Mechanics Of Solids
Prof. Priyanka Ghosh
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture - 47
Beam Transmitting both Shear Force and Bending Moment

Welcome back to the course Mechanics of Solids. So, in the last lecture if you recall we derived the different stress components under pure bending situation right. And there we have seen that all the stress components are 0 except sigma x and that sigma x is equal to minus My by I zz right so that we have derived. And that was only for pure bending situation right when your bending moment is constant throughout the beam.

And based on that we derived the expression for the deformation that is your strain and there also we have seen that how we can get all the I mean how we can get all the strain components normal strain components rather all the shear strain components are zero.

Now, today we will be talking about stresses in symmetrical elastic beams transmitting both shear force and bending moment.

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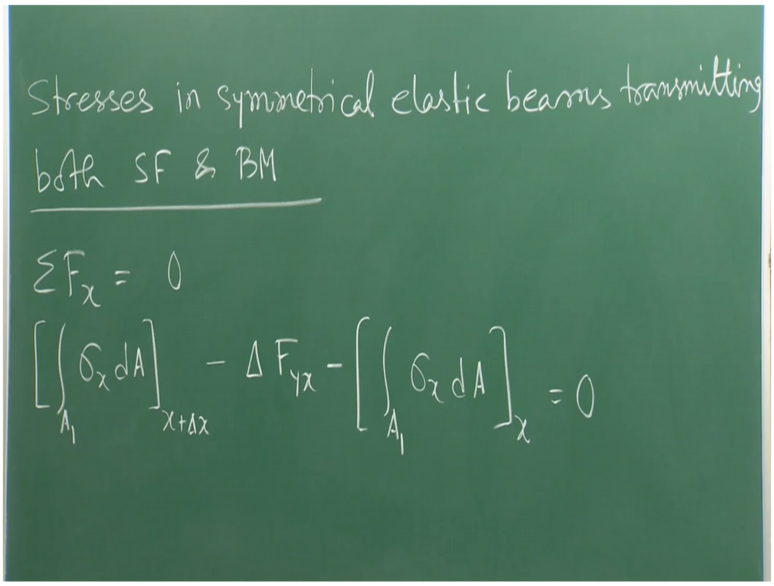

So, in the in the in the previous lecture whatever we have derived so that was based on pure bending situation. So there was no shear force acting on the beam, but here in this particular topic actually we will be discussing, if you have both shear force and bending moment then what will happen. Now we will actually we will make the assumption that the bending stress distribution whatever distribution you have seen that is minimum at the neutral axis and maximum at the top surface or the top fiber as well as the bottom fiber right. The extreme fibers will be expressing more I mean maximum bending stress right.

So, but at this moment we will make the assumption that the bending stress distribution is valid even when the bending moment varies along the beam; that means, if the bending moment varies; that means, shear force is present because you know d M d M b d x is nothing but v. So, earlier case in case of pure bending when the bending moment was constant. So, d M b d x was simply 0 right so that is why your shear force was zero, but now we are considering that bending moment is varying across the across the beam. So therefore, d M by d x is not zero; that means, it is a non 0 parameter. So therefore, your shear force is not a 0 component so that that is present very much present in the beam.

So, if you if you if you consider I mean this is a part of the beam.

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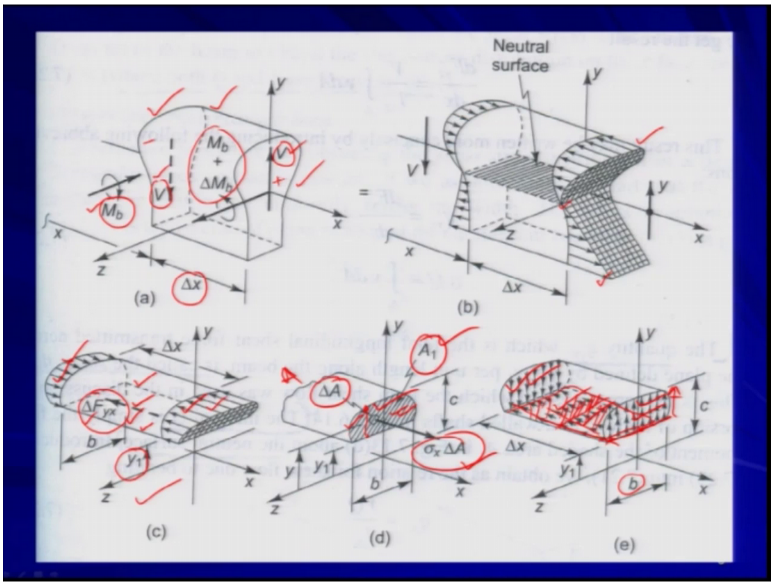

