Mechanics Of Solids

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Lecture - 54
Load Deflection Differential Equation

Welcome back to the course Mechanics of Solids. So, in the last lecture if you recall we were discussing about the deflection. We have established the relation between the bending moment and the deflection right. So, E I d 2 v d x 2 equal to M b; so that relation we have established in the last lecture.

And then basically we have discussed about the method of superposition and then we have given one we have seen one table and basically that table is giving you the readymade solution for different class of problems. And now by using those thing basically what do you can solve the deflection problem for different complicated systems. And those things these method of superposition can be used to find out the support reactions for the indeterminate structure that also we have seen in the last lecture.

So now today we will be talking about the load deflection differential equation and that whatever we have established that moment and deflection.

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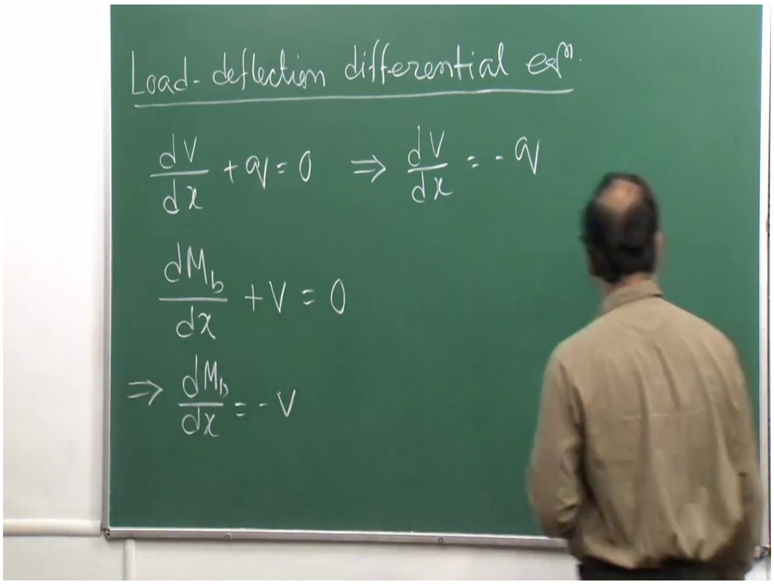

Relation bending moment and deflection relation from there we can we can establish this load deflection differential equation. So that means, if you if you know the load and I mean this differential equation will give you the deflection. So, let us let us establish that thing.

So, as you know from our previous discussion d v d x plus q where v is the shear force d v d x plus q equal to 0 that already we have seen when we talked about the bending moment and shear force and the differential relation right. So, from this I can simply write d v d x equal to minus q. Well and also we know from our earlier discussion d M b d x plus v shear force is equal to 0. That also we have seen. And from this we can write down d M b d x equal to minus v ok.

So, in place of this so, if we if we if we take the derivative with respect to x on both sides. So, what will get?

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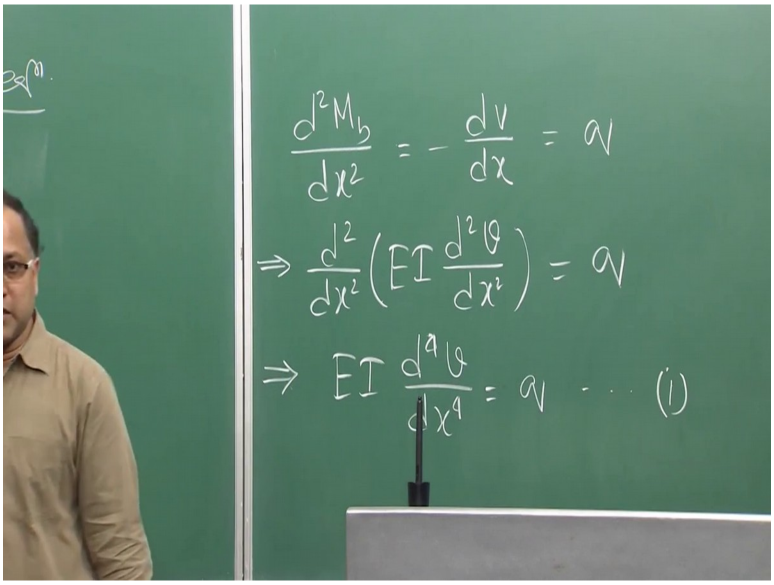

D 2 M b d x 2 equal to minus d v d x. Now minus d v d x is nothing but q. So now, in the last lecture if you recall whatever we have learnt. So, here in place of M b what we can write we can write E I d 2 v d x 2; now where v is the deflection from the neutral axis. So, that already we have established in the last lecture. So, this relation already we have got it that is equal to q. So, from there if I consider a homogeneous beam and if I consider the constant or the uniform cross section; then of course, E I is constant. So, I can take it out. So, I can write E I d 4 v d x 4 equal to q say equation say 1 ok.

And from this when we have established that relation basically. So, that relation gives me the differential relation between your load and deflection. This is your load say q is your load right. You have seen that is a uniform I mean distributed load over the beam and v is the deflection. So, this is the differential relation between the load and the deflection ok.

