Mechanics Of Solids

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Lecture – 43
Tutorial A

Welcome back to the course Mechanics of Solids. So, if you recall in that last lecture basically we were discussing about the torsion, and there we find out the development of different stress components for the circular cylinder member, when it is under torsion or the twisting moment. So, and there already we have seen that all the stress components are zero except tau theta z and we have derived the expressions to find out tau theta z and twist angle and all those things, if you go back to the previous lecture basically you will get you will be getting all those things. Now, today what we will be talking about will be looking at the stress analysis in case of torsion.

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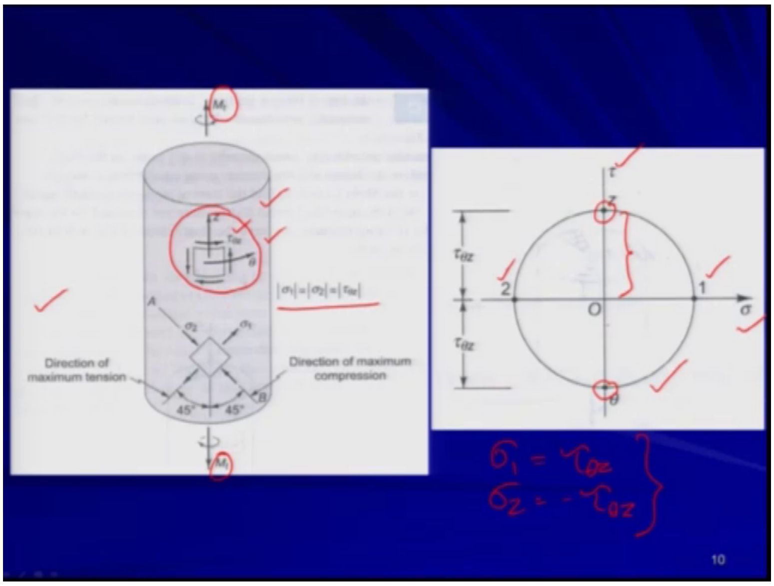

So, suppose I mean if you look at this figure basically so this circular cylinder member is under constant say twisting moment M t. And therefore, if you consider any say in square element on the tau theta z or on the z theta plane, basically you will be having only this is a kind of only pure shear. I mean there is no other kind of stress component available on this element only tau theta z is acting on z plane as well as on theta plane.

So, there is no normal stress acting on these planes. So, now, if I try to represent this stress state of stress with the help of the Mohr circle whatever we have learnt in sigma and tau space. So, basically it will look like. So if you look at this, so this tau theta z is having the positive signs; that means, the magnitude of tau theta z is positive as per our sign conversion or whatever you considered earlier. So, therefore, your theta will be corresponding to x-axis, and z is corresponding to y-axis. And so on x-plane you are having tau theta z there is no normal stress, so that is your theta plane on the Mohr circle and that is your z plane on the Mohr cycle.

So, if you compute the Mohr circle that will look like this. So, now, if you look at or if you try to find out the major and minor principle stresses right major principle stresses sigma 1 and minor principle stresses sigma 2. So, the magnitude of major principle stress and minor principle stress is nothing but tau theta z that is the magnitude means I am talking about the mod of sigma 1 is equal to mod of sigma 2 is equal to tau theta z. However, if you look at sigma 1 and sigma 2, so sigma 1 is nothing but your tau theta z from the Mohr circle or else sigma 2 is minus tau theta z. This is coming from the Mohr circle.

Now what you are getting from this, so tau theta z is nothing, but basically the magnitude now the magnitude. So, I mean this much of this is the radius, radius of the Mohr circle fine. So, now, if you look at this, so sigma 1 is giving you the tension that is the major principle stress is giving you the tensile stress. Whereas, the minor principle stress sigma 2 is giving you the compressive stress and that can be shown like that. So, if you try to rotate the axis because now another thing is that between theta axis and major principle stress axis that means, theta and one right this axis is making an angle 90 degree on the Mohr cycles. So, therefore, in the physical plane; that means, here it will be making 45 degree. So, this is your theta direction and this is your sigma 1. So, one direction, so 45 degree fine. So, now, here sigma 1 is acting and which is tensile in nature and sigma 2 is acting on this plane which is compressive in nature.

