Mechanicals Of Solids
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Lecture – 45
Stresses due to Bending

Welcome back to the course Mechanics of Solids. So, today we are going to start the new topic that is stresses due to bending. So, already you know what is bending and how it affects the slender member; already we have seen that is bending moment and shear force all those things how you can calculate over the whole span of the slender member. Now, we are in this particular chapter, we are going to discuss the stresses developed due to bending. Now, first we will be talking about geometry and deformation of a symmetrical beam subjected to pure bending.

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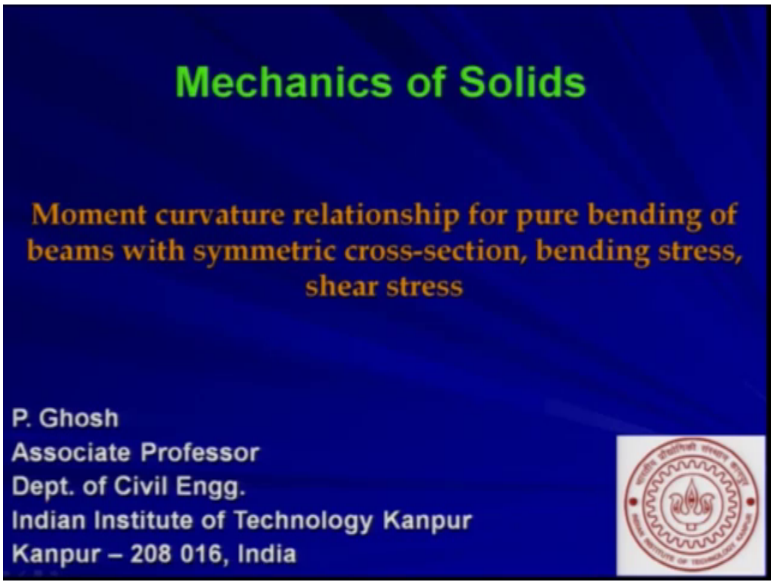

Now in this chapter if you look at will, so will be talking about the in the slide you can see that we will be discussing moment curvature relationship for pure bending of beams with symmetry cross section, bending stress and shear stress. So, first we will be talking about we will be concentrating on the bending stress determination and then we will be talking later on we will be talking about shear stress determination. Now, first what we are talking about we are going to find out the geometry of deformation. So, due to bending when the slender member is the action of bending moment when it bends. So, at the time what kind of say deformation and how the geometry of the deformation is taken place. So, of a symmetrical beam subjected to pure bending.

Now, what is this word pure bending? So, pure bending is nothing but a beam which transmits a constant bending moment is said to be in pure bending; that means, there is no shear force. So, it might happen right. So, if you consider a continue I mean a slender member a beam right which is subjected to the bending moment in such a way that you have the constant bending moment throughout the span without any shear force; the shear force is completely zero that kind of situation might happen in the beam. And under that situation we are going to find out or we are going to analyse the system for a stresses and deformation.

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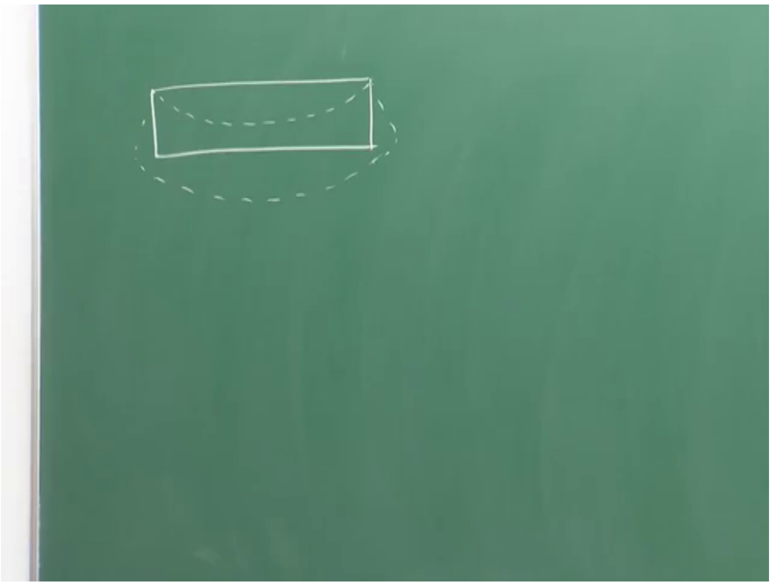

So, now since our originally straight, so originally straight beam, so you know that originally straight beam. So, this is your straight beam. So, originally straight beam and after the formation after bending it will take the shape like that. So, originally straight beam is now becoming the curved one is not it? Now, after deformation it takes the curved shape. Now, therefore, it is useful preliminary stripe to introduce the concept of curvature because that curvature will try to define the deformed shape of the beam.

