Mechanics Of Solids

Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture - 48
Shear Stress Distribution

Welcome back to the course mechanics of solids. So, in the last lecture if you recall we talked about the shear stress calculation or the expression for the shear stress we have derived. When your bending moment is not constant that is not the pure bending situation when bending moment is varying. So therefore, your shear force has exist and based on that you have got the shear stress on the on the cross section right ok.

Now, as you know that most of the times you will be getting the rectangular cross section of the beam that is more common. So, let us see how we can extend the same knowledge, same thing whatever we have learned to find out the shear stress. How it can be obtained for the rectangular cross section, or the square cross section or whatever I mean you generally get in the in the in the common practice ok.

So now already we know from our equilibrium condition that del sigma x, if you recall.

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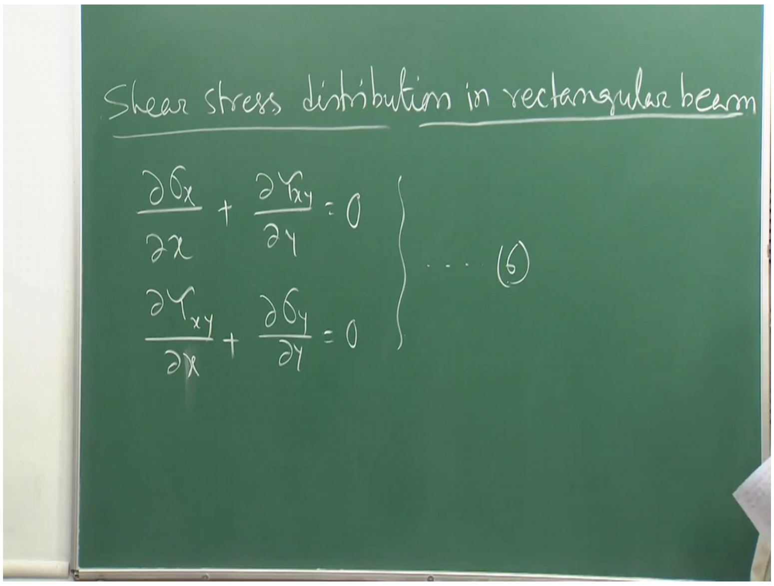

Del x plus del tau x y del y equal to zero, and del tau x y del y del x plus del sigma y del y equal to 0. So, that you know from your equilibrium equation right.

So now if the shear force does not vary, if the shear force does not vary with x as we have we have discussed in the last lecture the shear force, because there is no transfers load the shear force is not varying with x only the bending moment is varying, since bending moment is varying therefore, shear stress shear force exists, but they are not varying with x.

Then if shear force does not vary with x then the shear stress will be also independent of x, agreed or not? If there is no variation of the shear force then how the shear stress will be varying with x? So, shear stress will be always also independent with respect to x. So, then in the second equation, this part is 0 because there is I mean shear stress is not having any variation with x.

So, the first term of the second equation is simply zero, and we know from our previous discussion that in case of this type of problem this class of problem like bending and this shear force where sigma y is also 0. So, by default this equation that is the second equation is automatically getting satisfied. So, we need not to think about that.

Now, we will be only dealing with the first equation, which is more meaningful for us. So, the from the first equation we can write simply del tau x y del y is equal to del sigma x del x.

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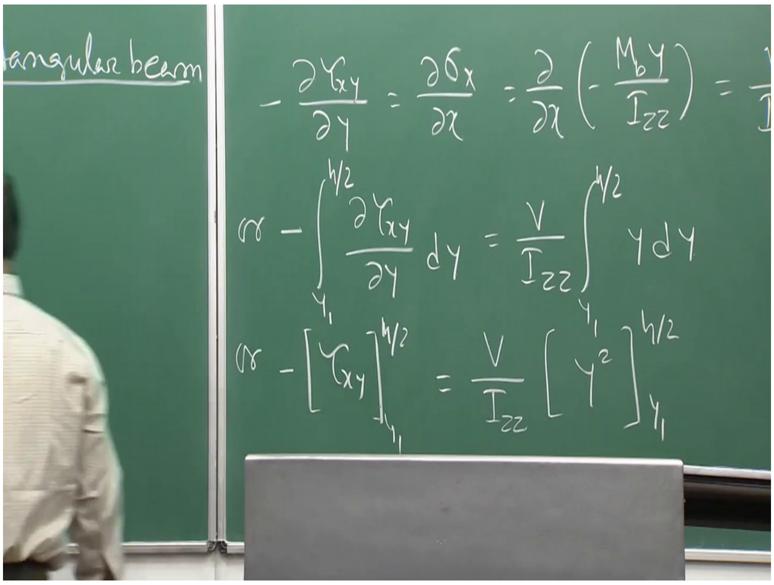