So, long standard beam. So, consider a length of delta x of a beam. So, consider a length of delta x of a beam, which is subjected to both bending and shear force. So, bending is shown here and shear force also right. Now no we are not considering any external transverse load no transverse load on the element, so that the v is independent of x; that means, v is not varying with x. So therefore, we are getting constant v on both the phases. So, this is your positive x phase and this is your negative x phase. So, on both the phases your v is same. So, v is not varying at all. So, we are because there is I mean there is no external transverse load if you do not have the change in force. So, transverse load then your shear force will not be changing it will be constant throughout the way.

Now, if you look at, but your bending moment is varying because if shear force has to be present then bending moment has to vary. So, bending moment is varying. So, on this phase on this side your bending moment is M b and, but on this side your bending moment is M b plus delta M b. So, a variation in bending moment with x is represented by this increment delta M b fine. Now consider the equilibrium of the segment of beam. So, if you come back to this and based on our previous discussion already we know that and we are making the assumption that whatever we have derived in the last lecture that holds good, even if you have the shear force present in the beam. So therefore, your distribution of the bending stress will be like that. So, maximum at the top fiber and the bottom fiber whether it is compression or attention that is that is different issue, but it will be maximum as per the magnitude wise it will be maximum at the top and the bottom fiber whereas, it will be simply 0 at the neutral surface fine.

So, consider the equilibrium of the segment of beam. So, this is one segment we are considering. So, in this figure. So, we are considering one segment. So, this is one segment where if you consider this is a this is a kind of bread we are we are cutting a slice along the x plane as sorry along the x axis ok. So, consider the equilibrium of the segment of beam which is obtained by isolating the part of the beam element above the plane defined by y is equal to y 1. So, the this plane. So, this is nothing but y plane this plane is defined by a distance y is equal to y 1 from the neutral surface. We are making y we are taking one slice, and now we are considering the equilibrium condition of this particular segment.

Now, one thing is very clear because this is this is a kind of a continuous say continuum body. Now each and I mean if you make n number of slices. So, every slice will be getting bonded with the adjacent slice, and you will be having some interface shear force or interface say force which will be which will be bonding these 2 these 2 say fibers together am I right? So, if you consider a top part of the beam and the immediately after that there is one adjacent part. So, these 2 parts will be getting bonded at the interface ok.

So therefore, you should expect you should expect one force which will be getting developed at this y plane along x direction, yes or no? Right, to I mean balance the forces acting on the x plane. On the x plane you are having the axial force. Now you are getting the variation in the axial force because your bending moment is varying am I right. Your if your bending moment is varying any particular section if your bending moment is varying as you know from our previous discussion that your sigma x will be varying because bending moment is varying. If sigma x is varying then who will try to balance this variation. The variation will be balanced by the force which will be getting developed on this y plane, so that is nothing but delta F y x on y plane along x direction.

So therefore, if I consider the equilibrium condition that means, summation of f x summation of all the forces in x direction is equal to say 0. So therefore, I can write simply A 1 I mean integration over the area A 1 sigma x dA at x plus delta x distance; that means, on positive x plane say minus delta F y x minus integration over the area A 1 sigma x dA at x equal to 0, so that means, what I am doing?

So, the first term that is integration over the area because this if you consider this is the area A 1 that is given here. On this area I mean if you consider small area say delta A or dA whatever on this area your sigma x into delta A is the force acting on this on the small area. And if you integrate over the whole area A 1, then you will be getting the force on that particular area A 1 so that is acting on this phase say so that the first term is acting on this phase which is x plus delta x distance apart ok.

Now, similarly the last term if you consider that is the integration over the same area because this area will be remaining same and sigma x into dA on this plane that is at x distance apart. And the delta F y x because what we are considering we are considering that all I mean this on this phase we are considering say tensile force I mean outward I mean towards the positive x axis. Similarly to balance it you will be having the force in this direction. So that is why it is negative and delta F y x is also acting opposite to x axis as shown by this arrow. So therefore, delta F y x is also coming as negative. So, these are the forces which are acting along x direction ok.