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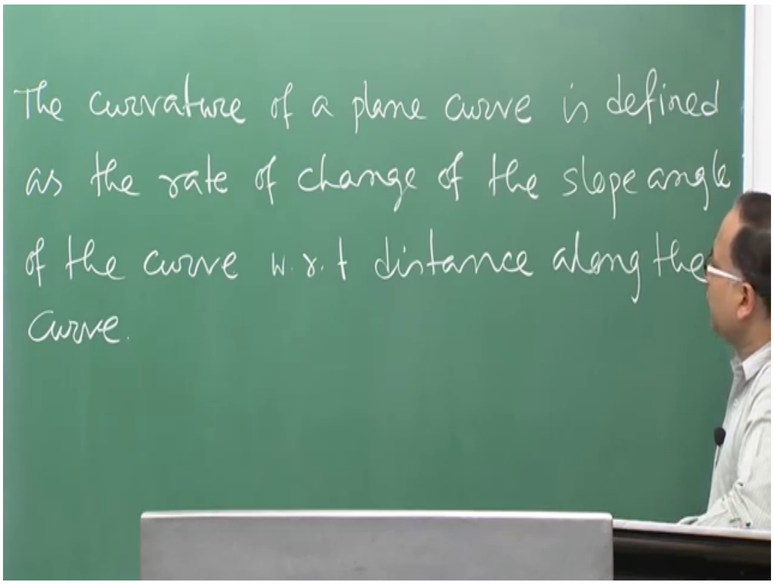

So, let us talk about the curvature first. So, what do you mean by curvature of a curve? So, as you know from your mathematics that the curvature of a plane curve is defined as the rate of change of the slope angle of the curve with respect to distance along the curve. So, this is the definition, proper definition of the curvature of a curve. The curvature of a plane curve is defined as the rate of change of the slope angle of the curve with respect to distance along the curve, very is simple definition.

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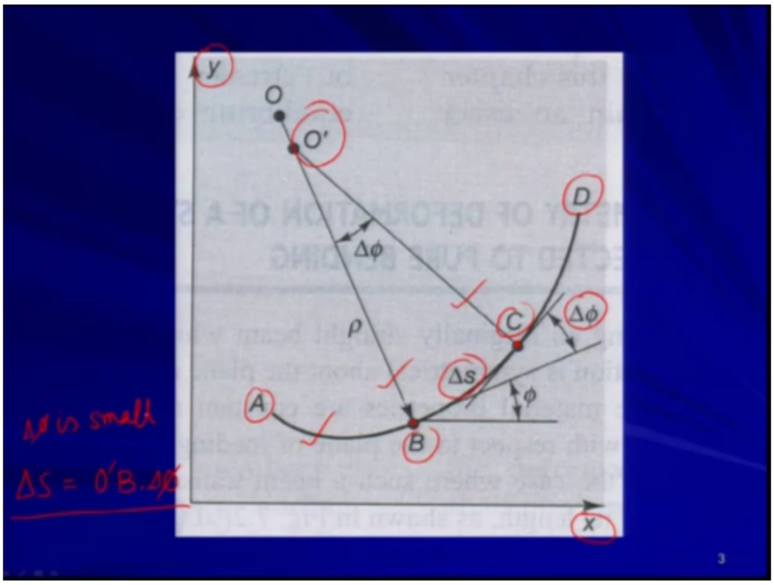

Now, if you look at this plane curve say ABCD is a plane curve. So, ABCD so that means, this is one plane curve and whose curvature is actually in x y plane this is the plane curve and the curvature is in the x y plane. And this B at this is this the normal to the curve at point B and normal to the curve at point C, they are intersecting at say point O prime. So, understood? So, normal at point B on the curve and normal at point C on the curve, they are intersecting at say point O prime. Now, this the change in the slope angle the change in the slope angle between B and C between this point and this point is nothing but say this is your delta phi that is the change in the slope angle between B and C. Because at each and every point I mean this curve is showing you that at each and every point your slope is changing. So, the change in slope between B and C is a delta phi.

