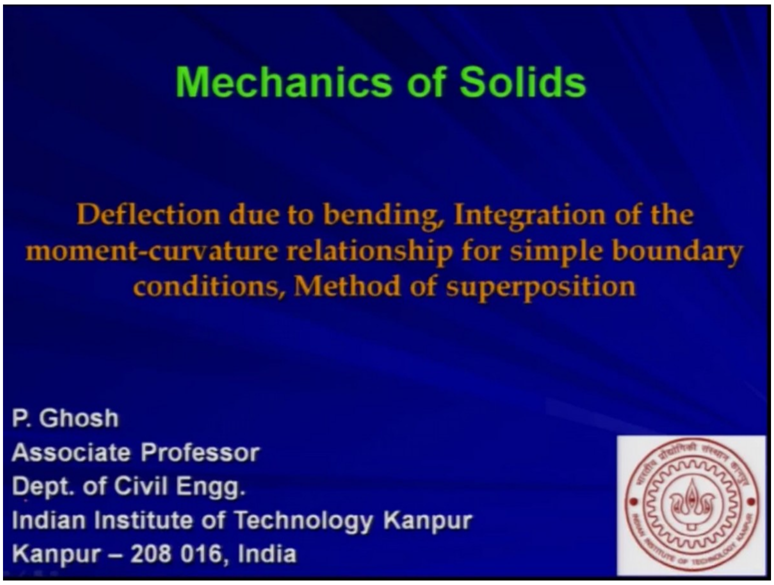
Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture - 51
Deflections due to Bending

Welcome back to the course Mechanics of Solids. So, as we discussed in the last class that will be starting a new topic or new chapter which will deal with the deflection due to bending as you can see from the slide.

(Refer Slide Time: 00:29)



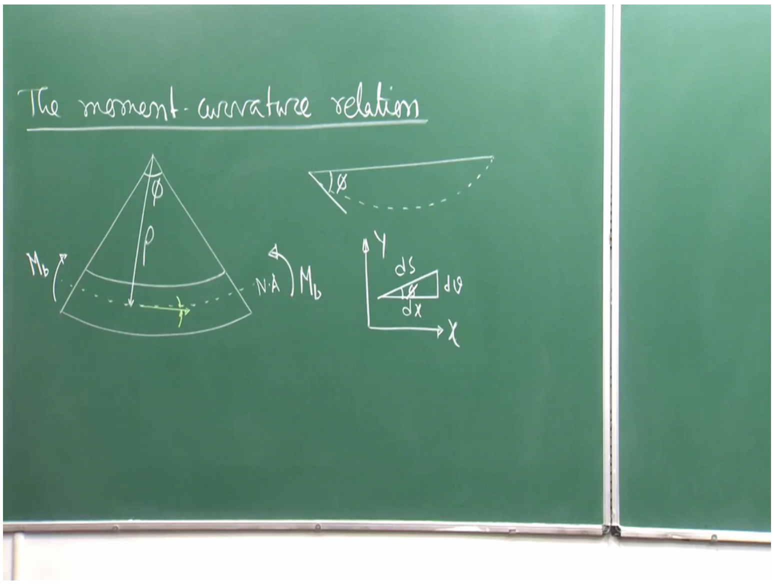
Deflection due to bending integration of the moment curvature relationship for simple boundary conditions and method of superposition. And then finally, we will be moving to the energy method that will be coming later on.

So now what is the, as you have seen when we are discussing about the bending theory then you. You have seen that the straight slender member you had and after bending it was taking this kind of shape where concave I mean of course, depending on the type of bending moment positive or negative. Depending on that you will be getting the curved shape of the slender member right.

So, because of this bending actually you are getting the deflection. Deflection means you had the straight beam and then it got deflected the from the neutral position or the from the original x axis to the final after deformation what is happening. So, this is nothing but the deflection right. So, that deflection can be estimated by using or by establishing the relation between the moment curvature ok.

So now let us see how we can establish that relation by which we can find out the deflection in the beam due to bending ok.