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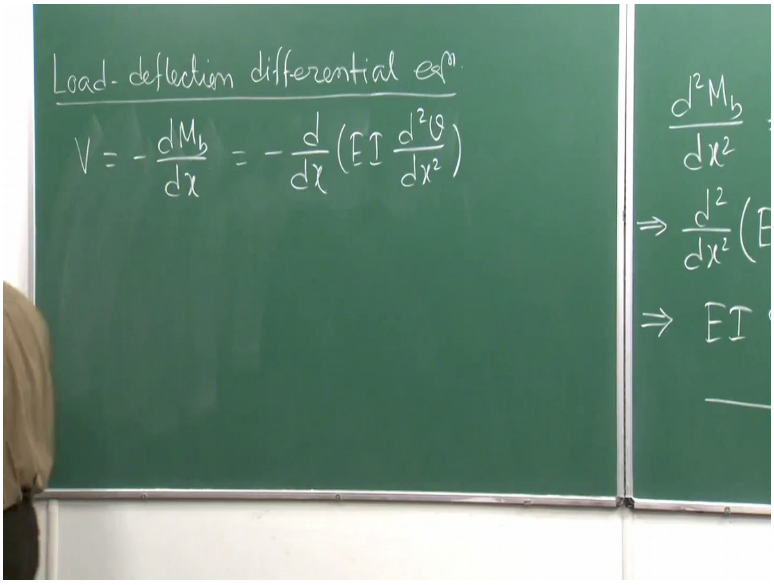

So now already we have seen that v equal to is nothing but minus d M b d x. So, in place of M b we can put E I d 2 v d x 2. So, I can write down d d x E I d 2 v d x 2. So, this is the relation between the shear force and the deflection ok.

So now this if you come back I mean this relation sometime we use that because if we know the shear force distribution. So, by using this differential equation we can find out the deflection of the beam or the cylinder member rather ok.

Now, if you look at this equation and if you want to solve this equation; from the mathematics you know you need 4 because this equation will be having I mean 4 integration constants because if you want to find out v. So, you have to perform integration 4 times. So, you will be getting or you will be accumulating 4 integration constants. Now how to get all those integration constants, right? That is a question. So, and you know already we have seen in the last lecture we solved one problem. And there we have seen that you have to introduce appropriate as well as suitable boundary conditions to obtain those integration constants ok.

So now with this we will be starting or we will be talking about another method by which you can find out this deflection quiet easily and efficiently. Because this is the relation I have already we have established, but as you have seen in the last lecture if you use this double integration or this integration method to find out the deflection basically it is it is a kind of laying the process it is not complicated, but it is laying the process. So, you have to solve this differential equation you have to get all the boundary conditions as well as the continuity conditions you have to impose those things and based on that you will be getting the integration constants and then finally, you will be getting the solution.

Now, whatever method, we are going to discuss now. By using that method you can easily get the solution. Solution means the deflection of the beam for a particular load application or I mean rather I should say, deflection corresponding to a particular load or rotation corresponding to a particular moment. And that method is popularly known as castiglianos method castiglianos theorem. So, we have to or we will we will we will see what the theorem says, but before getting to that that castiglianos theorem we should recapitulate few terms or few part we have to we need to refresh ok.

So, what are those things first one is work done, these things are known to you.

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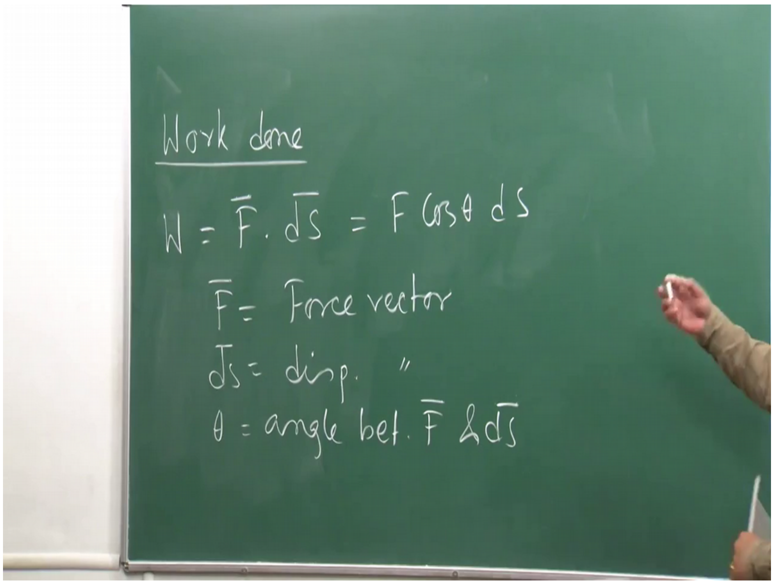

Just for recapitulation purpose we are discussing it again, work done. So, work done is if you say w is the work done then basically that is the dot product of the force as per the vectorial notation. Dot product of the force and the displacement vector right. So, that can be as you know that can be written as a cos theta d S that is nothing but your dot product, where F is your force vector d S is your displacement vector. And theta is the angle between force vector and displacement vector. This is knows to you just for I mean these things yu will be required when we are going to establish the castiglianos theorem ok.

