Mechanicals Of Solids
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Lecture – 52
Integration of Moment Curvature Relation

Welcome back to the course Mechanics of Solids. So, in the last lecture we were solving one numerical problem to understand the moment curvature relation, and there we have seen that we established four different I mean we established the differential equations and from there we got four integration constants, and we established 2 boundary conditions two continuity conditions by which we could find out the magnitude of all unknown integration constants.

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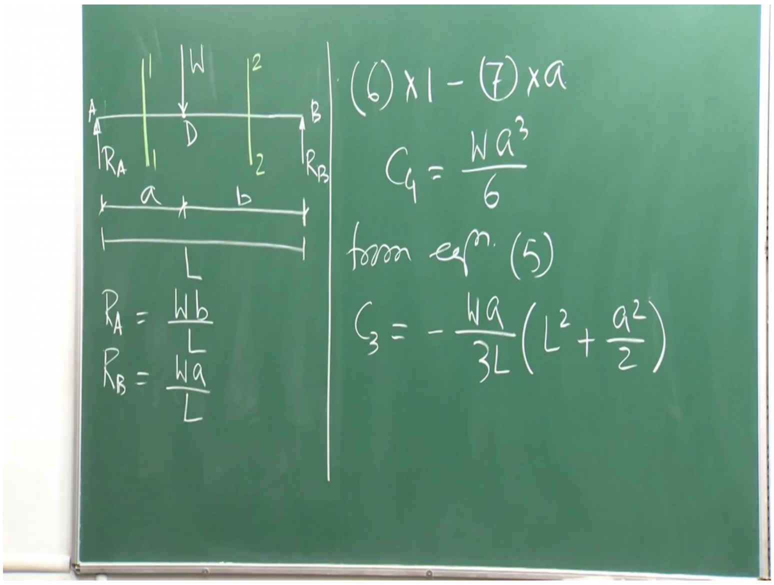

So, those things already we have done in the last lecture, and this was the problem was taken. So, our objective is to find out the deflection at point D the point where the concentration force is acting ok.

So, now from equation 6 if you operate if you do this operation equation 6 into 1 minus, equation 7 into a; if you do this operation then you will be getting simply C 4 equal to W a cube by 6, then from equation 5 you will get C 3 equal to. So, I am not writing all the steps you can find that thing by your own W a by 3 L, L square plus a square by 2 ok.

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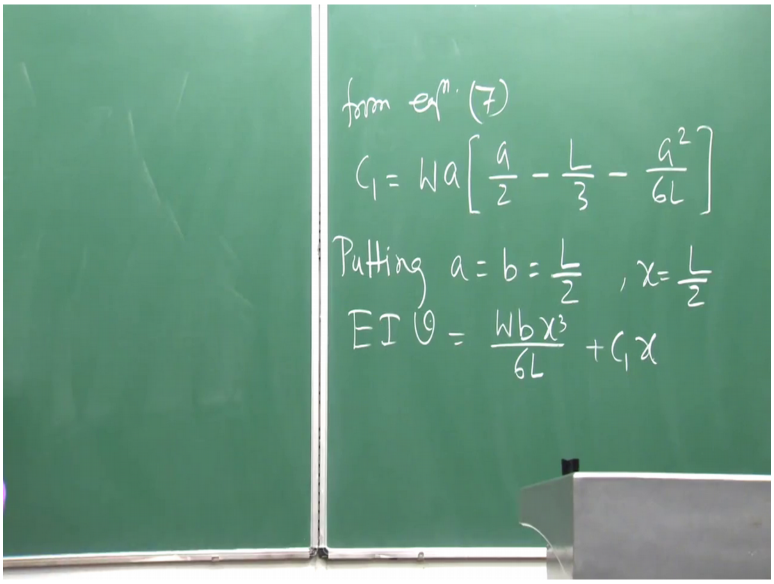

And then finally, you will get from equation 7 you will get C 1 equal to W a, a by 2 minus L by 3 minus a square by 6 L. So, you have got all the integration constants C 1, C 2 is already 0 already we have seen in the last lecture the C 2 was 0 then C 3 are C 4. So, all the integration constant have been obtained.

