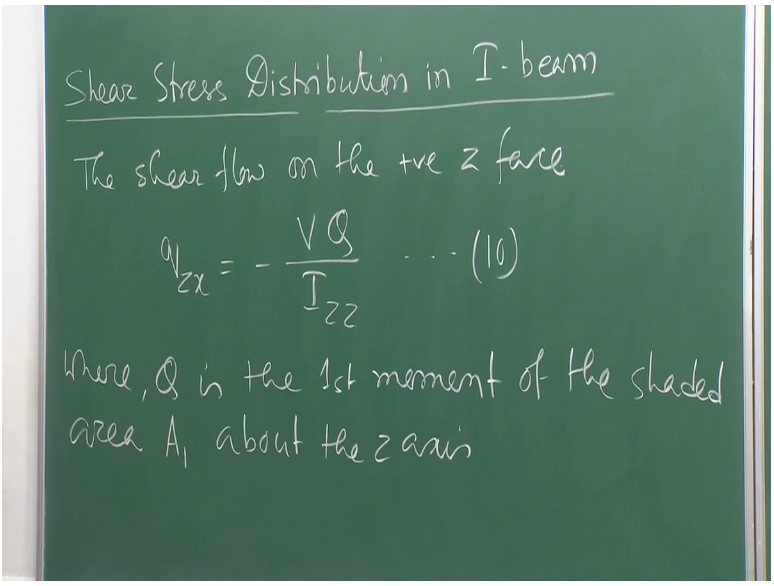
Mechanics Of Solids

Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 49
Distribution of Shear Stress in I Beam

Welcome back to the course mechanics of solids. So, in the last lecture we have seen we have we have derived rather the shear stress distribution due to bending. And there we have seen that how we can find out tau y x or tau x y both are same in the rectangular beam or in any kind of say symmetrical cross section with respect to the x y plane.

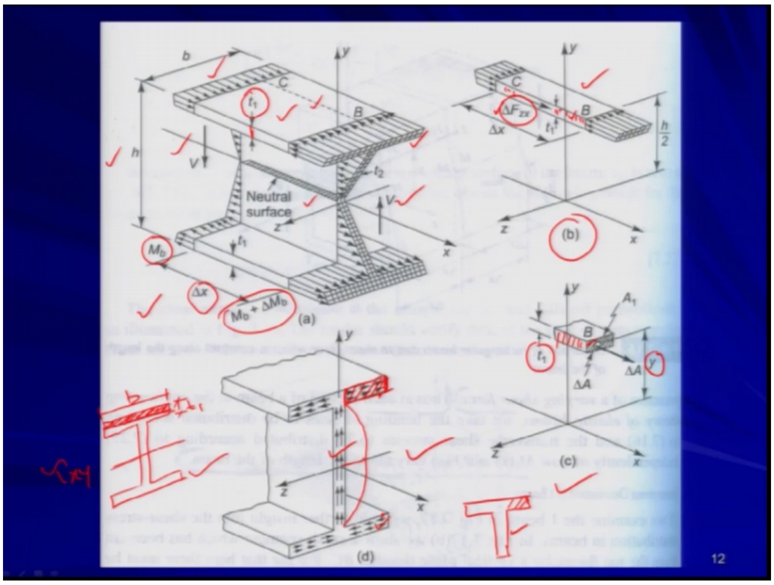
(Refer Slide Time: 00:47)



Now, today I will be talking about the shear stress distribution in I beam, as I told you adjust touched upon in the last lecture if you recall that I beam or t beam is having some specialty. So, the specialty is that it is not the solid say cross section like rectangular cross section it is having very thin say wave and white flange right. So, this kind of structure you need to find out or need to analyze the shear stress distribution in the flange particularly.

Now, why it is so? I mean if you will appreciate when we will be discussing this thing suppose I am considering one I beam like this ok.

(Refer Slide Time: 01:31)



So, this is the I beam. So, the consideration is very similar to the previous one like we are considering one section say the length of the section is delta x, on this section your bending moment is M b and here your on this side right hand side your bending moment is M b plus delta M b.