So now the total work done; so if I want to get the total work done, for all the forces whatever are applied in a particular system then the total work done Can be written as total work done is equal to integration F d s, quiet simple right.

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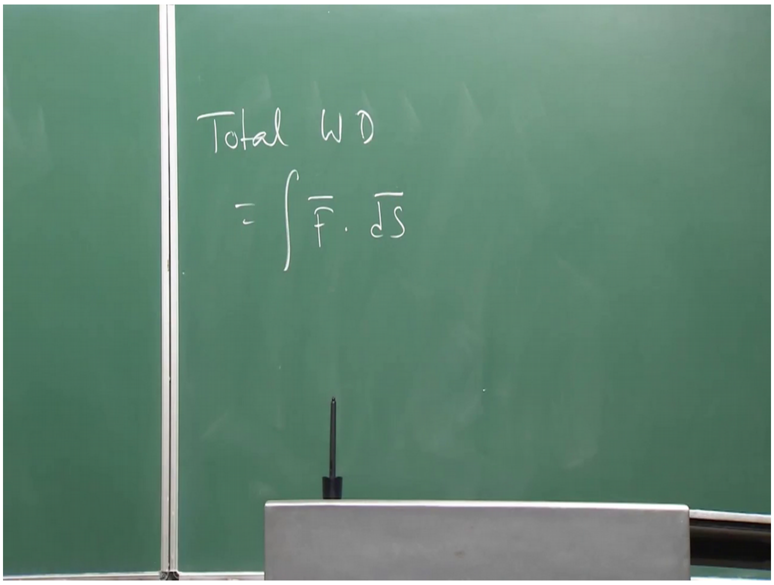

So, if you instead of one force you have multiple forces which are acting on the system. Then you will be getting the corresponding displacement vector and then if you sum them together then you will be getting the total vector that is quiet known factor form your physics ok.

Now, if I talk about the conservative system, what do mean by conservative system? So, to understand castiglianos theorem you need all those terms conservative system.

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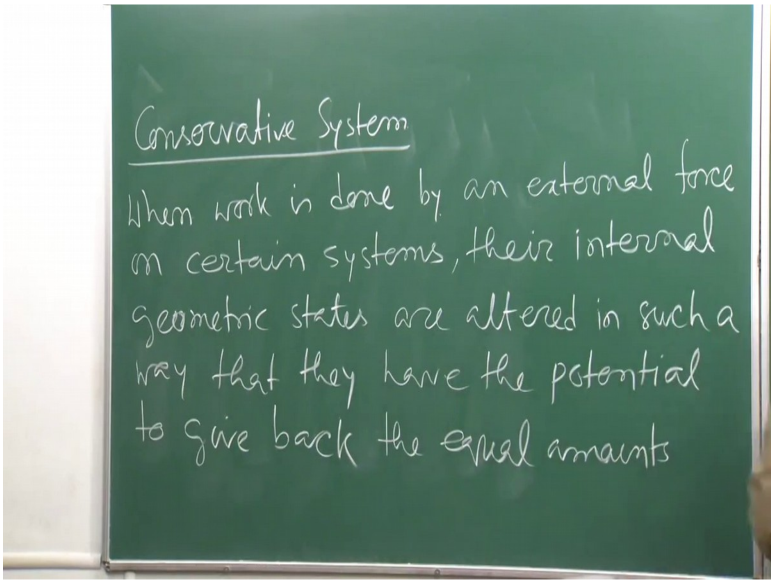

So conservative system if I write down the definition, when work is done by an external force on certain systems their internal geometric states are altered in such a way that they have the potential to give back the equal amounts of work whenever they are returned to their original configuration.

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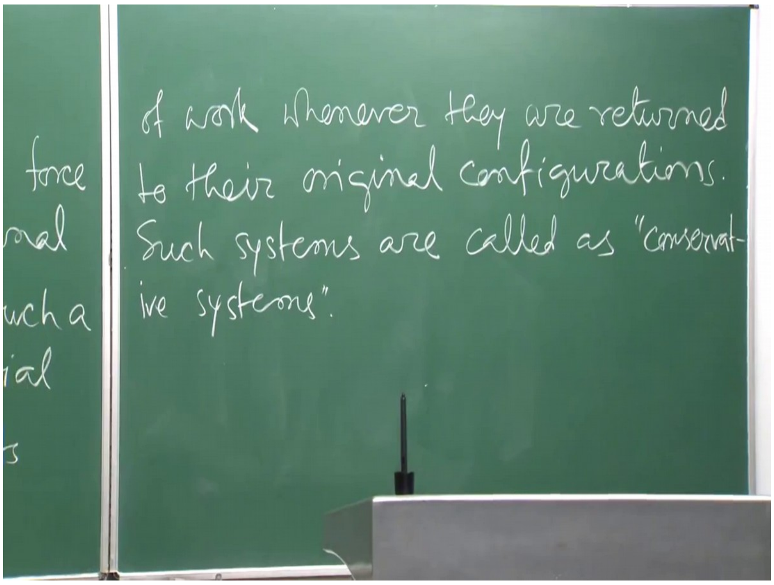

So, such systems are called as conservative systems ok.