(Refer Slide Time: 01:54)



The moment curvature relation. So now, as you recall so, this is so, this was your neutral axis. So, this is the beam after bending. And see this angle which is made at the centre say phi. And the radius of the curvature say rho. And this is under pure bending situation will be already we have seen that thing earlier pure bending situation say.

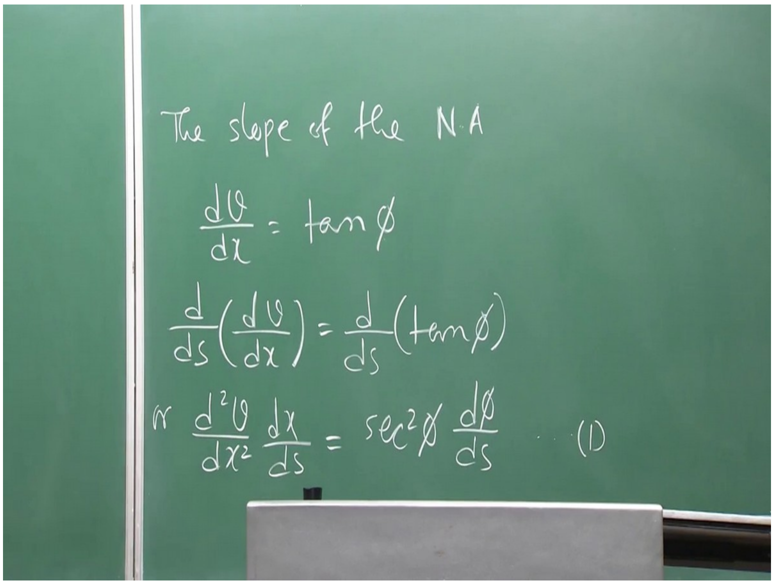
So, the original x axis was horizontal and this is the curve force. So, this angle is basically this angle; that means, the departure from the original x axis will give you the give you the relation between the deflection means the originally it was horizontal and now it has got the curved shape. So, this angle will establish the relation between your deflection and the curved shape ok.

So now so, if I draw the line diagram basically. So, this was the original position and then after deflection or after deformation after the application of the bending moment it takes the shape like that. So, this is nothing but your slope right? At any point if you consider at any point on the on the curved line you can find out the slope right. And that slope will be varying right. At each and every point of the beam the slope will be varying and that we can establish or we can derive or we can get some say some relation by which you can find out the slope at each and every point.

So now in the x y plane basically if you look at. So, this was the original configuration horizontal. Now it takes the shape like that. So, therefore, this much is your deflection. So, d v this was your say d x if I consider and this is your d x along the curved part ok.

So, along this curved part if I dash line or dotted line if I consider d S and this is the say d x. So, this much is a departure d v and this angle is say this angle automatically will be phi as for the definition there is the angle made at the centre. So now, the slope of the neutral axis.

(Refer Slide Time: 05:30)

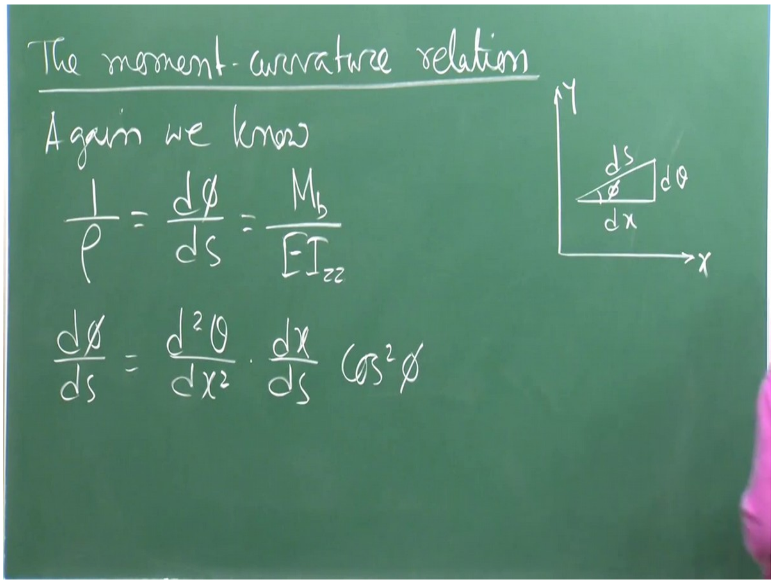


The slope of the neutral axis is nothing but d v d x equal to tan phi. Very simple because this is your say neutral axis. So, d v d x is nothing but the slope of the neutral axis ok.