It is can be written as del x minus M b Y by I zz that is expression for sigma x, which is nothing but b I zz y right V. Because del M b del x is nothing but V, that is the variation of bending moment with respect to x that is nothing but your shear force V. So, V Y by I zz right. V means minus V So, minus, minus will be becoming plus.

So, from this I can simply write; so therefore, if you come back to this figure actually.

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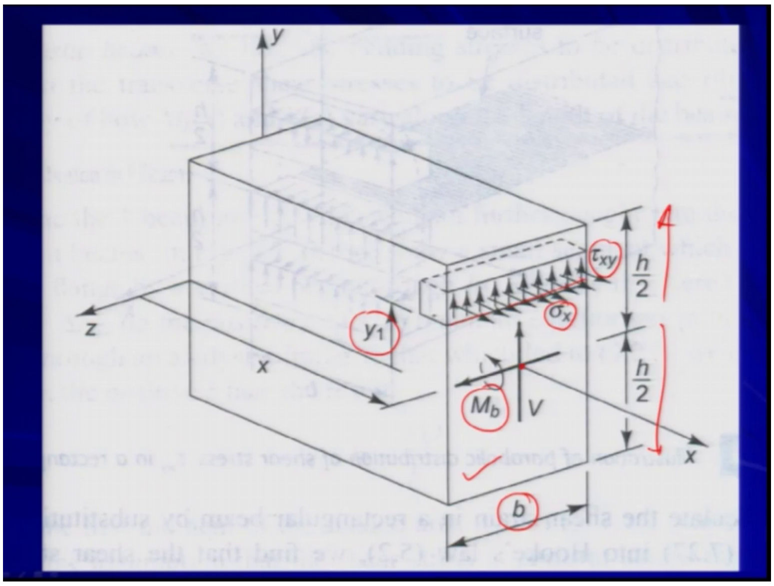

So, we are considering the rectangular cross section as we have decided, and this is the and you know that the neutral surface of the neutral axis will be passing through the centroid of the cross section. So, it will be passing through the centre of the beam. So, this is h by 2, this is h by 2, and on this plane if you if you consider one element sigma x is active due to bending you know, and now your because of your shear force your tau x y is also active fine.

And the bending moment on this section say M b and with of the beam is say b. And we are considering one section at say y 1. So, we will be finding out the variation of shear stress the magnitude of the shear stress from y 1 to the top surface, I mean at say at y 1 what is the magnitude of my shear stress. So, that we are going to find out fine.

So, from the basic principle or from the fundamental principle we are trying to obtain that thing. Later on you can you can use that whatever you have learns, right now the tau x is equal to V I mean in the previous class we have seen the tau x y is equal to VQ byv I zz. From there also directly you can get it, but we are coming from the basic principle fine.

So, from this I can write because we are interested to find out the shear stress, or the variation of shear stress, from y 1 to the top surface. So, y 1 to will integrate that top surface is h by 2. If you look at this figure you will get it So, del tau x y del y d y is equal to V I zz y 1 to h by 2 y d y, I can write that? There is no issue ok.

Now, from this I can get minus tau x y y 1 to h by 2, is equal to V I zz y square h by 2 y 1. So, which can be further written as, now your because here actually we know because tau x y is 0 at y equal to h by 2.

