Principles of Construction Management
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Lecture – 17
Uncertainities in duration of activities - Using PERT in Scheduling –

[FL] and welcome to this series of lectures on Principles of Construction Management and in this lecture today we will talk about uncertainties in duration of activities using the pert in scheduling.

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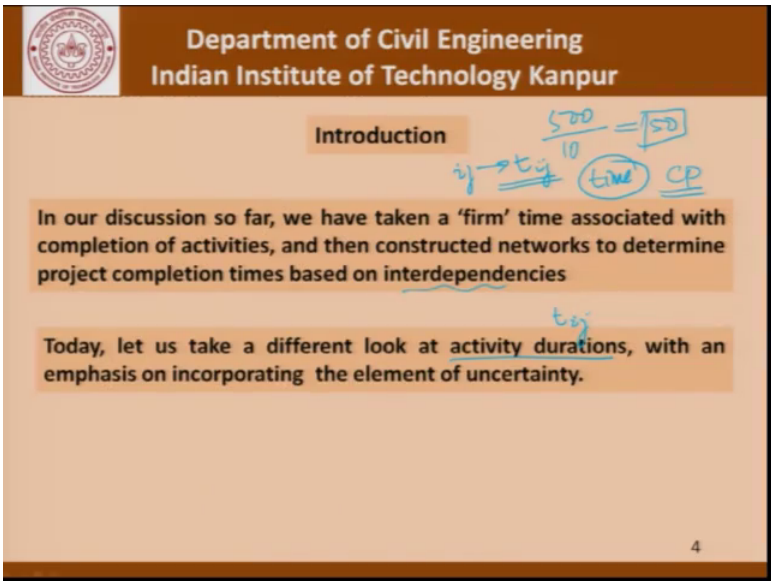

So far what we have said is that the time of completion of an activity is firm that is it is a given number and then we have constructed networks to determine project completion times based on interdependencies. So, you would recall that in a first example when we said there was 500 man days involved then we divided this by 10 which was a number of people and came up with 50 days of activity time. And then we have also constructed in the last class slightly different network more elaborate network and illustrated the concept of critical path that is that path which needs to be monitored more regularly, more continuously than the others and no slippage can be allowed.

But in any case when the interdependencies were taken into account that time associated was taken to be firm an activity ij it was known that this will be completed in a time of t ij. Now today what we will do is we will take a slightly different look at the activity duration that is this t ij with the emphasis of incorporating the element of uncertainty. How do you incorporate a situation when the t ij is not a firm number, but more like a probability distribution and activity may be completed in 10 days, but it might bellower to 12 days, if something could happens if we are lucky then it might be completed in just 5 to 6 days. So, this idea that the time associated with each activity is not necessarily a fixed number that is something which we will focus on today in our discussion.

So, how do we actually go about estimating the times for the different activities?

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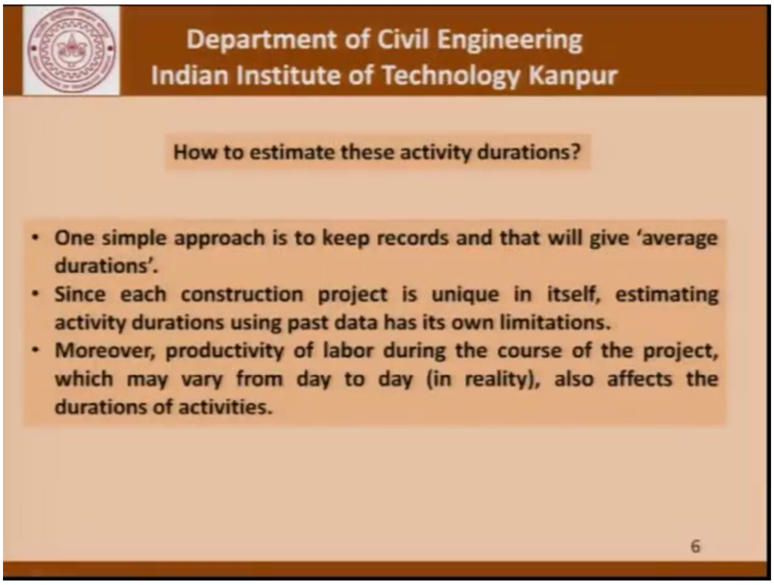

One simple approach is to keep records that will give us the average durations. So, a construction company keeps a record for erection of process and so on and finds out that in different projects what is the kind of time that it takes and from there we try to find out the productivities and then try to map that into a new project and try to estimate the time involved for that project.

We have talked about in the initial part of this course that each construction project is really unique. So, the bridge built at a particular place is surely different from may be a similar bridge, but built elsewhere. Also the productivity of labour during the course of the project changes and that could also affect the duration of activities. So, if there is an activity you put a gang of labour or a certain set of workers on day one, they may not be able to produce the maximum output, but has they learn the activity as they learn their role in the whole process their productivity improves. So, how do we keep track of these kind of things.

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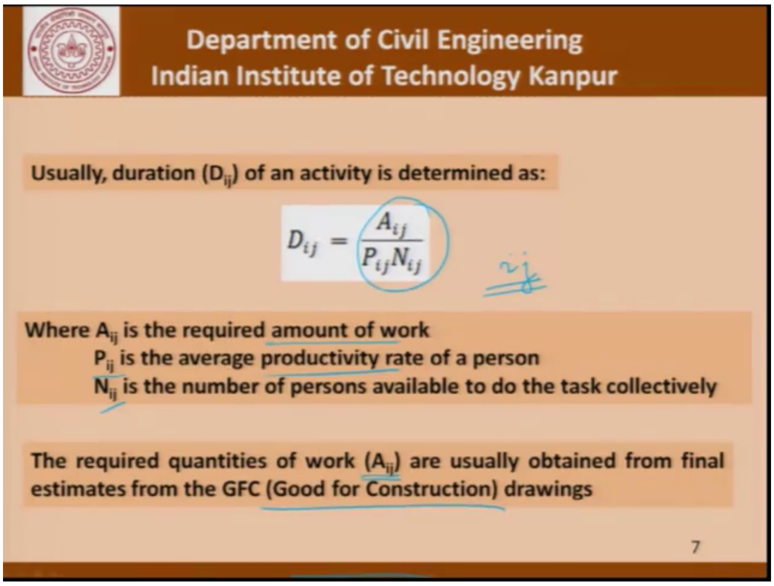

Going back to the basics usually the duration D ij of an activity is determined using this formula which is A ij upon P ij multiplied by N ij, where ij is the activity that we are talking about. A ij is the required amount of work, P ij is the productivity of the person which could possibly take care of the learning curve and so on, and N ij is the number of persons working on that job independently and together.

So, with this we can find out the D ij, but each of these parameters has their own variations for example, as far as A ij is concerned the only estimate that we can make from is the good for construction drawings it might happen, that there will be some changes in the total amount of work at site whether it is paid for or not it does not matter, but the fact is that if additional work is carried out or less work is carried out then it will affect the time duration.