So now from this I can simply write delta F y x equal to minus integration over the area A 1.

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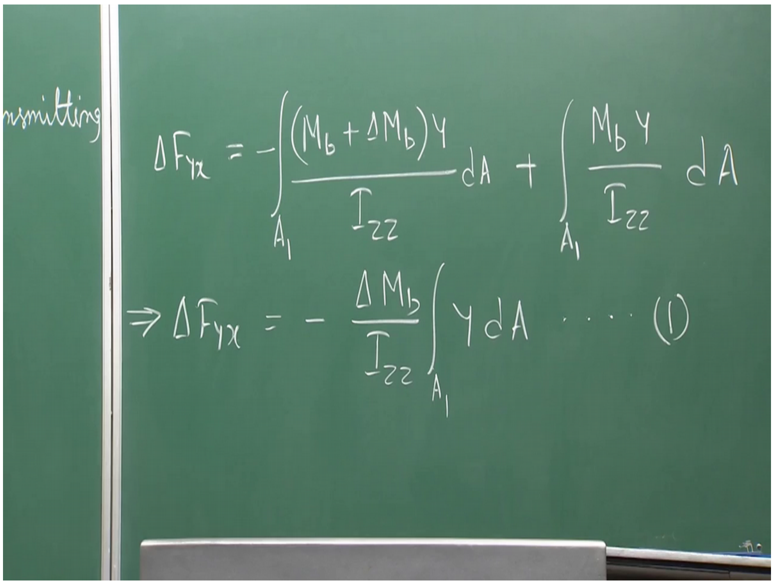

Instead of sigma x now we can write, as we know what is the bending moment at that particular section at x plus delta x distance? If you consider one section that that we are considering. So, what will be the bending moment there. So, delta M b y by I zz so that is the value of sigma x at x plus delta x into dA plus integration over the area A 1, and what is the bending moment is acting at x distance apart I mean on the section which is x distance apart that is M b simply M b y by I zz and dA.

So now further I can write delta F y x is equal to minus delta M b by I zz A 1 y dA say equation 1. Now dividing both sides by say delta x and taking the limit what we will get? So, what we are doing?

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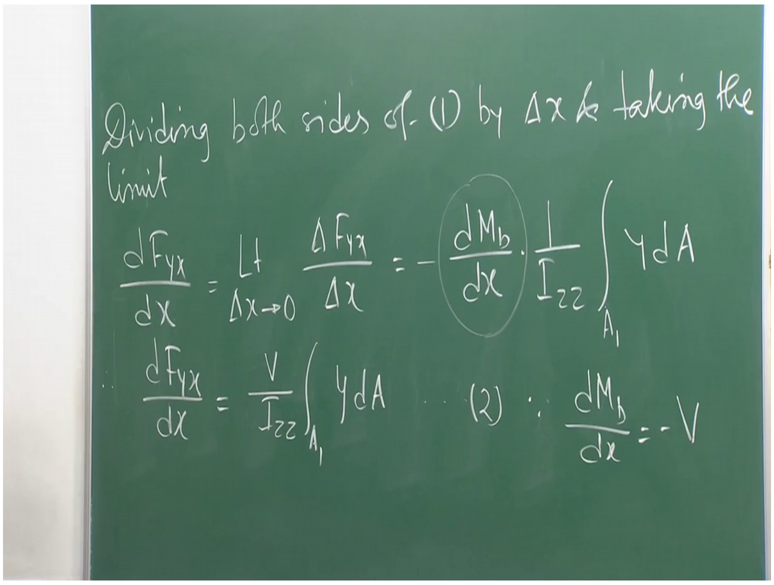

We are Dividing both sides of equation 1 by delta x and taking the limit. So that is nothing but d F y x d x is equal to limit del x tends to 0 delta F y x delta x is equal to d M b d x into 1 by I zz integration over the area A 1 y dA, agreed? So, just we are taking the limit and we are dividing both the sides of equation 1 by delta x and taking the limit so that is that is will be coming like that; so now, if you look at this. So, I can again write that d F y x d x equal to what is this? If you recall from our previous discussion that is nothing but minus a v the right shear force. So, I can simply write v by I zz into integration over the area A 1 y dA. So, this is my equation say 2, since your d M b d x equal to minus v that we know from our previous discussion ok.