So now if I differentiate both the sides with respect to the curved surfaced s. Then I can write d d S of d v d x equal to d d S of tan phi we are taking or we are doing the differentiation with respect to curved length d S on both the sides ok.

So, this can be written as d 2 v d x 2 d x d S is equal to sec square phi d phi d S say equation 1 agreed fine, So I have got this relation.

(Refer Slide Time: 07:05)

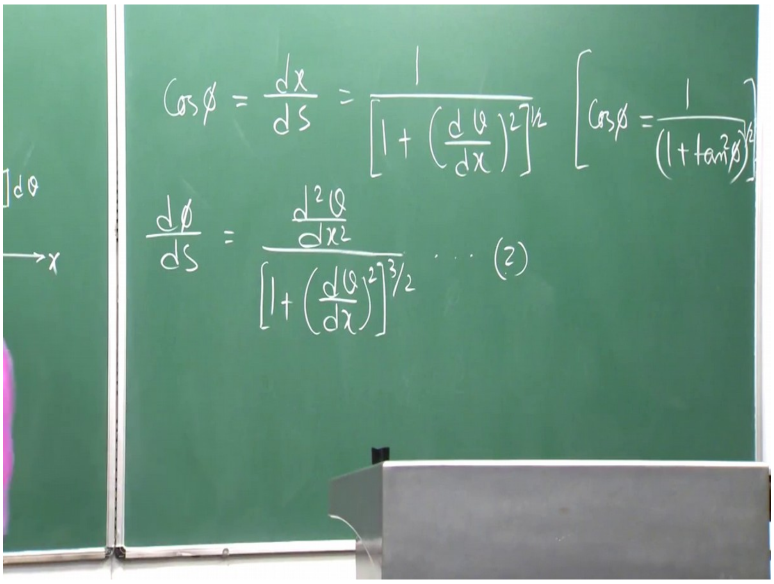


Now again we know from our earlier discussion that 1 by rho is equal to d phi d S is equal to M b by E I z z right. Already we have established this relation if you go back to the previous lecture you will see already we have established this relation ok

So now so, from one if I if I put this relation in equation 1 will get it. So, d phi d S is equal to d 2 v d x 2 d x d S cos square phi. Now from the figure whatever figure I had drawn the figure means that one. So, this is this was your d v this was your d x and this was your d S and this is phi in x y plane right.

Now, in that figure we can see cos phi is equal to d x d S.

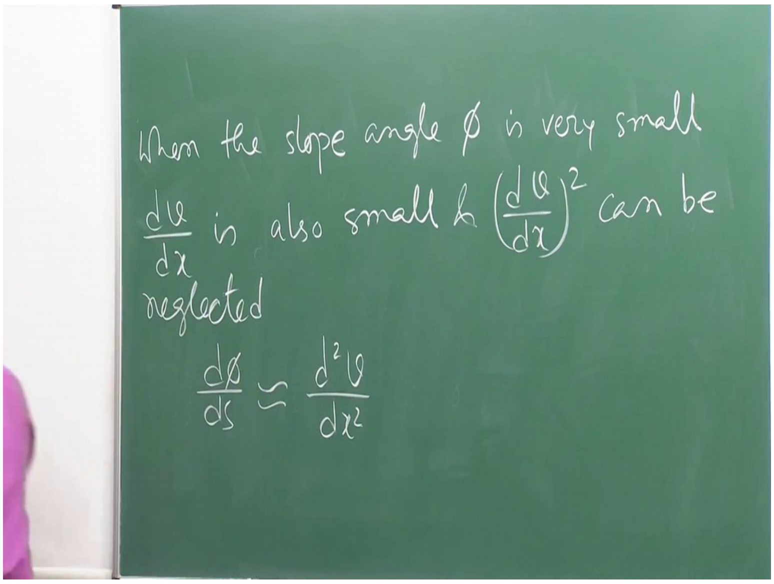
(Refer Slide Time: 08:32)



