Mechanics Of Solids
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Lecture - 44
Tutorial B

Welcome back to the course Mechanics of Solids. So, in the last lecture, we solved one numerical problem. So, today numerical problem on torsion and today we will be taking another numerical problem on torsion, ok.

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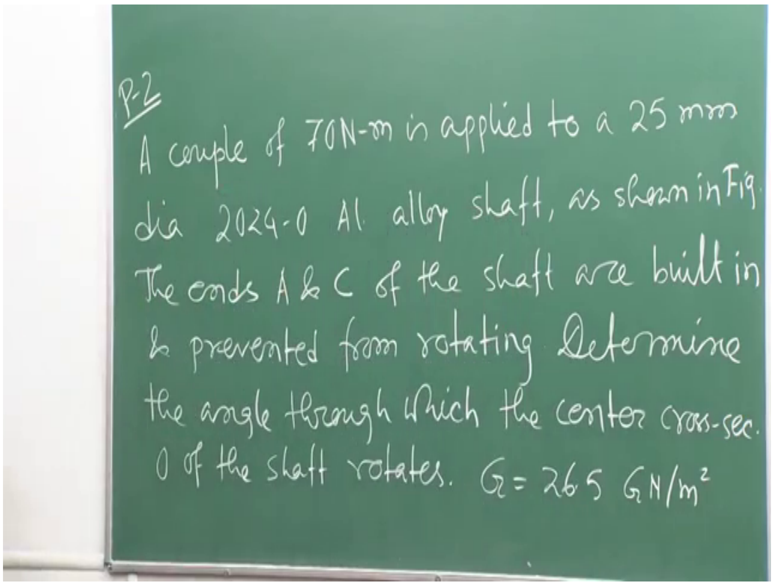

So, let us write down that problem. So, problem 2, the problem says a couple of 70 Newton meter is applied to 25 millimetre dia 2024 O aluminium alloy shaft as shown in figure is. I will draw the figure later. On the ends, A and V of the shaft are built-in and prevented from rotating determine the angle through which the center cross section O of the shaft rotates, given your g shear modulus value is 26.5 Giga Newton per meter square, ok.

So, let me draw the figure.

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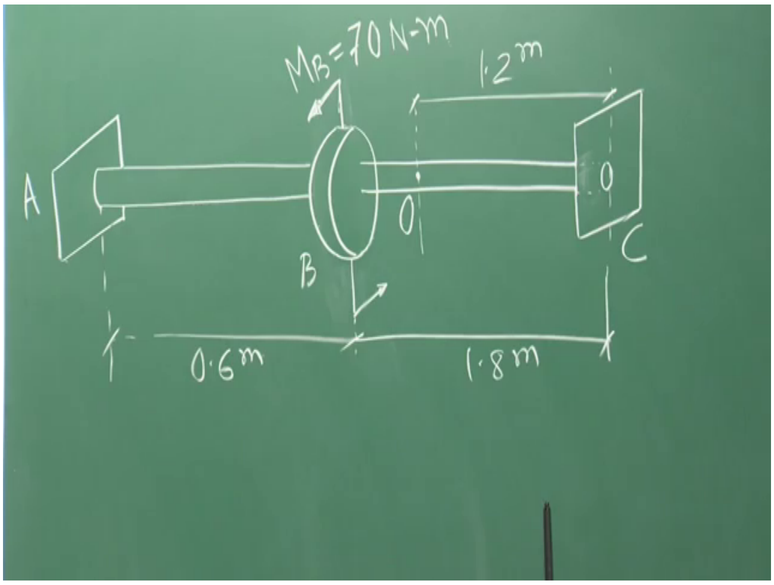

Then, I will explain this as the problem a couple of 70 Newton meter is applied to 25 millimetre diameter 2042 aluminium alloy shaft. So, this is the shaft at this point, at point B actually we are applying 70 Newton meter couple as shown in the figure that aims A and C aims of the shaft are built-in, ok.

So, they are camped, they are built-in with the support and prevented from rotating. So, it cannot rotate actually. So, you try to rotate because when the torque is applied, it cannot rotate at the end. So, the rotation is completely prevented the restricted at A and C. So, determine the angle through which the center cross section O is the center part of the shaft because the total length is 2.4 metre, the shaft length is 2.0 meter, this is 1.2 meter. So, center of the shaft though T is not looking I mean that I hope that you are understanding T is not to the scale. So, I mean O is the center portion of the shaft. We need to find out the angle through which the center cross section O of the shaft rotates the twist angle at O, you need to find out. So, g that is shear modulus is given as 26.5 Giga meter per meter square.