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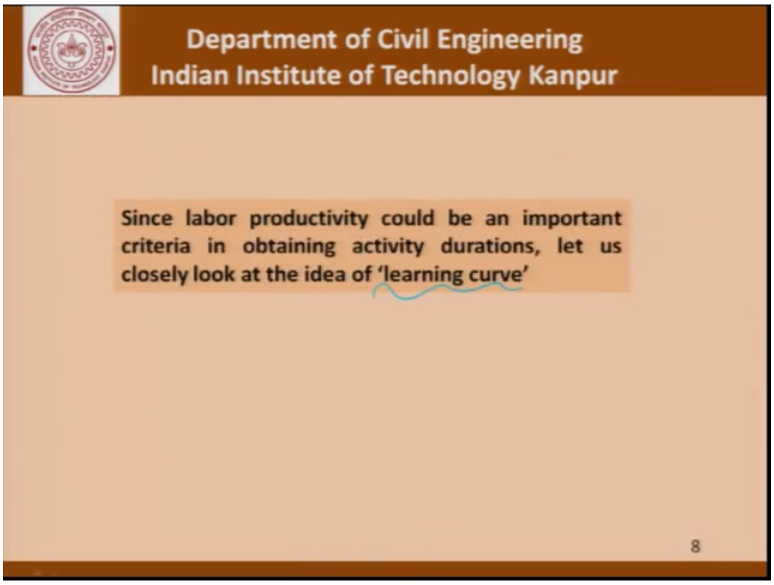

So, we coming to the next part which is labour productivity, this could also be an important aspect of determining the total time involved and that is where the learning curve becomes important.

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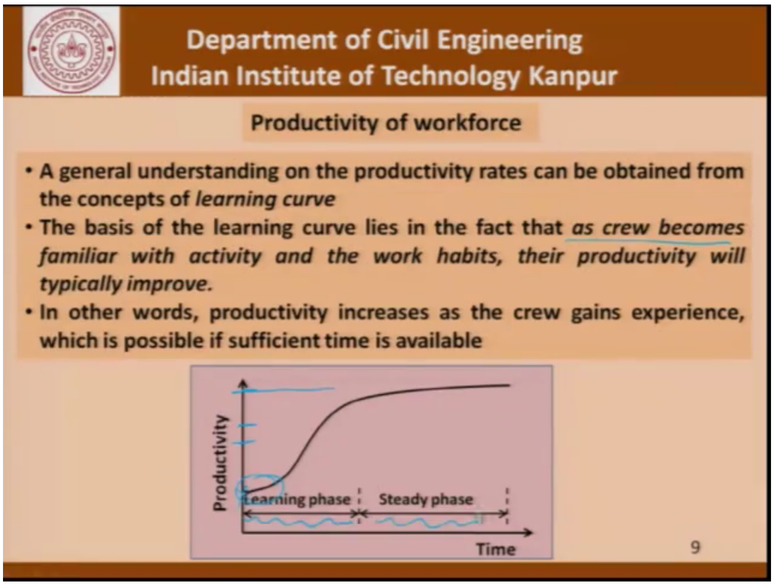

So, what is the learning curve? A general understanding of the productivity rates can be obtained from the concept of learning curve and the base of the learning curve lies in the fact that as crew becomes familiar with the activity and the work habits their productivity tends to improve of course, there is a limit beyond which it does not improve, there is a limit to the capacity of each worker. This capacity may be different for different workers, but sure enough there is a starting point and it goes through a learning phase and a study phase for each worker.

In other words productivity increases as the crew gains experience which is possible if sufficient time is available and most construction projects that is in (Refer Time: 05:32) typically the case.

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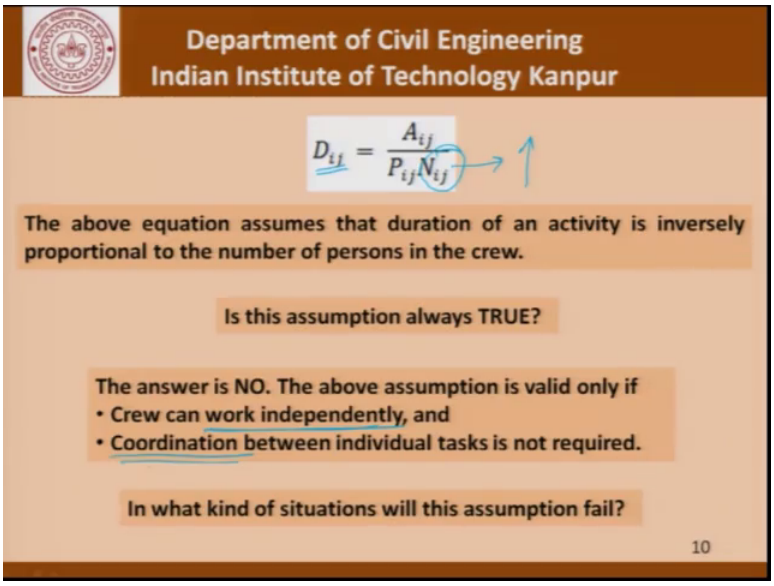

So, now another thing is the number of workers this equation here assumes that the duration of an activity is inversely proportional to the number of persons in the crew or the number of workers in the team. But is this assumption absolutely true is it true in the absolute sense, if we keep increasing this N ij is there no limit for the reduction in the duration that we get. There are two reasons why this assumption is not really true - one is the crew may not be able to work independently in a certain construction site there is a limit to the number of workers who can be accommodated without interfering with each other’s productivity. So, that is one part of it.

The second thing is coordination between individual task has to be ensured and that also becomes difficult. We all know the saying that too many cooks spoil the broth. So, if there are too many people trying to an activity that also has an adverse effect on the time duration. Apart from the workers working independent the coordination between individual task and the individual crew members has to be ensured.

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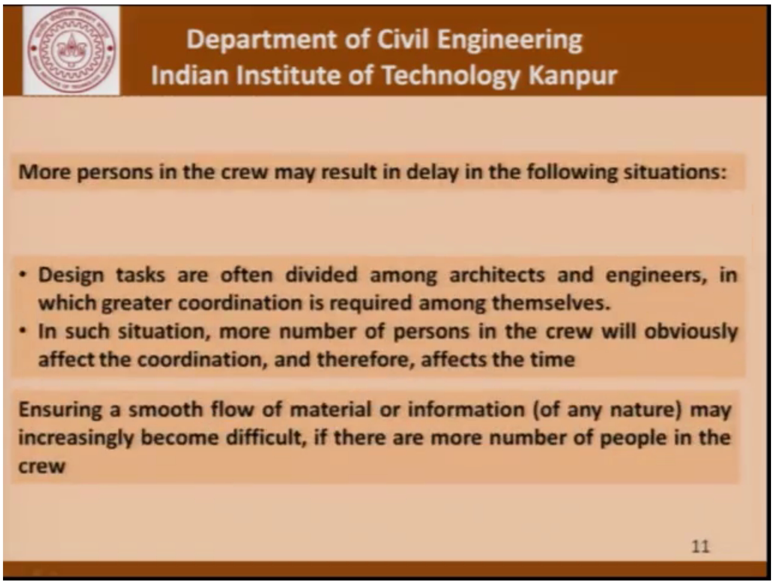

More persons in the crew may result in the delay in the following situations. An example is design task are often divided between architects and engineers in which greater coordination is required among themselves in such situations more the number of people in the crew will obviously, affect the coordination and therefore, it affect the time ensuring a smooth flow of material or information of any nature may become increasing the difficult if there is more number of people in the crew.