So, when delta phi is very small right at the time the arc delta S can be written as O prime B that is the radius of the curvature multiplied by the delta phi, can I write that. So, delta S is equal to O prime B into delta phi as delta S if delta phi is small right, so that is the condition. So, if delta phi is small the arc delta is approximately is equal to O prime B into delta phi.

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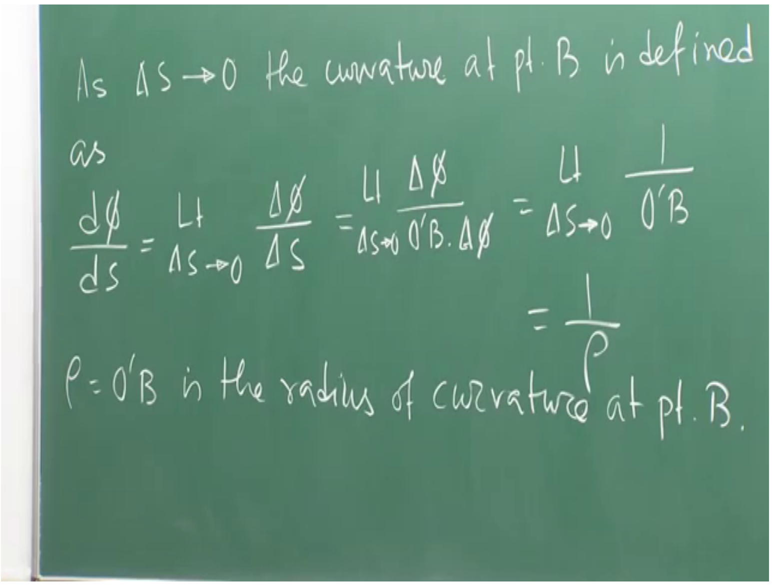

So, what we can write now we can write as delta S tends to 0, the curvature at point B is defined as your d phi d s because that is the definition that means is define as the rate of change of the slope angle of the curve with respect to the distance along the curve. So, d phi d s is your curvature which can be written as limit del s tends to 0, delta phi delta s agreed, so that can be further written as delta phi is very small. So, we can write down this thing as delta phi O prime B into delta phi. So, limit delta s tends to 0, we can right that. So, this can be further written as limit delta s tends to 0, this is O prime B and this can be written as 1 by rho, where rho is equal to O prime B is the radius of curvature at point B.

So, the definition of curvature is nothing but 1 by rho. So, 1 by rho, rho means the radius of curvature right, so that we got here. So, that this expression will be frequently used to define the stress due to bending that is why we are doing this exercise, so that some recapitulation also will happen; and at the same time you will appreciate that this one by rho term will be frequently used because the plane beam is now taking. So, this was the beam before bending and after bending it is taking it is taking the shape of some curve. So, therefore, radius of curvature is very, very say important parameter to define the deformation due to bend.

So, now it can be shown, it can be proved also, it can be shown due to I mean by following the symmetry argument, it can be shown that in pure bending in a plane of symmetry the plane cross sections remain plane even after deformation. So, you are starting with some plane cross section and in the plane of symmetry basically that that plane cross section will be remaining plane even after deformation so that can be shown.

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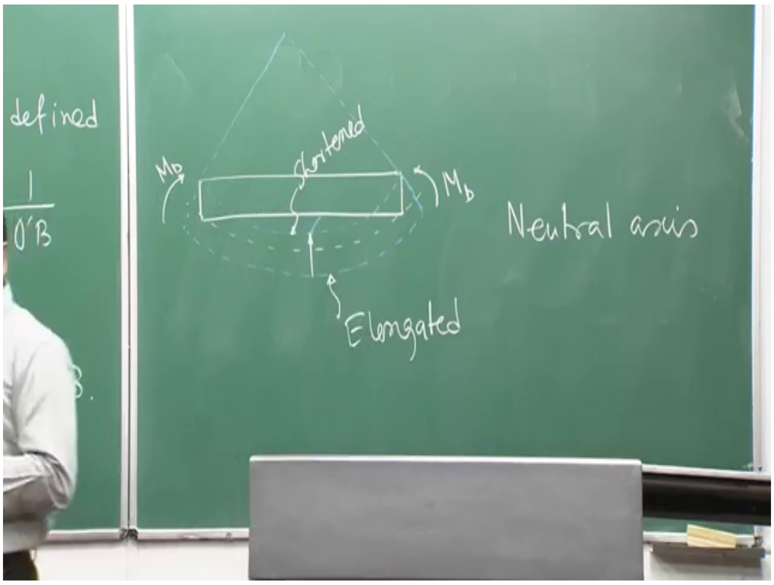

So, now basically if you look at the deform shape of the beam due to pure bending that means, I have the beam like that and say this is my deformed beam which is taking the circular arc fine and this is under the action of bending moment in beam because this is pure bending, so only bending moment constant. Now, if you look at this top fibre of the beam, it is getting shortened is not it, this concave because as you know this is I am showing the positive bending moment. So, the top fibre will be experiencing the compression that will be coming later on because it is getting shortened. So, when it will get shortened, if you compress it right. So, this fibre is basically getting shortened; whereas, the bottom fibre of the beam is getting elongated.