Which can be written as 1 by 1 plus d v d x whole square to the power half. So, where from where this is this relation is coming this is simple geometrical relation, because as you know cos phi from your trigonometry 1 by 1 plus tan square phi to the power half. That you know right. From trigonometry relation this thing is known to me ok.

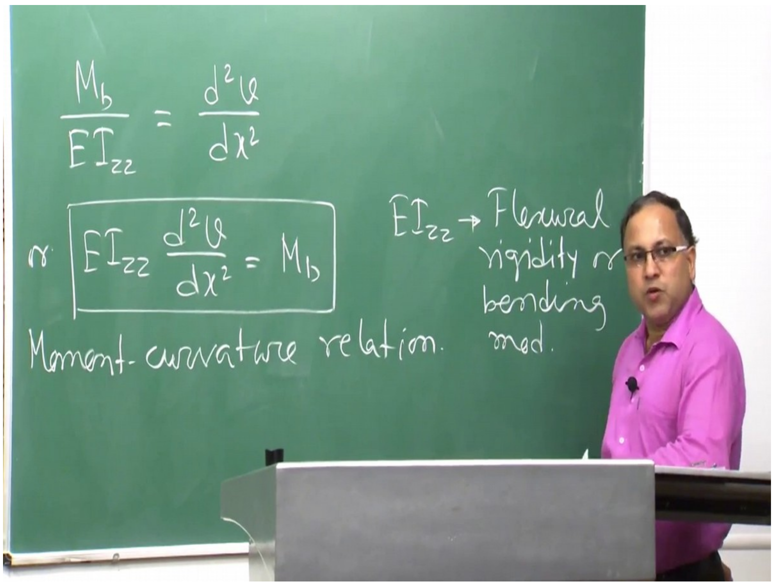
So, I can write that cos phi is equal to this now I am I am substituting that thing here. So, what I will get therefore, d phi d S is equal to d 2 v d x 2. Now d x d S cos square phi is nothing but cos q phi right. And cos q phi is nothing but cube of this. So, therefore, I can write 1 plus d v d x whole square to the power 3 by 2. So, this is equation say 2 ok.

(Refer Slide Time: 10:11)



Now, now when the slope angle phi is very small, because generally in this kind of slender beam or slender member right. You cannot I mean this phi is very small the slope angle is very very small right. So, therefore, we can say d v d x is also small. And d v d x whole square can be neglected therefore, from that equation 2 I can simply write d phi d S is approximately equal to d 2 v d x 2. Which can be further written as now what is this d phi d S this d phi d S is nothing but M b by E I z z is equal to d 2 v d x 2.

(Refer Slide Time: 11:28)



So, or we can write like this. So, E I z z d 2 v d x 2 is equal to M b right. This is nothing but your moment curvature relation.