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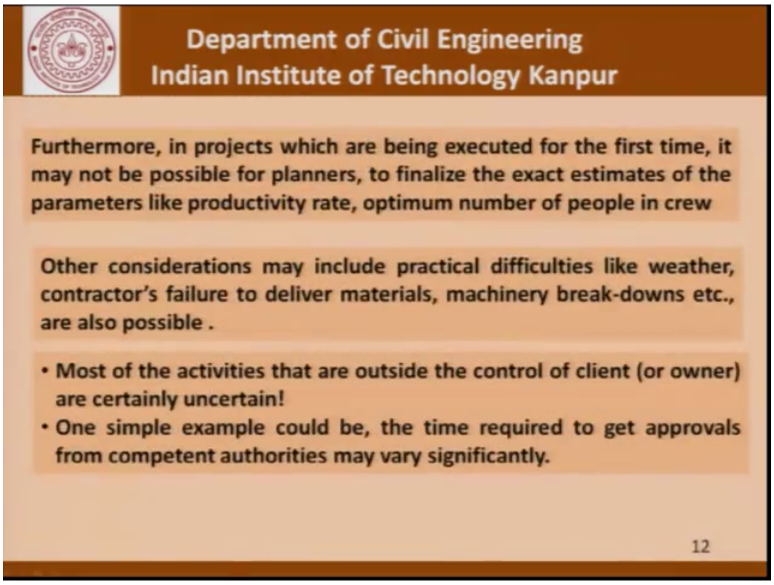

Further in projects which are being executed for the first time it may not be possible for planers to finalize the exact estimates of the parameters like productivity rates optimum number of people in the crew. Other considerations may include practical difficulties like the weather contractors failure to deliver materials, machinery breakdowns and so on. Also most of the activities that are outside the control of the client are certainly uncertain.

One simple example could be the time required to get a approvals from outside agencies and that could affect the project in different ways. What this competent authority means is for example, if there is a project which requires an approval from a pollution control board and such regulatory authorities that might be not within the control or within the powers of the client to be able to implement and that introduces an element of uncertainty as far as the completion of the project is concerned. And its not only the project, but also it has the affect; obviously, on individual activities and those individual activities in turn affect the project.

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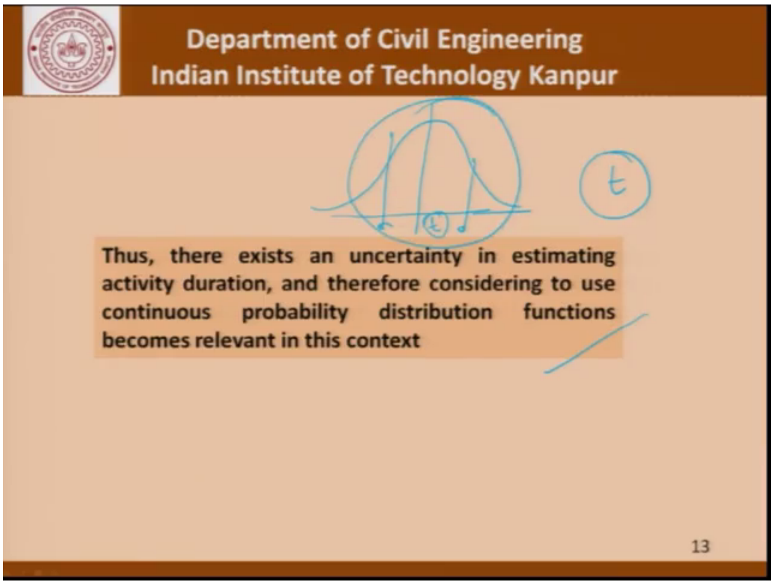

This underlines the need to understand that there exists an uncertainty in estimating activity duration and therefore, considering to use continuous probability distribution functions becomes relevant in this case. So, what we are saying in this sentence is instead of assigning a particular time t for any activity can we now talk in terms of a probability distribution, that is yes this activity will take some time, but that time could be may be little less may be a little more. So, whether this distribution should be normal or not is the next question that we need to answer. So, without going into a statistic of the whole issue let us try to understand the basic story of the normal distribution.

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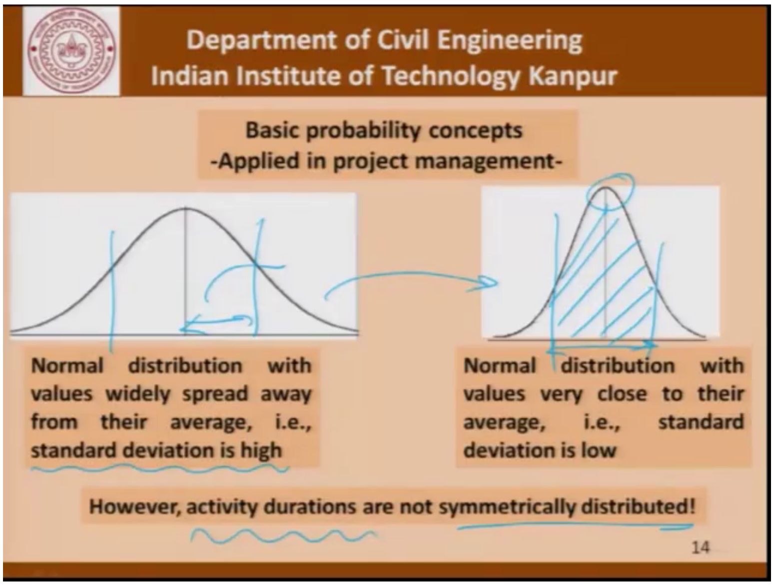

The normal distribution with values widely spread away from the average is the situation where their standard deviation is high. Compared to this picture this picture here is a situation where the peak is sharper which means that the area in this part is closer to the mean, if you want to go to the same area here we will go more standard deviations away from the mean; however, in both cases the normal distribution is symmetric.

So, the problem with our construction activities is that they are not necessarily symmetrically distributed.