Now if you look at this configuration. So, along this line a b along this line a b whatever plane is lying along this line a b on that plane basically your sigma 1 is acting that means, you major principle plane is nothing but along the line a b on which the maximum tensile stress is acting maximum tensile stress is nothing but your sigma 1. Similarly, this plane on which your compressive stress is acting and that is pure compression right that is your sigma 2. Now, you might have seen in several instances where suppose you are having very say some your Coca-Cola can or Pepsi can you might have seen and if you try to give the twist it will warp in particular direction. So, why it is happening like that have you ever thought about that?

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It is something like that if I do the similar kind of experiment with this chalk, you know the chalk is a brittle material as per previous definition the chalk is a brittle material it is very, very weak in tension, but it is very, very strong in compression. So, now, what I am doing I am just holding the chalk there is no other force acting on this chalk, and I am just giving the twist. Now, you see now if you look at this is the plane on which or rather this is the plane whatever I mean this is the lower part and this is the upper part. So, this plane I mean it has not got the failure or rapture along the cross section rather there is some angle along which so this is a angle of the plane.

That means, if you look at this figure, so as I told you this chalk is weak in tension. So, on this plane there must be some maximum tensile stress happened and therefore, this plane is becoming more vulnerable plane and that is why it is breaking or the failing on this plane and that is plane is nothing but your plane parallel to line a b understood.

So, now if you do this experiment this is very, very simple experiment. So, this chalk will be several times you do it you just keep your twist twisting movement, you will getting always failure like that and that is basically telling that because of chalk is weak in tension and you must expect the failure in the chalk where the maximum tensile stress is acting. And the maximum tensile stress is acting on this plane, because this plane is nothing but your ab plane as I showed in the figure. Similar kind of example or similar kind of thing you can do with the help of your Coca-Cola can or the Pepsi can right you just because that is very thin sheet right the can is made of with very thin sheet. So, if you keep the twisting moment you will see the failure along the plane on which maximum tensile stress is acting.

Similar kind of thing you might get say suppose you are making some cylindrical very thin cylindrical say member cylinder member with the help of paper. And you keep your twisting moment if you give that the paper will be will be folded or will be say warping along a particular plane on which the maximum shear stress is acting. So, this is the reason you might have experienced that thing you might have seen earlier, but you never thought that why it is happening like that why it is following that kind of plane on which you are getting this warping. Now you know that why it is happening like that because in case of torsion, the maximum tensile stress will be occurring on this inclined plane which is parallel to line ab fine.

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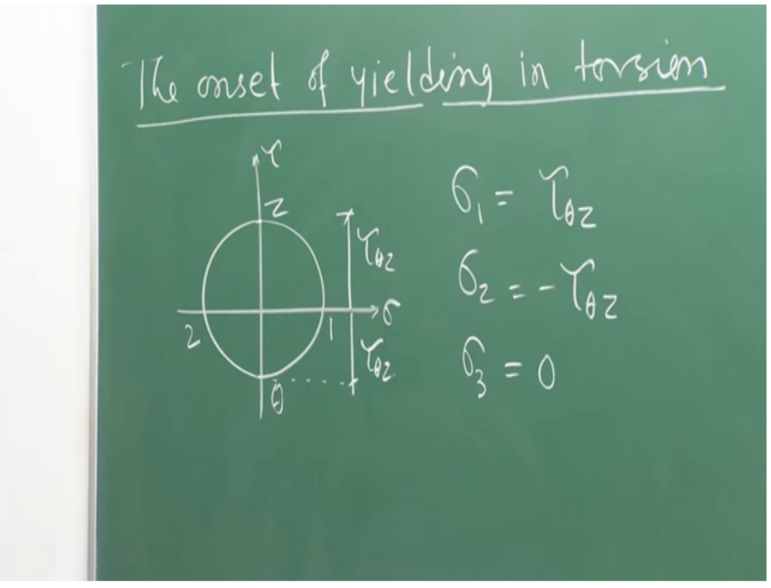

Now will come back to the yielding characteristics happened in case of torsion the onset of yielding in torsion now in the previous figure you have seen that if I try to get the Mohr circle for the state of stress developed in torsion so that will look like this right. So, this is my sigma tau axis, sigma tau, and this is the Mohr circle, this is 1, 2 and this your z and theta. I am just reproducing the previous figure whatever we just have seen well. So, now as we have written sigma 1 is nothing but tau theta z; sigma 2 is nothing but tau theta z because this is your tau theta z that is a radius of the Mohr circle; and sigma 3 is obviously 0, because the we are talking about the planar stress right. So, sigma 3 is zero.