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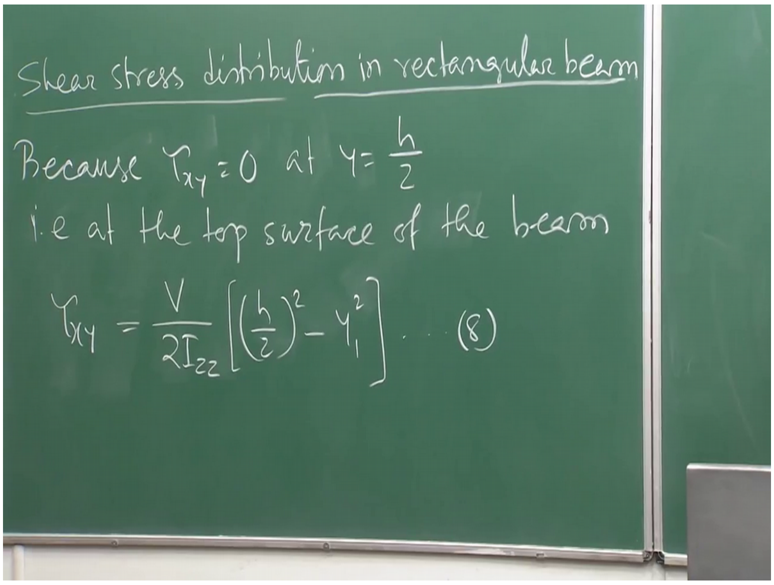

Already we have seen that the variation of your shear stress will be maximum at the neutral surface, and it will be 0 simply 0 at the top surface right top and bottom surface ok.

So, when y is becoming equivalent to x y 2; that means your reaching to the top surface. So, they are actually tau will say must be 0. So, that is at the top surface of the beam right. Y is equal to h by 2 is nothing but the top surface of the beam. So, if that is So, then we can write tau x y at say y 1, tau x y at y 1 whatever is our objective is equal to V by twice I zz into h by 2 whole square minus y 1 square equation 8.

So, you can verify that. If you put y 1 equal to h by 2. If you put y 1 equal to h by 2; that means, you are going to the top surface of the beam. So, if you are going to the top surface of the beam this equation will give you tau x y is simply equal to 0. So, that is satisfying the condition.

Now, if you put y 1 equal to 0, if you put y 1 equal to zero; that means, you are at the neutral surface, and then you will be getting the maximum value of your shear stress tau x y. And that is nothing but V by 2 I zz into h by 2 whole square, agreed? Now the this thing we have got from the basic principle right. Starting from the equilibrium equation and we have got it.

The same thing you will bee getting that you can do you can you can try by yourself. So, same thing you will get if you consider the equation whatever we have derived in the last lecture, that is tau x y is equal to tau x y is equal to VQ by b I zz.

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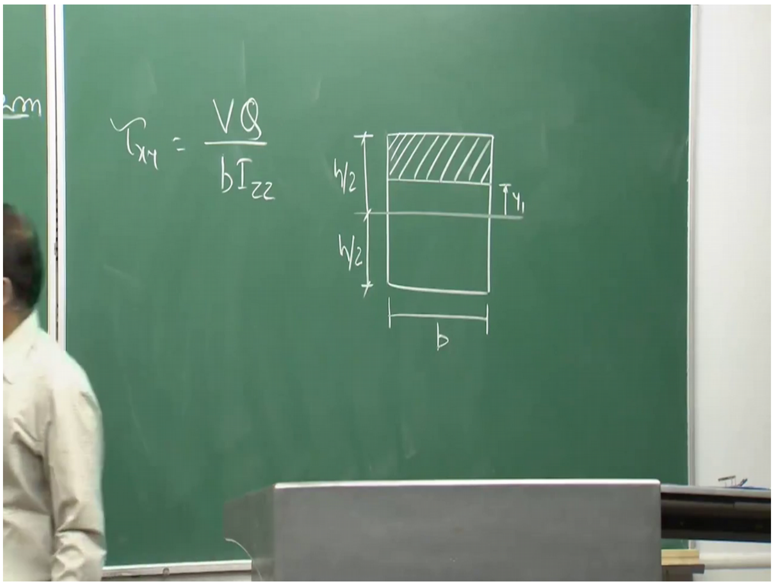

Where V is the width of the beam. So there basically this is your say cross section of the beam, rectangular this is the centre I mean, neutral axis I mean, neutral surface rather normal to the board is your neutral surface, and if you want to find out the shear stress at some distance y 1 from the neutral surface. So, then basically your Q will be the first moment of these shaded area right. First moment of the shaded area will be your Q, agreed? And this is your say h by 2 and this is also your h by 2 and this is your b ok.