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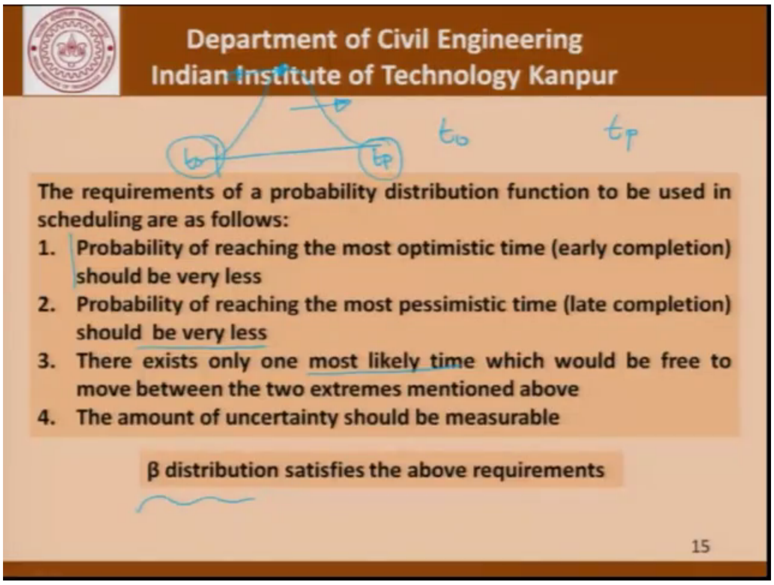

So, now what are the requirements that we need to impose in determining or finding the kind of distribution that helps or say that yes this is what really meets our requirements. The probability of reaching the most optimistic time which is early completion should be very less. So, this is the situation where we become lucky and we are able to achieve the target very quickly the probability of reaching the most pessimistic time should be very less that is there should be time where we will definitely be able to complete the project under most adverse conditions. Then there exist only one most likely time which would be free top move between these two extreme conditions.

So, what we are saying is that there is an optimistic time t naught and there is a pessimistic time t p between this t naught and t p we want to define a distribution which is not normal. So, either this most likely time will move towards the left in which case the pessimistic time becomes an outlier or it will move towards the right in which case the optimistic time becomes an outlier. So, this amount of uncertainty should also be measurable. So, once we understand these requirements we find that the beta distribution that is used in statistics satisfies these requirements for us.

So, we can use the properties of the beta distribution and move forward as far as determining the expected times of the activities and so on is concerned.

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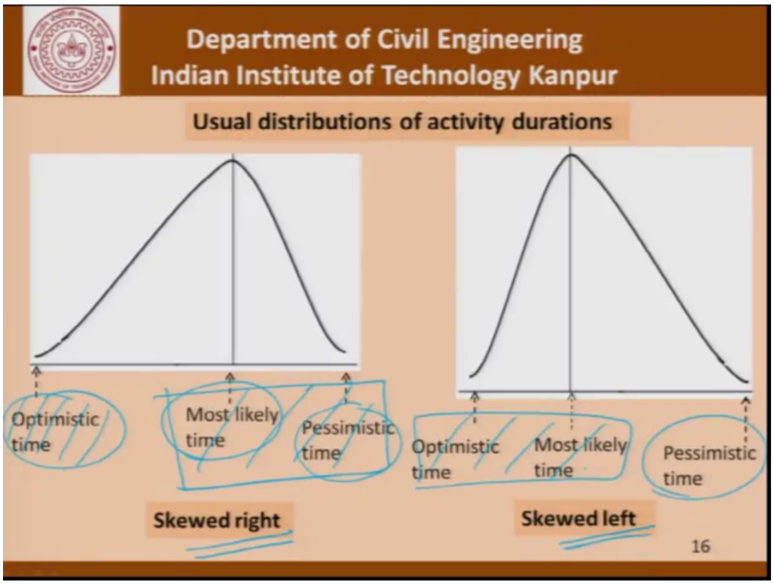

So, now what is a beta distribution? We assume that this is the most likely time this is the pessimistic time and this is the optimistic time and this distribution is skewed to the right. In this case however, on the figure on the right for the most likely time the distribution is skewed to the left. So, this is a situation where most of the times we will be able to complete the project here except if we become very lucky we might hit the jackpot and comes over here. In this case we will probability will able to complete the project somewhere here and only if we are not running in luck at all we will be able to complete the project within the pessimistic time. So, this is the kind of thought process and we move forward and try to see how the uncertainty in estimating activity times can now be incorporated in a scheduling process like PERT.

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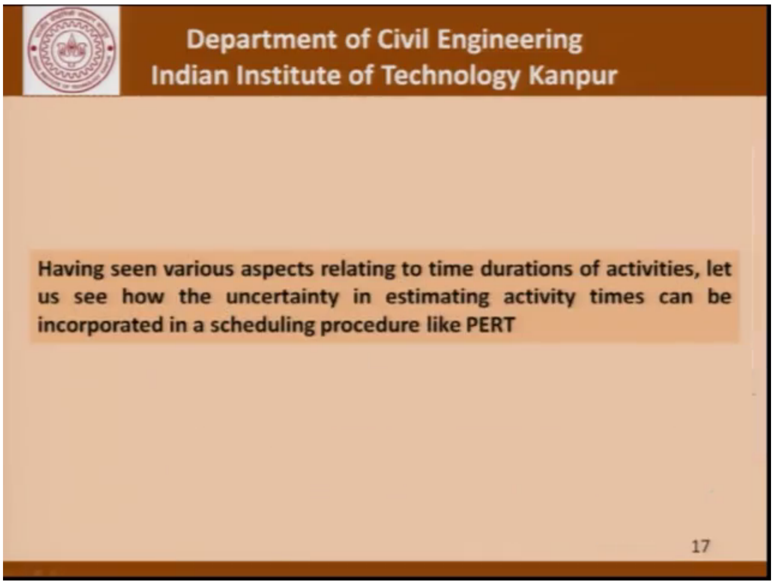

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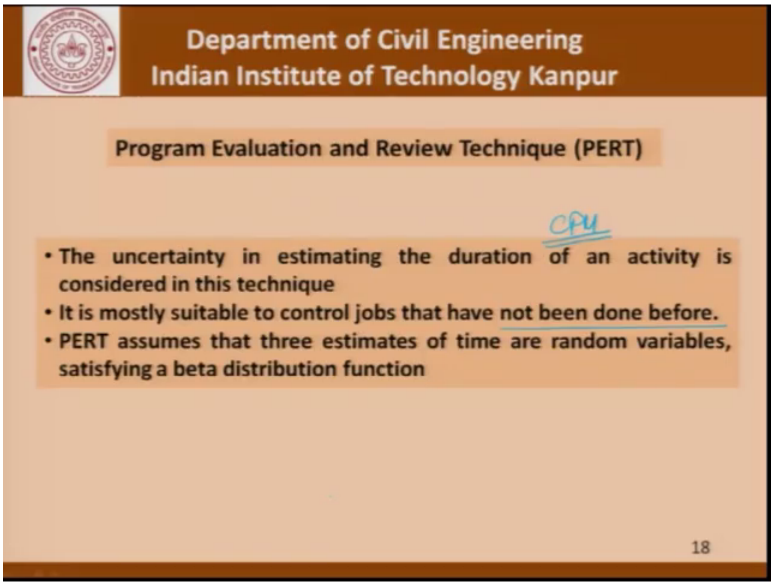

Pert is the program evaluation and review technique and the uncertainty in estimating the duration of an activity is considered in this technique as against the critical path method that we saw in the last class where the activities had a firm time of completion. It is more suitable to control jobs that have not been done before and there is more uncertainty in the activities.