So, from this at least we can say something from our intuition that the top fibre should experience some compressive stress and the bottom fibre is I mean is experiencing or should experience some tensile stress. So, whether that thing is coming from the analyses or not that we will we are going to say. So, now if you say the bottom fibre is getting elongated and the top fibre is getting shortened, then there must be one line in the plane of symmetry which has not changed in the length and that line is known as the neutral axis, understood. That means, if you move from bottom fibre to top fibre, there must be some line in between which will not experience any kind of deformation, no elongation, no contraction, because I mean because you are travelling from the elongation to the shortening right or the contraction. So, there must be one line in between why are you should not have any kind of deformation and that line is known as neutral axis. So, this is very, very important in this discussion. So, this is your neutral axis.

Now, let us try to try to define the neutral axis and now the location of neutral axis we do not know frankly speaking at this moment. We will try to find out the location of the neutral axis later on, but at this moment I mean from the discussion it is coming out that there must be one line where you not get any kind of deformation that line is neutral axis, it is complete neutral in terms of the deformation.

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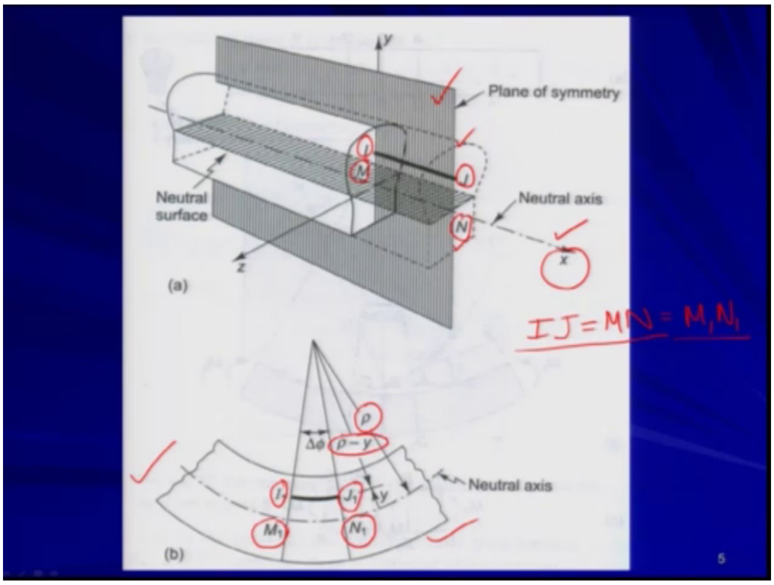

So, if you look at this figure actually so this is your neutral axis. This neutral axis is basically so this is your bottom fibre, this is your bottom fibre of the beam, this is your top fibre of the beam. And this x-axis we are saying the neutral axis, but we are not we are not talking about the location because location we do not know. We are just saying that x-axis is or we are forcefully we are making the x-axis of the neutral axis that we are doing. And this is your plane of symmetry fine, this is your plane of symmetry,

So, now we are considering two fibres; one is your IJ some distance above the neutral axis and one fibre say MN which is at the neutral axis, we are considering two fibres right. So, I mean before deformation basically IJ is nothing but MN because there is no deformation. So, all the fibres are equal. So, IJ is equal to MN because no deformation has happened now after deformation if you see the deformed configuration. So, this is your deformed configuration.

So, now here your earlier it was IJ equal to MN. Now, even we can say MN is nothing but equal to M 1 N 1 even after deformation because MN fibre lies in the neutral axis and m n fibre because it is lying on the neutral axis. So, MN fibre should not experience any kind of deformation like where no elongation, no contraction. So, even after deformation you are getting M 1 N 1 right. So, that must be equal to IJ and must be equal to MN. But at the same time, your IJ fibre will be experiencing some deformation and because neutral axis is not experience in the deformation, so anything above the neutral axis will be experiencing the contraction and below the neutral axis it will be experiencing the elongation. Because you are crossing from elongation or you are travelling from elongation to the contraction zone.

