Mechanics Of Solids

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Lecture - 46
Equilibrium Requirements

Welcome back to the course Mechanics of Solids. So, in the last lecture we were discussing about the stresses due to bending. And we have seen that you have I mean we defined a new kind of axis that is neutral axis and that what is the speciality of the neutral axis that neutral axis will not experience any kind of deformation right. So, above if you travel from the neutral axis to the top fibre then you will be getting the shortening I mean of course, under the under the positive say bending moment. Like your positive bending moment you know sagging. And if you move from neutral axis to the bottom fibre basically you will be getting the all the fibres will be expressing the elongation ok.

So, based on that what we can conclude that that this is your neutral axis this is your neutral surface in the beam.

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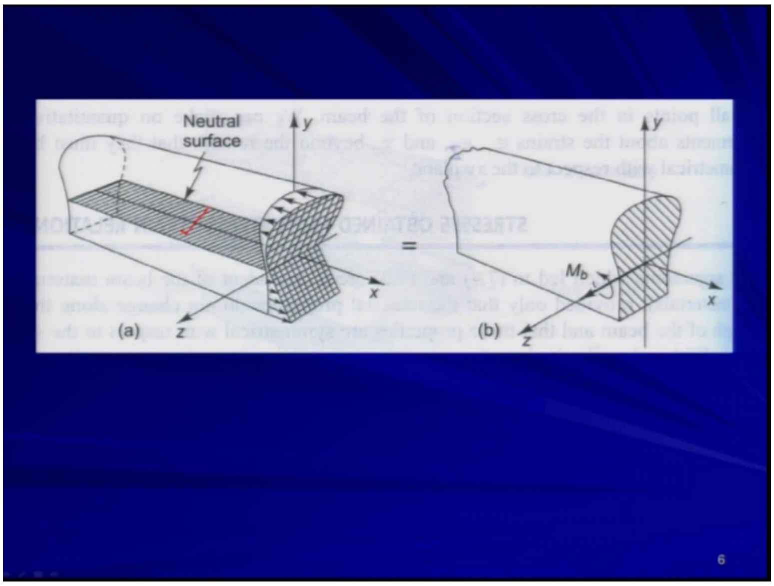

Which is coinciding with the x axis, but still we do not know; what is the location of the neutral axis that will be defined later on.

So, above the neutral axis. So, this if you move in this direction from the neutral axis to the top fibre. Then basically you will be observing the compression. That is why the fibres are experiencing the short range right. So, you will be getting the compression. So and that means, the sigma x will be compressive in nature, whereas if you move from neutral axis to the bottom fibre then sigma x will be tensile in nature.

Now still we do not know what is the magnitude of sigma x as well as sigma y and sigma z right. That we do not know at this moment. So, this development of stresses development of stresses means on this plane, whatever stress is getting developed that must be balanced by the bending moment M b, yes or no? It is very true right, because everything is under equilibrium ok.

So, now if I consider on this cross section on this is your say positively x plane.

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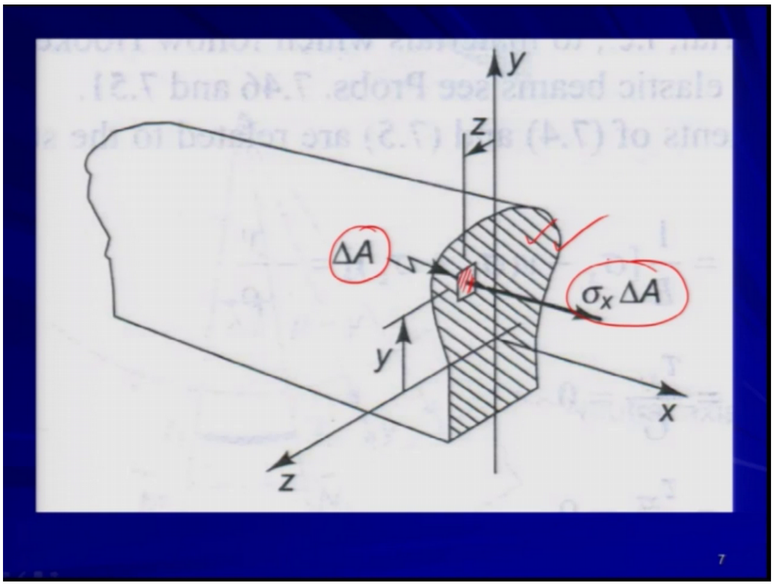

On this plane I am considering a small area say delta A. And on this area so, sigma x is acting on the whole surface sigma x is acting on the whole surface right. And due to the sigma x this small area is experiencing the 4 sigma x into delta A, isn't it?