PERT assumes the three estimates of time or random variables satisfying a beta distribution function. So, this is what we have been talking about and now let us try to see how we actually implemented on ground as far as activities are concerned.

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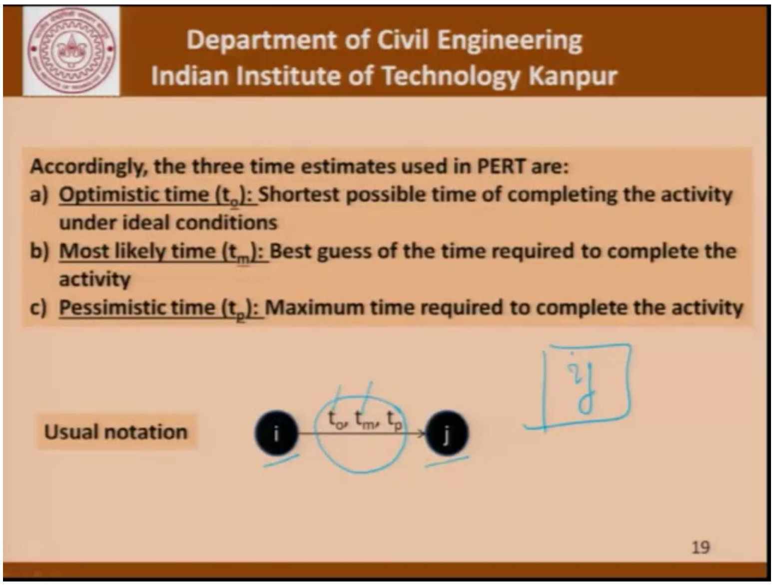

So, we have been talking off an activity i j all the time. So, here also we are saying that there is an activity i and j and we are talking of three time associated with it optimistic time to which is the shortest possible time of completing the activity under ideal conditions, the most likely time t m which is the best guess of the time required to complete the activity and the pessimistic time t p which is the maximum time required to complete this activity. So, usually the notation followed is given here for an activity i j we give to, t p and tm.

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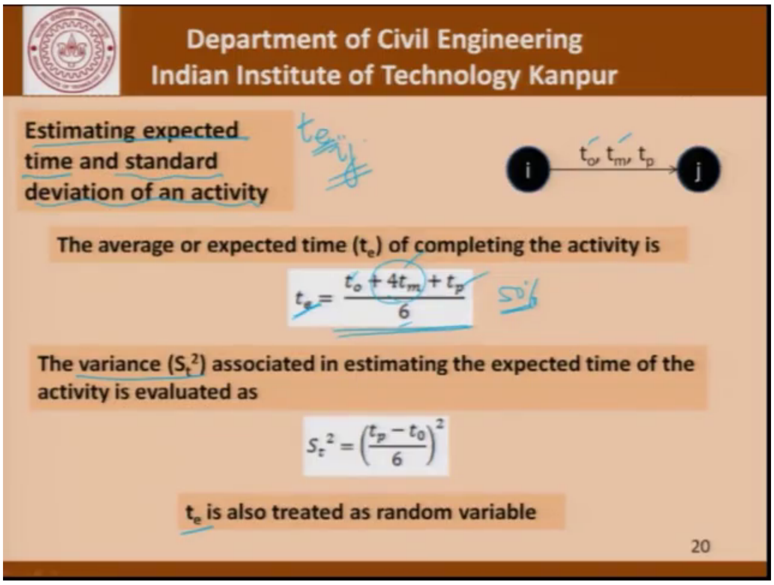

Now, from here the expected time for completion and the standard deviation in the activity we know the t o, t p and t m then how do we calculate the t expected of the activity or for the activity i j. Now this is calculated using this equation that is t o plus t p plus 4 times t m divided by 6. So, I am not getting into the derivation of this what it really says is the this on the pessimistic and optimistic times and we are giving a higher weightage to the most likely time and we are getting an estimate of the time at which or during which this activity will be completed.

Please remember that this expected time does not mean or does not give you an idea of the 50 percent time of completion. This time does not mean that the activity will be completed 50 percent of the time that would happen if the time of completion of that activity was normally distributed. Given that it is not normally distributed t e does not confirm to a 50 percent probability of completion of that activity.

Continuing the variance that is S t square associated in estimating the expected time of that activity evaluated as t p minus t naught divided by 6 squared and then we have must retreat the t e is taken as a random variable.

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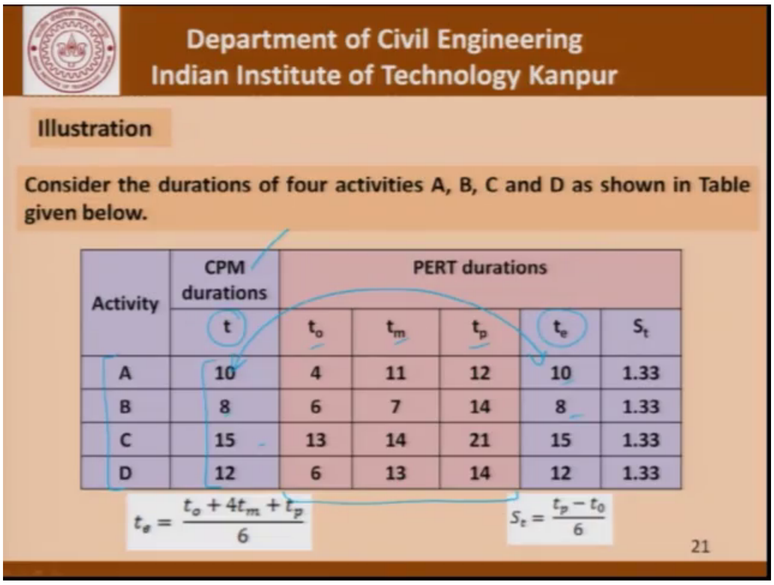

Activity	CPM durations	PERT durations				
	t	t_o	t_m	t_p	t_e	S_t
A	10	4	11	12	10	1.33
B	8	6	7	14	8	1.33
C	15	13	14	21	15	1.33
D	12	6	13	14	12	1.33

Moving forward let us try to implement this thought process in a example let us talk of four activities A B C and D and if these were the times that we used in a CPM calculation that is the firm times. Now this table here or this part of the table here gives you the t o t p and the t m for these activities and the way this t o, t p and t m have been chosen is to ensure that the time in CPM and the expected time of the activity are taken to be the same. So, activity has ten here it is t e here it is 8 here and 8 here, 15 here 15 here and 12 and 12. So, this effectively ensures that the t e is the same. Similarly using this equation here we have calculated the S t associated with each of these activities.