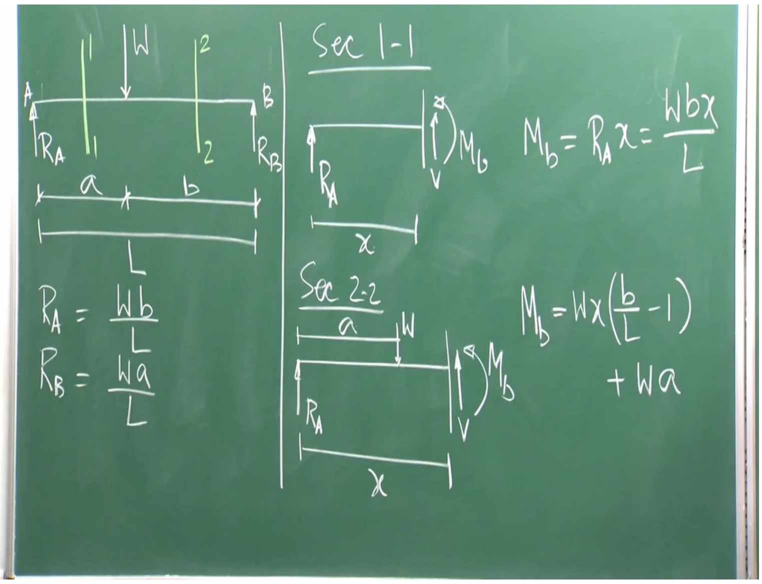
Now, what does it mean actually? What does it interpret? So, if you know the bending moment and though the equations all the equations are valid for your pure bending, but already if you recall in the previous lectures we agreed that whether it is pure bending situation or if you have the constant shear force or the varying shear force, this still whatever theory we have developed for pure bending that will be holding good right. So, based on that we can say that any situation where you have shear force also varying shear force also. Then in that situation also this equation is valid ok.

So now what does it say, it says that if you know the bending moment applied in the slender member. And if you know this E I z z. So, there is some special name for this E I z z. So, E I z z is nothing but your it is known as flexural rigidity or bending modulus. So, this is for particular section and particular material E I z z is constant right. If you see if you consider a constant construction of the slender member and if you consider constant I mean homogeneous material in the slender member then E I z z is constant. So, this is the flexural rigidity this is known as flexural rigidity in mechanics or bending modulus in some books that also mentioned ok.

Now So, this part you can calculate from the material property as well as the cross section of the beam or the slender member right. So, once you know this and once you know this how much bending moment is getting applied. You can find out or you can solve this differential equation to get v, what is v? V is the deflection that is the departure as I as I showed in the figure right. V is the departure from it is original horizontal position to the curved part. So, this v can be obtained from by solving this differential equation with proper boundary conditions ok.

Now, let us take one numerical problem or example. So, that you will be understanding also that how we can we can solve this kind of problem. Let us take that problem let us take one numerical problem. So, you will appreciate that how we can find out the deflection from the moment curvature.

(Refer Slide Time: 15:01)



The problem is like this very simple problem we are taking to understand this thing in a bitter way. You have a simply supported beam reaction R A and R B. So, this is point A point B you have one concentrated force W here, which is acting at a distance away from a and this is B the total length of the beam is say L and fine.

So, for this actually your objective is to find out the deflection as a slope at the point where the load is applied. So, your first, what is your first job? As already we have done several times your first step is to find out the external reactions R A and R B and you know how to find out that R A will be coming as W B by L and R B will be coming as W a by L.

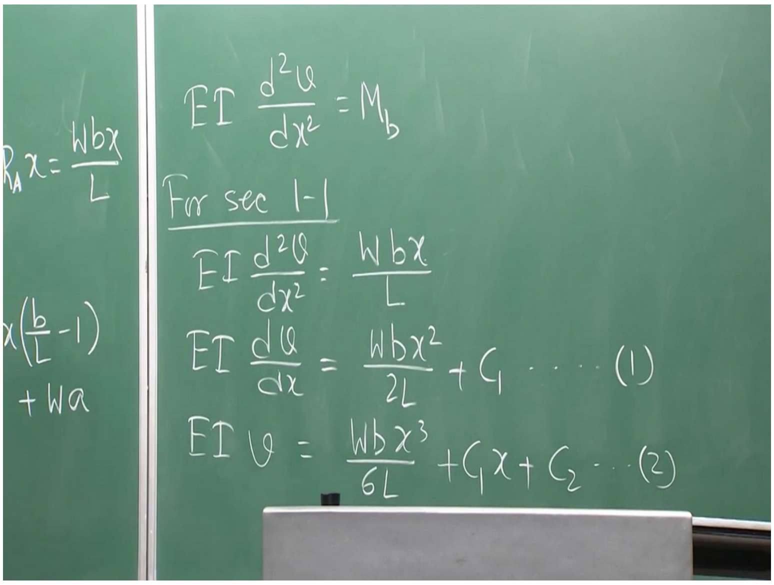
Now, to define this whole beam how many minimum sections you need to take? 2 because there is one load. So, from this point to this point there is no variation of load. Then from this point to this point there is no variation of load. So, we are taking 2 sections section 1 1 and section 2 2 ok