However the shear force is we are considering shear force is constant this is not varying with x, the bending moment is only varying. And the depth the total depth of the beam is say h and this is called flange as I told you in the last lecture and this is called wave.

So, the neutral surface this is your neutral surface which will be passing through the centroid of the whole say I section. So, the I section will look like this very similar to like this. Now this is a normal stress distribution coming from the bending on this face and this is the distribution of normal stress on the other face.

Now, what we are doing here. So, in figure b if you see in figure b, a small segment has been cut from the top flange by a vertical plane through b c. So, this is your line. So, through this b c we are just cutting one small segment of flange and that is shown here. So, this small part we are taking out from the beam and then we are trying to analyze the small part, let us see how the shear stress is getting distributed over the small part.

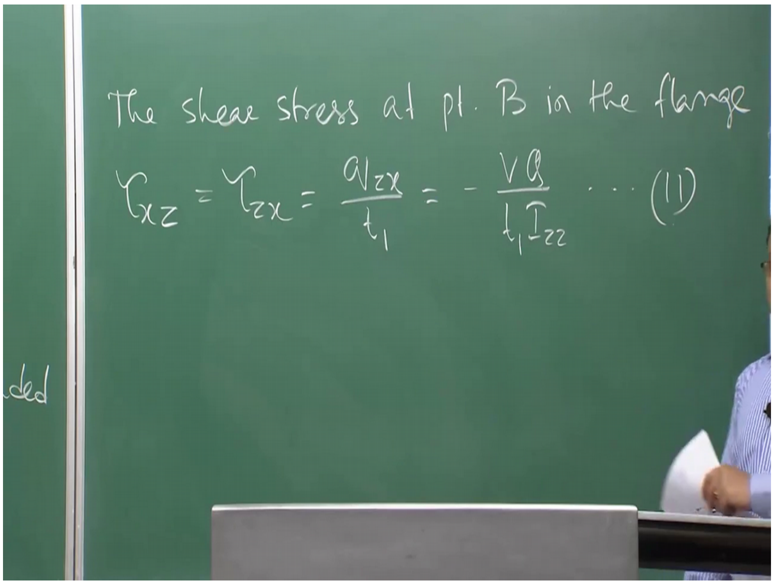
Now, there must be a shear force delta F z x on the positive z face. So, this is your positive z face. So, this face is your positive z face, cut face is your positive z face. On these positive z face there must be a shear force delta F z x to maintain the equilibrium in x direction which is pretty similar to the previous one where, you had f delta F y x on the y plane. So, that force was balancing the satisfying the equilibrium, but in this case there is no other face which will try to because the bottom face is not continuous right the bottom part is hanging kind of thing. So, this is the wave, this is the wave and there you have the flange. So, the flange is kind of hanging. So, the bottom surface is not participating in the in balancing or in satisfying the equilibrium. So, this is the surface on which this force delta F z x is developed to balance or to maintain the equilibrium in the x direction.

Now, if you try to maintain the equilibrium in the x direction, then the shear flow on the positive z face will be very similar to the shear flow as discussed earlier on the positive Z face will be simply q z x which will be equal to minus VQ by I z z, say this is equation 10 in continuation with the previous equations ok.

Now, what is this Q where Q is the first moment of the shaded area, A1 about the Z axis. now if you come back to this figure again, if you see this figure C. So basically A1 is this shaded areas as shown this area. So, this is the part which is defined by A1. So, Q capital Q is the first moment of this area with respect to your Z axis; that means, first moment of area means this is the y distance apart I mean this area is lying y distance apart from the z axis and this area if you know you can find out or you can determine the first moment of that area right ok.

Now, and this thickness of the flange is say T 1 this is a thickness so as shown here also. So, T 1 is the thickness of the flange.