So, therefore, I 1 J 1 this fibre is getting compressed or the contraction or the shortening is happening in this fibre. So, if I define the radius of the this is this is a circular curve right. So, this arc is circular because it is the pure bending case. So, it will take the circular shape because it is everything is symmetry. So, the radius of the curvature is a rho fine and this fibre IJ or I 1 J 1 is lying at some rho minus y distance from the centre. So, this is the centre of the curve fine. So, this is this distance is say y that is given here fine. So, what I can write epsilon x equal to and this is happening is contraction in IJ fibre is happening in x direction.

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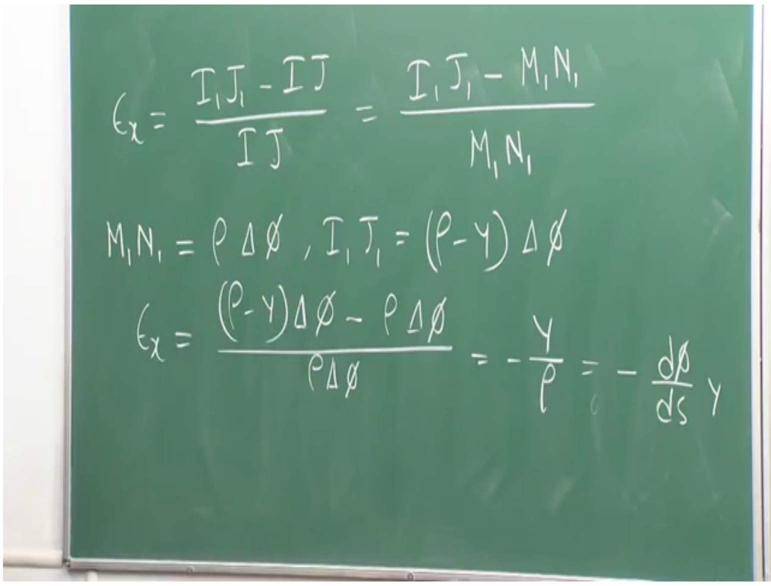

So, epsilon x can be written as I 1 J 1 minus IJ that is the difference in length divided by the original length that is happening in the IJ fibre. So that can be written as I 1 J 1 fine minus M 1 N 1 because IJ is equal to M 1 N 1 by M 1 N 1. I can write that. So, now if you look at the figure m one n one is nothing but rho into delta phi. So, rho is the radius of the curvature and delta phi is the angle made that is the angle made at the centre. So, I can write that and your I 1 J 1. In the similar argument, we can write rho minus y that is the radius of curvature for that fibre multiplied by delta phi. So, therefore, your epsilon x can be written as rho minus y delta phi minus rho into delta phi by rho into delta phi. So, this is nothing but your minus y by rho. So, what is your y by 1 by rho, 1 by rho already we have seen that is d phi d s. So, I can write d phi d s into y. So, this is very, very important relation epsilon x is equal to minus y by rho.

Now, why this minus sign is coming into the picture, because this indicates that there is a shortening above the neutral axis as are we are talking about right. This minus sign is basically indicating that you are getting shortening above the neutral axis; if you go below the neutral axis, you will be getting plus sign; that means, elongation is happening. So, neutral axis is the point or the boundary between elongated fibre and the shortened fibre that is creating the boundary between these two.

Now, we can conclude from the symmetry argument. So, this is the elongation happening in the fibres. So, if I consider different fibre in the beam, so each fibre will be experiencing some elongation or contraction depending on its position with respect to the neutral axis that is that we have got now. But from the symmetry argument because the beam will be bending symmetrically right the plane of symmetry is there and it should bend symmetrically because you have the constant bending moment throughout the beam. So, the symmetry arguments will required, the plane sections to remain plane already we have discussed already we have decided, it can be shown also though I am not showing that thing, but any textbooks you will see the plane section should remain plane by following the symmetry argument. Symmetry argument already we have seen in case of torsion if you call right that was the symmetry argument that is very, very strong argument at least in mechanics.