Now, how to solve this problem, how to proceed with this problem? So, if you look at this problem, and if you try to draw the free body diagram of the shaft.

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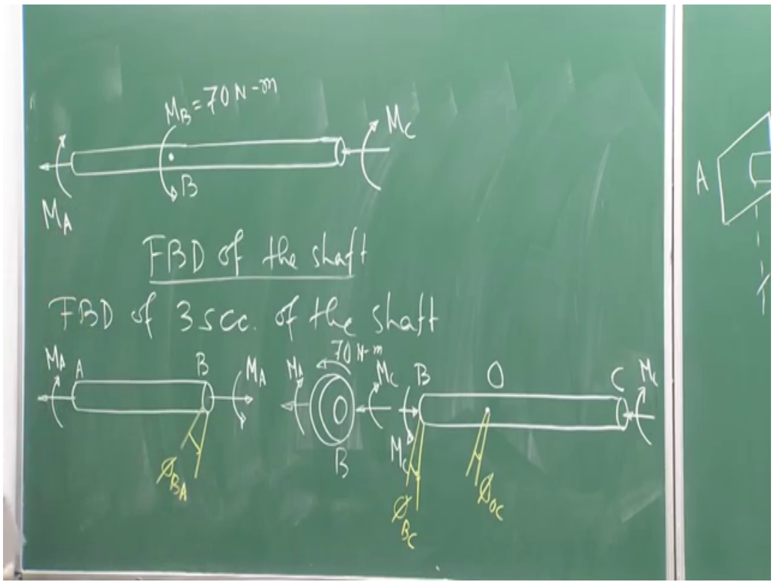

So, basically this is the shaft, this is the whole shaft. So, it is rigidly built in the support. So, therefore rotation is restricted. So, you will be getting some moment, reaction will be getting developed. So, that is nothing, but say MC which is not known to me that is a support reaction and at point B, you have a moment MB that is equal to 70 Newton meter and similarly at support A, you will be having a moment due to the reaction at the support and that is nothing, but MA. Now, if you look at this free body diagram of the shaft, this is your free body diagram of the shaft. Can you solve? Because how many unknowns are there? There are two unknowns MA and MC, both are unknown, right.

So, can you determine those two unknown movements MA and MC. Why because this is to determine structure. So, how many equations available with you as per your equation, equilibrium equation because there is no force. So, those F summation of F x summation of F y, all those things are not coming to the picture. So, only summation of moment equal to 0, only one equation is available with you. One equilibrium equation is available that is summation of moment equal to 0. Now, summation of all moments equal to 0 you can apply, but I mean you have two unknowns only on equation is with you. So, therefore you cannot solve this. So, this is a classical example of statistical in determinant structure. So, therefore if I want to solve it, then I need to impose some geometric compatibility and then only I can solve it as I told you previously. So, now if I try to draw the free body diagram of the part AB and BC separately and let see I mean how we can get the free body diagram and from there whether we are getting some clue or not, ok.

So, we are going to draw the free body diagram of three sections of the shaft. So, free body diagram of three sections of the shaft, now we are going to do that. So, this is the first part, first part means a b part. So, the part which is on the left side of this wheel, we are separating out that part and this is the free body diagram because MA is anyways acting at this end. So, therefore it is this shaft is under torsion or the twisting moment. So, if I cut the shaft at point B and if I consider the left side of the shaft, whatever we did for the bending moment calculation and shear force calculation and basically this moment is acting at point A. So, this is your point A. So, therefore to balance this you should have the same amount of moment in A, at this point B and therefore, you are getting some twist compared to this point A, this is a twist phi BA, agreed.

So, this is the twist over this whole length phi BA and then, I can draw the free body diagram of this wheel MC. This is at B. Some talk some couple is acting 70 Newton meter that is externally applied couple. This is your MA. So, this is the free body diagram of that wheel on which that couple is acting and then, the next part is from B to C, right. That is another part of the shaft, ok.