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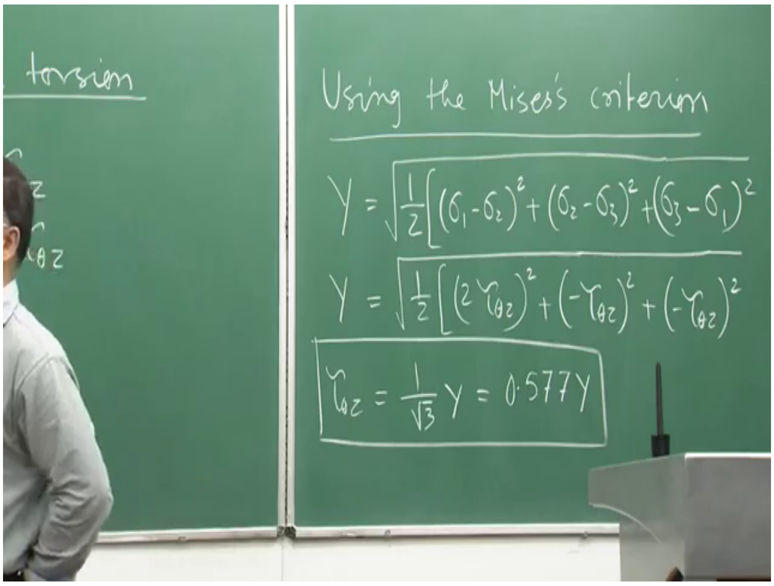

So, now, if as per your Mises's criteria, so using the Mises's criterion your Y that you know that is a that is your yield strength is given by this equation already we have derived that half sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square. Already we have seen that this is your von-Mises's criteria. So, as per that if you put the values of sigma 1, sigma 2, sigma 3 whatever you got here, so you will be getting half 2 tau theta z whole square plus minus tau theta z whole square plus minus tau theta z whole square. So, from this I can get tau theta z is equal to 1 by root 3 Y that is nothing but 0.577 Y. This is as per Mises's criteria; that means, if your yield strength of the material, so the material with which your circular cylinder member is made of. So, if the yield strength of the material is say Y, so you can go up to maximum this much of shear stress to avoid von-Mises's I mean to avoid the failure as per von-Mises's yield criteria.

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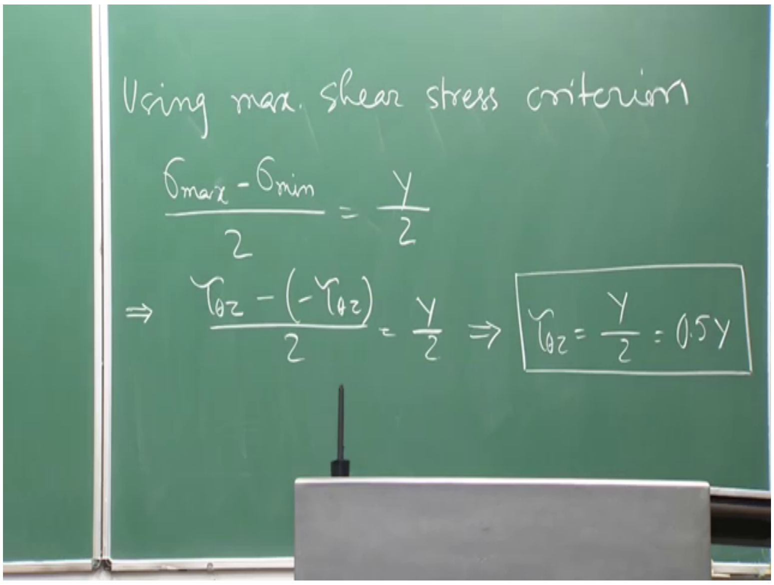

Now, let us see what is happening with the maximum shear stress criterion whatever we have learnt earlier. So, let us talk about that as per maximum shear stress criteria that is your shear stress criterion. So, using maximum shear stress criterion your sigma max that is your shear stress criteria maximum shear stress sigma max on a sigma mean by 2 is equal to Y by 2 that was your maximum shear stress or that is the criterion. So, what is your maximum value of sigma that is sigma max that is nothing sigma tau theta z that is nothing but your sigma 1.