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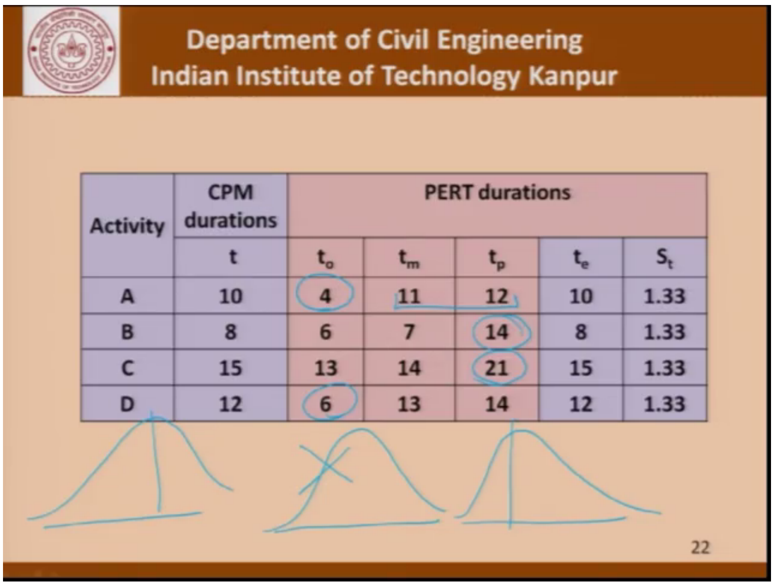

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C	15	13	14	21	15	1.33
D	12	6	13	14	12	1.33

So, if we examine this table closely we find that for activity A this t naught is a little bit of an outlier. These two times are fairly close to each other, but t naught is the optimistic time. In the case of B this number here that is the pessimistic time is the outlier and for activity C again this is the outlier and this is the outlier here.

So, what we have try to do through this arrangement of numbers is to covey to you that these activities are not normally distributed and sometimes they are skewed to the left or they are skewed to the right. And we have already talk before that this skew to the left and skew to the right has a different meaning when it is come to interpreting the time duration associated with an activity.

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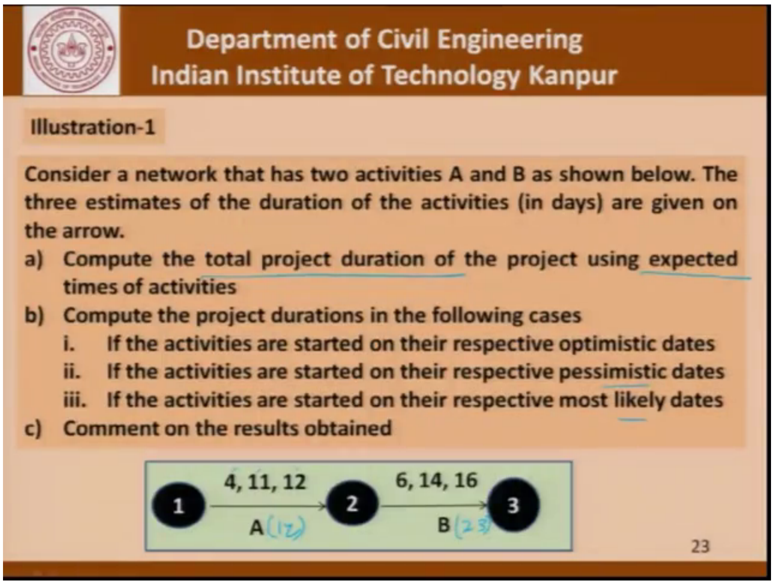

So, now let us actually consider some numbers let us consider a network that has two activities A and B as shown here. The three estimates for the duration of these activities and days is given on the arrow. So, here is the t o, t m and t p for activity A which is 1 2 and t o, t m and t p for activity B are 6 14 and 16 which is 2 3 that is the activity B. So, what we are required to do is to compute the total project duration using the concept of expected times of activities as we have defined earlier and compute the project durations in the following cases. If the activities are started on the respective optimistic dates pessimistic dates and the most likely dates and then of course, we can discuss most results a little bit.

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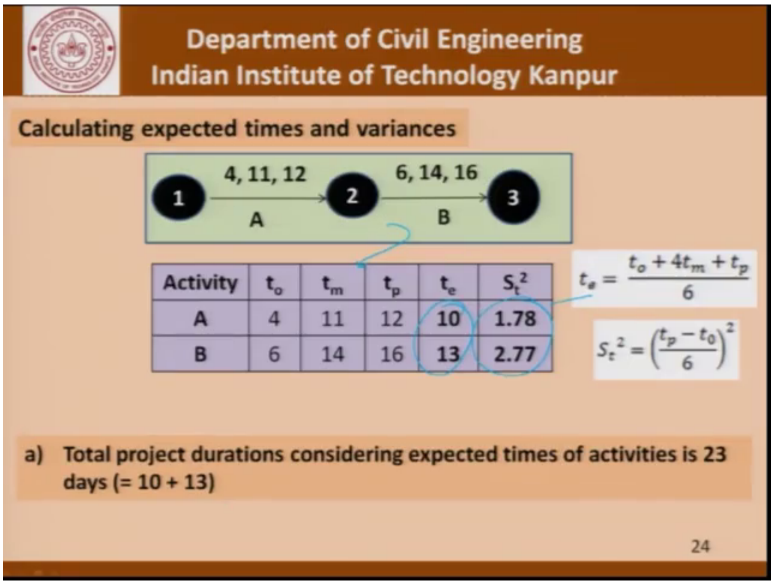

So, working out the t e and the standard deviations we find that for activities A and B we convert this information to this table here the using these formulae we calculate the expected times of completion of the activities and the variances. So, the total project duration considering the expected time of the activities is 23 that is 10 and 13.

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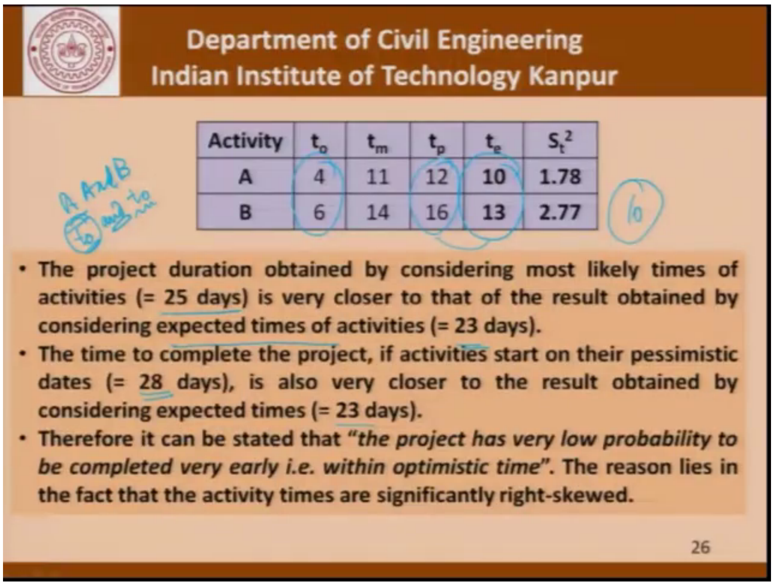

Now, coming to the second part of the problem which was to say that well if activities are completed at the optimistic times then of course, if we go back to our representation on the time access and try to say that a is completed in 4 days then B can immediately follow from here and in 10 days the project will be over. However, if they started the pessimistic dates the project will be over only in 28 days that is this will take twelve days and then this will gone for 16 days. As far as the most likely dates are concerned then we are talking of 11 and 14 which is giving us a number of 25.