So, in the same process you can try you can give a try. So, this V by I zz will not be varying I mean that is that will be remaining same. If you try to find out Q that may is the first moment of the shaded area if you try to find out, and there if you put in this equation you will be getting the same expression whatever you are getting here. But that we because this thing you can do at any time because already we have derived this equation and you can use this equation to find out that, but what I wanted to do wanted to show you that from the basic principle also we can arrive this thing at least for the rectangular beam ok.

So now if you look at the variation of your shear stress for the rectangular beam as I told you. So, this is the variation.

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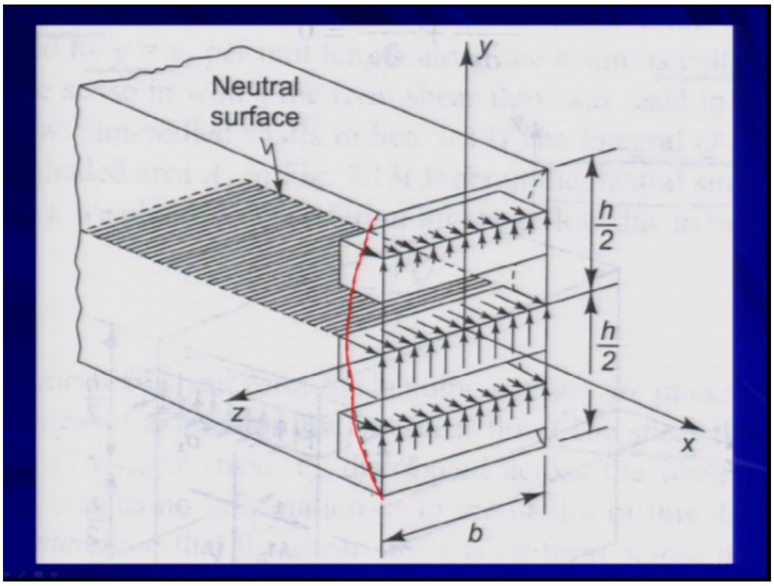

So, this is the variation of your shear stress, 0 at the top and bottom surface and maximum at the neutral surface.

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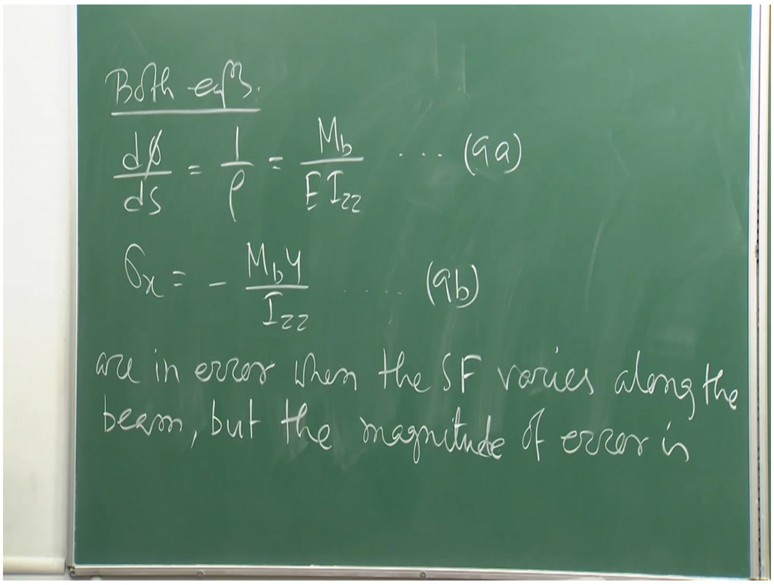

So, both equations whatever you have got say, d phi you recall that d phi d x is equal to one by rho is equal to M b e I zz, say equation say 9 a. And you have got sigma x equal to minus M b Y by I zz. So, equation 9 b. Both these equations are in error. When the shear force varies and which is pretty common actually right, along the beam.