Now, based on that we are going to find out the bending moment and shear force. So, section 1 1. So, that is nothing but this this is your v this is your M b, this is your R A and this is your x. So, from this I can find out M b equal to R A x is equal to W B x by L fine.

Similarly, section 2 2 if we consider we are considering this part left hand side part. You have R A you have W which is at a distance a from a and you have shear force here bending moment. And this distance is say x now. For that your M b can be calculated I am not going through the stepwise calculations that you know how to find out M b. So, M b will be coming as W x B by L minus 1 plus W a ok.

So, I have got M b for section 1 1 and M b for section 2 2 and that is both equations are defining the total variation of bending moment over the all whole beam right. Over the whole beam.

(Refer Slide Time: 18:43)



So, as we know This is my moment curvature relation. I am not writing I z z every time because it is quite understood. So, I am just writing simply E I. So, for I mean now onward I will not be writing I z z again and again fine.

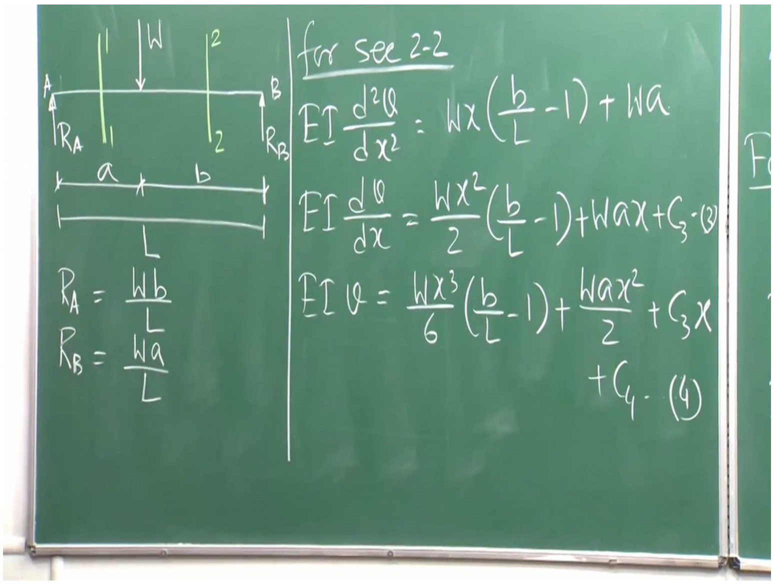
So, this is my moment curvature relation. So, for now for section 1 1 if I apply this moment curvature relation I will get E I d 2 v d x 2. This is equal to what bending moment what is the bending moment in section 1 1 that is the bending moment. So, that is W B x by L very simple ok.

So now if I integrate this E I d v d x is equal to W B x square by twice L plus some integration constant C 1 say equation 1. Similarly I will be getting one more integration E I v that is the deflection I have got that is W B x cube by 6 L plus C 1 x plus C 2 is equal to equation 2 ok.

Now, C 1 C 2 they are not known to be and they can be found out from the boundary conditions suitable appropriate boundary conditions. So, this is the expression which will give you the deflection for section 1 1; that means, if you are if you are travelling from point A to this point. Within this point your deflection can be defined by this equation. And similarly what is this is nothing but your slope, isn't it? This is nothing but your slope ok.

So now similarly we will do the same exercise for section 2 2.