(Refer Slide Time: 06:26)



Now if I want to find out the shear stress; the shear stress at point B as shown in the figure at point B in the flange. So, that is nothing, but. So, that is on the Z plane along x direction that is nothing, but tau x z which must be equal to tau z x, which will be equal to the shear flow divided by the thickness that is the shear flow is nothing, but the shear force per unit length of the beam on that. So, this is varying over the whole plane that is T 1 right.

So, this as I told you this F z x basically, this shear flow is happening on this plane. So, you will be getting your I mean the way we calculated tau y x or tau x y in the previous lecture, the similar argument is applicable here also. So, that is nothing, but q z x by t 1 over the thickness over that plane that is nothing, but VQ by t 1 into I z z. So, this is equation 11 ok.

Now, if you look at figure d. So, if you come back to this this is your figure d. So, the figure d if you look at in each flange tau x z varies linearly from a maximum at the junction. So, because if A1 is becoming maximum, then you will be getting the maximum amount of Q because vies constant for that particular section right I z z is constant for a particular cross section of the I beam.

So, therefore, they are not varying they are constant, but only thing is that is varying. So, Q is becoming maximum when you are considering the junction because then in that case the whole area will be participating in calculating the magnitude of q. So, that in each flange the tau x z varies linearly from a maximum at the junction with the wave to zero at the edge. So, it will be zero at the edge, while in the wave the stress tau x y has a parabolic distribution, but in case of wave you have the parabolic distribution as we have got.

So, what I mean to say in this flange, flange part because there is no material here. So, from this point to this point there is no material. So, this is something like your over hanging part so, in the flange as you are have seen tau x z or tau z x I mean that will be more critical. Where as in case of wave this is your wave in the wave your tau x whatever you have calculated previously. So, tau x y will be more critical where you will be having the maximum at the neutral surface and that is the parabola distribution and minimum at the edge mean at the top surface as well as the bottom surface.

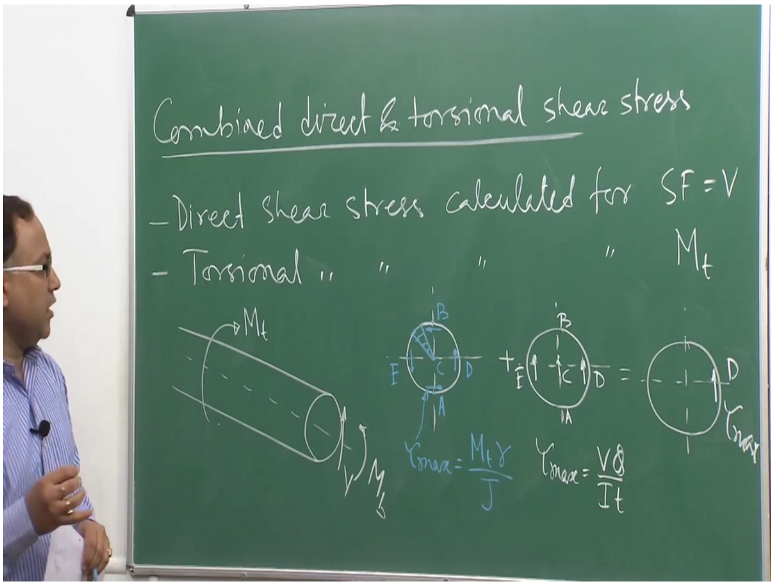
Now, frankly speaking there are also tau x y in the flanges. So, in the flange also will say suppose if this is the flange, there also you will be getting the tau x y say suppose if you consider this section. So, this is the area which will be on this plane you can find out tau x y right, but that tau x y will be very very small as compared to your tau x z, because this this thickness of the flange will be I mean very very small because this this part this thickness of the flange will be very very small as compared to the I mean flange with. So, this is say flange width say b.

Now, if you considered the first moment I mean if you try to find out the tau x y value in your flange, then basically this is the area which will be participating in calculation of q the first moment of this shaded area. So, that area will be very very negligible because that b into t 1 by I mean if you I mean b into t 1 or maybe less than t 1 right. So, that is very very negligible.