So, the project duration obtained by considering the most likely times of activities is 25 is very close to the results obtained by the expected times that is 23 that is this number here. The time to complete the project if activities started on their pessimistic date that is 28 days is also pretty close to this result, so this is 28 and this is also close to this result of 25. However, the project being completed in these two dates the fact that both of these were outliers, only shows that the project can be completed yes in 10 days what we understand that the probability of being able to complete this project in that time is indeed very low.

So, in the most simple terms what we are saying is that we become lucky not only once, but twice not only the activity A is completed in the optimistic time, but activity B is also completed in optimistic times. So, we all know the basic probability kind of questions that A and B has to happen in this case. We have to have a being completed in its optimistic time and we have to have B also being completed in its optimistic time of course, there are other combinations that A is completed in its optimistic time and B in its pessimistic time and so on. So, that is the kind of range in the time estimation that is important or that become relevant as far as construction management is concerned.

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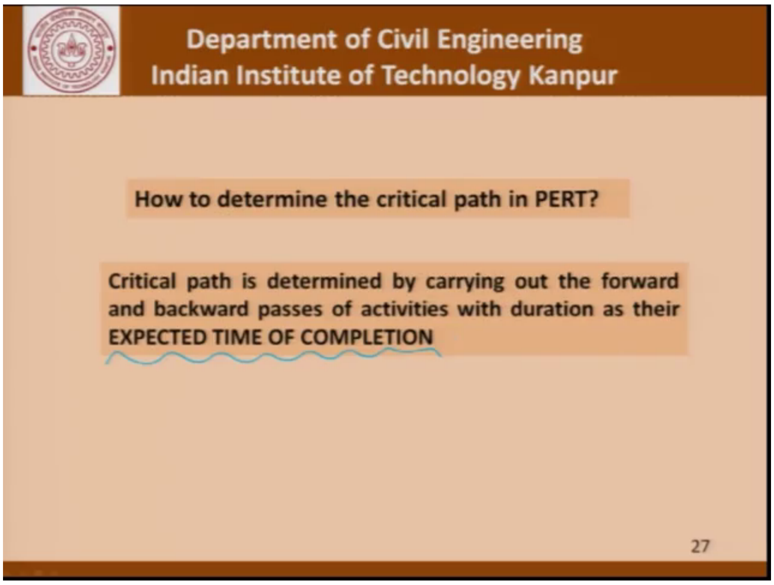

So, continuing our discussion further how do we determine the critical path in this method? The critical path is determined by carrying out the forward and backward passes of activities with durations being taken as the expected time of completion. So, of course, we take the expected time of completion and then we use probability concepts too find out what are the probability of completing the project in that time and so on.

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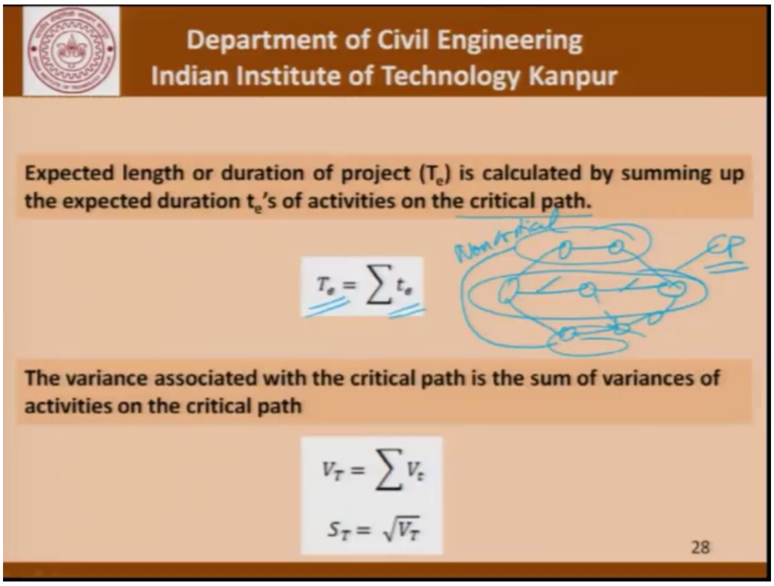

So, expected length or duration of the project capital t e is calculated by summing up the expected durations small t es of activities on the critical path. So, what we are saying is that if we say that this is the critical path we are not denying that there are no other paths there can be other paths, but this is the critical path. So, for this critical path we take the activities on this path and try to find out the expected time of the project using only the expected times of the activities on the critical path. These activities which are non critical will have to be dealt be separately.

And the variance associated with the critical path is the sum of the variances of the activities on the critical path and we can find out the standard deviation of finding the square root of this variance. So, this is the statistical concept which we are not deriving in this course and I am leaving it to you to take a look at some of the statistics text books and move forward.

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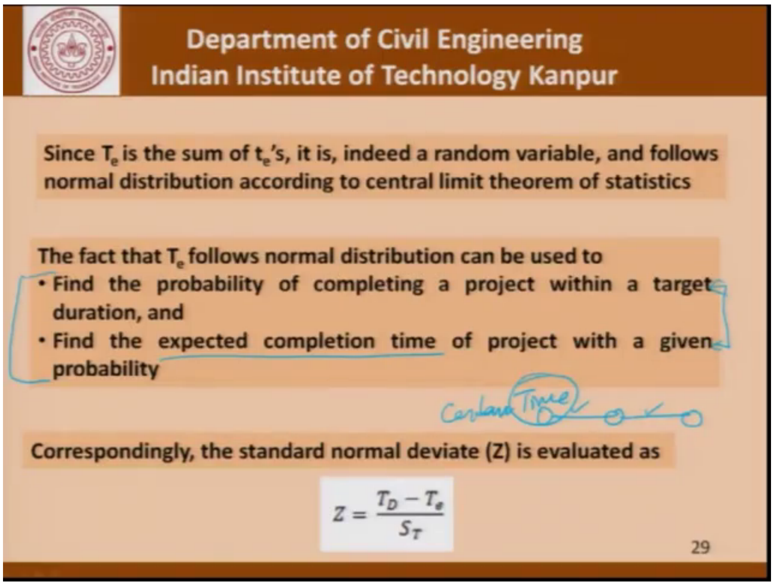

Now, since the capital t e is the sum of small te s it is indeed a random variable and follows normal distribution according to the central limit theorem of statistics. The fact that capital T e follows normal distribution can now be used to do these two things find the probability of completing the project within a target duration if there are two activities we can ask the question that well there are some time associated with these two activities what is the probability that this project will be completed within a certain time whatever that number is, that we will see (Refer Time: 22:47) example later on. But also we can find the expected completion time with a given probability basically these two questions are just two sides of the same coin. Correspondingly the standard normal deviate Z is evaluated as T d minus T e upon S T and this is something which comes in very handy when we are trying to answer questions such as these.