So, what I can write? Your effects because everything should be balanced right. So, if a summation of f x.

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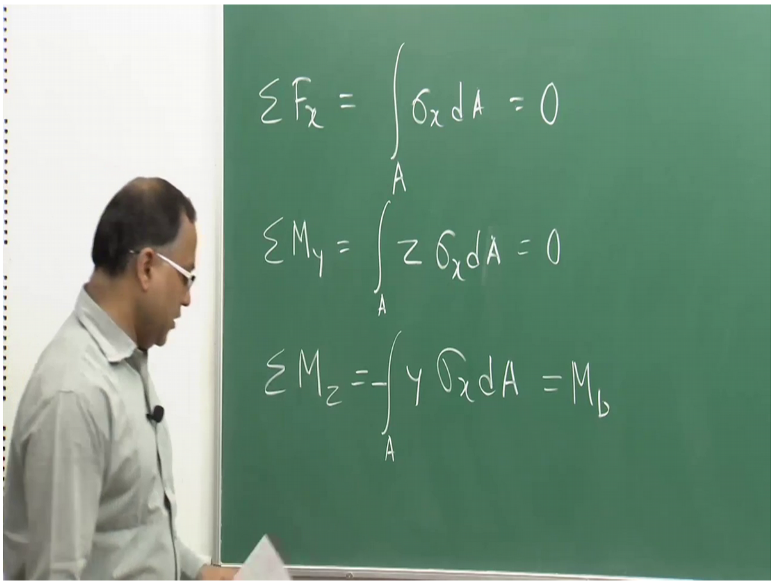

That means summation of all forces along x direction is how much, because there is no force. There is no actual I mean externally applied force in the in the x direction right. So, they must be sigma x dA must be 0. Integration over the whole area, integration over the whole area sigma x dA must be equal to 0 as per our this figure right. I am considering a small element and I am integrating over the whole area. That is the expression for your or that is rather your equilibrium condition along x direction, mean force equilibrium condition.

Then similarly you will be having summation of My it is also 0, but how we will get 0 here? I mean, what is the I mean what force will try to try to contribute to this moment? Moment with respect to the y axis that is nothing but, integration over the whole area z into sigma x dA right.

If you look at this figure what is z? Z is this right; that means, the distance from the y axis to this element. So, z into sigma z into sigma x dA is nothing but the moment with respect to y axis and then you integrate over the whole area you will be getting the summation of My. And that is also 0, because no development of the moment happened with respect to the y axis ok.

Then what about M z? Summation of M z that must be equal to the bending moment, because that is applied. But what should be the expression for that if you integrate over the whole area y and that should be minus if you see as per our sign if I mention clockwise moment is positive. Clockwise moment is negative and anticlockwise moment is positive. So, as per that this is negative. So, this is the expression of expression coming from the summation of M z ok.

So, now if I look at if I take any Say now ware going to establish the stress deformation in the symmetrical elastic beam subjected to pure bending stress and deformation.

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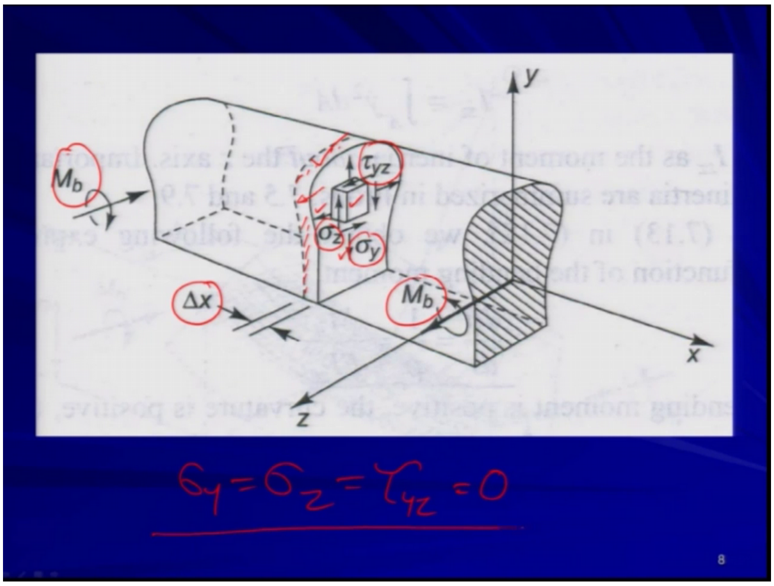