Now, we will be considering one special case or the special situation when D is at the midpoint of the beam; that means, say some special condition we are considering putting a equal to b equal to L by 2, and in the in the bending equation. So, in the moment curvature relation your x is equal to L by 2; that means, section 11 is varied from 0 to L by 2, and section 2 2 is varied from L by 2 to L ok.

So, if I put that. So, from our earlier equation whatever we have derived for section 11 I can write W b from section 1 1 I had that equation like this, W b x cube by 6 L plus C 1 x if you recall right C 2 was 0. So, from this I can simply write by putting x equal to L by 2 in this equation ok.

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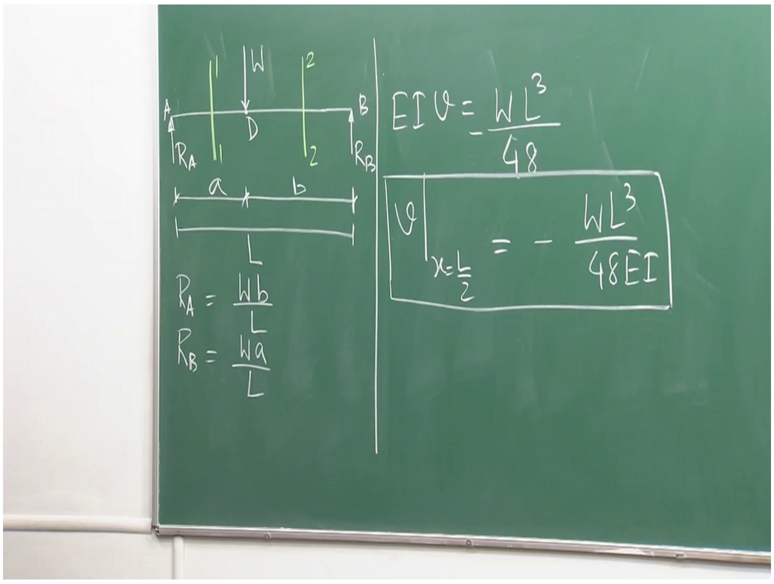

I can get EIV equal to WL cube by 48 of course, minus ok.

Therefore v at x equal to L by 2 is equal to minus WL cube by 48 EI. So, minus sign is for fees downward; that means, y is upward v that is the deflection is downward due to the application of the load, that is why it is the minus sign is coming and so what does it mean? That means, the central displacement of the or the deflection of the beam under this load W, when a equal to b equal to L by 2 at the time that deflection will be coming as WL cube by 48 EI.

So, here you can see as your W increases your deflection also will increase, as your length increases the deflection will be increase in the cubic I mean term in the cubic power, and EI that is the flexural rigidity if it increases then your deflection will be getting reduced. So, that that is why this is nothing, but your it is a mirror of rigidity; that means, how rigid the beam is ok.

So, anyway whatever I wanted to say that by establishing or by using your moment curvature relation, you have got the deflection at the midpoint of the beam like this. Similarly at any point because you know all the integration constants; at any point you can find out the deflection of the beam by using the proper equation and the boundary conditions ok.