So, what does it say? It says that when work is done by an external force some external force is applied on a particular system. And some work is done by that force their internal geometric states; because we are we are dealing with the deformable bodies. So, when you are applying some force on a particular system. So, their internal states will I mean the I mean geometric states basically the deformation is nothing but the geometric states are altered in such a way that they have the potential to give back the equal amounts of work whenever they are returned to their original configurations. And such system is known as conservative system. This is quiet known phenomena for you for you all from the from the physics point of view right.

So, conservative system; that means, the force is doing some work and that work will be stored. So, whenever it gets some opportunity to come back to it is original position then that work would be I mean given back. So, that system is known as conservative system.

Now, if you if you see this classical example of the spring if you consider the classical example of spring that means You have the spring.

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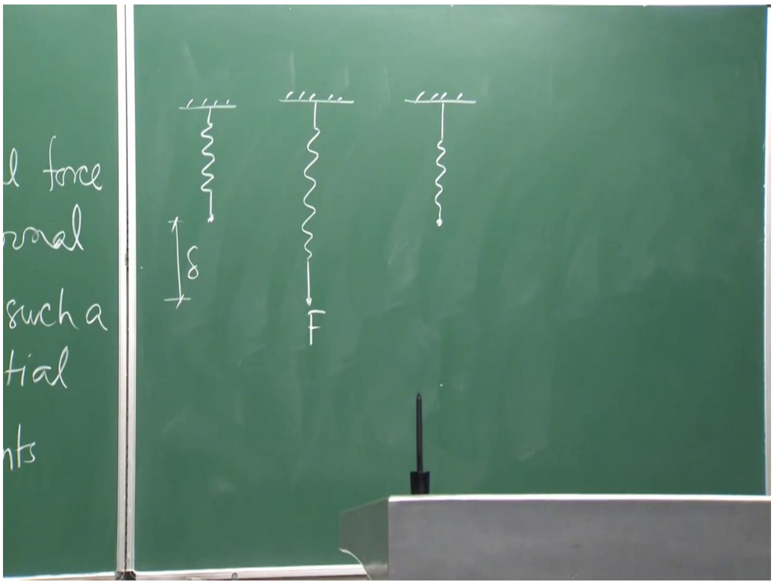

Now you are applying some force on the spring. So, this much of displacement can be observed due to the application of the spring. So, this is the change or the alteration in the geometric state. So, the spring will be stretched out ok.

Now, if you remove the force it will go back to it is original position; that means, the same amount of work has been given back. So, this is the classical example if I consider this system this system can be called as the conservative system. And if you look at this.

So, this displacement and this force, they together are doing some work, isn't it? As per the definition of the work done ok.

Now, then on the contrary what is your non conservative system? So, on the contrary what is your non conservative system. So, this is the conservative system that you are applying force some geometric alteration is happening once you remove the force basically it is coming back to it is original position by giving the same amount of work back.

Now, if you consider the non Conservative system.

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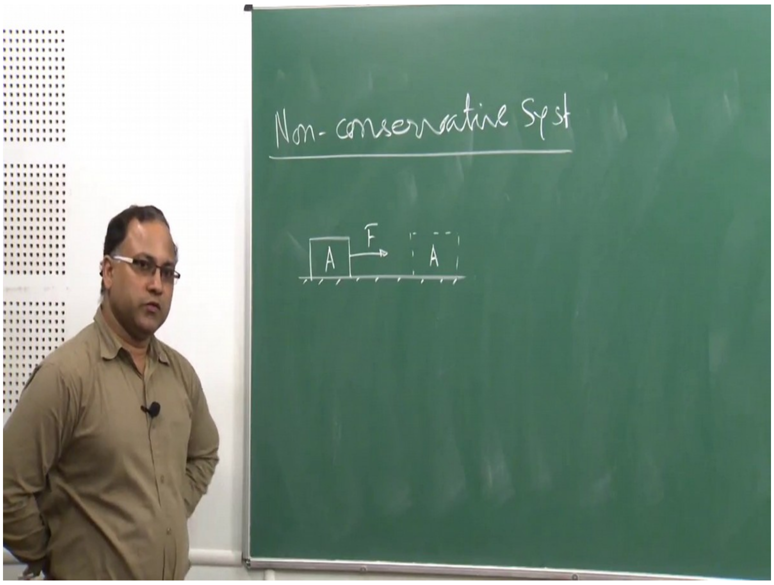

So, what does it mean? So, suppose work done is not recoverable. So, here actually work done is recoverable, in the conservative system work done is recoverable, but if you if you happen to get a system where the work done is not recoverable. So, that system is known as conservative non conservative system ok.

So, suppose for example, you have one surface on that surface you have a block A you are applying some force F. So, this is some table top or may be ground or whatever you want I mean you are having one one, one, one block which is resting on that surface. Then you are pulling that block. So now, that block is changing the position here. So, this force has done some work isn't it? This force has done some work. This because of that work the system is changing the position from here to here. Now if you remove the force will it come it back come back to it is original position, no. It is not recoverable. So, this kind of system is known as non conservative system and you know this ok.

Now, we will be talking about the potential energy, because these are the basic things will be required to define the theorem now potential energy.