Now, here what is the minimum value of your sigma that is nothing but your sigma 2 that is minus tau theta z, so minus of minus tau theta z. So, this gives me tau theta z is equal to y by 2 that means, 0.5 Y. So, that means, in case of von-Mises's or the Mises's criterion how much you are getting tau theta if tau theta z is becomes equal to 0.577 Y then you are getting the yielding. Now, if you look at the maximum shear stress criterion as per that when theta z is reaching 0.5 Y basically then you are getting the yielding. So, this as per maximum shear stress criterion you are getting much lower value of tau theta z to initiate your yielding.

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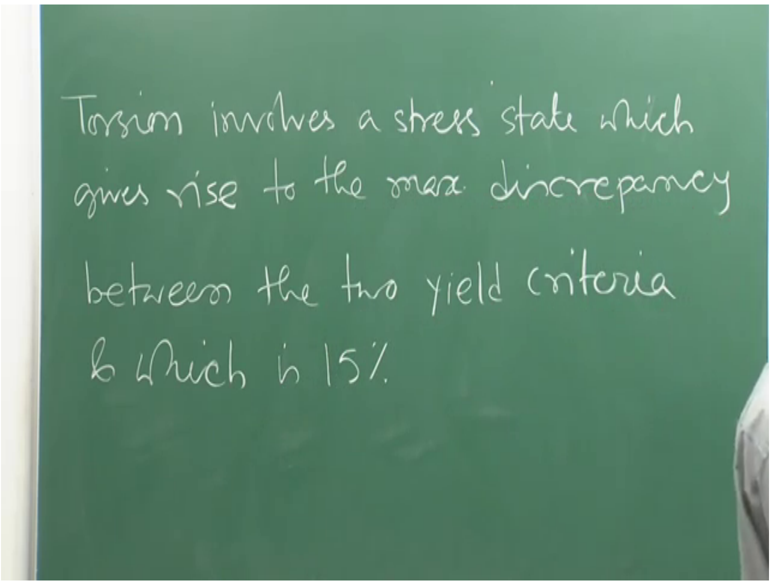

So, therefore, you can see that you can get it, you can write down torsion involves a stress state which gives rise to the maximum discrepancy between the two criteria you yield criteria rather two means one is von-Mises's, another one is maximum shear stress that is your nothing you stress state, and which is 15 percent. This gives a maximum discrepancy that means, if you go by Mises's criterion or if you go by your maximum shear stress criterion, and if you compare this both the criteria basically then basically you are getting the 15 percent discrepancy between these two failure criteria that is happening in torsion. So, in one case you are getting 0.577 Y; in another case you are getting 0.5 Y and that gives a discrepancy between these two criterion. So, with this I am concluding your torsion chapter. Now we will take couple of numerical problems and then will move to the next chapter.

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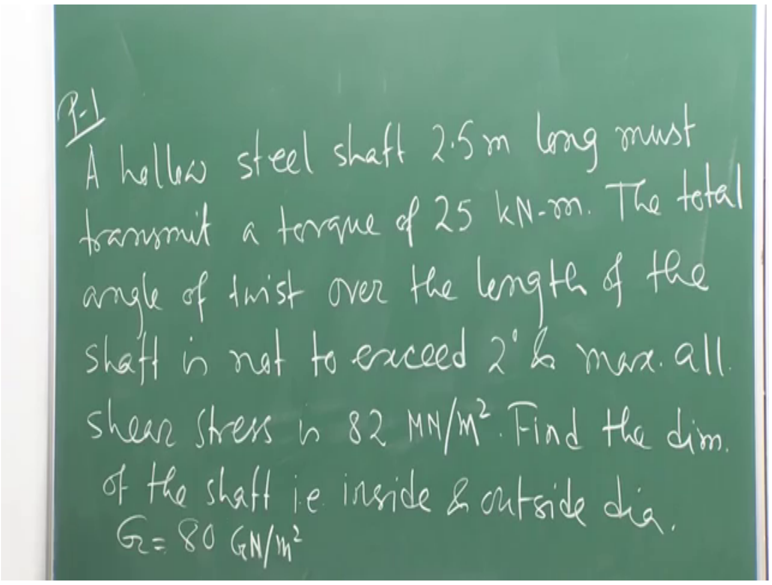