What is happening? Now we have we have seen what I mean what is the expression for deformation now we are going to try going to find out the relation between stress and deformation, now we are going to establish the relation ok

Now, if you look at this figure this is under pure bending this beam is under pure bending. Now I am considering one slice some elemental slice. So, this is this is some elemental slice we are considering in between the beam this is a kind of your bread if you consider this is a bread and there is a slice of the bread we are considering that slice. And we are considering one element in the slide which is experiencing the stresses, define stress components like sigma y in y direction sigma z in z direction. So, this is your sigma z this your sigma y and of course, tau y z it is experiencing same for example.

Now, if you consider and thickness of the slice say delta x that is the thickness of the slice. So, this is element you are considering inside the slice. Now if you move towards the surface of the. So, this is your surface of the slice right. The surface is completely stress free yes or no. So, you are what you are doing you are coming from the code of the slice to the surface, and you are that is there say suppose if you consider any beam. So, the beam surface outside surface will be always stress free, because why it will be stress because there is no externally applied for I mean that fibre will be always stress free right.

So, if that external fibre I mean that outside surface external surface of the slice is completely stress free; that means, there is no normal stress there is no shear stress, then the sigma y sigma z and tau y z must vanish in the surface. So, if they vanish in the surface then why it will be there because to satisfy the boundary conditions, you should have sigma y equal to sigma z equal to tau y z equal to 0 right.

So, this we are getting from our I mean logical argument. So, what we can write? We can write sigma y is equal to sigma z equal to tau y z must be 0.

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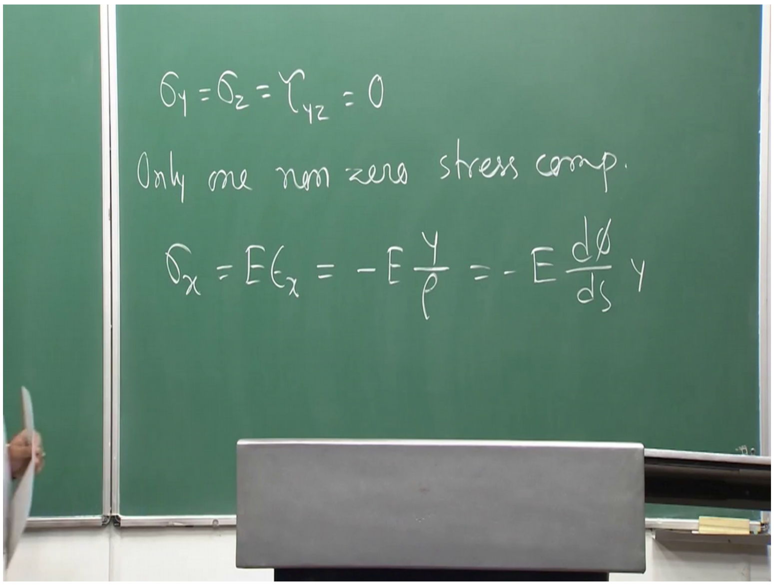

Because your external surface is stress free. So, that should not be different from your code right. So, if the code is stress ethic external surface should have also experience them because this element we to have considered at the code. I can consider the similar element at the surface, then it should be stress free. So, this is a kind of ambiguity right. So, that ambiguity it should be removed only if your sigma y sigma z and tau y z all are 0 ok.

Now, only one non 0 stress component remains. Already we have seen that tau x z and tau x y they were 0, agreed? In the previous lecture if you recall your tau x z and tau x y were 0. Now you have got tau y z 0 and also at the same time you have got sigma y sigma z both are 0 right.

So, only one non 0 stress component is remaining and that is nothing but your sigma x right. Which can be written as E into epsilon x as you know, from the stress and relation. Which is equal to E into y by rho, because I we know that epsilon x is nothing but minus y by rho. This can be further written as minus E into d phi d s into y in terms of curvature ok.

So, now we have already seen here from these 3 equations right. Already we can we have seen that Summation of f x.