So now from this basically if we put this d F y x As q y x d F y x d x as q y x.

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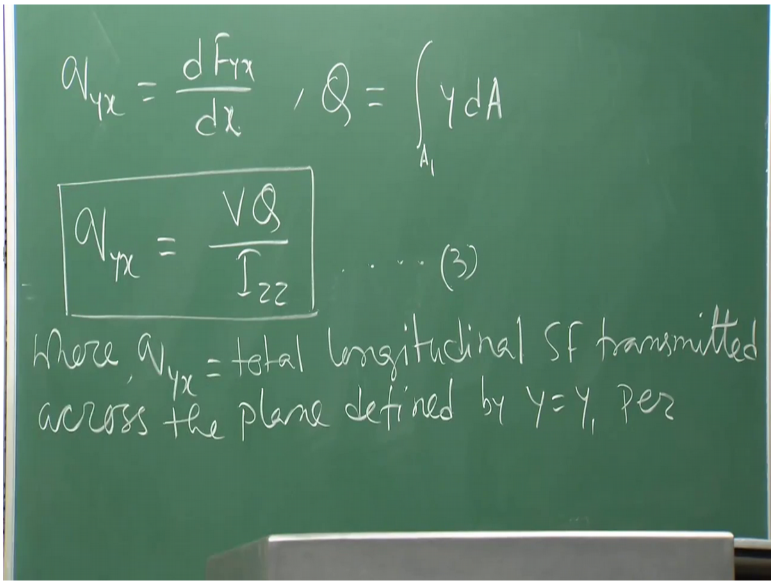

And Q capital Q is equal to integration y dA if you put like that then I can simply write q y x equal to v Q by I zz. This is very important relation we have established so that is equation say 3. Now where your q y x, what is your q y x? If you look at q y x that is given by d F y x d x right. So that is nothing but total longitudinal shear force transmitted across the plane defined by y is equal to y 1 per unit length along the beam and is called shear flow very important term ok.

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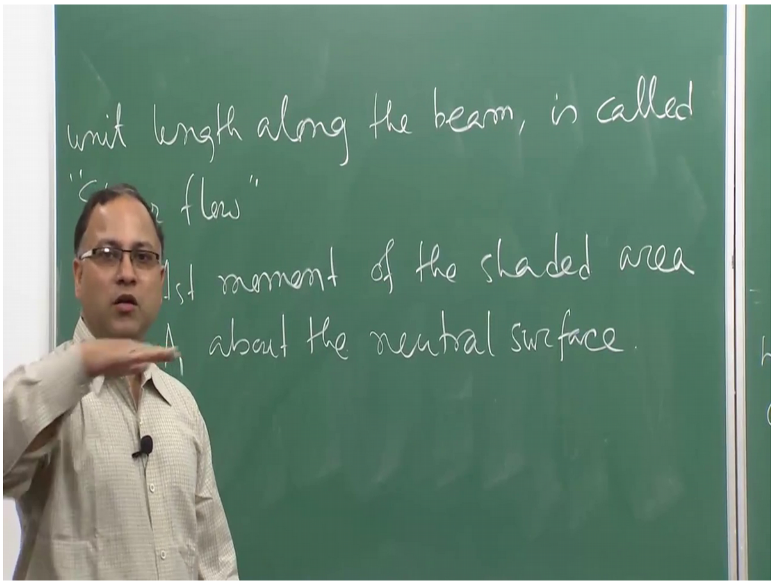

So, what is q y x? That is a total longitudinal shear force transmitted across the plane defined by y is equal to y 1. So, at y is equal to y 1 you are defining one y plane say you are defining one y plane on that plane your shear force is acting that is delta F y x is nothing but the shear force is acting on that plane fine.

So, basically q y x is that shear force per unit length along the beam along the beam, your x is here. So, d F y x d x is nothing but the shear force acting on that particular y plane per unit length of the beam along x direction. And is called shear flow that is very important. So that is the shear flow on that particular plane per unit length per unit length if you move along the beam that is the shear force acting. And your capital Q is nothing but the first moment of the shaded area A 1 that is shown in the figure about the neutral surface, agreed?