Let us take some numerical problem say problem one. A hollow sheet hollow steel shaft 2.5 meter long must transmit a torque of 25 kilo Newton meter. The total angle or twist over the length of the shaft is not to exceed 2 degree and maximum allowable shear stress is 82 mega Newton per meter square. Find the dimensions of the shaft that is inside and outside diameters given is G equal to that is a shear modulus is given as 80 giga Newton per meter square. So, this is the problem.

So, a hollow steel shaft is a hollow circular shaft whatever we have discussed earlier. So, hollow steel shaft 2.5 meter long must transmit a torque of 25 kilo Newton meter. The total angle of twist over the length that means from the bottom if you compare from the bottom to the top the total over the total length the twist angle is not to exceed 2 degree and maximum allowable shear stress is 82 mega Newton per meter square. So, these are your limits. Find the I mean you are basically going to design this kind of shaft under this restriction. So, the total twist should not exceed 2 degree and shear stress should not exceed 82 mega Newton per meter square. So, find the dimensions of the shaft that is inside and outside the diameter, so that is nothing, but your shaft. So, you are going to design that where G that is your shear modulus is given as 80 giga newton per meter square.

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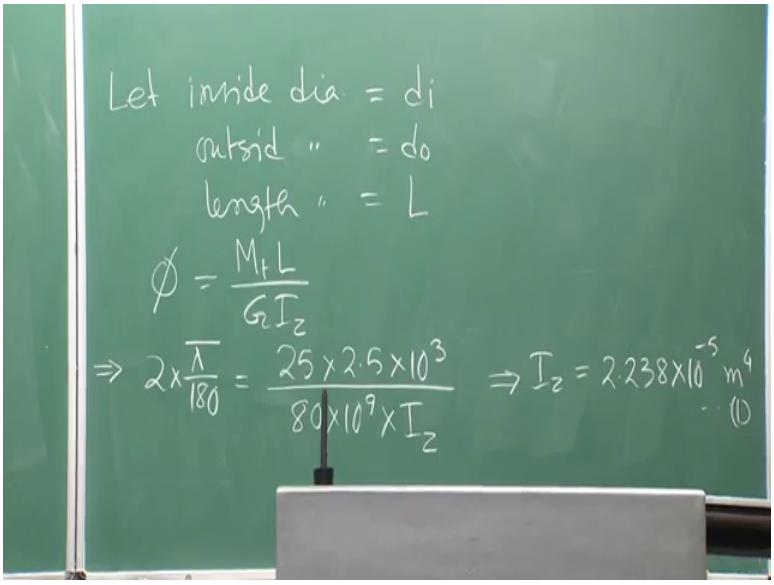

So, let us solve that problem. So, let your inside diameter equal to say d i and your outside diameter say d o. So, now and your length that is also not given, so length we are considering say L. So, as per our derivation whatever we have done earlier, so the twist angle is given by M t L by G I z if you recall right no issue. So, here actually we can put because our restriction is 2 degree that is a maximum twist we can have in the shaft. So, 2 into pi by 180 radian is equal to M t, M t is nothing but the torque applied that is given that is 25 kilo Newton meter, and L is given 2.5 meter long then L is given as 2.5 meter long. And G value is given as 80 giga Newton per meter square, so that is nothing but 80 into 10 to the power 9 Newton per meter square.

So, this 25 into 10 to the power 3 is basically Newton meter and I z z because, you do not know the dimensions. So, you cannot find out the polar moment of inertia. So, I z z is not known to you. But however, from this numerical values you can find out I z z which will becoming as from this I can simply get I z equal to 2.238 into 10 to the power minus 5 that is your meter to the power 4 say equation 1. That is a value of your. So, you need a shaft which will give you this much of I z to get to satisfy other parameters fine.