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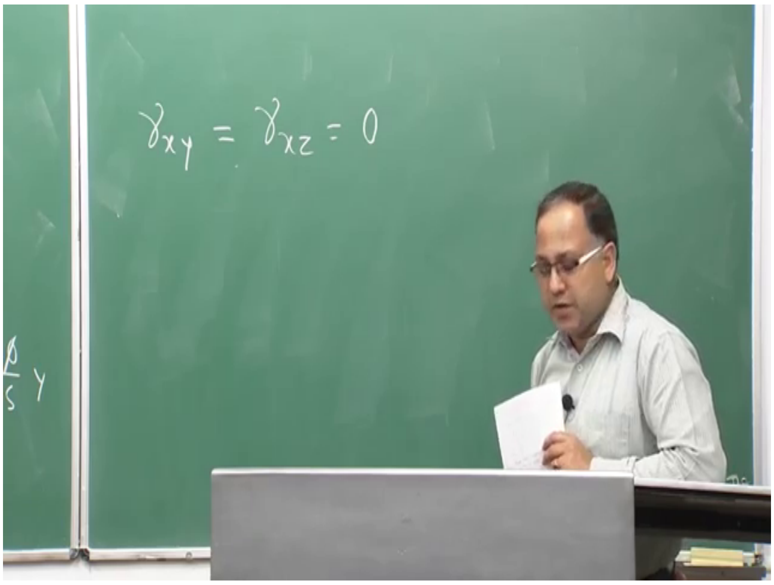

So, by following the symmetry argument I can simply write the shear stress is on that plane basically must be 0; otherwise you will not be satisfying your symmetry. By following symmetry argument your gamma x y must be equal to gamma x z must be equal to 0. So, what you have got you have got on x plane normal strain you have got that is equal to minus y by rho, on x plane in y direction shear strain is 0; and on x plane on in z direction that is gamma x z is also 0 to satisfy the symmetry condition

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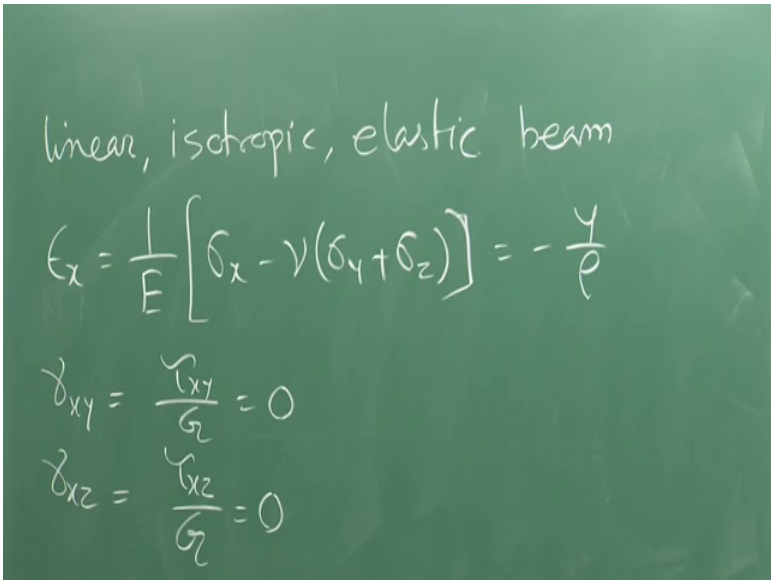

So, now if you if you recall your stress strain relation everything is within your elastic limit. If you recall your stress strain relation for linear isotropic elastic beam, so for this kind of beam your epsilon x can be written as 1 by E sigma x minus nu into sigma y plus sigma z which is nothing but minus y by rho already we have got it. Just now we have derived. Gamma x y is equal to tau x y by g is equal to 0, therefore your shear stress is also must be 0, because the shear strain is 0. Similarly, gamma x z is equal to tau x z by g equal to 0. So, tau x z that is the shear stress on x z plane is also zero to satisfy these shear strain condition.

So, the shear stress components tau x y and tau x z, they must vanish in pure bending that is I mean biggest say finding from this discussion. So, only thing is that you will be, but still I do not know about sigma x sigma y and sigma z, the norm I mean we cannot say anything about a normal stress we will we have to we have to do little bit more say analysis to talk something about to tell something about sigma x, sigma y and sigma z. But at this moment only we can conclude that the shears tress tau x y and tau x z must vanishing pure bending that is coming from these two equations But from this equation we cannot say anything about sigma x sigma y and sigma z. So, we have to do little bit more exercise to tell something about this stress components.

So, I will stop here today in the next lecture we will discuss more about these things. And there from there we will try to find out this stress components because if you know the stress components then basically you can find out I mean if you know the strain or if you know the stress basically you can find out the actual deformation pattern and the stress developed due to bending. So, I will stop here.

Thank you.