(Refer Slide Time: 21:04)

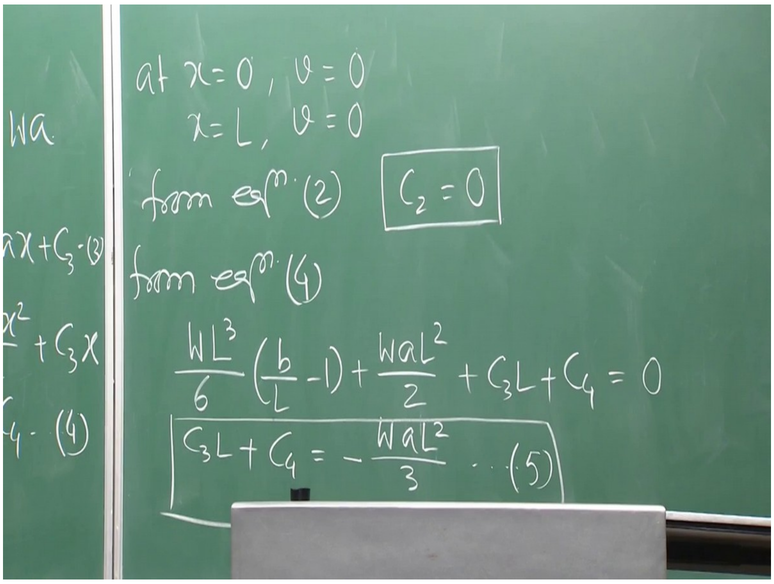


For section 2 2, E I d 2 v d x 2 is equal to bending moment, that is M b for section 2 2, please try to remember. So, that already we have calculated W x B by L minus 1 plus W a.

So now I integrate once that will give me W x square by 2 B by L minus 1 plus W a x plus C 3 say equation 3 say. And then one more integration will give me E I v W x cube by 6 B by L minus 1 plus W a x square by 2 plus C 3 x plus C 4 equation say 3. So, I have got 4 equations to be solved. First we need to find out all the integration constant C 1 C 2 C 3 C 4. And you know how to find out from simple mathematics ok.

So, let us find out those constants by putting the suitable boundary conditions.

(Refer Slide Time: 22:45)



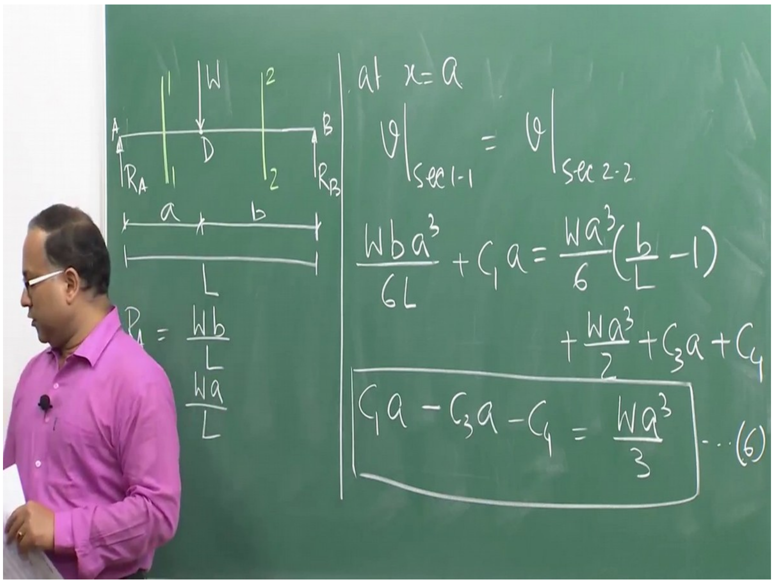
So What are the boundary conditions I can think of. So, you need at least 4 boundary conditions right. 4 boundary or the continuity conditions you need to find out all 4 unknown integration constants.

So, first thing is that at x equal to 0 I can say v equal to 0 that is the displacement is 0 simply at x equal to 0 for that B whether it is a support condition right. At x equal to 0 means at point A point A there is no deflection of the beam. Similarly at x equal to L your v equal to 0 right.