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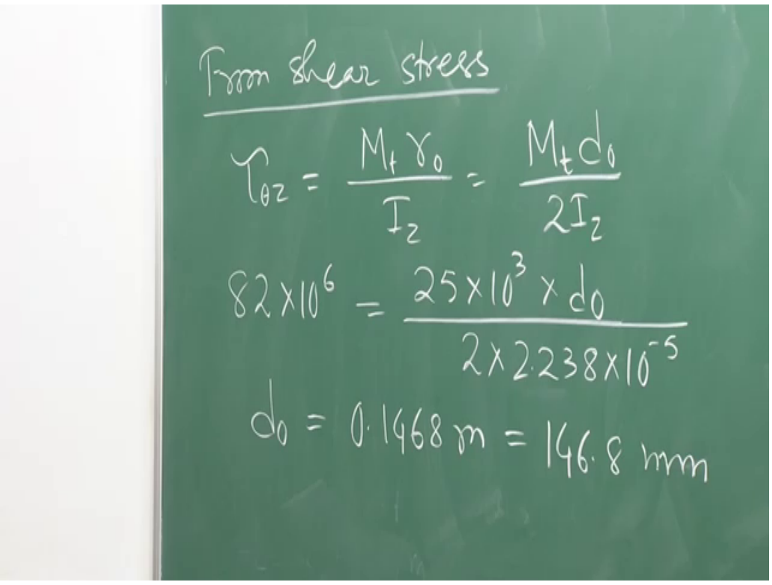

So, from shear stress your tau theta z is equal to as you know M t r naught by I z which can be written as in terms of your diameter d naught, r naught is the outside radius, so d naught twice I z fine. So, here your upper limit of shear stress, which is getting developed in the shaft that is given in the problem that is nothing but 82 into 10 to the power 6 mega Netwon per meter square is equal to 25 into 10 to the power 3 that is Newton meter that is the twist M t. And d naught obviously, is not known d naught d o rather d o is not known to you, and 2 into I z is now known to you from equation 1 that I am putting 2.238 into 10 to the power minus 5. So, from this I can find out d o is equal to 0.1468 meter that is nothing but 146.8 millimetre. Now, you have got at least the outside diameter of the shaft and you know the polar moment of inertia of the shaft.

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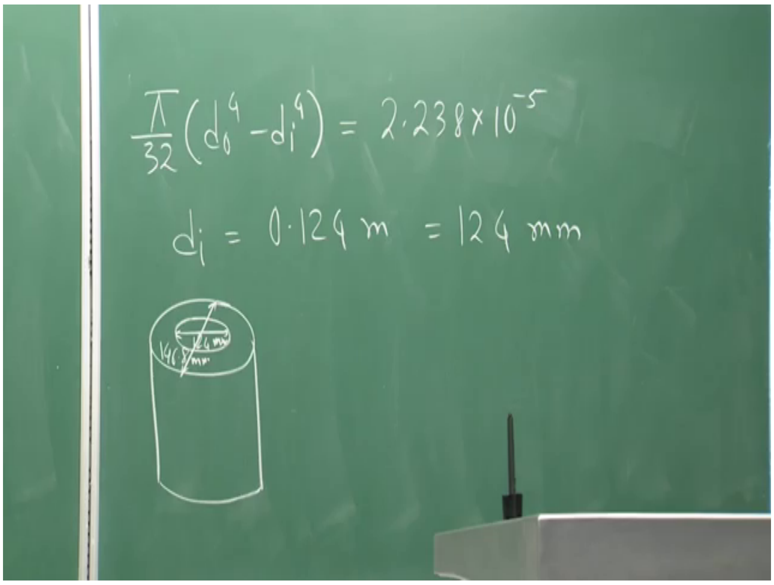

So, from that you can find out the other dimensions that is the inside diameter of the shaft so that means, from this, this is the expression for I z of a hollow circular shaft d already we have seen that. And you know from your earlier say knowledge 2.238 into 10 to the power minus 5. So, from this you will be getting d i, because now d o is known to you that is given here. So, d i can be obtained as 0.124 meter and that is nothing but 124 millimetre. So, you need a hollow circular shaft to satisfy the conditions this is your inner diameter 124 millimetre and this is your outer diameter 146.8 millimetre. So, this can be used which will satisfy all the I mean limiting conditions like your limiting condition of the stress shear stress, limiting condition of your twist and all those things will be taken care if you choose I mean hollow circular shaft like this.

So, I will stop here today. So, in the next lecture, we will be taking one more numerical problem and that will be basically concluding this chapter on torsion.

Thank you very much.