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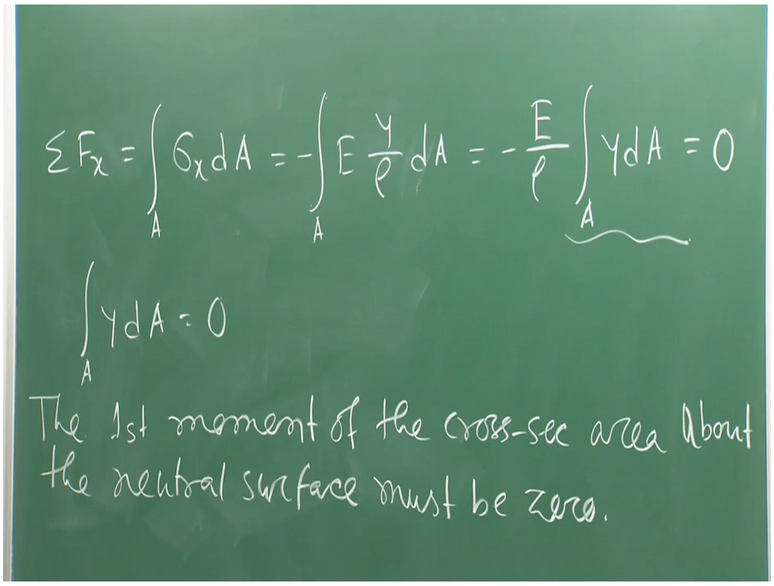

Already we have seen that integration over the whole are sigma x dA. Which is nothing but integration over the whole area, now I am putting instead of sigma x I am putting E into y by rho. So, E into y by rho dA that can be further written as E by rho integration over the whole area y dA equal to 0 right. Summation of f x was 0 earlier already we have seen from the equilibrium condition ok.

Now, your E by rho is not 0 right. E by rho is a non 0 component, because you are dealing with some feasible material. So, E by rho cannot be 0. So, cannot be 0 for a particular material. So, E by rho is not 0. So, if E by rho is not 0 then what is 0 this part is 0 right. So, we can say y dA is 0. Now what is this? Can you recognise this expression? So, this is nothing but the first moment of the cross sectional area about the neutral surface must be 0. So, that is your first moment of the cross sectional area with respect to the neutral axis. Because y, y is measured from the neutral axis, isn't it; because neutral axis is coinciding with the x axis. So, y is measured from the neutral axis. So, that is nothing but the first moment of the cross sectional area.

Now, if this first moment of cross sectional area is 0 with respect to the neutral axis, then what we can say? We can say then conclude that a neutral axis must pass through the centroid of the beam right. That is the conclusion because So far we didn't know about the location of the neutral axis. We are just talking about the neutral axis is coinciding with the x axis, but still we didn't know that what is the location. Now we have got the location by satisfying this.

So, neutral axis we can write down that that is very, very important.

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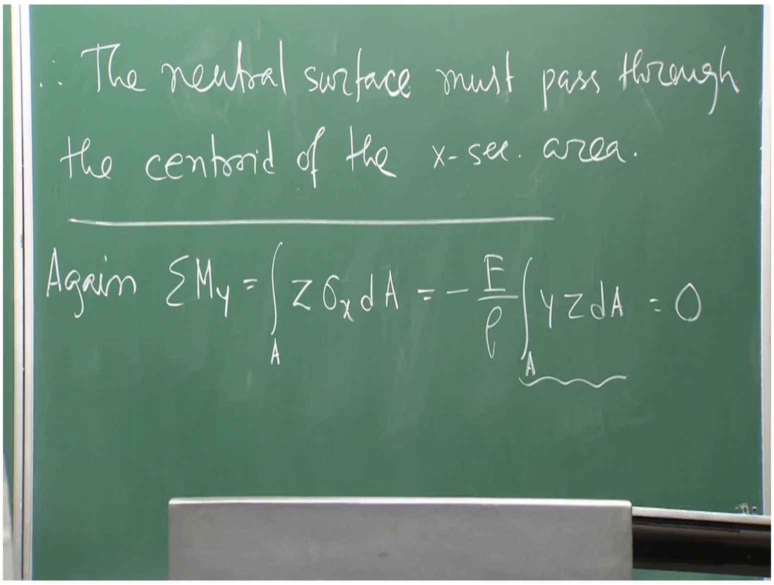

So therefore, the neutral axis NA or rather neutral surface we can write at this moment, the neutral surface must pass through the centroid of the cross sectional area. That is very, very important conclusion; that means, we have established the location of the neutral axis.