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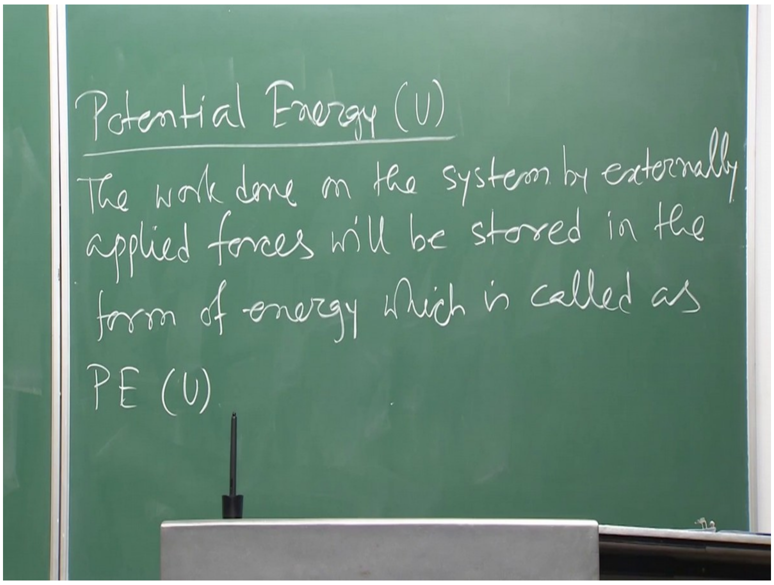

Defined by say u. Now what is the definition of potential energy? As you know from your earlier knowledge right. The work done on the system by externally applied forces will be stored in the form of energy, which is called as potential energy u ok.

So, the work done on the system by some externally applied forces, will be stored in the form of energy. So, we are now we are not talking about the non conservative system, we are dealing with the conservative system; that means, if you load a cylinder member suppose if you deflects if you remove the load it will give the work back.

So, when the system is under the work done of the externally applied forces basically that, that work done will be stored in the form of energy. And that energy is known as the potential energy you know the definition of the potential energy. Now this is just for sake of completeness of our discussion ok.

Now, if you if you take the same example whatever we just took to define the conservative system the spring system.

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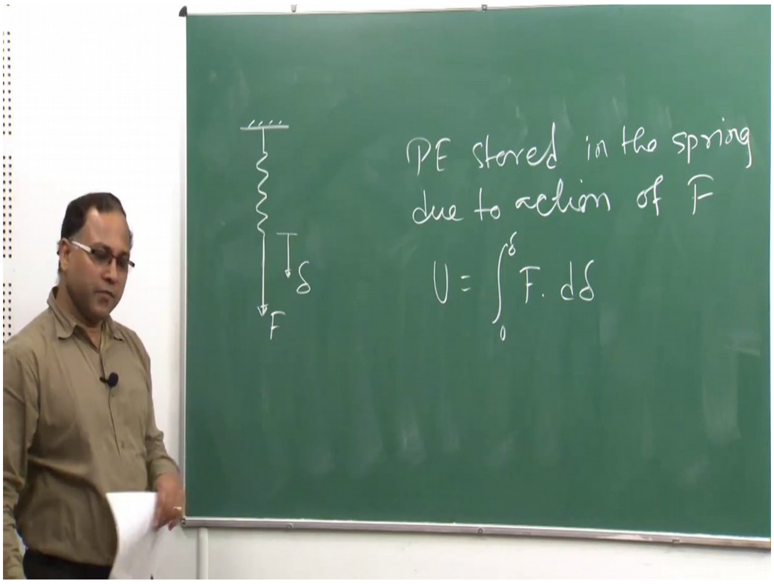

This is the spring, you are applying force here. And because of that say you are getting some deflection delta. Now it may be linear spring or non-linear spring it could be anything right. So, for the time being we are considering it is non-linear. The force deformation radiation is non-linear that is the most general situation. But we are considering the elastic system, and we are considering non-linear system for the time being. So, because this conservative system could be linear or non-linear and for both the systems you will be having or this potential energy is applicable there ok.

So therefore, your potential energy stored in the spring due to action of force P F is nothing but u is given by 0 to delta F into d integration that you know; that means, you are giving the load. So, incremental displacement is happening and if you if you integrate over the over the whole range; that means, 0 to delta that F into d delta is nothing but the incremental work done and that is getting stored in I mean in each state and then finally, if you add them together if you take the integration basically you will be getting the total potential energy stored in the system due to the action of force F right.

So, if you try to draw the force deformation plot, let us draw the force deformation plot.