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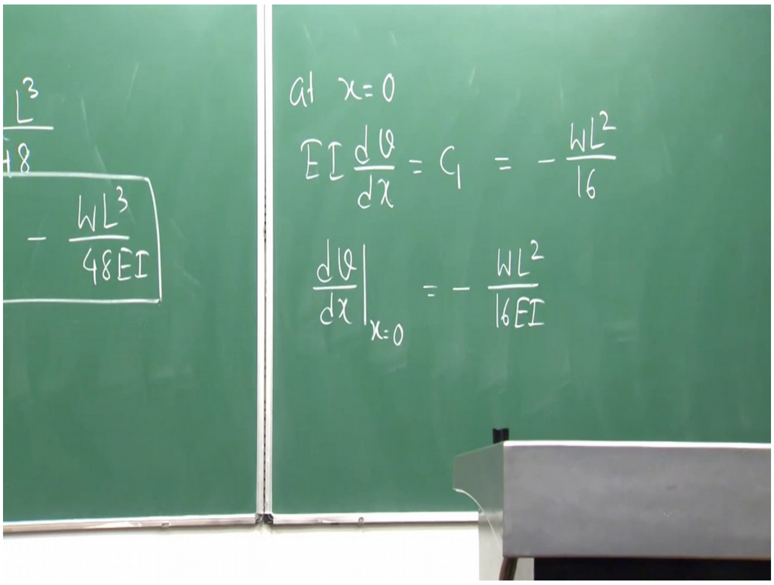

So, now we can find out the slope also, we can find out the slope at x equal to 0, your EI d v d x equal to C 1, at x equal to 0 means at point a, you want to find out the slope; slope is nothing, but d v d x and if you recall your section 1 1 equation valid for section 1 1 from there I can get simply EI d v d x equal to C1.

So, from there I can putting a equal to b equal to l, I can simply get minus WL square by 16. So, therefore, I can write that slope at x equal to 0 is equal to minus WL square by 16 EI that is the slope, that is the slope of your deflection deflected curve ok.

Similarly, you can find out d v d x at x equal to L, d v d x at x 3 equal to L by 2 at any point you can find out your slope by using the proper equation. I hope you have understood this, now this is one workout example for understanding the moment curvature relation and how to find out the deflection at a particular point in the slender member under the action of some bending moment.

Now, we will see later on some energy method, by using this energy we using that energy method, very simply you will get this kind of or you can solve this kind of problem the deflection problem or the slope problem. So, that will be discuss later on.

But at this moment we are trying to find out the deflection from our moment curvature relation, and because your for getting your v you are doing what you are performing integration twice right. So, that is why sometimes in some books it is also known as double integration method. So, nevertheless this is the problem based on your moment curvature relation.

Now, we will look at the method of superposition, which is valid for our system also now let see what is that. So, as you have seen that by using the moment curvature relation, the problem though the problem was very simple, but it was little bit say lengthy right not complicated I would not say complicate complicated, but it is little bit lengthy. So, that is why people if you know the solution for some known loading conditions, if you know the solution means slope as well as deflection maximum deflection for some known structure or the known loading condition, then you can super impose that thing to get the deflection or the slope for a complicated system. So, that is idea of method of superposition ok.

Now, what does it mean? The method ofsuperpositionis valid for the linearly elastic system, which exactly we are talking of in this particular course.

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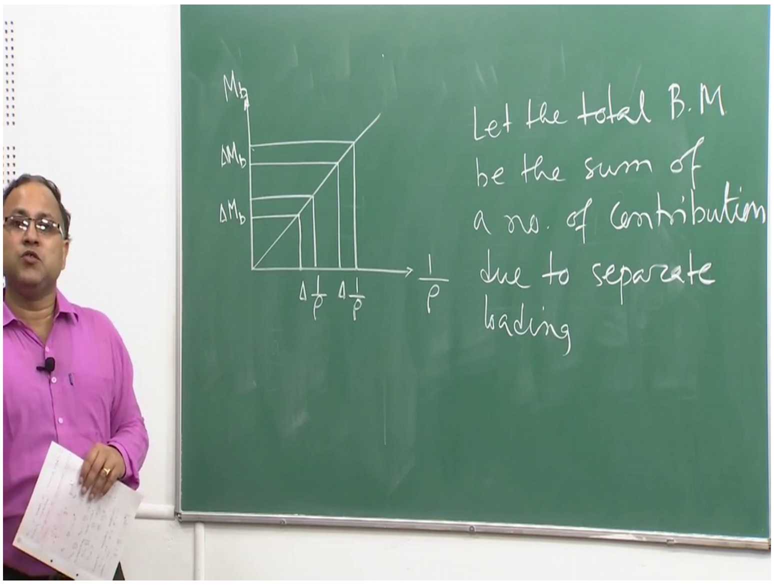

So, it means. So, along that we are writing 1 by rho; 1 by rho as you know that is d phi d s and along the y axis we are plotting M b.

