Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 53
Superposition Continued

Welcome back to the course Mechanics of Solids. So, as we have seen in the last lecture that the method of superposition right and we have seen this table also if you come back to this table. So, we have seen this table also, where you have got some readymade solutions for some say basic problems ok.

(Refer Slide Time: 00:34)

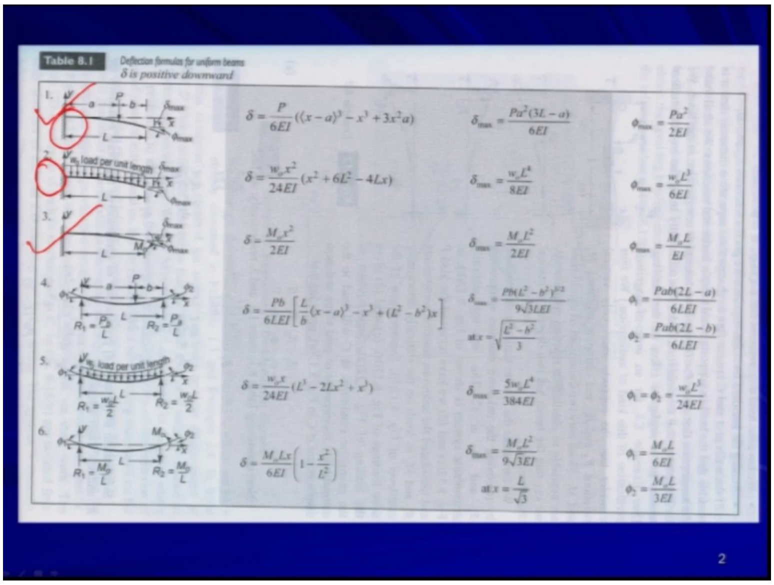
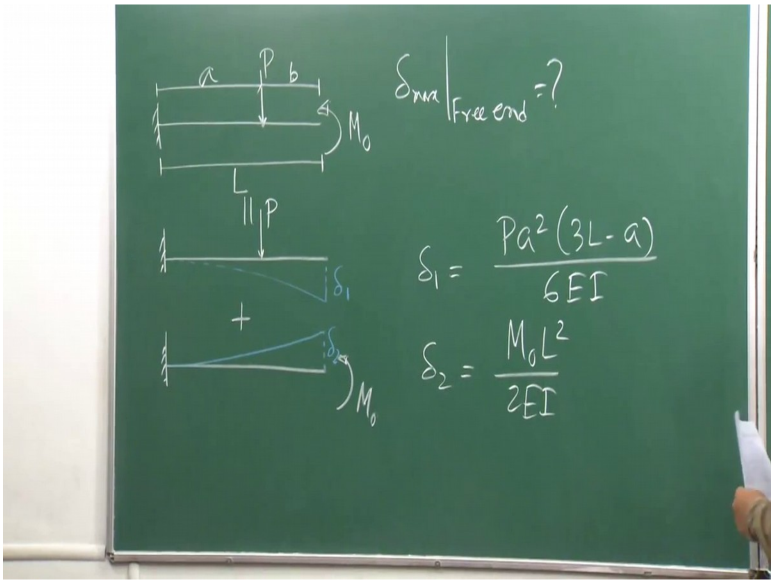


Table 8.1 Deflection formulas for uniform beams
δ is positive downward

1.	$\delta = \frac{P}{6EI}(\langle x-a\rangle^3 - x^3 + 3x^2a)$	$\delta_{max} = \frac{Pa^2(3L-a)}{6EI}$	$\phi_{max} = \frac{Pa^2}{2EI}$
2.	$\delta = \frac{w_o x^2}{24EI}(x^2 + 6L^2 - 4Lx)$	$\delta_{max} = \frac{w_o L^4}{8EI}$	$\phi_{max} = \frac{w_o L^3}{6EI}$
3.	$\delta = \frac{M_o x^2}{2EI}$	$\delta_{max} = \frac{M_o L^2}{2EI}$	$\phi_{max} = \frac{M_o L}{EI}$
4.	$\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}\langle x-a\rangle^3 - x^3 + (L^2-b^2)x\right]$	$\delta_{max} = \frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}LEI}$ at $x = \sqrt{\frac{L^2-b^2}{3}}$	$\phi_1 = \frac{Pab(2L-a)}{6LEI}$ $\phi_2 = \frac{Pab(2L-b)}{6LEI}$
5.	$\delta = \frac{w_o x}{24EI}(L^3 - 2Lx^2 + x^3)$	$\delta_{max} = \frac{5w_o L^4}{384EI}$	$\phi_1 = \phi_2 = \frac{w_o L^3}{24EI}$
6.	$\delta = \frac{M_o Lx}{6EI}\left(1 - \frac{x^2}{L^2}\right)$	$\delta_{max} = \frac{M_o L^2}{9\sqrt{3}EI}$ at $x = \frac{L}{\sqrt{3}}$	$\phi_1 = \frac{M_o L}{6EI}$ $\phi_2 = \frac{M_o L}{3EI}$

So, now by using these basic problems we can solve how; we can solve this critical problem or this complicated problem that we will see by using method of superposition.