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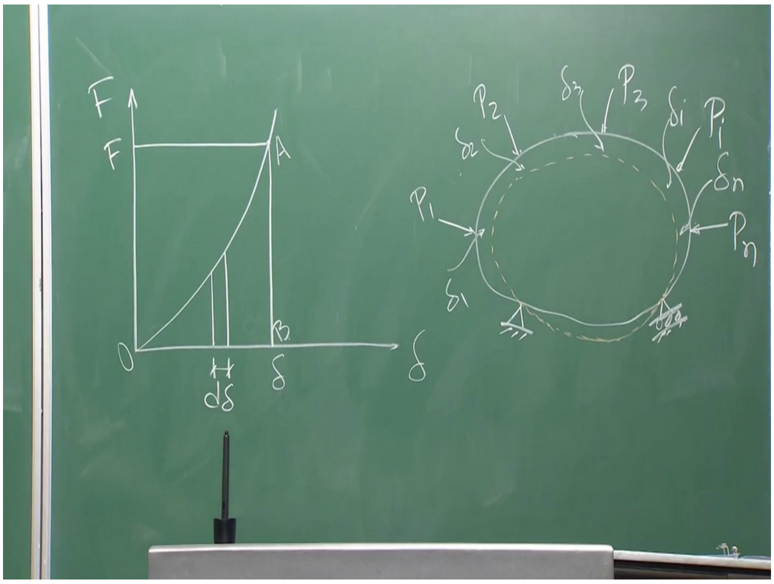

So, this is your force along say y axis F and along x axis say delta. So, as we are considering non-linear thing. So, this is the force deformation say plot. So, your potential energy suppose this is the force say and this is say final delta. So, your this is your d delta. So, this part now basically the area under this curve if I say this is O A O A area under this curve will give you the potential energy. As per definition graphically we are getting that. So, area under this curve O A this point say if you say b. So, area under the curve and the area O A b will give you the potential energy ok.

Now, let us talk about this thing. If I consider a conservative system like this is one conservative system supported like. That under the action of different forces, say this is say P 1 P 2 P 3 P i and P n, say you have n number of externally applied forces which are acting which are applied on this conservative system. And because of that you are getting the deformation. So, may be this is your deformed shape and that deformation you are getting for P 1 you are getting say delta 1 for P 2 you are getting delta 2 for P 3, you are getting delta 3 for P i you are getting delta i and for P n you are getting delta n ok.

Now, your potential energy can be written as your potential energy for this system can be written as per definition 0 to delta 1 P 1 d delta 1.

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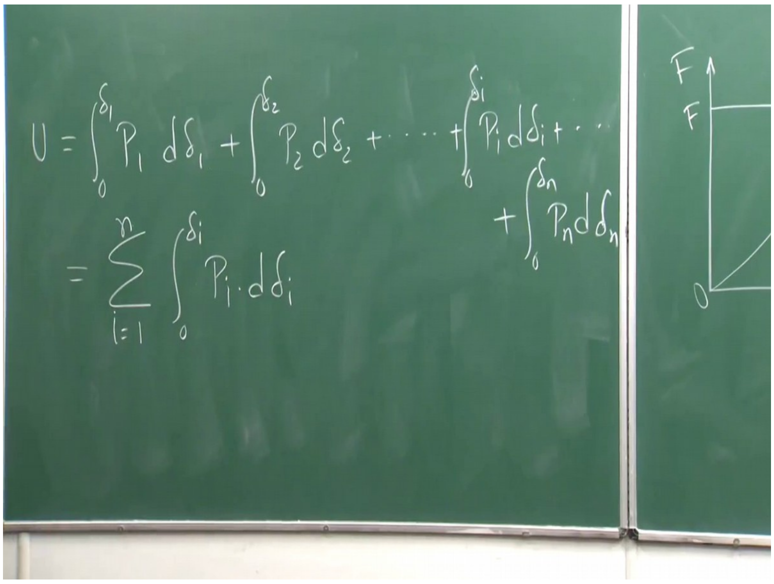

So, this is the potential energy contribution for force P 1, plus 0 to delta 2 P 2 d delta 2, this is the potential energy contribution towards the potential energy by force P 2. And similarly 0 to delta i P i d delta i and finally, you will be getting 0 to delta n P n d delta n.

So, this is your total potential energy of the system under the action of n number of forces applied on the conservative body conservative system. So now, it can be written as summation of So, this part is continuous. So, that is why we can express that thing with respect to the integration sign. But when we are adding all this I mean discrete say potential energy contribution for different forces then we can I is equal to 1 to n 0 to delta i P i d delta i. So, this is the total potential energy of the conservative system shown here ok.

So, if I say this is my figure one and this is my figure 2. So, whatever is shown in figure 2 the potential energy for that system is given by this agreed. So there is no issue.

Now, we are going to define one more different type of energy that is nothing but your complementary energy.

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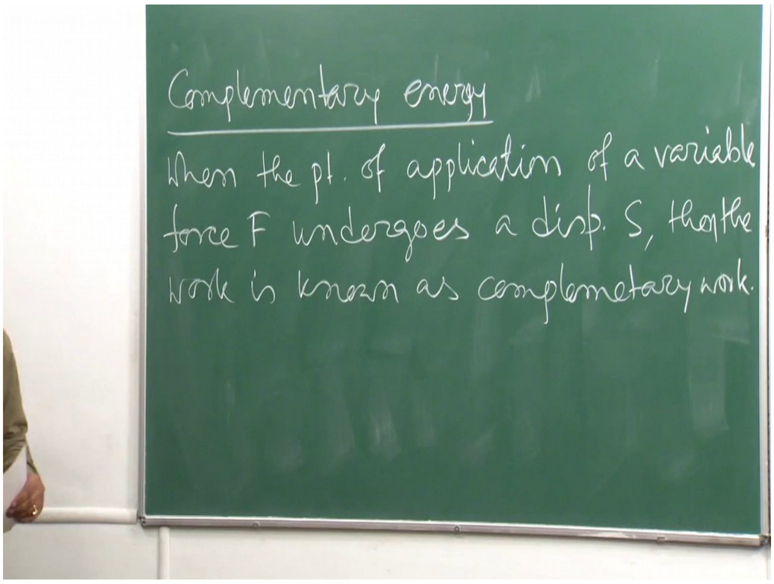

Complementary energy. So, what is the definition when the point of application of a variable force F undergoes a displacement S then the work is known as complementary work.