So, therefore, the tau x y will be coming very very small as compared to tau x z. So, therefore, I mean in case of t beam even you I mean this is this this analysis will be valid for your t beam also where you have flange on the top side, and then you have the wave. So, there also whenever you are having this kind of slender say overhanging part beam or I beam. So, there actually your tau x z will be more critical than your tau x y ok.

So, I hope that you have understood the concept. So, whenever you will be dealing with this kind of flange and wave system in the beam or beam cross section, then you need to find out the shear stress separately for the flange and for the wave; because in both the components flange as well as in the wave your shear stresses the critical shear stresses will be different ok.

(Refer Slide Time: 12:21)



Now, we will be talking about combined stresses. So, combined say you have combined direct and say torsional shear stress. So, this is not very uncommon in several occasions you may get this kind of situation, where the slender member is experiencing the direct shear stress; that means, the shear stress coming from your shear flow I mean these bending moment and all, and at the same time you are applying the torsional moment twisting moment because of that also the cross section is suffering from shear stress. Now if you try to combine these two then how it will look like. So, that is the question.

So, if you have direct shear stress calculated for shear force V, just now we have calculated tau x y right we have seen that that is that we are calling as direct shear stress calculated for shear force, then you may have torsional shear stress calculated for M t that is the twisting moment, the slender member is experiencing shear force as well as twisting moment together then what will happen? Then you need to combine the shear stress then combine; that means, you have the slender member like this, if suppose this is the slender member you are applying M t, and at the same time you have shear force as well as bending moment this right happen.

So, if this happens then what will happen to your shear stress distribution? So, this is the cross section. So, here you have shear stress distribution for your torsion that is tau max equal to M t r by J right already we have J is a polar moment of inertia already we have discussed. So, this is the equation for your shear stress when you are dealing with the torsion ok.

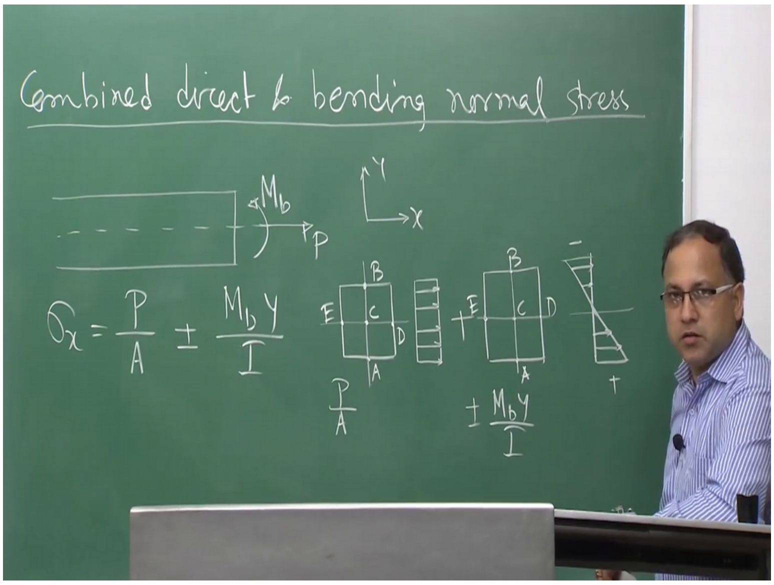
So, I am defining 4 points B A this is my C this is my D and this is my E. So, therefore, the shear stress will be distributed like that. So, this is at the periphery you are considering this points. So, they are actually as you know from your theory of the torsion that will be maximum shear stress right. So, and distribution will be like this triangular. So, this will be is 0 at the center and maximum will be at the circumference.

Similarly, for direct thing same thing we are considering A point B point this is your D E. So, you have tau max which is nothing, but VQ by I say t. So, this is your thickness or you know this expression already you have seen that.

So, now if you combine these two both are shear stress, this will also give you shear stress distribution this will also give you shear stress distribution. So, ultimately, this at point d it will be additive at point e basically it is in this direction it is in this direction. So, it will be subtracted and all other points will be having different thing right.