Now, I mean what we consider, we started with when we derived this equations we considered pure bending situation; that means, shear force was not there. Then we have assumed that even if you have the shear force and if the shear force is not varying along with the x, if shear force is constant, with respect to x then, but your bending moment is varying with x in that situation also this equations hold good actually.

But these both these equations truly frankly speaking are in error when the shear force varies and that is very, very pretty common actually you mean you may have different transfers loading condition along the x direction; that means, along the length of the beam along the span of the beam you may have the addition of shear force, that is pretty common actually. So, if that situation arises, then these equations are in error. Along the along the beam, but the magnitude of error is small for long slender beam and consequently Q y x is equal to VQ by I zz; that means, shear flow represents a good estimate even in the presence of varying shear force ok.

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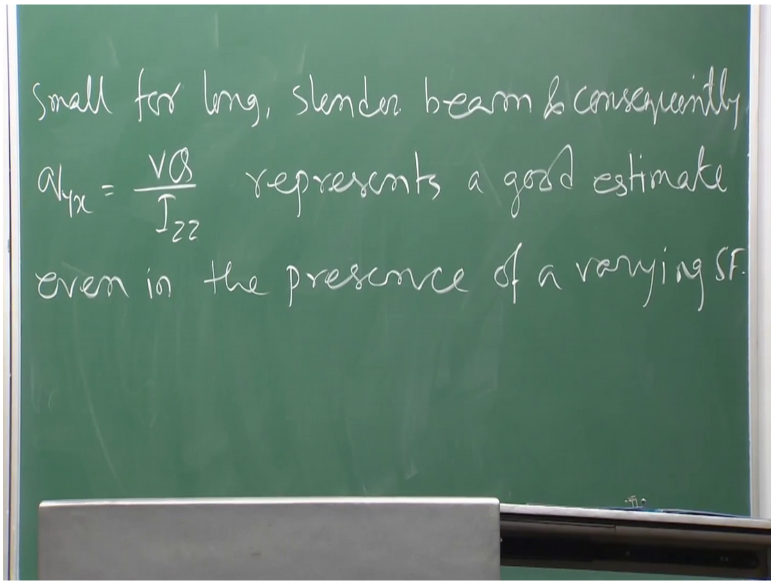

So, what I said? The most general thing is that your bending moment is also varying shear force is also varying along the length of the along the span of the beam. If that situation arises which is pretty common, then these both the equations are in error when the shear force varies along the beam. But the magnitude of error is pretty small for long slender beam whatever we generally considered in the in our in our different structures or different kind of say (Refer Time: 15:48) structures or any kind of I mean say structure if we if we use this beam long slender beam for those beams actually these equations I mean, whatever error will be developed or generated by these equations, that will be pretty small. And consequently that Q y x z is the shear flow whatever we are calculating based on that represents a good estimate even in the presence of varying shear force ok.

Even because this equation was derived considering that your shear force is not varying with x shear force was constant, but even if the shear force is varying with x, but still this equation estimates I mean, mean it gives good estimation of the shear force.

Now, what I mean to say, that if you consider the very general situation; that means, shear force and bending when both are varying with x still we can use this equations to find out the bending stress and the shear stress. That will not create much or significant error in the analysis, fine ok.

So now thus at each section actually.

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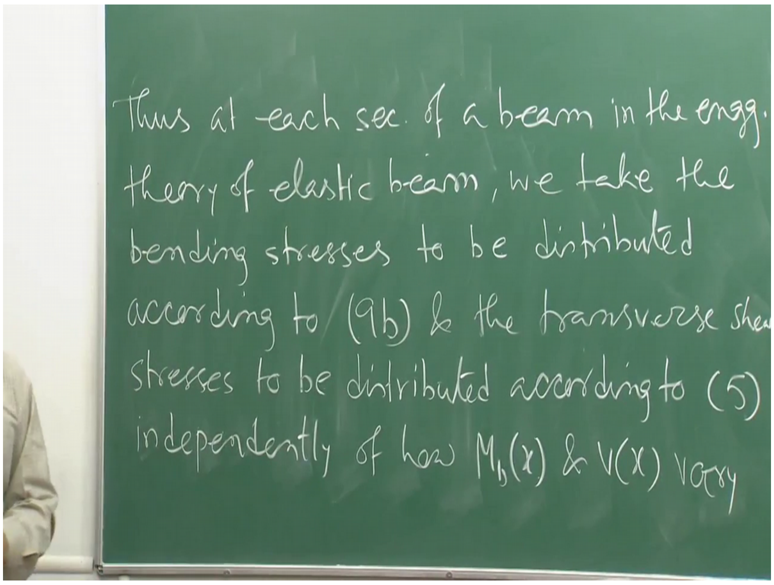