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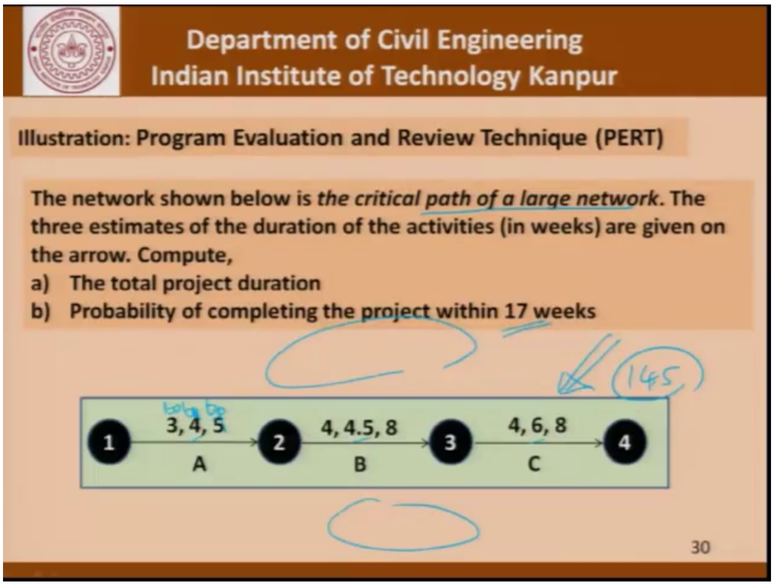

So, now to use an illustrative example for pert let say that there is a network which is given here, which is a part of a larger network. So, there is a lot of activities here and here, but we are talking only of this part because that is what is our critical path. So, for this critical path the durations are given here for that activities A B and C this is the t o, t m and t p for all these activities. So, what we are trying to find out is the total project duration and the probability of completing this project within 17 weeks.

Now 17 weeks please see is if we do a very simple calculation 4, 4 and half and 6 will give us 14.5. So, what we are saying is that if we just take the most likely times we did that in an example just now that if we just take the most likely times it turns out to be 14 and half. So, what we are talking about is that what is the probability associated with completing this project not an 14 and half or 15 weeks, but an 17 weeks. So, what the probability should be what we need to calculate.

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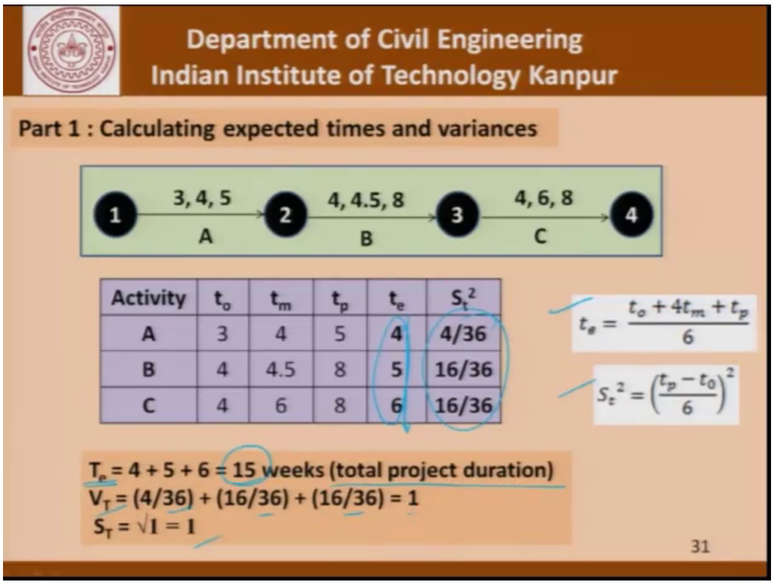

So, moving forward we first calculate the t es and the S t squares using this formulae and we found out the capital T which is the total project duration by summing up these three small t es and we find 15.

Similarly, the variance which is associated with the project is the sum of the variance of the activities which is one in this case, so the standard deviation is 1. So, we have this calculation carried out.

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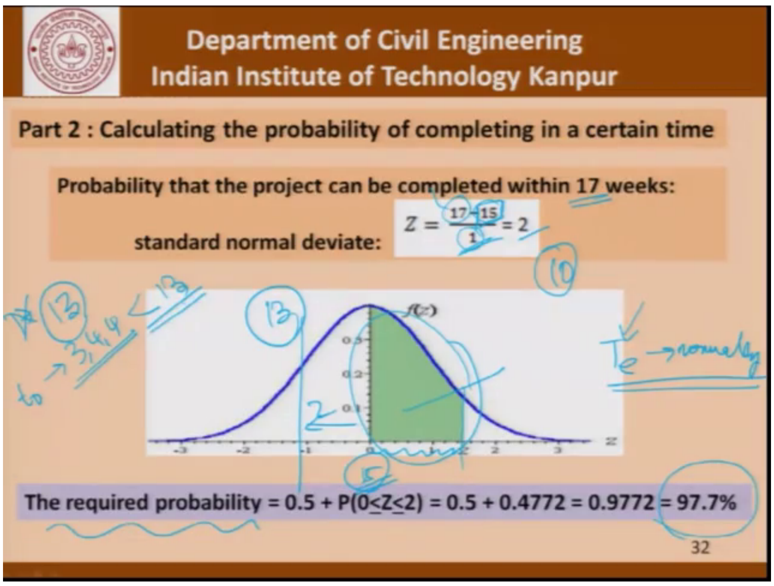

Now, with this calculation we go to the charts and tables relating to finding out how this area is related to the number of standard deviations that you go away from the mean. So, if you do this exercise for this particular project here we have 15, here we have 17 and here we have 1. 17 is our target we want to find out what is the probability associated with completing the project in 17 days which is higher than 15 which is the most likely time of completing this project.

So, this number here which has been arrived at using the expected times is 15 and we are trying to find out what is the probability associated with completing the project in 17 and this is the standard deviation. So, the Z value is 2 and with this if we calculate the required probability we will find that the answer is 97.7 percent. So, what we can see is that the probability of completing this project in 17 weeks is 97.7 percent here we can say that yes the capital T e represents that time which has a 50 percent probability of completion.

So, with this we have introduced the concept of pert we have introduced the concept of uncertainty in the activity durations. Now suppose instead of 17 we were talking of 13 if we go back to the data given here the t naught for the activities is 3 4 and 4.

So, the idea is that instead of 15 which represent the 50 percent probability of completion we are talking of going minus 2 here at 13, so what will be the probability of completing this project in 13 days. Of course, there is a finite possibility because the optimistic time associated with these three activities is indeed less than 13. This is something which I am leaving to you as a for thought you try to do this example on your own and I am sure you will get the answer very quickly. And I hope that you now understand how to handle these concepts and be able to calculate or estimate project durations find out the probabilities associated with completing a project in a given time and so on.

With this we will just give you the list of references which I always do at the end of a lecture and look forward to seeing you again.

Thank you.