So, let us draw that. So, from B to C this is MC and this is the point ON which is our point of interest, this is B, this is C and you are getting the twist over the whole length at O. This is the twist say phi OC and this is the twist you are getting at this end. So, that is phi BC absolutely fine. Now, still I do not any MC or all those thing, right. If we apply the geometric compatibility, then only we will be coming to or will be getting some clue. Let us see how we can get all those clue, but at this moment what I have done is, I have drawn all fewer diagram, all the parts of the shaft whatever I have shown or whatever I have in the actual problem, ok.

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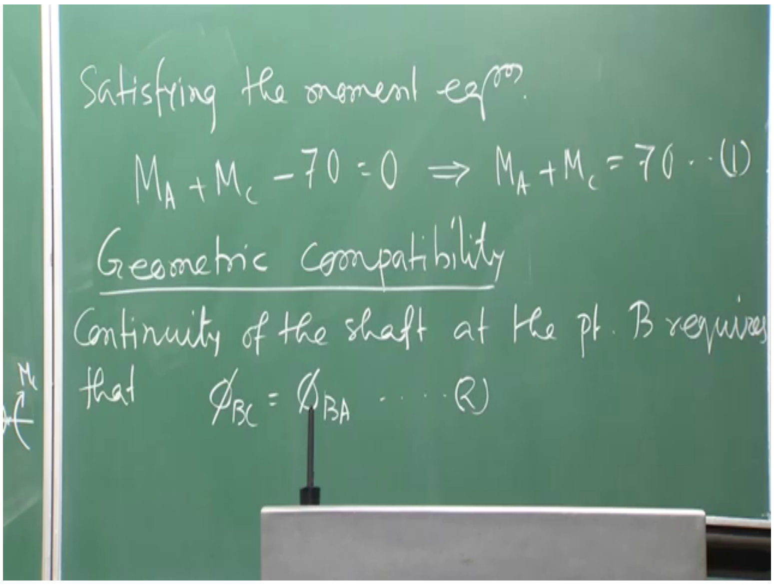

So, now if iI try to solve now satisfying your moment equilibrium, we can simply write MA plus MC minus 70 equal to 0 as per this figure. As per this figure I can write this and what MA and MC, both are unknown to me and this can be further written as MA plus MC is equal to 70 which is equal equation 1, ok.

Now, coming to the geometric compatibility, what is the possible geometric compatibility? What you can think of? Now, if you go back to the actual figure, basically that wheel is the connector between these two parts part BC and part AB. Both are getting connected through this wheel. Yes or no? Yes. Now, therefore I can say that whatever twist you are getting at this point B of this shaft, the same amount of twist you must get at point phi, I mean point B of this part of shaft to satisfy the continuity. So, phi BC must be equal to phi BA. Isn't it?

So, continuity of the shaft and without that clue actually you cannot proceed because this is in deterministic structure. You cannot go ahead. So, continuity of the shaft at the point B requires that phi BC equal to phi BA. So, equation 2 agreed. So, this biggest clue or the biggest information we have extracted from the problem.

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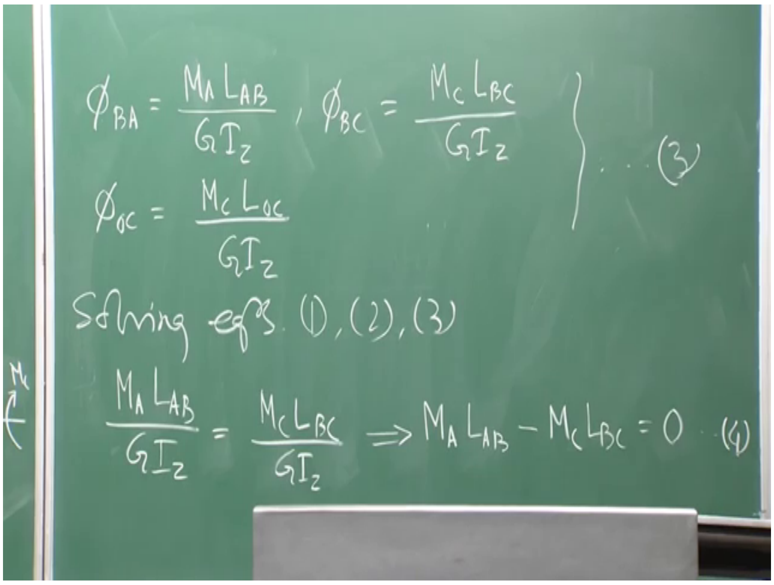

So, now from your load deformation relation whatever we have derived earlier, we can write phi BA is equal to MA LAB GI Z. I add that yes. Of course, I can write it that is the expression for angular twist, right. How are the length LAB, but already we have derived, right. Similarly, I can write phi BC is equal to MC LBC by GI Z. Similarly, I can write phi OC is equal to MC LOC by GIZ, agreed. So, these are expressions of twist at different locations of the shaft phi BA phi BC phi OC over the length of your concern, ok.