Now, the question is that always the neutral axis will be passing through the centroid of the cross sectional area? No. So, the answer is no, all the times it I mean I mean, in the general case neutral axis will pass through the centroid of the cross sectional area, but in some special cases neutral axis will not pass through the centroid of the cross sectional area.

Suppose if you consider some linear elastic beams of more than one material right. You are considering one linear last elastic beam of more than say one material 2 materials or 3 material. So, something like that. In that case neutral axis will not pass through the centroid. Or if you consider the beams whose material behaves in a non-linear way. So, we are we were right now we are talking about linear elastic is tropic material. If you deal with non-linear material whose stress and behaviour is non-linear already you have seen when we talked about the stress and relation. If you have that kind of material whose stress and behaviour is non-linear in that situation also neutral axis will not passed through the centroid of the cross sectional area ok.

But neutral surface, then how we locate the neutral surface in those cases? So, neutral surface can be located by following the same thing that summation of f x is equal to 0. That is valid for any kind of situation whether the whether you are dialling with 3 4 types of materials or whether you are dealing with non-linear material whether you are dealing with linear material, in any case your summation of f x equal to 0, and based on that you can locate the neutral axis for those special cases. But this is the backbone this is the basic equation which will be valid for all the cases. So, by satisfying that condition for those special cases when you are dealing with I mean different types of materials more than one material or non-linear material in those situations you can locate the neutral surface fine ok.

Now, now again the second equation that is summation of My is also 0 right. Already we have seen that is a second that was a second equation. So, that was integration over the whole area z sigma x dA. Which can be written as E by rho integration over the whole area A y z d at which was equal to 0 earlier already we have seen right.

Now, it is this part E by rho again it is not 0 it is non 0 component. So, this part is 0 only. So, it is 0 because of the symmetry of the cross section with respect to the x y plane. So, it is automatically satisfied, if you consider the symmetric cross section as we have seen with respect to the x y plane right. If you consider the x y plane right, if you recall this figure ok.

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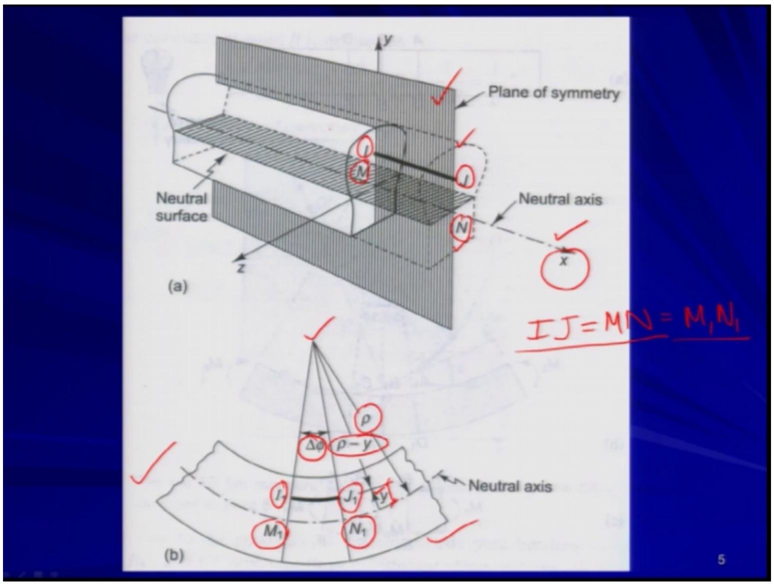

So, this is the plane of symmetry. So, with respect to this x y plane basically your cross section was symmetry right. And because of that it is already satisfied. This is automatically satisfied, because you are dealing with the symmetric cross section with respect to the x y plane. Now the third equation you are having that is summation of M z equal to M b right. Already we have seen summation of M z is equal to your bending moment M b. So, from that what we get?