So, maximum shear stress you will be getting at point D and that will be your tau max if you combine algebraically. So, that says that if you can have this kind of situation when bending shear force as well as torsion all the things are working together, and method of superposition holds good because we are dealing with linear elastic systems. So, therefore, we can just combine these two to get our required say state of states. So, this is my combined I mean state of stress fine.

(Refer Slide Time: 17:59)



Similarly, you can have the combination with combined direct and bending normal stress. You can have that also when you are combining the direct normal stress and normal stress due to your bending. So, at the time what will happen let us see.

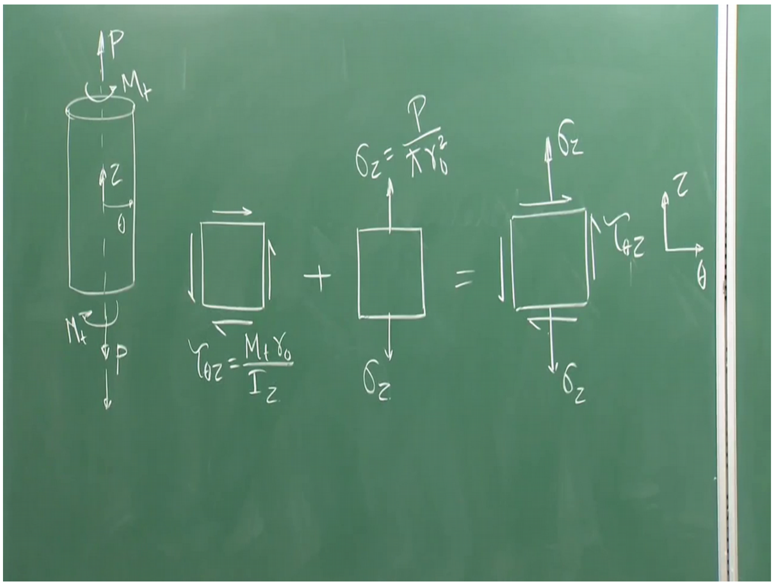
So, what does it mean? So, this is your slender member this is the axis you are having axial force P as well as bending moment M b. So, both are acting together and as you know from the discussion that bending moment if you apply bending moment basically it will create only the normal stress in this direction this is my x if I define this is my x direction. So, in the x direction both P will be I mean P will be developing some normal stress as well as M b whether the bending moment will be also developing some normal stress fine.

So, if you combine these two. So, therefore, sigma x will be if the cross sectional area is A. So, P by A is the direct normal stress fine always it will be tension. So, positive always it will be positive there is no issue, but for bending it will be plus minus; that means, minus is for compression it depends if the upper fiber. So, you now the variation right it will be in the triangular distribution, maximum at the top and minimum at the I mean zero at the neutral surface right.

So, that will be M b Y by I. So, if you try to draw the distribution. So, this is your say cross section say. So, this is say point B C A D E. So, for that your stress distribution is always positive that is tensile that is for P by A distribution, that is the direct normal stress plus stress due to bending A B C D E. So, that is for plus minus M b Y by I, for that your distribution is something like this, this is positive this is negative. So, you combine these two and then you get the combined normal stress ok.

Now, similarly you may have the situation where you have direct say loading something like your P the axial loading as well as torsion, you may have that kind of situation also let s see what will happen for that.

(Refer Slide Time: 21:55)