So, now what we are doing. So, this is your linear elastic system. So, if you apply a an incremental bending moment delta M b, for that you will be getting some incremental change in 1 by rho. Similarly in the next increment if you increase there also you will be getting some incremental change in 1 by rho.

Now, the idea is that the total let the total bending moment be the sum of a number of contribution due to separate loading.

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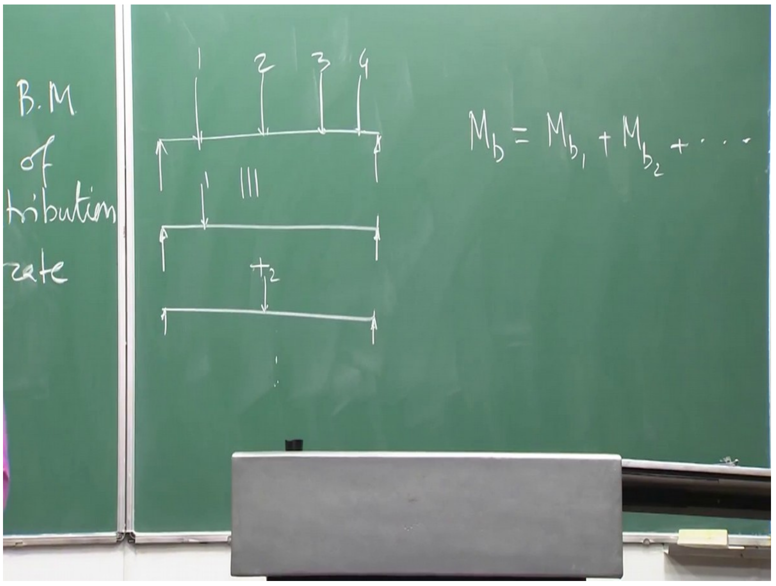

That means suppose I have this is one slender member under the action of different loads 1, 2, 3 and 4 there are different loads are acting on the slender member.

Now, I this is equivalent to say this is the same beam with this load, plus this is the same beam with this load, this is one this is two and so on. So, if you add them together you will be getting a same effect. So, if you find out the bending moment for this whole beam at a time or if you find out the bending moment of each individual beam and then you if you add them together sum them together you will be getting the same magnitude of the bending moment at a particular section, that is the idea and that is known as method of operation. So, that let the total bending moment be the sum of a number of contribution due to separate loading as I was telling you.

Then I can similarly write M b that is the total bending moment is equal to M b 1 plus M b 2 plus so, on up to n number of beams ok.

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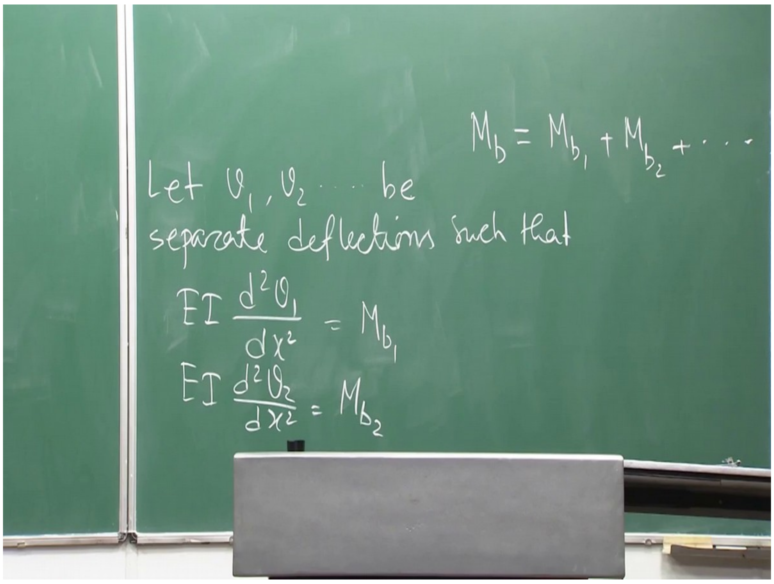