Thus At each section of a beam in the engineering theory of elasticity, elastic beam engineering theory of elastic beam, we take the bending stresses to be distributed according to 9 b, just now I have shown you equation 9 b, and the transverse shear stresses to be distributed according to equation 5 whatever we have seen earlier, independent, independently of how your M b and your V shear force vary along the length of the beam ok.

So, thus at each section of a beam in the engineering theory of elastic beam; so now, this is not the I mean I should not say this is the actual theory or the correct theory. But this is the engineering theory and we are doing the engineering approximation to find out the bending stress and shear stress. So, we take the bending stresses to be distributed according to 9 b; that means, in sigma x equal to minus M b Y by I zz. So, that equation holds good, and the transverse shear stresses to be distributed according to equation 5 that is tau x y is equal to VQ by b I zz. Independently of how M b and V vary along the length of the beam, whether M b varies along the length of the beam or whether b varies along the length of the beam.

So, that is quite immaterial for the timing, because we are talking about on the elastic theory of the I mean sorry, engineering theory of the beam. So, we are doing little bit of approximation, but that approximation holds good I mean that that will not create significant difference in the result ok.

Well. So, in the in the next I mean sometimes you will see in your different structural component, you generally use instead of using say your rectangular beam, you sometime use T beam or I beam you might have seen that, it is particularly very much say valid for your steel structure design like this kind of beam you might have use.

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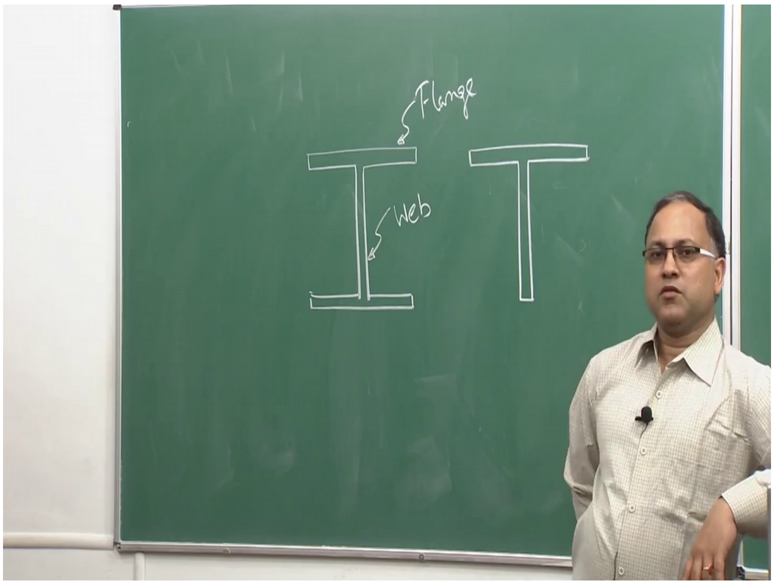

I mean you might have seen, I beam or T beam. This is your I beam, this is your say T beam you might have seen this kind of beam pretty commonly in your steel structure design or something like that.

So, this kind of beam actually if you try to analyse and if you try to find out the shear stress distribution, on this beam how they will look like or I mean what will be the variation and what kind of shear stress will be predominant in this particular type of section. It is not the rectangular section right? Rather it is having so, this is known as web and this is known as flange right? These are the technical names.

So, if you come across this kind of I beam or T beam, then how your shear stress distribution will be will be varying. Or how I mean which type of shear stress will be predominant. So that thing will be addressing in the next class. So, I will stop here today in the next class we will continue with this kind of beam, and we will try to conclude this chapter and then we will take couple of numerical problems.

Thank you very much.