So, this is a equation say 3. Now, solving equations 1, 2 and 3, you will be getting because already you have seen in equation 2 that is phi BC is equal to phi BA, right as per your geometric compatibility. So, I can simply right MA LAB GIZ is equal to MC LBC GIZ with the same material, same cross-section of the shaft. So, GIZ will be remaining same. So, from this I can get MA LAB minus MC LBC is equal to 0 and that is my equation say 4. So, LAB and LBC, both are known to me. Yes or no? Yes of course. LAB is given as 0.6 meter and LBC was 1.8 meter. So, that is known to me. So, now I have got two equations. This equation is the relation between MA and MC and another equation if you recall the equation 1 that is also MA and MC. So, these two equations basically can be solved and from those two equations, we can get the magnitude of MA and MC.

So, let us do that.

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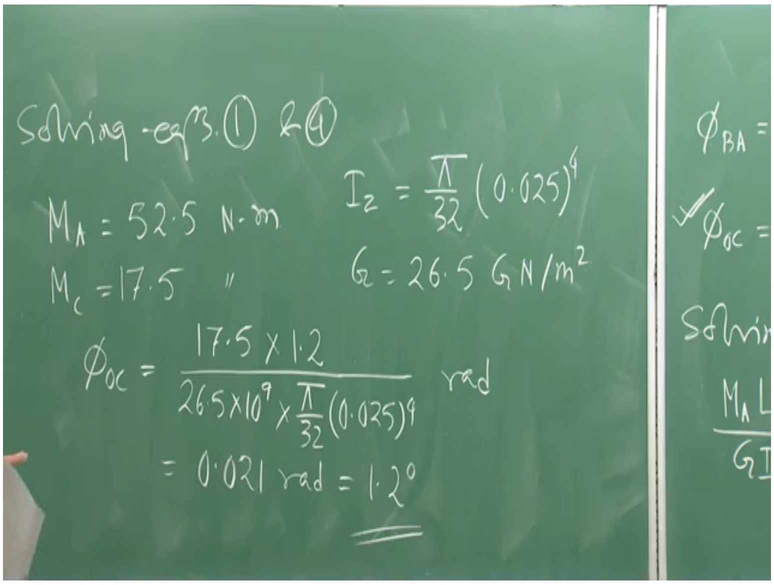

So, I am rubbing this figure. So, solving equations 1 and 4, we will be getting MA equal to 52.5 Newton meter and MC is equal to 17.5 meter. So, two unknowns now are found out. So, unknowns are MA and MC. Now, we have found out those two unknowns and now, once you have found out these unknowns actually, then you can find out phi OC because this is you basically objective to obtain phi OC. Now, you have got MC. You know LOC G is given I ZZ. You can calculate because the cross section of the shaft is given. So, let us calculate I ZZ rather phi by 32 into 0.25 to the power 4, right. So, that is your IZ because the shaft diameter is given. So, that is your IZ and G is given 26.5 Giga Newton per meter square. Therefore, phi OC is equal to 17.5 that is MC value into LOC, that is 1.2 meter 26.5 into 10 to the power 9. That is G value in terms of Newton per meter square into IZ phi by 32.025 to the power 4 radial.

So, that will come as 0.021 radial. That is nothing, but 1.2 degree. So, this is the twist at the center cross section or the shaft. I hope you have understood that how I have analyzed this problem and this kind of problem should be analyzed like that. Any in determinant problem, you cannot solve directly by using your equation of equilibrium.

So, what you have to do is, you have to find, put or you have to extract the information about the geometric computability. So, once you have picked up the geometric compatibility, then rest of the problem is solved, but it is straight forward.

So, I will stop here today. So, in the next lecture, we will be starting the new chapter and then, basically will move on with the discussion with I mean now remaining part is nothing, but your bending and shear stress depletion and finally stability. So, these three topics are remaining. So, we will be looking at those topics gradually.

Thank you very much.