So, now let v 1 v 2 be separate deflections, such that your EI d 2 v 1, d x 2 is equal to M b 1 fine. If v 1 is caused due to the bending moment M b 1 so, this is the moment curvature relationship for bending moment M b 1 and v 1.

Similarly, your EI d 2 v 2, d x 2 is equal to M b 2 finally, I can write then the sum v equal to v 1 plus v 2 plus so on.

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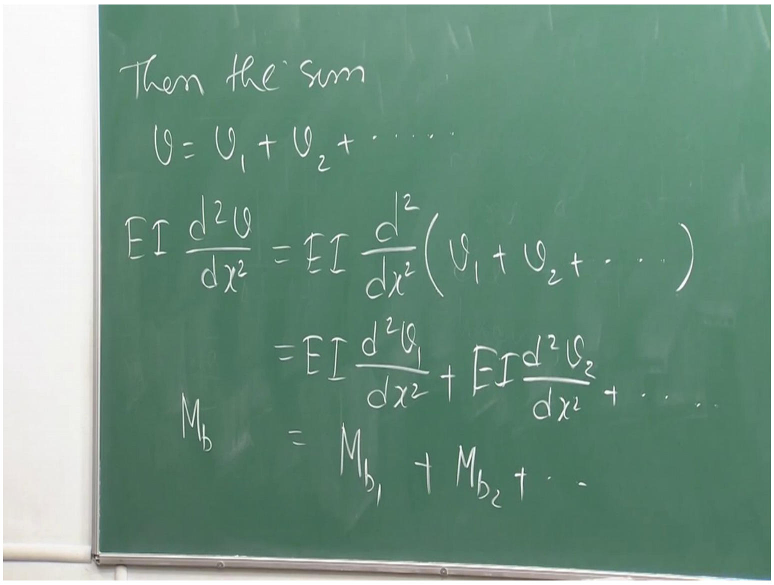

That means, the deflection in the actual beam is equal to the deflection of each individual beam I mean that is the summation of all the deflection together will give you the total deflection right as for the method of superposition ok.

So, this is nothing, but EI, I can write EI d 2 v d, x 2 right this is the I am going to establish the moment curvature relation for the original beam where all the loads are there, is equal to I can separate it like this because EI is constant. So, EI d 2 d x 2 in place of v I can write v 1 plus v 2 plus so on. So, from this I can write EI d 2 v 1, d x 2 plus EI d 2 v 2, d x 2 so on.

Now, what is this? This is nothing but M b 1, this is M b2 and so on and what is this? This is nothing, but your total bending moment in the original beam and that is equal to the summation of all the individual bending moment caused by individual loading system, as I was showing in the figure right. Load 1, 2, 3, 4 similarly you can find out that. So, this is nothing, but your method of superposition ok.