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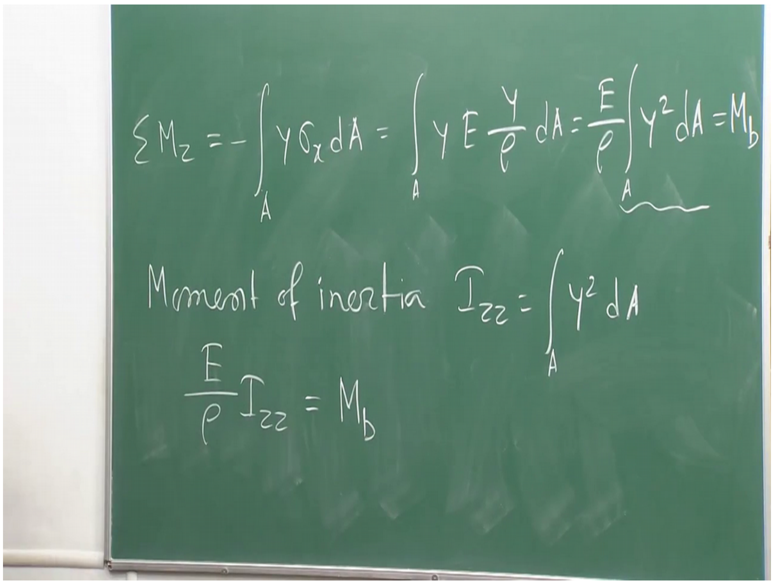

So, summation of M z is equal to I can right. Already we have seen that y into sigma x dA minus. So, if I put instead of sigma x E by minus E by rho then we can right. Y E into y by rho d at which can be written as E by rho y square dA which is equal to M b. Now can you recognise this part. From your physics or from your earlier knowledge about the mechanics, what is this? This is your second moment of area; that means, your moment of inertia. So, this is your moment of inertia right. Moment of and you know how to find out the moment of inertia of a particular cross section right.

Therefore we can write E by rho I zz is equal to M b right. So, from this I can write 1 by rho is equal to d phi d s is equal to M b by E I zz.

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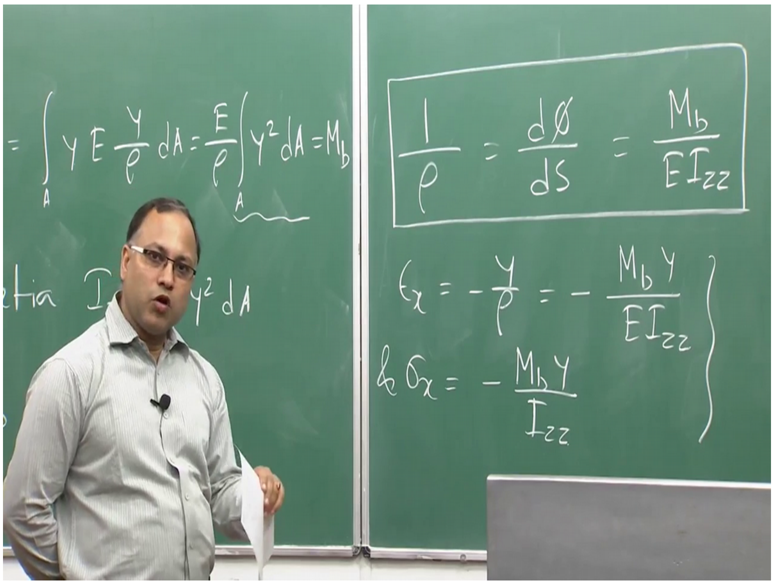

This is very, very important relation moment curvature relation in mechanics. So, when your bending moment is positive; that means, sagging bending moment right. Bending moment is positive; that means, your sagging kind of it (Refer Time: 21:29). So, the top fibre is always under compression and that will take the concave surface right. Concave top surface and convex bottom surface right. In case of positive bending moment.

Now, if I try to find out epsilon which is equal to minus y by rho. Which is nothing but minus M b from this expression M b y by E I zz. And your sigma x is equal to minus M b y by I zz. So, these 2 are the relations between the strain there is a deformation strain and the bending moment. And this is the relation between stress and the bending moment. Other stress components are 0 already we have seen and already we have seen, that if the shear stress is 0 therefore, shear strain will be also 0; that means, gamma x y gamma x z gamma y z all are 0 all shear strain components are 0. Epsilon axis equal to this, but still I do not know what is the M magnitude of epsilon y and epsilon z. They will have some finite value because of your poison effect. So, that I will come to that point later on.

Now, from this if you look at this expression sigma x equal to M b y by I zz. If you are at neutral axis if you are at neutral axis what is the value of y? Simply 0 right. Because neutral axis is coinciding with the x axis. So, y is 0 at the neutral axis. And you are getting sigma x equal to 0, right? Epsilon x is also equal to 0; that means, you satisfied the condition of the neutral axis. Neutral axis is not or will not experience any kind of deformation, and it is completely stress free.