So, what that is say? When the point of application of a variable force to F. So now, earlier case when we are talking about the potential energy your displacement was variable. Now your force is variable force undergoes a displacement S then the work is known as the complementary work ok.

So now we are going to define the complementary energy. So, once this is the complementary work definition then in this process whatever energy will be stored that will be known that will be called as complementary energy.

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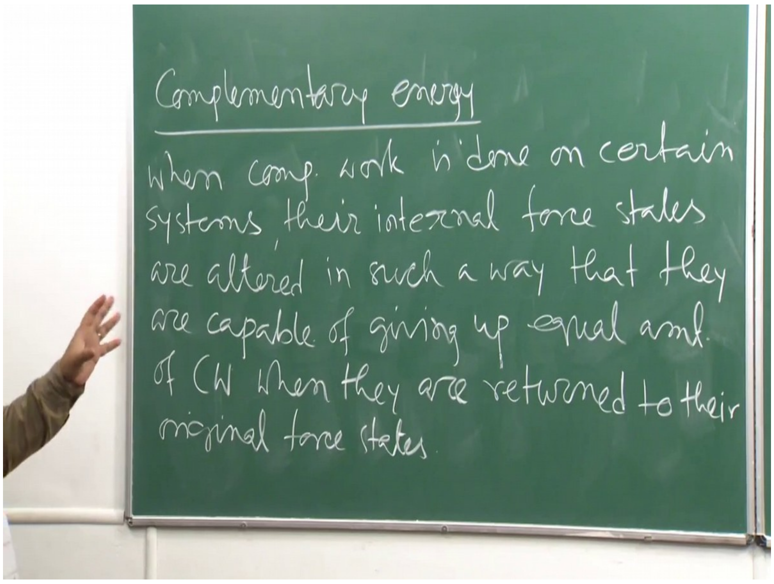

So, the definition of complementary energy is, when complementary work is done on certain systems, their internal force states are altered in such a way that they are capable of giving up equal amount of complementary work when they are returned to their original force states ok.

So, when the complementary work is done on certain systems. Their internal force states are altered because when you do the complementary work basically you are using the variable force and the internal force states will be altered gradually your internal force will be getting built up. In such a way that they are capable to capable of giving back equal amount of complementary work when they are returned to their original force states; that means, you are applying the force variable force gradually your system is doing some complementary work now once you remove that force. So, system will give the same amount of complementary work what it has been done by that force when it has it was applied right. So, the energy thus stored in this kind of process is known as complementary energy ok.

Now, if you look at the figure here. So, this is the variable force say d F and the area under this O A say this is you can say C. So, area under this curve I mean above this curve O A; that means, O A C this area will give you the complementary energy u star, agreed?

So now if we want to write down this complimentary energy or if you want to express the complimentary energy in the mathematical expression whatever we have done for potential energy, then we can write down U star is equal to 0 to F delta into the variable force d F.

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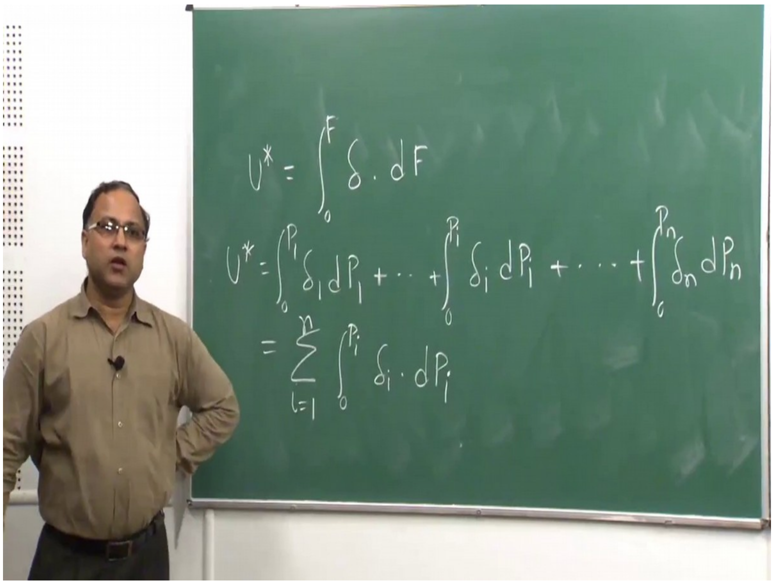

So, considering the whole body whatever you have seen here. So, considering this body now if I want to write down the complimentary energy, I can write simply equal to 0 to P 1 delta 1 d P 1 plus 0 to P i delta i d P i plus 0 to P n delta n d P n ok.