(Refer Slide Time: 00:41)



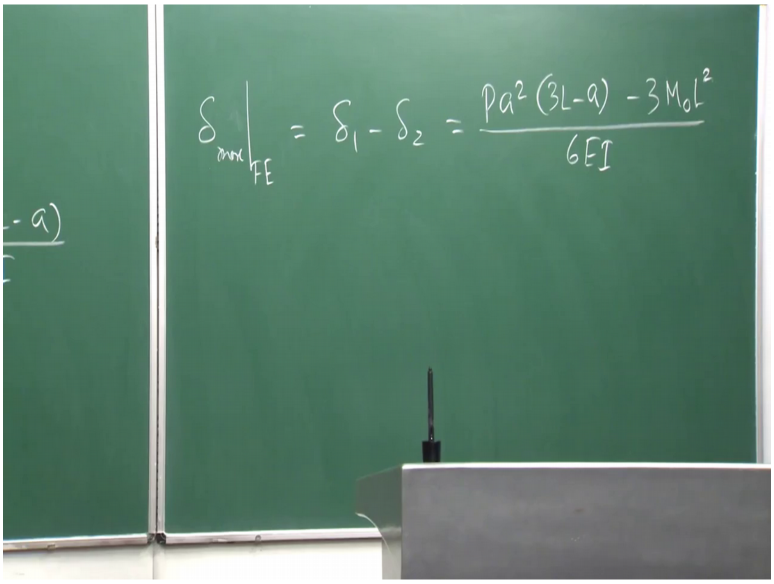
So, there is a cantilever beam. So, you have a force P here. So, this length is a this length is say b, and a moment is applied at the free end M naught the total length of the beam is say L ok.

So, this beam actually has to be solved by using the method of superposition, we are not going to solve that thing by using the double integration method and all. So, let us see how we can solve it. So, this is equivalent to there is a cantilever beam, I have only the concentrated force P and which is deflecting like that. So, our objective is to find out the maximum deflection at free end ok.

So, our objective is to find out delta max at free end that is our objective. So, this is the deflected shape under this loading; that means; only one load is there. So, that is say delta 1, and this part can be added. So, this is your cantilever beam, and you are applying only the moment here for that your beam is getting deflected like that say this is my delta 2 say ok.

So, from that table if you look back if you come back to this table, if you look at this case and this case; so, these are two cases which are shown here right. This is your case 1 this is your case 3. So, for this your delta 1 as per the table already you have seen 3 L minus a by 6 EI, and for this your delta 2 is equal to M naught L square by twice EI that is given in the table.

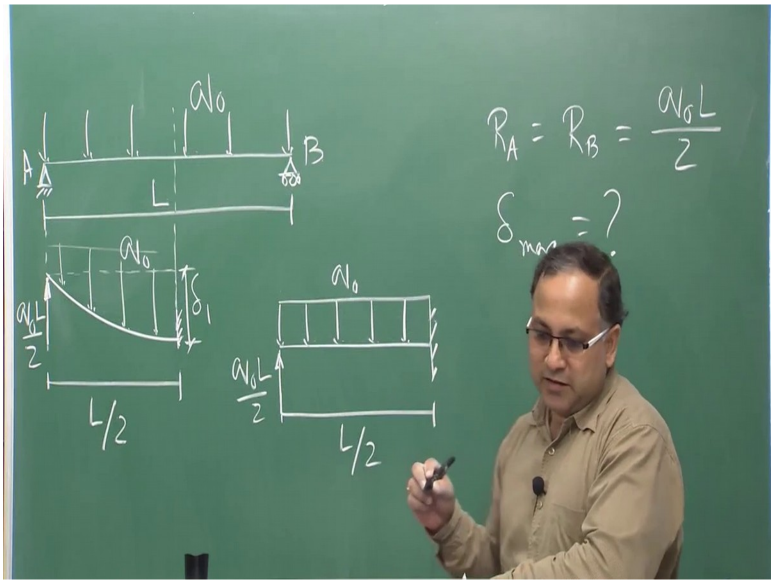
(Refer Slide Time: 03:26)



now using these two things ok we are going to find out this delta max; that means, the maximum deflection at the free end. So, delta max at say free end if you want to get it free end is nothing, but delta 1 minus delta 2, which will be P a square, 3 L minus a minus 3, M naught l square by 6 EI. Very simply you have got it just 2-3 steps I hope that you are you are enjoying this thing. Because if you use the double integration method here, it will you can there is no issue at all because that is the basic method you can solve it, but that will be little bit lengthy ok.