So, the stress is maximum at the top surface and at the bottom surface. If you go to the in case of positive bending moment, the top surface will be explanation experiencing the maximum compressive stress, because this is the negative sign y is positive at the top surface. So, it will be experiencing maximum compressive stress the bottom surface; that means, y is negative, there your bottom surface will be experiencing maximum tensile stress. So, that if you try to plot earlier already we have seen right. So, that if you plot then you are getting 0 at the neutral axis and you are getting the maximum on the top surface that is maximum compressive stress whereas, maximum tensile stress at the bottom surface, agreed?

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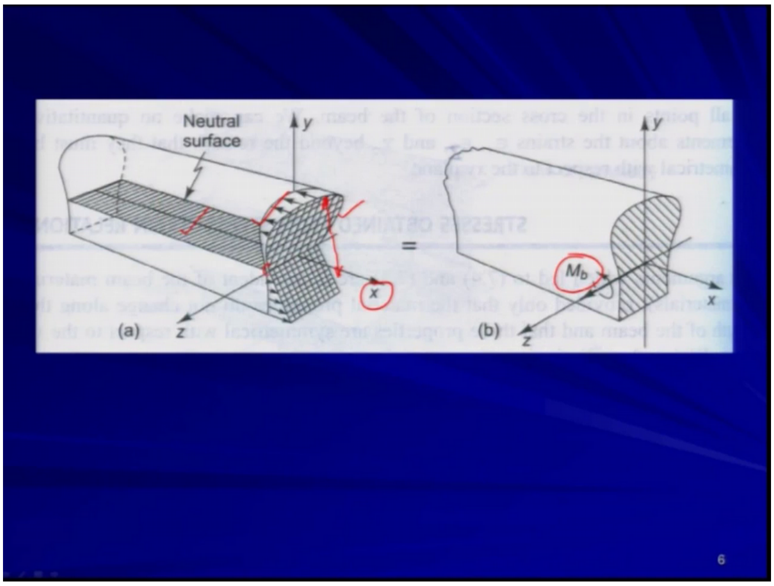

So, that you need to you need to remember that that due to the bending and moreover due to the bending basically you are only getting the normal stress, no other stress only the normal stress along x direction ok.

So, now for the sake of completion of your discussion we need to find out the other 2 strain components, that is epsilon y and epsilon z, that can be obtained by knowing epsilon x.

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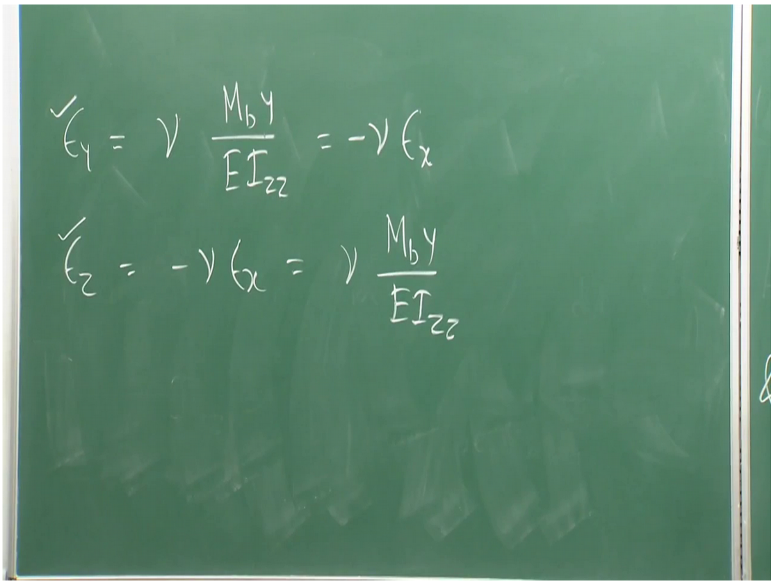

So, epsilon y is simply poison ratio into M b y by E I zz. So, that is my poison ratio into sorry epsilon x right. Similarly epsilon z also your into epsilon x that is nothing but say M b y b y zz right. And of course, gamma z is 0 because tau y z is 0.

So, these are the strain components; that means, in case of bending you will be only getting the normal strains. Epsilon x epsilon y and epsilon z whereas, you will not be getting any shear strain. And everything we are discussing based on pure bending situation please try to understand. And only stress component is sigma x other stress components are simply 0. So, I will stop here today. We will continue with the shear stress development due to bending in the next lecture.

Thank you very much.