Now, by putting this from your equation 2 I can simply get C 2 equal to 0. At least one integration constant has been found out. So, same process if we perform for other equations from equation 4 we can write down this W L cube. Equation 4 means this equation, this equation will be giving me the deflection from this point to point B in between that it will give me the deflection.

So, if I put x equal to L here. So, I will get the deflection at point B which is nothing but 0. So, I can write W L cube by 6 B by L minus 1 plus W a L square by 2 plus C 3 L plus C 4 equal to 0. So, from this if I if I do little bit of simplification finally, I will get C 3 L plus C 4 equal to minus W a L square by 3 say equation 5. So, that I am getting. I am getting one relation between C 3 and C 4.

(Refer Slide Time: 25:14)

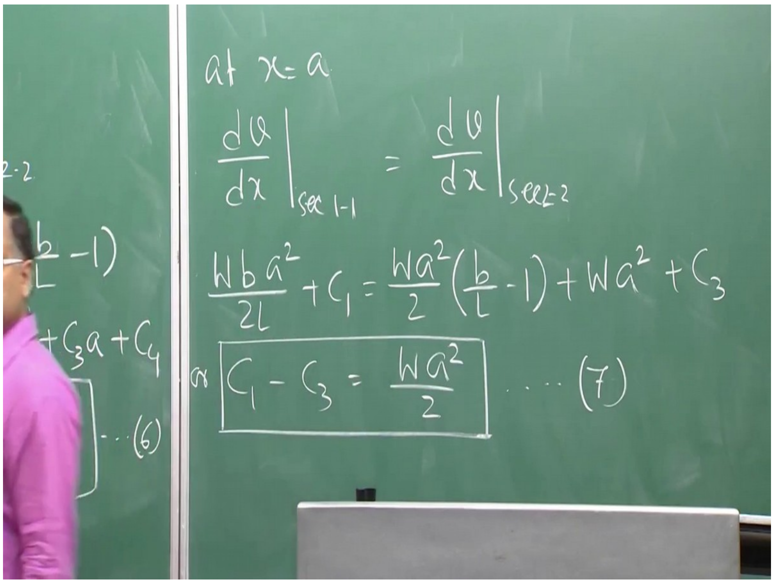


Now at x equal to A, at x equal to A means you are at this point say this point say d at x equal to A; that means, when you are at point d. Then basically v coming from section 1 1; that means, if you if you solve the equation of v v means your deflection for section 1 1. That should be equal to the v calculated from section 2 2, that is the continuity thing because at the same point the beam bending is continuous right. At the same point you will not be getting 2 different defection of the beam.

So, whether you come from section 1 1 or from section 2 2 at that particular point that is the common point d at that point your deflection should be same. So, that I have write it. So, v coming from section 1 1 and v coming from section 2 2 at x equal to A must be same. So, if I put the values of v for section 1 1 at x equal to A is nothing but W a cube by 6 B by L minus 1 plus W a cube by 2 plus C 3 a plus C 3. So, from this I will be getting one relation C 1 between C 1 and C 3 C 3 a minus C 4 I am getting the relation among these 3 constants equal to W a cube by 3 say this equation say equation 6 ok.

Now, even by following the similar argument I can say at x equal to A.

(Refer Slide Time: 27:26)



Your slope of the curve must be same whether you come from section 1 1 or section 2 2. So, what I can write d v d x calculated from section 1 1, is equal to d v d x calculated from section 2 2. They must be same, deflection may has to be same at point d whether you come from one or 2 as well as your slope will be same.

So, if you put the equations for that W B a square by twice L plus C 1 is equal to W a square by 2 B by L minus 1 plus W a square plus C 3. So, by simplifying this thing I will be getting C 1 minus C 3 equal to W a square by 2 say equation 7. So, we have got 4 equations and 4 unknowns. So, from which you can find out the magnitude of your integration constants.

So, I will stop here today in the next class or the next lecture we will be solving all the or we will be finding out the magnitude of all the integration constants. And then we will impose some special condition to find out the slope and deflection at point d. So, I will stop here today.

Thank you very much.