So, if I want to express that thing in the summation procedure 1 to n 0 to P i delta i d P I. So, this is your complementary energy. So, this is your complimentary energy. Now if you look at this system; so in this figure 2 whatever system you have seen now what we are considering.

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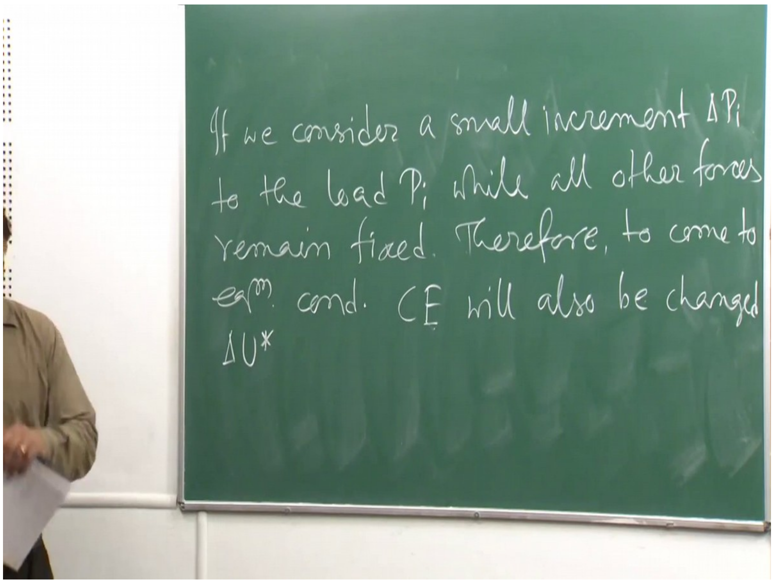

If we consider a small increment say delta P i; so along the line of action of P i if I if I consider a small increment of load delta P i to the load P i, while other all other forces all other forces remain fixed, we are not changing other forces. We are just giving one small increment to P i remain same therefore, to come to equilibrium condition the complimentary energy C E will also be changed to say delta u star. Because you have given some small increment to P I. So therefore, your complimentary energy will be also having some increment say delta u star ok.

So therefore, what we can write. So, what we can write?

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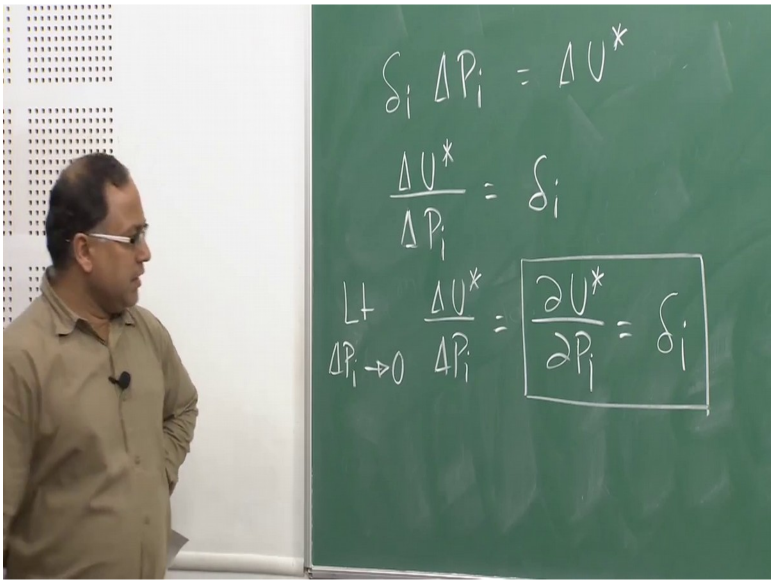

We can write delta i delta P i because delta i is the displacement for load P i is nothing but delta u star because that is the displacement that is the increment of force P I. So therefore, that is giving you the increment in complimentary energy. So, from there we can simply write delta u star by delta P i is equal to delta i can I write that or not, I can right.

So, in the limiting condition if I consider the limiting condition limit delta P i tends to 0 delta u star delta P i is equal to del u star del P i is equal to delta i. Now this partial derivative has been introduced instead of total derivative you are you are using partial derivative to show that only you are altering in the force P i. Other forces are remaining fixed or remaining same. So therefore, this partial derivative is coming into the picture.

Now, what you have got from this? So, you have got some indication, that if you manage to get the or manage to calculate the complimentary energy for a conservative system, any conservative system whatever system we are considering if that system is conservative and if we can manage to get the complimentary energy for that particular system the total I mean total complimentary energy, then if I take the partial derivative of that complimentary energy with respect to some force P i, then that partial derivative will be giving me the deflection along that force P i right. What is delta? Delta is the deflection along that force P i?

So, if I manage to get u star that is the complimentary energy whole total complimentary energy of the whole system and then basically if I take the partial derivative with respect to P i that is giving me the deflection along that line of load P i right. Now based on that castigliano has proposed his theorem.

Well, I will stop here today. So, in the next lecture basically we will be talking about the castiglianos theorem. And based on this derivation we will look at the castiglianos theorem ok.

Thank you very much.