So, integration y dA is nothing but your Q and that is nothing but your first moment of this shaded area A 1 the agreed. So, this equation is giving you. So, what you are getting from this? This Q is basically defined by the plane on which you are interested to find out the shear flow. I zz is nothing but the cross sectional property as you know. So, if you know the shear force then you will be able to find out how much shear flow how much shear flow per unit length of the beam is acting on the beam along, I mean on the on the beam along your x direction; that means, on the y plane ok.

So now if you try to find out the; so therefore, this is nothing but your shear force I mean shear flow is nothing but the shear force per unit length of the beam. So, you can find out. So, your q y x Is basically is the resultant of a shear stress tau y x on the y plane along the x direction.

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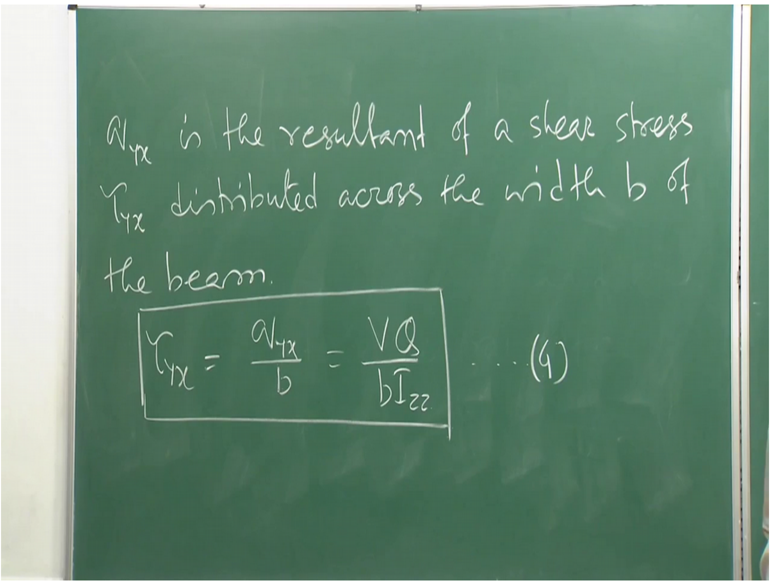

So, tau y x distributed across the width b of the beam, what does it mean? So, this shear flow is acting on this plane. So, this is the plane on which shear flow is acting as you see this is the plane y plane.

Now, if you this tau y x is basically giving you that is per unit length whatever shear force is acting. Therefore, it is a resultant of a shear stress tau y x distributed across the width b. So, this is the width b. So, so this if you consider this whole part basically this whole part is giving you the Q x q y x. So, q y x divided by b is nothing but the shear stress acting on that particular plane fine. So, if we can say that then immediately I can write down tau y x equal to q y x by b is nothing but equal to v Q by b I zz very important equation. So, if you if you are having the varying bending moment across the beam; that means, if shear force is present then basically your shear stress will be also getting developed. And that tau y x that is the shear stress is nothing but equal to v Q by b I zz.

So, and we know from our earlier discussion that tau y x is nothing but tau x y which is again v Q b I zz.

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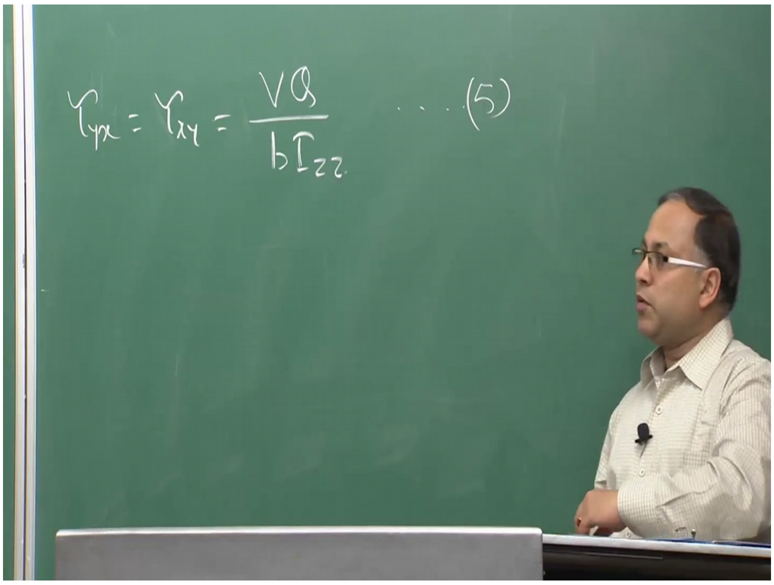

So now, you can say you can you can see the variation or I mean distribution of tau y x on this plane. So, this is your tau y x this is your tau x y right. Both are same and both are equal to v Q by b I zz. Now if you look at this expression. So, what we can write basically.