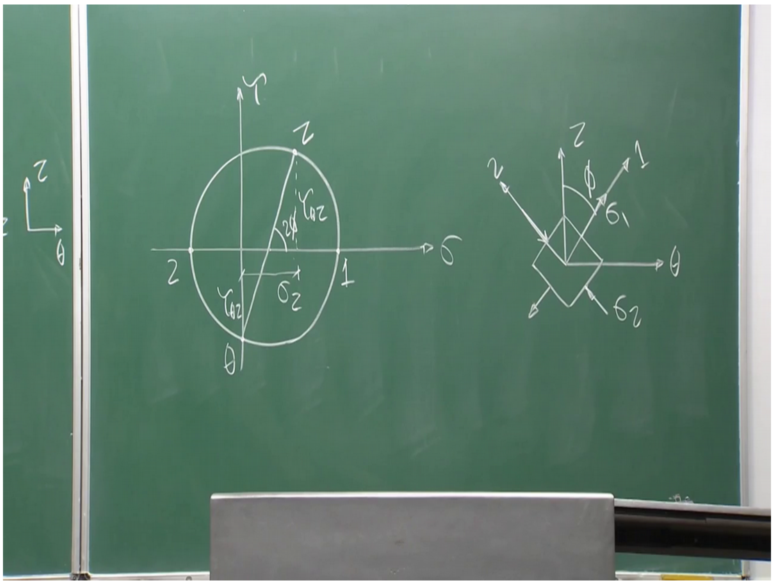
So, what I am saying you have a slender member like this, you have the twisting moment as well as axial force P. So, you have the tensile force axial force as well as the twisting moment. If you see the situation so the twisting moment will try to develop some shear stress on the cross section as you know right which will be minimum at the center and maximum at the periphery, and the axial force will try to develop some normal stress; normal tensile stress on the cross section ok.

So, if you try to combine these two say what will happen. So, if you try to draw the state of stress, this is your tau theta z develop due to your torsion, M t r naught by say I z. I z is nothing, but the polar moment of inertia previously I wrote j is nothing, but the polar. So, with the one of the notation you can use that is not a. So, polar moment of inertia of course, you know from the previous discussion.

So, this is the normal stress getting developed due to twisting moment, only pure shear right there is no other stress available there right. Already if you recall from the torsion chapter only shear stress will be developed and we are talking about the maximum shear stress; that means, we are we are just dealing with the periphery peripheral point. So, maximum shear stress will be developed at the periphery. Now it will be combined with the. So, this is the state of stress due to your axial force that will give you the sigma z. So, this is your z direction say this is your z, I mean this is your theta, this is your say z ok.

So, sigma z is nothing, but P, this is the circular area cross sectional area. So, I can write per naught square this is your sigma z. So, the combined state of stress will be. So, you can draw the state of stress for this things also right, the combined state of stress of will be you will be having shear stress and you will be having normal stress sigma z. So, this is your tau theta z. So, this is your combined state of stress due to this kind of combination of force axial force and the twisting moment.

(Refer Slide Time: 25:10)



Now, if you try to draw the mohr circle for that how it will look like? If you try to draw the mohr circle for this, this is your sigma this is your tau. So, on the z plane you are having both sigma z that is normal stress as well as the shear stress where as on the theta plane because this is your theta z called in a system, on the theta plane basically you are having only shear stress ok.

So, I can get this is the point theta, this is the point z. So, this is your tau theta z this is your sigma z, this is also tau theta z. Now this is your point 1, this is your point 2. So, you can draw. So, this was your Z theta called in a system we are not doing anything new everything is known to you only thing is that we are just representing this combined state of stress we are representing in the mohr circle ok.

So, this is your major principle stress axis, this is your minor principle stress axis, and if you look at this figure this mohr circle, major principle stress is positive that is tensile in nature and minor principle stress is negative that is compressive in nature. So, therefore, I can show this is my sigma 2 and this is my sigma 1 and if this angle is say twice phi then this angle is also phi agreed. So, if you have you can have this kind of situation because if you deal with deformation parts, deformation parts can have both the things together torsion as well as axial stress right. So, in that situation you can you need to deal the problem like this, you have to find out the combined state of stress ok.

So, now with this I conclude this chapter, I hope that you have understood the concept of stress develop due to bending because I wanted to because this is the thing which we are taking from the previous chapter, but I wanted to tell about the combined stress when you know thus development of stress in bending shear as well as torsion, then you will appreciate that how we can combine the state of stress fine.

So, with this I will conclude this chapter, now we will be taking some numerical problems on this chapter, and in the next class basically we will be solving couple of numerical problems, dealing with the stress calculation in presence of bending and shear force.

Thank you very much.