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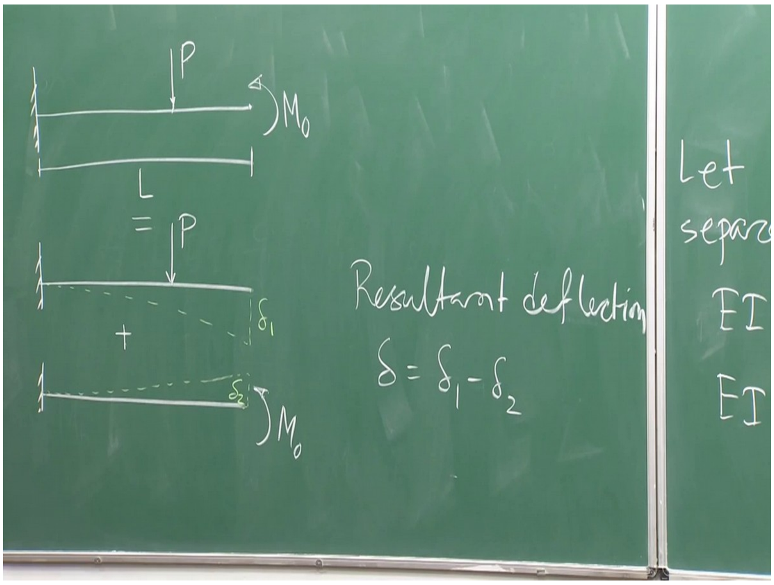

So, now what I mean how it will help me? Let see that how it will say; suppose this is one cantilever beam, there is a load concentrated force applied here, the total length of the beam say L, and there is one concentrated moment applied at the free end. I want to find out the deflection at the free end that is my job.

Now, you can apply the moment curvature relationship in the actual beam or the original beam, and then you can find out by taking two sections and whatever you have seen in the last numerical problem, in that way you can proceed. So, that will be little bit say lengthy process, I should not say complicated, but lengthy process.

So, what we will do here intelligently, we will separate the loading system; that means, this is equal to this system is equivalent to two systems one system is this where only the concentrated force is acting, and another system where only the concentrated moment is acting. I have separated two systems, now if I add these two these two things together, I will getting the original system that is your method of superposition.

Now, here in the deflected shape under this loading the deflected shape probably deflected shape is like this. So, this is your say delta 1; that means, the deflection at the free end due to the application of thee concentrated P at this point, and this moment we will try to deflect the beam like that and this is say delta 2. So, your actual or the resultant deflection under the action of both the things together is nothing, but say delta which is nothing, but delta 1 minus delta 2, agreed just algebraic summation.

Now, if I consider this is one basic system and this is another basic system, and if I know the solution of course, by following the same process like your moment curvature relationship, if we know if I have a cantilever beam, if I have a load concentrated force on the cantilever beam like this, the delta 1 value if I know priory. I mean beforehand if I know this delta 1 value, and if I know the delta 2 value under this kind of situation beforehand, then I just put this thing here to get the resultant delta, which is applicable for the system understood.

So, this is the say intelligent way by which you can solve different complicated systems. You separate them for different systems, and then you go to the basic system and those basic system solution should be known to you prior handm and then based on that you can find out the resultant deflection.

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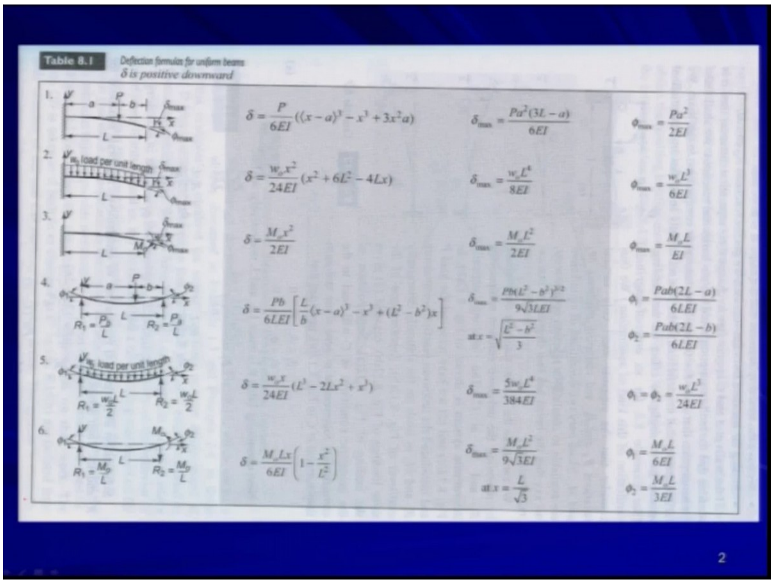