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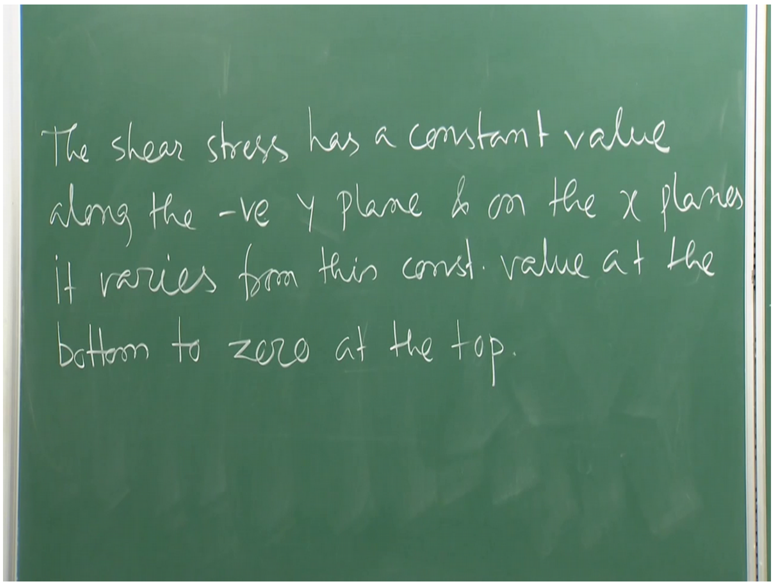

The shear stress has a constant value along the negative y plane and on the x planes. It varies from this constant value at the bottom to 0 at the top. So, what does it mean? The shear stress has a constant value along the negative y plane, so on this plane. So, this is the negative y plane ok.

This is your negative y plane on that plane your shear stress value is constant because there actually your Q is constant v of course, is constant on that particular section and then b I zz all those things are constant. So, you are getting the constant shear force on that particular plane, but when you are talking about the shear stress on the x planes; that means, when you are talking about the shear stress on this planes then basically it varies from this constant value ok.

So, the constant value will be at the bottom. So, this is the constant value because here at this is the junction of y plane and the x plane. So, along y plane you are having constant tau y x and along this say point, along this line on the x plane you will be having the constant value, but as you move to the top your constant value will be going to 0 because your Q is becoming 0 Q what is Q? Q is the first moment of this area. So, as you move from this point as you move from this point to the top surface your Q is getting reduced right. Isn’t it? Your Q is getting reduced that is the first moment of that shaded area is getting reduced. So, ultimately when you will be going to the top surface your Q will be becoming simply 0. So, once Q is becoming 0 then tau x y will be becoming 0. So, this is the variation of tau x y from the So, what does it mean then?

That means, in case of bending stress that is sigma x when you calculated that was simply 0 at the neutral surface and it was maximum at the extreme fibers right. Extreme surfaces, but here in case of your shear stress it is just reverse it will be maximum at the neutral surface, because at the neutral surface if you consider you will be getting maximum magnitude of q, but as you move towards the extreme surface or the extreme say fibers your shear stress will be becoming simply 0 at the top. And bottom your shear stress will be becoming 0 and in between it will be varying and it will make taking the maximum value at the neutral surface ok.

So now next we will be talking about, now I will stop here today. So, in the next class we will be talking about the stress distribution in the. So, so far we have considered the section in such a way that it is it is symmetrical with respect to your x y plane, but now we will be considering the rectangular cross section and then we will which is which is more common I mean generally you see this kind of rectangular beam right. The cross section is rectangular or square whatever so that kind of say cross section if you have then how your shear stress will be getting I mean how much will be your shear stress on that particular rectangular cross section that we are going to find out in the next class. So, I will stop here today.

Thank you very much.