And this is the intelligent way to solve this, because you have this table I mean this table has been provided to you. So, you can solve like this ok.

(Refer Slide Time: 04:41)



Similarly we can take another problem say if you take another problem the problem is say something like this, this is the simply supported beam under the action of uniformly distributed load q naught per metre length, and the total span of the beam is say L, this is say A this is B. Now if you want to find out the reaction at A and reaction at B both are same and this is the symmetric problem right. So, that will be nothing, but q naught L by 2.

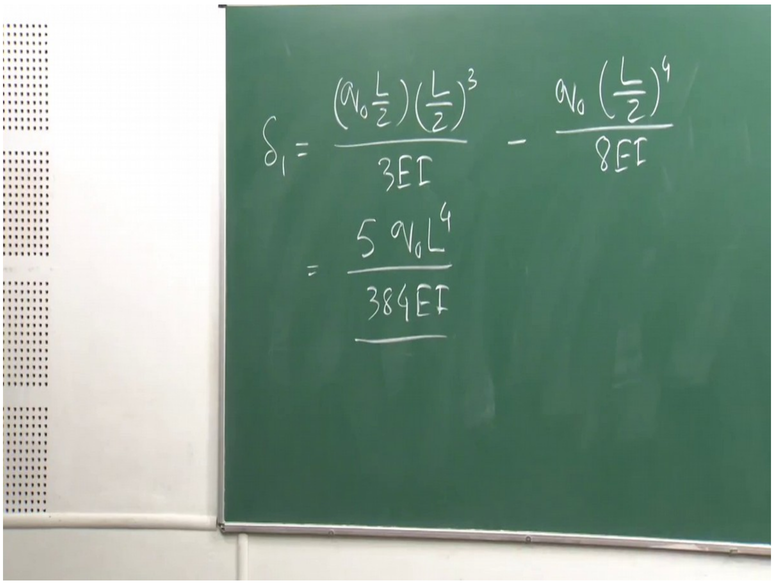
Now, what will do some intelligent way will be considering here, what will be considering see this is the saymid span of the beam this is the mid span of the beam, this is your say L by 2. Now at the mid span of the beam under this kind of symmetric loading, this quite expected that you will be getting the deflection which is maxima; you will be getting the maximum deflection agreed. If you get maximum deflection then your d v d x must be 0 what does it mean? That means, at the midpoint of the beam your slope must be 0. Now if your slope is 0 then you can think of a cantilever beam which will be under this kind of loading at this end you have q not L by 2, and you have say uniformly distributed load, and this is your maximum deflection delta 1 because my objective is to find out delta max, and that is happening at the midpoint of the beam that is now objective is to find out ok.

So, this is your delta 1. So, if I manage to get delta 1 then basically my job is done delta 1 is nothing, but delta max and where the delta will be maximum your slope must be 0 and this kind of support condition fix support condition is giving you that condition that issue I mean that kind of say configuration, where slope is 0 that will satisfy that kind of support condition.

So, now here you have q naught. So, what is happening basically, you have one cantilever beam this is one cantilever beam under this is your cantilever beam which is having the span L by 2 and you are having one concentrated force q naught L by 2 and this is q and you want to find out the deflection at the free end; because free end deflection is nothing, but equal to the deflection at this point because this is the line ok.

So, if you find if you want to find if you can find out the deflection at the free end, then basically your job is done and this kind of configuration you have in this table. So, suppose if I say this is the case, this is case 2 and you have the concentrated force at the end. So, that can be extracted by doing some manipulation from this problem, from this readymade problem ok.

(Refer Slide Time: 09:13)



So, let us do that. So, your delta 1, 1 thing is coming from this q naught l by two; that means, this case where the load is applied at the end, in this case your load is applied at the at some point right. If you consider this problem the load is applied here, but I can manipulate I can consider the load is applied here and I can find out the deflection here right. So, if I do that then basically my delta 1 will be q naught L by 2 that is the load, and instead of. So, this is your solution. So, they are actually if you put the L by 2 whole cube divided by 3 EI. So, that is the cantilever problem considering a equal to L by 2. So, a equal 2 L by 2 if you consider. So, you will be getting that minus. So, that is going up and this will be giving you the minus sign. So, that is obtained as q naught L by 2 to the power 4 by 8 EI. So, that comes from this problem here L is nothing, but L by 2.

So, if you do that you will be getting 5 q naught L to the power 4 by 384 EI that is your central deflection of the beam very simply you have got it just using your method of superposition. Otherwise you can use the double integration method then proper boundary conditions all those things you put and then you can find out, the same same value will be getting, but here you see just within few steps you have got it that is the intelligent way of solving this kind of problem.

So, I will stop here. So, in the next lecture we will be taking or we will be discussing about the another method of solving or another method of getting the deflection problem that is your (Refer Time: 11:46) theorem.

Thank you very much.