Table 8.1 Deflection formulas for uniform beams

δ is positive downward

1.	$\delta = \frac{P}{6EI}((x-a)^3 - x^3 + 3x^2a)$	$\delta_{max} = \frac{Pa^2(3L-a)}{6EI}$	$\phi_{max} = \frac{Pa^2}{2EI}$
2.	$\delta = \frac{w_o x^2}{24EI}(x^2 + 6L^2 - 4Lx)$	$\delta_{max} = \frac{w_o L^4}{8EI}$	$\phi_{max} = \frac{w_o L^3}{6EI}$
3.	$\delta = \frac{M_o x^2}{2EI}$	$\delta_{max} = \frac{M_o L^2}{2EI}$	$\phi_{max} = \frac{M_o L}{EI}$
4.	$\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3 - x^3 + (L^2 - b^2)x\right]$	$\delta_{max} = \frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}LEI}$ at $x = \sqrt{\frac{L^2-b^2}{3}}$	$\phi_1 = \frac{Pab(2L-a)}{6LEI}$, $\phi_2 = \frac{Pab(2L-b)}{6LEI}$
5.	$\delta = \frac{w_o x}{24EI}(L^3 - 2Lx^2 + x^3)$	$\delta_{max} = \frac{5w_o L^4}{384EI}$	$\phi_1 = \phi_2 = \frac{w_o L^3}{24EI}$
6.	$\delta = \frac{M_o Lx}{6EI}\left(1 - \frac{x^2}{L^2}\right)$	$\delta_{max} = \frac{M_o L^2}{9\sqrt{3}EI}$ at $x = \frac{L}{\sqrt{3}}$	$\phi_1 = \frac{M_o L}{6EI}$, $\phi_2 = \frac{M_o L}{3EI}$

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Similarly, let us see some basic systems. So, these are some basic systems as you see these are some basic systems, this is some system one as this problem is given. So, if P is applied here. So, you can find out delta max phi max at the free end. So, that value is given already ok.

Similarly, if you have the system like this, this is your delta max this is your phi max which will be happening at the free end similarly this is your third system four system fifth system sixth system. So, all these systems; the all these systems if you consider the basic system and if you try to separate or try to say make your complicated system or if you try to separate your complicated system or the original complicated system into different basic systems, and then if you try to obtain the magnitude of the deflection or the slope and then finally, you will get the resultant thing for the original complicated system. So, that is the idea of your method of superposition.

So, now this method of superposition can be used for indeterminate structure also, let see how we can use that.

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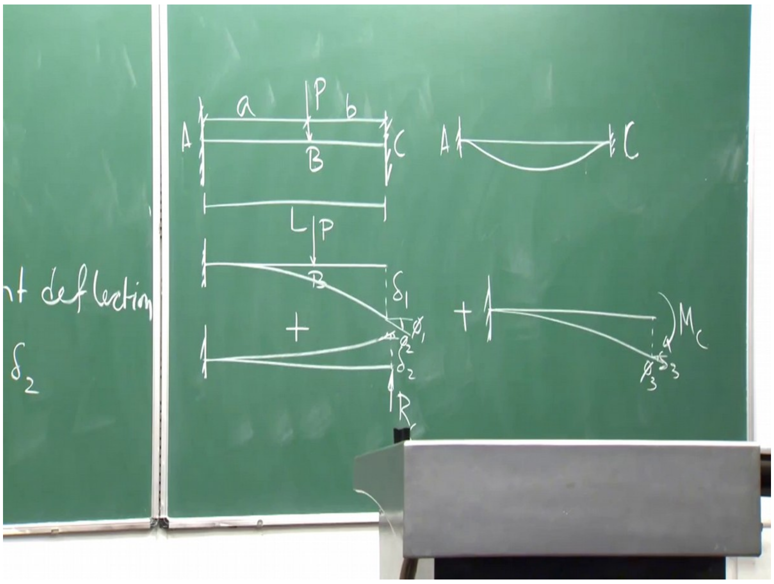

Suppose you have both ends are fixed. So, this is; obviously, your indeterminate structure why? Because here you will be having two unknowns because I do not consider any axial force here, you have two unknowns one vertical reaction and another moment here also you have two unknowns one vertical reaction and one moment right. So, you have four unknowns, but you have only two equations right sum I mean 3 equations right. So, basically you cannot solve it. So, this is your indeterminate structure as whatever we have discussed previously ok.

