, but the algorithm that has been laid out, that is we are trying to understand, re calculate the cost slopes that enables us to take care of any change in the cost slopes should that information be made available to us.

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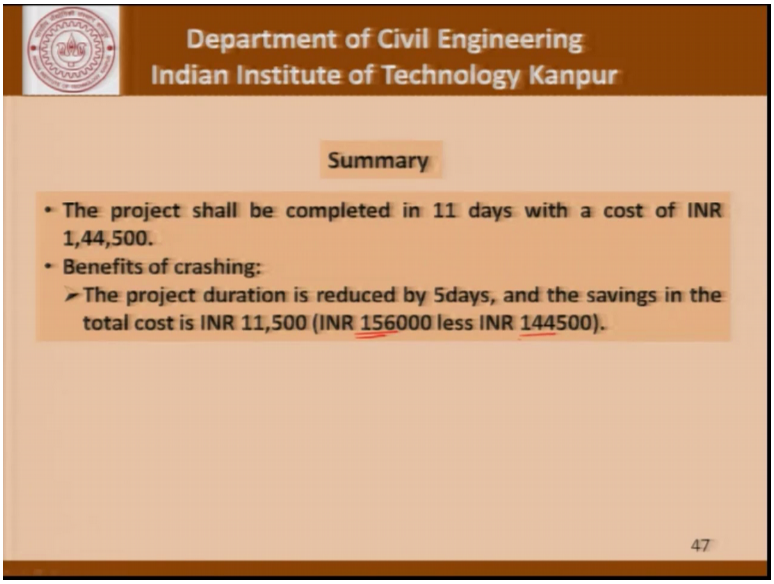

So now, with this here is the summary of a our discussion the projects shall be completed in 11 days with the cost of 144.5 and the benefit has been that we have been able to crash from 156 to 144.5 and reduce the duration by 5 days, we shall be started with 16 and now we are able to complete the project in 11. So, it is a win-win situation, it is a shorter project duration and a cheaper cost.

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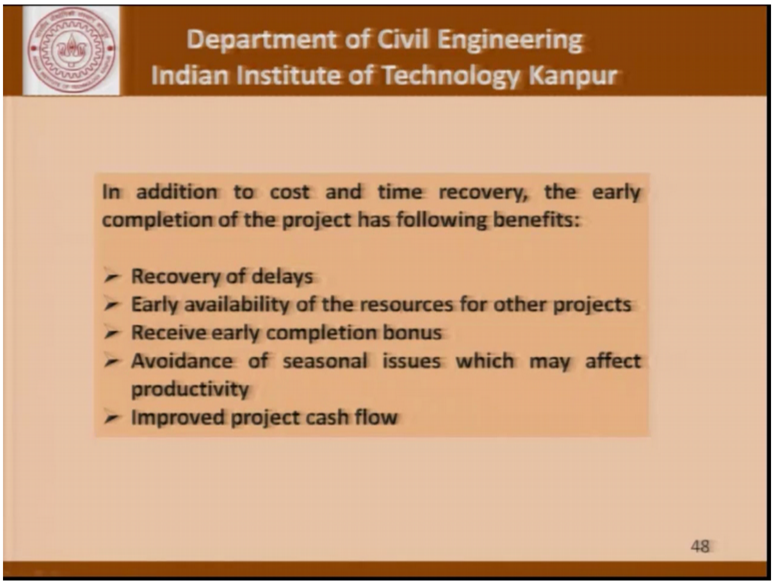

So, in addition to the cost and time recoveries, the early completion of the project has also got the following benefits; recovery of delays, early availability of resources for other projects, receive early completion bonuses if that is applicable for a project, avoidance of seasonal issues which may affect productivity at a later point in time, improve project cash flows.

So, with this we come to an end of our discussion today and as usual this is the list of references which might help you understand the subject a little better and I look forward to seeing you at another lecture.

Thank you.