So, now this on this beam I am applying some load P. So, this is say a distance this is your b distance then total length of the beam say L. So, for this system one thing is known to me because these are 2 fixed support. So, here your slope will be 0 do you remember here your slope will be 0 because moment is there right, you have the moment reaction. So, the slope must be 0 at this 2 ends in A and in C. So, this is your A and this is your C and this point say B.

So, this can be separated in three basic systems, first one is when you have only the cantilever beam where you have the concentrated force acting at this point B, this is one basic system. So, this will be deflected this much. So, this is my say delta 1 and this is your say slope phi 1 ok.

Similarly, next basic system will be another cantilever beam. So, I will be having the vertical reaction upward vertical reaction. So, that is say R c under this action the deflected shape will be like this. So, this is my say delta 2 and this is the angle phi 2 plus what else is left out the moment reaction. So, for that another basic system I am considering, for that say M c is acting here and your deflected shape is like this. So, this is your say delta 3, and this angle say phi 3 that is the slope.

So, therefore, your resultant deflection now, these three are all your basic systems now these basic systems solution are known to you, now you get the delta 1 delta 2 delta 3 value from the basic systems, and phi 1 phi 2 phi 3 all are from basic systems.

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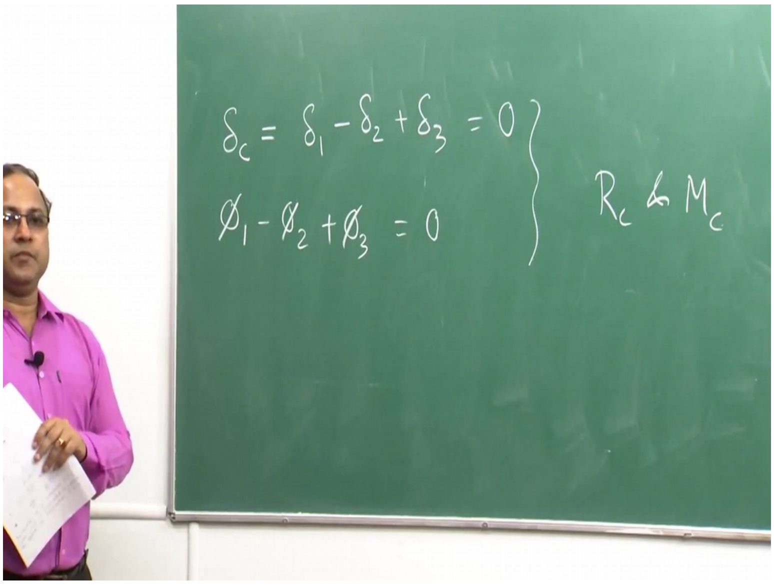

And then basically your if you look at the deflection at point c is nothing, but delta 1 minus delta 2, plus delta 3 and what is the magnitude actual magnitude as for the boundary condition what is the magnitude of your delta c? That is the fix support there is no displacement or the deflection at point c that must be 0.

Similarly, phi 1 minus phi 2 plus phi 3 that is the resultant slope at point c, that also must be 0 because that is the fix support. Now from these two equations you get two unknowns R c and M c, they are unknown to you. So, in that process actually in this method of superposition process you can even find out the unknown reactions because this R c and M c are not known to you because this is the indeterminate structure, they can be found out by solving in this fashion. So, I will stop here today in the next lecture we will be continuing with the discussion regarding this.

Thank you very much.